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CP violation in the Σ^0 decay

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"Men are mortal. So are ideas. An idea needs propagation as much as a plant needs watering. Otherwise both will wither and die."

B.R. Ambedkar

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Abstract

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The non-trivial structure of the QCD vacuum gives rise to a P and CP violating term in the QCD Lagrangian. The fact that we do not see an observable CP violation in the strong interaction despite this CP violating theta term is called the Strong CP Problem. In this thesis, we analyze an observable consequence of this theta vacuum term in the decay of the ground state neutral Sigma hyperon. Due to the SU(3) flavor symmetry, the current upper bound of the neutron electric dipole moment can be translated to an angular asymmetry in the decay distribution of the Σ^0 particle. The self-analyzing weak decay of the Λ hyperon means that any P violation in the initial $\Sigma^0 \rightarrow \Lambda\gamma$ decay will result in an asymmetry in the angular distribution of the final decay products. Studying the Sigma and anti-Sigma hyperon decays, we get an idea of C and CP violation in the decay chain. The effect of the production process of the Σ^0 hyperon on the angular distribution of the final products is also worked out. A significant angular asymmetry in the decay will mean not only physics beyond the Standard Model, but also physics beyond the CP violating term in the QCD Lagrangian.

ലളിതമായ സംഗ്രഹം മലയാളത്തിൽ

നിരീക്ഷിക്കാവുന്ന പ്രപഞ്ചത്തിൽ ആന്റി-മാറ്റർ എന്നതിനേക്കാൾ കൂടുതൽ മാറ്റർ ഉണ്ട്. ഈ അസമത്വത്തിൻറ്റെ കാരണം ഇപ്പോഴും നിലവിലുള്ള സിദ്ധാന്തങ്ങൾ വിശദീകരിച്ചിട്ടില്ല. നമ്മുടെ പ്രപഞ്ചം "ഡിസ്ക്രീറ്റ്" സമമിതികളെ ലംഘിക്കുന്നു എന്നതാണ് ഒരു സിദ്ധാന്തം. ഭൗതികശാസ്ത്രത്തിൻറ്റെ നിയമങ്ങളിൽ മാറ്റം വരുത്താത്ത ഏതൊരു രൂപാന്തരീകരണവും ഒരു സമമിതിയാണ്. ഭൗതികശാസ്ത്രത്തിലെ ഒരു തുടർച്ച അല്ലാത്ത സമമിതിയെ ഡിസ്ക്രീറ്റ് സമമിതി എന്ന് വിളിക്കുന്നു.

"പാരിറ്റി" അല്ലെങ്കിൽ മിറർ-റിഫ്ളക്ഷൻസ്, ചാർജ്ജ്, ടൈം റിവേഴ്സൽ എന്നിവ മൂന്നു സ്വഭാവസവിശേഷതകൾ പ്രകൃതിയെ അനുസരിക്കാനാണെന്ന് കരുതപ്പെട്ടിരുന്നു. സംയോജിത ചാർജ് ആൻഡ് പാരിറ്റി ("സിപി") സമമിതിയുടെ ലംഘനം ഈ പ്രശ്നത്തിന് ഒരു പരിഹാരമാണ്. എന്നിരുന്നാലും, നമ്മുടെ നിലവിലെ സൈദ്ധാന്തിക ചട്ടക്കൂടിൽ ("സ്റ്റാൻഡേർഡ് മോഡൽ") സിപിയുടെ ലംഘനം ഈ പ്രപഞ്ചത്തിലെ മാറ്റർ-ആന്റിമാറ്റർ അസമത്വത്തിന് കണക്കുകൂട്ടാൻ കഴിയില്ല. നമ്മുടെ നിലവിലുള്ള "സ്റ്റാൻഡേർഡ് മോഡൽ" ചട്ടക്കൂടിനു പുറത്തുള്ള ഭൗതികതയെ നമ്മൾ കണക്കിലെടുക്കണം എന്നാണ് ഇതിനർത്ഥം. സ്റ്റാൻഡേർഡ് മോഡൽ എന്നത് പ്രാഥമിക കണികകളും അവരുടെ പെരുമാറ്റത്തെ നിയന്ത്രിക്കുന്ന പരസ്പരപ്രവർത്തനങ്ങളും വിശദീകരിക്കുന്ന സിദ്ധാന്തമാണ്.

സ്റ്റാൻഡേർഡ് മോഡലിൽ വീക്ക് ഫോഴ്സ് മാത്രമേ സി.പി. സമമിതി ലംഘിക്കുകയുള്ളൂ. "ക്വാണ്ടം ക്രോമോഡിനാമിക്സ്" എന്നത് "സ്ട്രോങ്ങ്" ഫോഴ്സ് ശക്തിയുടെ സിദ്ധാന്തമാണ്. ഉദാഹരണമായി, സ്ട്രോങ്ങ് ഫോഴ്സ്, ന്യൂക്ലിയസിൻറ്റെ ഘടകങ്ങൾ തമ്മിലുള്ള ശക്തിയാണ്. സ്ട്രോങ്ങ് ഫോഴ്സിൽ സിപി സമമിതിയുടെ ലംഘനം ഇതുവരെ കണ്ടിട്ടില്ല.

എന്നിരുന്നാലും, സ്റ്റാൻഡേർഡ് മോഡൽ എന്നതിനു പുറത്തുള്ള ഭൗതികതയെ കണക്കിലെടുക്കുമ്പോൾ, സിപി നിയമം ലംഘിക്കുന്നതിനെ സിദ്ധാന്തം അനുവദിക്കുന്നു. ഇതിൻറ്റെ ഒരു പരിണിത ഫലമായി നമ്മൾ ശക്തമായി ഇടപെടുന്ന (സ്ട്രോങ്ങ് ഫോഴ്സ്) സൂക്ഷ്മ കണങ്ങളുടെ ("ഹാഡ്രോൺ") ശോഷണ ഉത്പന്നങ്ങളിൽ ഒരു കോണീയ അസമത്വം കണ്ടെത്താനാകും. ഈ കൃതിയിൽ, നമ്മൾ "ന്യൂട്രൽ സിഗ്മ" (Σ^0) കണികയുടെ ശോഷത്തെ നോക്കുകയാണ്. ന്യൂട്രോൺ ഇലക്ട്രിക് ഡൈപ്പോൾ മോമെന്റ്റ്ൻറ്റെ പരീക്ഷണ തലത്തിലുള്ള പരിധി ഉപയോഗിച്ചു നമുക്ക് ശോഷണ വിതരണത്തിൽ ഈ കോണീയ അസമത്വം കണക്കാക്കാം. ന്യൂട്രോൺ, ന്യൂട്രൽ സിഗ്മകണികകൾ തമ്മിലുള്ള നിലവിലുള്ള "ഫ്ളേവർ" സമമിതി ഉപയോഗിച്ച് നമുക്ക് ഇത് ചെയ്യാൻ കഴിയും.

പൂജ്യം അല്ലാത്ത "ഇലക്ട്രിക് ഡൈപ്പോൾ മോമെന്റ്" ഞങ്ങളുടെ സിദ്ധാന്തത്തിലെ സി.പി. ലംഘനത്തിൻറ്റെ ഒരു സൂചകമാണ്. ചാർജ് സമമിതി വിശകലനം ചെയ്യാൻ "ആൻറ്റി സിഗ്മ" ($\overline{\Sigma^0}$) കണികയുടെ ശോഷവും ഈ തിസിസ് ഉൾക്കൊള്ളുന്നു.

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Contents

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Chapter 1

Introduction

The study of discrete symmetries helps us understand the scope and limitations of our current theories. More specifically, the transformation properties of a physical law under discrete symmetries reveal a certain preference (or lack thereof) within the laws of nature. When a discrete symmetry is violated, it points towards a bias in nature. One such bias is the apparent abundance of matter over antimatter in the observable universe [1]. In fact, one of the solutions to this longstanding problem was thought to be caused by a violation of discrete symmetries. In 1967, A. Sakharov proposed three conditions that could help solve the puzzle of matter-antimatter asymmetry [2]. He postulated what are now famously called the Sakharov conditions, which attribute the baryon-antibaryon imbalance to dynamic processes rather than to initial conditions beyond the scope of physics. These are [2]:

1. Baryon number violation must occur in reactions.
2. C (Charge) and CP (Charge-Parity) symmetries must be violated.
3. Interactions must proceed outside thermal equilibrium.

Understanding how the known forces of the Standard Model behave under discrete symmetry transformations will thus get us closer to unraveling the mystery of baryogenesis. Discrete symmetries also provide clues to physics beyond the Standard Model. The gauge principle forms an important part of any relativistic quantum field theory, and is a necessary feature of a nontrivial and renormalizable field theory. The insertion of discrete symmetry requirements in *local* gauge theories results in ABJ anomalies, a consequence that points towards Beyond Standard Model effects [3]. The focus of this work, however, is on the violation of *global* discrete symmetries due to effects not explained by the Standard Model. In the strong sector this would mean studying the non-trivial topological structure of the QCD vacuum and any discrete symmetry violations that may result from it. A thorough investigation into the nature of these violations will be undertaken later in this thesis, but for now it merely serves to illustrate the importance of discrete symmetries as a probe for new physics.

Discrete symmetries have occupied a pivotal position in the development of quantum field theories. Charge conjugation invariance (C), parity symmetry (P), and time reversal invariance (T) constitute the discrete symmetries that nature was largely thought to obey [4]. However, starting from 1956, when Lee and Yang first proposed a test for parity symmetry in the weak interaction [5], these prized assumptions held dear by theoreticians were put under experimental scrutiny. The discovery of P violation in the beta-decay of Cobalt-60 in 1956 by Wu et al., followed by Christenson's discovery, in 1964, of the combined effect of charge and parity, namely CP, violation in the decay of neutral kaons laid the foundation for a new paradigm in particle physics [3]. The violation of CP symmetry came as a great shock to physicists at the time. Due to the CPT theorem, which states that all local relativistic field theories must be invariant under the combined action of charge (C), parity (P) and time-reversal (T) transformations, a violation in CP would invariably imply a violation in time-reversal [4]. The violation of T symmetry prompted a complete rethink of the fundamental assumptions governing the Standard Model. It was not until nine years later, in 1973, that the discovery of CP violation had a theoretical postulation. The minimal implementation of CP violation in the Standard Model came via complex phases in Yukawa couplings of the weak interaction in what is known as the Kobayashi-Masakawa (KM) ansatz, named after Makoto Kobayashi and Toshihide Maskawa [6]. They postulated the existence of a third family of quarks, an extension to the earlier Cabibbo matrix, which was required to corroborate this theory. In 1995 this was validated by the discovery of the top quark, the final member of the third family of quarks [3][6].

While the study of CP violation in the electroweak sector has yielded important results, it still does not account for the large matter-antimatter imbalance we see in the universe [1]. The other candidate for possible CP violation is the strong interaction. Quantum Chromodynamics (QCD) is the gauge theory of quarks and gluons, and its behavior under discrete symmetry transformations is less understood. The experimental consensus so far indicates no P and CP violation in the strong sector. This is problematic since a non-trivial QCD vacuum gives rise to P and CP violating terms in the QCD Lagrangian. The fact that there seems to be no discernable CP violation in the strong interaction when its Lagrangian includes natural terms that violate this symmetry is called the Strong CP problem. A more detailed discussion on this topic will follow, however, it should be stated that the best experimental estimate we have for CP violation in hadrons is given by the upper bound of the Neutron Electric Dipole Moment.

In this thesis we will deal primarily with the possibility of P and CP violation in the decay of the neutral Sigma hyperon, i.e. a radiative decay $\Sigma^0 \to \Lambda\gamma$ and a subsequent weak decay $\Lambda \to p\pi^-$. Hyperons are baryons with one or more strange quarks. Hyperon physics provides an opportunity to investigate, among other things, the structure of matter, the spin dynamics in hyperon decays, and the physics underlying hyperon production [7]. The first of these - the structure of matter - has a long history in the study of form factors. Electromagnetic and transition form factors in nucleon to baryonic resonances have improved our understanding concerning the charge distribution and magnetic properties of certain hadrons [8][9]. The upcoming experiments at FAIR (Facility for Anti-proton and Ion Research) in Germany will help us probe the corresponding properties for hyperons [7]. FAIR could thus serve as a bridge between studying the fundamental structure of hyperons and any effect that its discrete symmetry properties has on this structure.

Working within the framework of Heavy Baryon Chiral Perturbation Theory, it is possible to relate the three-body decay of this Σ^0 hyperon to the current upper bound of the Neutron Electric Dipole Moment [10]. As we will see later, the dynamics of this decay will help us parameterize any P and CP violation that may occur in the initial decay as an angular dependence of its final decay products. An investigation into the corresponding antiparticle decay will also be carried out. This will give us an idea about the experimental viability of looking for angular dependence as separate tests for P and CP violation in decays of neutral hyperons.

Outline

The thesis is structured as follows. Chapter 2 deals with an overview of discrete symmetries and their violation in quantum field theory. In this vein, we will discuss the CKM ansatz which introduces CP violation in the weak sector. The crucial role played by final state interactions via the introduction of a complex phase will also find a mention in this chapter. Chapter 3 examines the violation of discrete symmetries, specifically P and CP, in the strong sector. The QCD Lagrangian has an excess global $U(1)_A$ symmetry which is not realized in the hadron spectrum. This anomalous global $U(1)_A$ symmetry is ultimately tied to the topological structure of the QCD vacuum and leads us to formulate the Effective Vacuum Angle. An illustration of the non-perturbative topological effects of the QCD vacuum will follow. This will help us identify the theta vacuum angle as a crucial component of the Strong CP problem. An observable effect of this effective vacuum angle is the Neutron Electric Dipole Moment. Since this study is carried out within the framework of Heavy Baryon Chiral Perturbation Theory, a section of this chapter will be devoted to this topic. Chapter 4 introduces the most general Lorentz invariant transition form factors for baryons and their utility in studying discrete symmetry properties. Chapter 5 provides the motivation for the study of the decay of the neutral Sigma hyperon. The kinematics of the three-body decay and the observable consequences of P and CP violating effects in this decay are laid out in this chapter. We also discuss the production process for the Σ^0 decay and investigate its effect on the angular dependence of the final decay products. Chapter 6 consists of the relevant calculations and results. Lastly, Chapter 7 presents the conclusions drawn from this study and a brief summary of the thesis.

Chapter 2

Discrete Symmetries in the Standard Model

Symmetries play a central role in physics.[1] Put simply, a symmetry transformation is a change in the observer's point of view that does not change the outcome of the experiment or the 'observable' [4]. Symmetries of a physical theory can be exact or approximate. The study of symmetries has been formalized and explored using the theory of groups. More concretely, the properties of a symmetry transformation relate to the representations of the group to which that transformation belongs [12].

The theories describing strong, weak and electromagnetic interactions between fundamental particles are classified by the symmetry group respected by each theory. In fact, the $SU(3) \times SU(2) \times U(1)$ gauge theory is just another way to describe what physicists call the Standard Model [13]. Additionally, our quantum field theories are invariant under the Lorentz and translational group of transformations. The fact that quantum field theories respect these symmetries has important consequences. The relationship between continuous symmetry transformations and conservation laws, given by Noether's theorem, is one such consequence and is a cornerstone of any field theory [13]. Furthermore, the fact that these symmetries can be broken leads

[1] Following Castellani, we say that symmetries can be attributed to physical systems or to physical laws. This work focuses on the latter, i.e. symmetries based on invariance principles of the interaction under study [11].

to intriguing consequences. A broken symmetry, quite apart from reducing the beauty of a theory, reveals something deeper about it. In group theoretic terms, a broken symmetry implies that the original symmetry group has been broken into one of its subgroups. We can then describe symmetry breaking as the relation between transformation groups, namely the initial unbroken symmetry group and its subgroups [11].

A symmetry can be broken in two ways, spontaneously or explicitly. Spontaneous symmetry breaking refers to those symmetries of the action that do not leave the vacuum state invariant [14]. The spontaneous breaking of the approximate global $SU(2)_L \times SU(2)_R$ symmetry of the strong interaction led to the identification of low mass spinless particles (pseudo-Goldstone bosons), the pions. This was followed by the discovery of spontaneous breaking of the exact local $SU(2)\times U(1)$ symmetry of the weak and electromagnetic interactions. This spontaneous breaking of gauge symmetries is now called the Higgs mechanism as it gives rise to helicity zero states of vector particles, which then acquire mass [14].

Similarly, discrete symmetries play a fundamental role in the Standard Model. Space inversion or parity, charge conjugation and time reversal are the discrete symmetries that are relevant in any discussion of quantum field theories [3]. Unlike continuous symmetry transformations (translational or Lorentz) where we first consider an infinitesimal transformation about the identity, and then proceed to study finite transformations by compounding several infinitesimal operations, discrete (from the Latin *discretus* meaning "separated") symmetries are non-continuous and cannot be treated in the same way [15]. Therefore it is possible for a quantum field theory to be Lorentz invariant (under the proper orthochronous subgroup) while not respecting a discrete symmetry [16].

In this chapter, we will first explore the three discrete symmetries in quantum field theories and discuss their relevance to our decay. In the subsequent section we will briefly review the violation of these discrete symmetries in the Standard Model, focusing on the CKM mechanism which minimally implements CP violation in the SM. In the last section, we will discuss final state interactions and their importance in any study based on P or CP violation.

In the process we will analyze the discrete symmetry properties of an interaction Lagrangian that characterizes the $\Sigma^0 \to \Lambda\gamma$ decay.

2.1 Parity

Parity refers to the operation of space inversion. In other words, it refers to a mirror reflection followed by a rotation of 180° around an axis perpendicular to the mirror [15]:

$$\vec{x} \xrightarrow{P} \vec{x}' = -\vec{x}. \tag{2.1}$$

The coordinate axis after a parity operation is projected back through itself and stands inverted. We thereby transform a right-handed coordinate system into a left-handed one and vice versa. Parity transformations in classical dynamics manifest themselves as a change in sign for *polar vectors* like position (Eq. (2.1)) and momentum [3]:

$$\vec{p} \xrightarrow{P} \vec{p}' = -\vec{p} \tag{2.2}$$

and no change in sign for *axial vectors* like angular momentum:

$$\vec{l} = \vec{x} \times \vec{p}, \tag{2.3}$$

$$\vec{l} \xrightarrow{P} \vec{l}' = \vec{l}. \tag{2.4}$$

Further, we have *scalars* like $S = \vec{p_1}.\vec{p_2}$ that do not change sign under parity transformations

$$S \xrightarrow{P} S \tag{2.5}$$

and *pseudoscalars* like $\mathcal{P} = \vec{p}.\vec{l}$ that do [3]:

$$\mathcal{P} \xrightarrow{P} -\mathcal{P}. \tag{2.6}$$

In non-relativistic quantum mechanics, the parity transformation is defined by a unitary operator P which acts on the complex Hilbert space spanned by state vectors. Since it is a unitary operator it satisfies the condition $P^\dagger P = \mathbb{1}$. If the Hamiltonian operator commutes with the parity operator, we say that the process is parity invariant, or $P^{-1}HP = H$. This can be understood as saying that the total energy of the system, for a parity symmetric potential, remains unchanged after an inversion of the coordinate axis. Notice that if H and P commute, the Schrödinger equation:

$$i\hbar \frac{\partial \psi(\vec{x}, t)}{\partial t} = H\psi(\vec{x}, t) \tag{2.7}$$

tells us that both $\psi(\vec{x}, t)$ and $P\psi(\vec{x}, t) = \psi(-\vec{x}, t)$ represent possible solutions, as does any combination of these two solutions. That is, for a spherical (parity-even) potential, we can express all solutions as eigenstates of parity [3]. In order for the above to hold true, we see that the condition

$$P^{-1} i P = i \tag{2.8}$$

must be satisfied. This shows us that parity is a linear operator. Further, based on the correspondence principle we require that parity and the rotation operator commute (for an explicit derivation of this, refer to [15] and [17]). This in turn implies that parity commutes with the infinitesimal rotation operator, and therefore with the angular momentum operator $\vec{J}$:

$$[\vec{J}, P] = 0. \tag{2.9}$$

We refer to Eq. (2.4), where we discussed the invariance of angular momentum under parity due its being an axial vector ($\vec{l} = \vec{x} \times \vec{p}$). This is the same as saying:

$$[\vec{L}, P] = 0 \tag{2.10}$$

where $\vec{L}$ is now the orbital angular momentum operator.

However, unlike rotation, space inversion does not commute with the position operator. This can be understood (again, from the correspondence principle) as saying that the expectation value of the position operator changes sign under parity. That is [3]:

$$P^{-1}\vec{X}P = -\vec{X}, \quad \text{or} \quad \{\vec{X}, P\} = 0. \tag{2.11}$$

The same holds true for the momentum operator $\vec{P}$, which is the generator of (infinitesimal) translation:

$$P^{-1}\vec{P}P = -\vec{P} \quad \text{or} \quad \{\vec{P}, P\} = 0. \tag{2.12}$$

One of the strongest consistency checks for Eq. (2.9), Eq. (2.11), and Eq. (2.12), comes from the fundamental quantization conditions of quantum mechanics:

$$[X_i, P_j] = i\delta_{ij}, \tag{2.13}$$

$$[J_i, J_j] = iJ_k\varepsilon_{ijk} \tag{2.14}$$

where we have set $\hbar = 1$. On using the properties of unitarity and linearity, we find that the parity operation leaves the above two fundamental conditions invariant. We now have a strong argument for the unitarity of the parity operator, $P^{\dagger}P = 1$ or $P^{\dagger} = P^{-1}$.

Parity considerations serve as a powerful tool while studying reactions. For instance, the angular solutions to the Schrödinger equation for a spherically symmetric potential well are the spherical harmonics [15]:

$$Y_{lm}(\theta, \phi) = (-1)^m \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}} P_l^{|m|}(\cos\theta)e^{(im\phi)}. \tag{2.15}$$

These spherical harmonics transform under parity as follows:

$$Y_{lm} \xrightarrow{P} (-1)^{l+m}(-1)^m Y_{lm} = (-1)^l Y_{lm}. \tag{2.16}$$

This means that s, d, g (and so on) waves have even parity while p, f, h (and so on) waves have odd parity. For a reaction where parity is conserved, we will have $[H, P] = 0$, where H is the Hamiltonian of the process. We can then analyze the orbital angular momentum as given in Eq. (2.16) along with the *intrinsic* parity of the particles involved in the reaction. A discussion on the intrinsic parity of the particles in our decay chain will be undertaken below.

The requirement of Lorentz invariance in addition to the postulates of quantum mechanics leads us to quantum field theories. The introduction of discrete symmetries like parity and time-reversal takes us out of the proper orthochronous Lorentz subgroup of transformations (those which are continuously connected to the identity) [16]. In this section we will deal with discrete symmetry properties as they appear in quantum field theory, focussing on spin-1 and spin-1/2 fields, these being relevant to the Σ^0 decay under study. A more detailed discussion of discrete symmetry transformations in QCD will follow in Chapter 3. The present section follows discussions in [3],[4],[13] and [18].

The Lagrangian that leads to the Lorentz covariant form of the Maxwell equation ($\partial_\mu F^{\mu\nu} = eJ^\nu$) is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - eJ^\mu A_\mu \tag{2.17}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ being the field strength tensor and J^μ being the current density. In QED, the current density takes the form of the fermion bilinear $J^\mu = \bar{\psi}\gamma^\mu\psi$ given by the conserved Dirac vector current. It is found in the interaction term of the Lagrangian that couples the photon field to the Dirac field. That is,

$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} + \mathcal{L} \tag{2.18}$$

with $\mathcal{L}_{Dirac} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$ [13]. The parity transformation of the Dirac field will be covered in the next section. Now, for this simplistic theory to be invariant under parity, we have to find the parity transformation of Eq. (2.17) that leaves the action invariant. We must also ensure that the quantization postulates of the theory are invariant under parity. The invariance of the quantization postulates is covered in any quantum field theory text book, and will not be shown here (see for example [13], [15]). In order to demonstrate the invariance of the Lagrangian under parity, we require the following (corresponding to the transformation properties of classical fields) to hold true under a parity operation [3]:

$$P^{-1}A_\mu(t,\vec{x})P = \begin{cases} A_0(t,-\vec{x}), & \mu = 0, \\ -A_\mu(t,-\vec{x}), & \mu = i, \end{cases} \tag{2.19}$$

$$P^{-1}J^\mu(t,\vec{x})P = \begin{cases} J^0(t,-\vec{x}), & \mu = 0, \\ -J^\mu(t,-\vec{x}), & \mu = i, \end{cases} \tag{2.20}$$

and lastly, we should keep in mind that under a parity operation P, the derivative becomes:

$$\partial_\mu^x \xrightarrow{P} \begin{cases} \partial_0^{\mathcal{P}x}, & \mu = 0, \\ -\partial_i^{\mathcal{P}x}, & \mu = i. \end{cases} \tag{2.21}$$

These conditions when applied to Eq. (2.17), yield the following conclusion:

$$\mathcal{L}(t,\vec{x}) \xrightarrow{P} \mathcal{L}(t,-\vec{x}). \tag{2.22}$$

Since we can change the sign of the integration variable, we see that the parity transformation of the Lagrangian as shown above leads to the action being parity invariant:

$$S = \int d^4x\mathcal{L}(t,\vec{x}) \xrightarrow{P} \int dt\, d^3(-\vec{x})\mathcal{L}(t,-\vec{x}) = S. \tag{2.23}$$

In order to see the importance of parity symmetry in our decay, we make a brief digression to discuss the helicity states of a photon. For a massless particle (i.e. a particle with no available rest frame), we can measure its polarization along the direction of motion [4]. In such cases, it makes sense to define a helicity, h, such that:

$$h \equiv \vec{S}.\vec{z} \tag{2.24}$$

where $\vec{S}$ is its polarization direction and $\vec{z}$ is its direction of motion. This helicity h is a pseudoscalar. Now, an on-shell photon has two possible helicity states, $h = \pm 1$ (for a detailed derivation as to why massless states have only two helicity states refer to [4]).

This fact has important consequences for the study of P and CP violation in our decay of the Σ^0 hyperon. As we will see in Chapter 5, the initial helicity of Σ^0 is sufficient to determine the helicity states of Λ and the photon. As discussed above, an on-shell photon can have only one of two possible helicity states, and if the decay $\Sigma^0 \to \Lambda\gamma$ is parity conserving, then it does not discriminate between either of the two possible helicity states of the Σ^0 hyperon. If the photon were virtual, then this would not be true as it would have three possible helicity states to choose from. A more detailed discussion of this will be carried out in Chapter 5.

Now, coming to spin 1/2 particles, the Lagrangian for a free spin 1/2 field is given by [13]:

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\partial\!\!\!/ - m)\psi \tag{2.25}$$

where ψ is the four-component spinor field. The equation of motion for this Lagrangian is the Dirac equation:

$$(i\partial\!\!\!/ - m)\psi = 0. \tag{2.26}$$

The solution to the Dirac equation, Eq. (2.26), is a superposition of plane waves. The spinor field can then be expressed by its Fourier components in momentum-space:

$$\psi(t, \vec{x}) = \sum_{s=\pm} \int \frac{d^3p}{(2\pi)^3 2E_p} (a(\vec{p}, s)u(\vec{p}, s)e^{-ip.x} + b^\dagger(\vec{p}, s)v(\vec{p}, s)e^{+ip.x}) \tag{2.27}$$

where the operators $a(\vec{p}, s)$ and $b^\dagger(\vec{p}, s)$ denote the annihilation operator (defined in Fock space) for particles and creation operator (again, in Fock space) for antiparticles, respectively [13]. Since $\psi(t, \vec{x})$ is a solution of the Dirac equation, the four-component spinors u and v must satisfy:

$$(\not{p} - m)u(\vec{p}, s) = 0 \tag{2.28}$$

and

$$(\not{p} + m)v(\vec{p}, s) = 0. \tag{2.29}$$

For the Lagrangian in Eq. (2.25) to be invariant under parity transformations, we make the ansatz [18]:

$$\psi(t, \vec{x}) \xrightarrow{P} \gamma_0 \psi(t, -\vec{x}). \tag{2.30}$$

Then we have

$$\overline{\psi}(t, \vec{x}) \xrightarrow{P} \overline{\gamma_0 \psi}(t, -\vec{x}) = \overline{\psi}(t, -\vec{x})\gamma_0. \tag{2.31}$$

This ensures that the Lagrangian transforms under parity as:

$$\mathcal{L}(t, \vec{x}) \xrightarrow{P} \mathcal{L}(t, -\vec{x}) \tag{2.32}$$

which in turn leaves the action invariant. Note that if $\psi(t, \vec{x})$ satisfies the Dirac equation, then so too does the parity transformed spinor $\gamma_0 \psi(t, -\vec{x})$. Likewise, the quantization conditions are also invariant under parity as is easily demonstrated using the known transformation properties of the four-component spinor (see for instance [16], [15]).

The discussion above has implicitly assumed that a parity transformation does not induce a phase. However, this need not be true. For an elementary field we could define a more general parity transformation than described in Eq. (2.30) as follows [4]:

$$\psi(t, \vec{x}) \xrightarrow{P'} \eta_p \gamma^0 \psi(t, -\vec{x}). \tag{2.33}$$

Since we require $P'^2 = \mathbb{1}$, we have $\eta_p = \pm 1$ [15]. Parity could very well be redefined using known conserved internal quantum numbers like baryon number B, electric charge Q and lepton number L [15]:

$$P' = Pe^{i(aB+bQ+cL)} \tag{2.34}$$

with P as defined in Eq. (2.30) and a, b and c being real numbers. Both P and P' can be considered as the parity operator as long as these internal quantum numbers are conserved in an interaction. Using these 'superselection rules' (baryon number conservation, lepton number conservation etc.), one can assign an intrinsic parity of $+1$ to protons, neutrons and electrons after adjusting the values of a, b and c [15]. This is largely a matter of convention. For a more detailed discussion on intrinsic parity and superselection rules, see [15],[17].

Fermions and anti-fermions carry opposite intrinsic parity as can be demonstrated when performing the parity operation on the solution to the Dirac equation Eq. (2.27). This must satisfy the condition given in Eq. (2.30) and so we obtain the transformation properties of the operators $a(\vec{p}, s)$ and $b(\vec{p}, s)$ under parity [3]:

$$P^{-1}a(\vec{p}, s)P = a(-\vec{p}, s) \tag{2.35}$$

$$P^{-1}b(\vec{p}, s)P = -b(-\vec{p}, s) \tag{2.36}$$

Now, a photon field operator in Fock space is given in terms of its creation and annihilation operators, $d(\vec{p}, s)$ and $d^{\dagger}(\vec{p}, s)$, as follows:

$$A^{\mu}(t, \vec{x}) = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_{s=\pm} [d(\vec{p}, s)\varepsilon^{\mu}(\vec{p}, s)e^{-ip.x} + d^{\dagger}(\vec{p}, s)\varepsilon^{\mu *}(\vec{p}, s)e^{ip.x}] \tag{2.37}$$

where $\varepsilon^{\mu}(\vec{p}, s)$ is the polarization vector for a photon with momentum $\vec{p}$ and 'spin' s. For an on-shell photon traveling in the z-direction, the polarization vector can be written as:

$$\varepsilon^{\mu}(\vec{p}, \pm) = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0). \tag{2.38}$$

Keeping in mind the transformation properties of the photon field (Eq. (2.19)) and the knowledge that the photon polarization transforms under parity like spin (an axial vector), we see that the creation operator, d, is odd under parity [3]:

$$P^{-1} d(\vec{p}, \pm) P = -d(-\vec{p}, \pm) \tag{2.39}$$

This could be interpreted as saying that a one-photon state carries an odd intrinsic parity with $\eta_p = -1$.

Historically, physicists have *assigned* an intrinsic parity to the electron and the proton (and their respective antiparticles) as a matter of convention [15]. As we saw above, the intrinsic parity of the photon, on the other hand, was *calculated* to be -1. For particles created or decaying in parity conserving reactions, we can determine their intrinsic parity by analyzing orbital angular momentum (see Eq. (2.16)), i.e. the angular distribution of the particles [15]. Following this method it was possible to establish that pions have negative intrinsic parity, i.e. they are pseudoscalars. In addition, if symmetries imply a multiplet structure, then the intrinsic parities of all the members comprising this multiplet can be determined. In particular, one finds that Σ^0, Λ, the proton and the neutron all have the same intrinsic parity of $+1$ due to flavor symmetry.

For example, in the weak decay of $\Lambda \to p\pi^-$, the total angular momentum of the initial state, in the rest frame of Λ, is $1/2$. Since the proton is a spin $1/2$ particle, the relative orbital angular momentum for the final state is $l = 1$ or $l = 0$. As stated above, the relative intrinsic parity of Λ and p is $+1$, and therefore, from Eq. (2.16), the parity of the final state is given by [15]:

$$(-1)^l \eta_\pi = +1. \tag{2.40}$$

Now, because the pions are pseudoscalars ($\eta_\pi = -1$), we see that when $l = 1$

(p *wave* final state) parity is conserved, and when $l = 0$ (s *wave* final state) parity is violated in the process. Since the weak interaction does not respect parity symmetry, we will have both s and p wave contributions to the decay width.

Similarly for our initial decay, $\Sigma^0 \to \Lambda\gamma$, angular momentum conservation tells us that the decay matrix element can have contributions from even and odd partial waves. With the photon having a negative intrinsic parity and both the baryons involved in the decay having a positive intrinsic parity, we see that (following Eq. (2.16)) odd partial waves conserve parity while even partial waves violate it. Now, the only non-trivial (complex) *phase* that could arise in our study of the Σ^0 decay is due to final state interactions and will be discussed in detail in the last section of this chapter.

Going back to our discussion on spinor fields, from Eq. (2.30) and Eq. (2.31) we see that the combination of the two spinors, $\overline{\psi}\psi(t, \vec{x})$, in addition to being Lorentz invariant, transforms as a scalar under parity [18]:

$$\overline{\psi}\psi(t, \vec{x}) \xrightarrow{P} \overline{\psi}\psi(t, -\vec{x}). \tag{2.41}$$

In like manner, we can study the properties of composite objects $\overline{\psi}\Gamma^\mu\psi$ called fermion (or spinor) bilinears, where Γ^μ is any 4×4 matrix that is compatible with Lorentz invariance. Fermion bilinears will play a crucial role in determining the parity transformation properties of our decay. Consider for example the fermion bilinear $\overline{\psi}\gamma^\mu\psi$. Under parity this transforms as a vector [18]:

$$\overline{\psi}\gamma^\mu\psi(t, \vec{x}) \xrightarrow{P} \overline{\psi}\gamma^0\gamma^\mu\gamma^0\psi(t, -\vec{x}) = \begin{cases} \overline{\psi}\gamma^0\psi(t, -\vec{x}), & \mu = 0, \\ -\overline{\psi}\gamma^i\psi(t, -\vec{x}), & \mu = i. \end{cases} \tag{2.42}$$

Note that i in the above equation runs from 1 to 3. Similarly, $\overline{\psi}\sigma_{\mu\nu}\psi$ transforms as an anti-symmetric tensor under Lorentz transformation, where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. On applying a parity transformation it behaves as follows

[18][13]

$$\overline{\psi}\sigma_{\mu\nu}\psi(t,\vec{x}) \xrightarrow{P} (-1)^{\mu}(-1)^{\nu}\overline{\psi}\sigma_{\mu\nu}\psi(t,-\vec{x}), \tag{2.43}$$

with $(-1)^{\mu} \equiv 1$ for $\mu = 0$ and $(-1)^{\mu} \equiv -1$ for $\mu = i$.

We can also construct bilinears using a matrix conventionally defined as follows:

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{2.44}$$

Further, we notice:

$$\{\gamma_5, \gamma^{\mu}\} = 0. \tag{2.45}$$

We can now form a Lorentz scalar and another Lorentz vector [19]:

$$\overline{\psi}\gamma_5\psi \quad \text{and} \quad \overline{\psi}\gamma_5\gamma^{\mu}\psi.$$

They transform under parity as:

$$\overline{\psi}\gamma_5\psi(t,\vec{x}) \xrightarrow{P} \overline{\psi}\gamma^0\gamma_5\gamma^0\psi(t,-\vec{x}) = -\overline{\psi}\gamma_5\psi(t,-\vec{x}), \tag{2.46}$$

$$\overline{\psi}\gamma_5\gamma^{\mu}\psi(t,\vec{x}) \xrightarrow{P} \overline{\psi}\gamma^0\gamma_5\gamma^{\mu}\gamma^0\psi(t,-\vec{x}) = \begin{cases} -\overline{\psi}\gamma_5\gamma^0\psi(t,-\vec{x}), & \mu = 0, \\ +\overline{\psi}\gamma_5\gamma^i\psi(t,-\vec{x}), & \mu = i, \end{cases} \tag{2.47}$$

a pseudoscalar and an axial vector, respectively [19]. In like manner, the bilinear $\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi$ transforms as [13]:

$$\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi(t,\vec{x}) \xrightarrow{P} -(-1)^{\mu}(-1)^{\nu}\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi(t,-\vec{x}). \tag{2.48}$$

We therefore have 16 possible bilinears transforming under parity and Lorentz transformations that result from the four-component spinors. These comprise of 1 scalar, 1 pseudoscalar, 4 vectors, 4 axial vectors and 6 anti-symmetric tensors. Any other bilinear that can be constructed must be expressed in terms of the above bilinears. We will see this more explicitly in Chapter 4, where we construct Lorentz invariant form factors using combinations of these fermion bilinears. The knowledge of these bilinear transformations helps us pin down the parity violating terms in the Lagrangian as a potential source of Electric Dipole Moments.

Consider, for example, an interaction Lagrangian for the Σ^0-Λ transition:

$$\mathcal{L}_{\Sigma^0-\Lambda} = \frac{e\, c_B}{(m_\Sigma + m_\Lambda)} \overline{\Lambda} \gamma_5 \sigma_{\mu\nu} \Sigma^0 F^{\mu\nu} + \frac{e\, \bar{c}_B}{(m_\Sigma + m_\Lambda)} \overline{\Sigma}^0 \gamma_5 \sigma_{\mu\nu} \Lambda F^{\mu\nu} \tag{2.49}$$

where c_B and $\bar{c}_B$ are complex numbers for the neutral Sigma decay and its charge conjugated process, respectively. The reasons behind the Lagrangian assuming this form will become clear in Chapter 4 and Chapter 5. For now we are interested in the parity symmetry properties of this interaction Lagrangian. If Σ^0 and Λ transform under parity in the *same* way, then the above Lagrangian violates parity (see Eq. (2.48)). If, however, there is a *relative* parity of -1 between Σ^0 and Λ, the above Lagrangian conserves parity. As we mentioned earlier, Σ^0 and Λ belong to the same multiplet structure, and therefore both transform under parity in the same way. Thus the interaction Lagrangian in Eq. (2.49) describes a P violating process. In Chapter 5 we will see how this manifests itself experimentally in the decay chain $\Sigma^0 \to p\pi^-\gamma$.

Parity also inverts the 'handedness' or chirality of a spinor. If we use the projection operator, we can express this property in terms of the four-component spinor:

$$\psi_R = \frac{1}{2}(1+\gamma_5)\psi, \qquad \psi_L = \frac{1}{2}(1-\gamma_5)\psi \tag{2.50}$$

where under a parity transformation we observe:

$$\psi_R(t, \vec{x}) \xrightarrow{P} \psi_L(t, -\vec{x}). \tag{2.51}$$

That is, a right handed spinor transforms into a left handed one and vice versa [19]. In a subsequent section of this chapter we will discuss the parity violating weak interaction, and how this results in the left and right handed particles being treated differently by the weak interaction. A theory which treats ψ_R and ψ_L differently is referred to as a *chiral* theory whereas one that treats them on an equal footing is called a *vector-like* theory [19]. The electroweak theory is an example of a chiral theory. QCD, on the other hand, is a vector-like theory since it consists of both right and left handed quarks coupled in the same way to the gluon field.

QCD possesses an approximate chiral symmetry [14]. This has interesting consequences when studying P and CP violating effects in the strong sector. The approximate $SU(3)_{L-R}$ symmetry is spontaneously broken to yield an octet of pseudo-Goldstone bosons [14]. The axial component of the additional $U(1)_{L-R}$ group, however, is not realized as a symmetry for reasons that will be discussed in Chapter 3. This axial $U(1)$ problem, as we will discover in Chapter 3, has deep implications for any P and CP invariant theory of the strong interaction.

2.2 Charge Conjugation

Classically, charge conjugation is a rather straightforward idea, wherein we replace the positive charges with negative ones and vice versa. Maxwell's equations are invariant under charge conjugation, i.e. when the sign of the charge density (ρ) is reversed, $\rho \xrightarrow{C} -\rho$ [15]. This results in the current, electric field and magnetic field being odd under charge conjugation:

$$\vec{j} \xrightarrow{C} -\vec{j}, \tag{2.52}$$

$$\vec{E} \xrightarrow{C} -\vec{E}, \tag{2.53}$$

$$\vec{B} \xrightarrow{C} -\vec{B}. \tag{2.54}$$

The concept of charge conjugation is not well defined in quantum mechanics, especially because antiparticles are an alien notion in non-relativistic physics. In one sense we can say that the Dirac theory predicted the existence of a particle with opposite charge with the same mass and that charge conjugation as a symmetry became well defined only in relativistic quantum physics [15]. Like parity, charge conjugation is a unitary operator:

$$CC^{\dagger} = 1. \tag{2.55}$$

Its effect on a single particle state is as follows:

$$C\,|(\vec{p}, s, Q)\rangle = \eta_c\,|(\vec{p}, s, -Q)\rangle \tag{2.56}$$

where Q is the charge of the particle with momentum p and spin s, while η_c is the phase induced by such a transformation. As with parity transformations, applying C twice restores the original state, i.e. $C^2 = \mathbb{1}$. This implies ${\eta_c}^2 = 1$ or $\eta_c = \pm 1$. η_c is called charge-conjugation parity or C parity of the particle [4]. As was the case with parity, for any operator C satisfying Eq. (2.56) we can define another operator with a different η_c using other internal symmetry phase transformations such as the one given in Eq. (2.34).

The only particles which have a well defined C parity are the neutral particles like pions and photons [4]. These carry no other conserved quantum numbers and are their own antiparticles. Now, in the electromagnetic and strong sectors, the Hamiltonian for a given interaction commutes with the charge conjugation operator as we expect a particle and its corresponding antiparticle to have the same energy:

$$[H, C] = 0. \tag{2.57}$$

This means that both the interaction potential and the free Hamiltonian must have charge conjugation as a symmetry. As a consequence, the S-matrix is invariant under charge conjugation [15]:

$$C^{-1}\mathcal{S}C = \mathcal{S}. \tag{2.58}$$

For a reaction where both the initial and final states are only neutral particles, Eq. (2.58) tells us that C parity for both the initial and final state must be the same [4]. In the reaction $\pi \to \gamma\gamma$, if the intrinsic C parity of the photon is -1, then the decay implies $\eta_{\pi^0} = 1$. This also makes clear that the decay $\pi \to 3\gamma$ is forbidden by charge conjugation symmetry.

In the Lagrangian for the photon field shown in Eq. (2.17), we notice that the theory remains invariant under charge conjugation transformation when we consider the transformation properties of the current and the electromagnetic four-potential [3]:

$$CA_\mu(t, \vec{x})C^\dagger = -A_\mu(t, \vec{x}), \tag{2.59}$$

$$CJ^\mu(t, \vec{x})C^\dagger = -J^\mu(t, \vec{x}). \tag{2.60}$$

And so $\mathcal{L}(t, \vec{x}) \xrightarrow{C} \mathcal{L}(t, \vec{x})$. One can see from the above equations that C conjugation does not affect anything related to the Lorentz group (like t, $\vec{x}$ and μ). The quantization conditions too can be shown to be invariant under charge conjugation, see for instance [16] and [3]. The photon field operator discussed in Eq. (2.37) is composed of the polarization vector $\varepsilon(\vec{p}, \pm)$ and the creation (annihilation) operators. The transformation property of the former under charge conjugation is the same as that of spin, i.e. it is unaffected. For Eq. (2.59) to hold true, the charge conjugation transformation of the annihilation operator must be

$$Cd(\vec{p}, \pm)C^\dagger = -d(\vec{p}, \pm). \tag{2.61}$$

This can be interpreted as saying that a photon state has an odd intrinsic C parity with $\eta_c = -1$.

The operation of charge conjugation on the free Dirac Lagrangian Eq. (2.25) transforms a fermion field into an antifermion field and vice versa. Thus, we consider a transformation of the field which should obey the transformation property of the vector current, J^μ, under charge conjugation (again using the analogy from classical dynamics) [3]:

$$C\psi(t, \vec{x})C^\dagger = \mathcal{C}\overline{\psi}^T(t, \vec{x}) \tag{2.62}$$

with $\mathcal{C}$ being a 4×4 matrix we have to identify. Note that the phase η_c

has been set to 1. We can now cross-check the transformation of the current $J^\mu = \overline{\psi}(t,\vec{x})\gamma^\mu\psi(t,\vec{x})$ given in Eq. (2.60):

$$\begin{aligned} CJ^\mu(t,\vec{x})C^\dagger &= \overline{\psi^C}(t,\vec{x})\gamma^\mu\psi^C(t,\vec{x}) \\ &= \psi_\alpha(t,\vec{x})[\gamma^0\mathcal{C}^\dagger\gamma^0\gamma^\mu\mathcal{C}]_{\alpha\beta}\overline{\psi}_\beta(t,\vec{x}) \end{aligned} \tag{2.63}$$

with $\psi^C = C\psi(t,\vec{x})C^\dagger$ and α,β being the spinor indices. In order for the last line to equal $-J^\mu(t,\vec{x})$, we require:

$$\gamma^0\mathcal{C}^\dagger\gamma^0\gamma^\mu\mathcal{C} = \gamma^{\mu T}. \tag{2.64}$$

That is, if $\mathcal{C} = i\gamma^2\gamma^0$ the above condition is satisfied. With the usual choice of phase $C^2 = \mathbb{1}$ and consequently $C^\dagger = C^{-1}$, the fermion field transforms under charge conjugation as [3]:

$$\psi(t,\vec{x}) \xrightarrow{C} i\gamma^2\gamma^0\overline{\psi}^T(t,\vec{x}). \tag{2.65}$$

The study of fermion bilinears is just as crucial here as it was for the case of parity. A detailed derivation of these transformations for bilinears can be found in [13], [19] and it follows the same line of reasoning as was illustrated in the case for parity. We reproduce below their transformation properties under C symmetry [13]:

$$\overline{\psi}\psi \xrightarrow{C} \overline{\psi}\psi, \tag{2.66}$$

$$i\overline{\psi}\gamma_5\psi \xrightarrow{C} i\overline{\psi}\gamma_5\psi, \tag{2.67}$$

$$\overline{\psi}\gamma^\mu\gamma_5\psi \xrightarrow{C} \overline{\psi}\gamma^\mu\gamma_5\psi, \tag{2.68}$$

$$\overline{\psi}\sigma_{\mu\nu}\psi \xrightarrow{C} -\overline{\psi}\sigma_{\mu\nu}\psi, \tag{2.69}$$

$$\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi \xrightarrow{C} -\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi. \tag{2.70}$$

The invariance of the S-matrix under charge conjugation, shown in Eq. (2.58), implies that a decay process for a particle must have the same rate when replaced by its anti-particle as long as the interaction observes charge conjugation symmetry [4]. The experiments conducted in 1957 showed that C

was not conserved for the theory of weak interactions as laid out by Lee, Oehme and Yang [20]. On the other hand, the Lagrangian for the strong and electromagnetic interactions conserve charge parity.

In addition to analyzing the decay of $\Sigma^0 \to p\pi^-\gamma$, we will also study the corresponding antiparticle decay $\overline{\Sigma}^0 \to \overline{p}\pi^+\gamma$. Using the Lorentz invariant transition form factors to be discussed in Chapter 4, it will be fruitful to check whether the two decays produce identical angular distributions. If there is a disparity, this would point to an apparent non-conservation of charge conjugation symmetry.

Let us now consider the charge conjugation properties of the P violating interaction Lagrangian discussed in Eq. (2.49). Following Eq. (2.70), we note that the interaction Lagrangian in Eq. (2.49) under charge conjugation becomes:

$$\mathcal{L}_{\Sigma^0-\Lambda} \xrightarrow{C} \frac{e\, c_B}{(m_\Sigma + m_\Lambda)} \overline{\Sigma}^0 \gamma_5 \sigma_{\mu\nu} \Lambda F^{\mu\nu} + \frac{e\, \bar{c}_B}{(m_\Sigma + m_\Lambda)} \overline{\Lambda} \gamma_5 \sigma_{\mu\nu} \Sigma^0 F^{\mu\nu} \qquad (2.71)$$

The hermiticity condition when applied to Eq. (2.49) yields $\bar{c}_B = -c_B^*$. Therefore, for C symmetry to be preserved in the $\Sigma^0 \to \Lambda\gamma$ decay, we must have $c_B = \bar{c}_B = -c_B^*$. In other words, if the decay is invariant under charge conjugation, this implies that c_B must be purely imaginary ($c_B + c_B^* = 0$) and CP is broken. If, on the other hand, CP is conserved, then c_B is purely real (we will see this explicitly in the next section) and C symmetry is violated. Note that if c_B is neither purely real or imaginary, then both C and CP symmetry is not respected in this decay.

As we will see in the section dealing with final state interactions, c_B can be written in terms of a modulus and a phase due to the fact that it is a complex number. This phase, taken together with the phase due to final state interactions, will give us a more complete picture regarding C and CP symmetry properties of the Σ^0 decay.

2.3 Time Reversal

Time reversal symmetry is the symmetry of a theory under the transformation $t \to t' = -t$. The operator that enables such a theory, say T, generates a reversal of motion. Just like in classical dynamics, we see that in quantum mechanics T does not affect the position operator but certainly reverses the momentum, both linear and angular [3]:

$$T\vec{X}T^{-1} = \vec{X}, \tag{2.72}$$

$$T\vec{P}T^{-1} = -\vec{P}, \tag{2.73}$$

$$T\vec{J}T^{-1} = -\vec{J}. \tag{2.74}$$

On inspecting the fundamental commutation relation between the position and momentum operators $[X_i, P_j] = i\delta_{ij}$, one finds something strange. In order for this commutation relation to be invariant under time reversal, we notice that the following condition must be fulfilled:

$$T^{-1}iT = -i. \tag{2.75}$$

This is what Wigner called the anti-linear property of the T operator [15]. Further, we also demand that the time-dependent Schrödinger equation:

$$H\psi(t,\vec{x}) = i\frac{\partial}{\partial t}\psi(t,\vec{x})$$

remains invariant under time reversal. However, we notice that $\psi(-t,\vec{x})$ does not satisfy the above equation. Instead, following Wigner, we state that there are two ways to preserve the norm $\langle\psi|\psi\rangle = \langle T\psi|T\psi\rangle$. The first when T is a unitary operator such that [15]:

$$\langle\phi|\psi\rangle = \langle T\phi|T\psi\rangle \tag{2.76}$$

and the second when T is an anti-unitary operator:

$$\langle \phi | \psi \rangle^* = \langle \psi | \phi \rangle = \langle T\phi | T\psi \rangle \, . \tag{2.77}$$

The first case is forbidden given that we know $\psi(-t, \vec{x})$ does not satisfy Schrödinger's equation. The only conclusion we can draw is that T is an anti-unitary operator:

$$T\psi(t, \vec{x}) = \psi^*(-t, \vec{x}). \tag{2.78}$$

The effect of the T operator on the S-matrix is as expected. It interchanges the initial and final states to give us the inverse $\mathcal{S}$ operator [15]:

$$T\mathcal{S}T^{-1} = \mathcal{S}^\dagger = \mathcal{S}^{-1}. \tag{2.79}$$

The anti-unitary (and anti-linear) property of T has interesting consequences. The foremost among them is Kramer's degeneracy which tells us that for any energy eigenstate ψ of an odd number of spin 1/2 particles, there is an orthogonal eigenstate of the same energy in the absence of an external magnetic field (so long as $T^2 = -1$) [4]. The details and implications of this theorem will not be carried out here and can be found in [4].

The Lagrangian for the photon field, given in Eq. (2.17), transforms under time reversal as follows:

$$T\mathcal{L}(t, \vec{x})T^{-1} = \mathcal{L}(-t, \vec{x}). \tag{2.80}$$

Since the action is given by the integration of the Lagrangian over all space-time coordinates, a simple change of variable ensures that the action is invariant under time reversal [3]. Eq. (2.80) is achieved by the following transformation properties of the current and the electromagnetic four-potential:

$$TJ^\mu(t, \vec{x})T^{-1} = \begin{cases} J^0(-t, \vec{x}), & \mu = 0, \\ -J^i(-t, \vec{x}), & \mu = i, \end{cases} \tag{2.81}$$

$$T A_\mu(t,\vec{x}) T^{-1} = \begin{cases} A_0(-t,\vec{x}), & \mu = 0, \\ -A_i(-t,\vec{x}), & \mu = i. \end{cases} \tag{2.82}$$

For the free Dirac theory (2.25), the transformation properties are a little trickier. Time reversal not only reverses the momentum of a particle but also its spin as can be seen from Eq. (2.74) for angular momentum. This implies that a time reversal operator must be one that flips the spinor, i.e. the sign associated with its spin state [13]. The spinors with their signs flipped and momentum reversed take the form:

$$u(\tilde{p},-s) = -\gamma^1\gamma^3[u(p,s)]^*, \tag{2.83}$$

$$v(\tilde{p},-s) = -\gamma^1\gamma^3[v(p,s)]^*. \tag{2.84}$$

This leads us to the following transformation property of fermion annihilation operators:

$$T a(\vec{p},s) T^{-1} = a(-\vec{p},-s). \tag{2.85}$$

Considering the above transformation properties and that of the spinors $u(\vec{p},s)$ and $v(\vec{p},s)$, the transformation of the fermion field under time reversal obtained from Eq. (2.27) is [13]:

$$T\psi(t,\vec{x})T^{-1} = \gamma^1\gamma^3\psi(-t,\vec{x}). \tag{2.86}$$

We can now derive the transformation properties of fermion bilinears as was done for parity and charge conjugation. These are [3]:

$$\overline{\psi}\psi(t,\vec{x}) \xrightarrow{T} \overline{\psi}\psi(-t,\vec{x}), \tag{2.87}$$

$$i\overline{\psi}\gamma_5\psi(t,\vec{x}) \xrightarrow{T} -i\overline{\psi}\gamma_5\psi(-t,\vec{x}), \tag{2.88}$$

$$\overline{\psi}\gamma_\mu\psi(t,\vec{x}) \xrightarrow{T} \begin{cases} \overline{\psi}\gamma_0\psi(-t,\vec{x}), & \mu = 0, \\ -\overline{\psi}\gamma_i\psi(-t,\vec{x}), & \mu = i, \end{cases} \tag{2.89}$$

$$\overline{\psi}\gamma_\mu\gamma_5\psi(t,\vec{x}) \xrightarrow{T} \begin{cases} \overline{\psi}\gamma_0\gamma_5\psi(-t,\vec{x}), & \mu = 0, \\ -\overline{\psi}\gamma_i\gamma_5\psi(-t,\vec{x}), & \mu = i, \end{cases} \tag{2.90}$$

$$\overline{\psi}\sigma_{\mu\nu}\psi(t,\vec{x}) \xrightarrow{T} -(-1)^\mu(-1)^\nu\overline{\psi}\sigma_{\mu\nu}\psi(-t,\vec{x}), \tag{2.91}$$

$$\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi(t,\vec{x}) \xrightarrow{T} -(-1)^\mu(-1)^\nu\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi(-t,\vec{x}). \tag{2.92}$$

The CPT theorem respected by quantum field theories tells us that a violation of T symmetry is also a violation of CP symmetry. A non-vanishing electric dipole moment of the neutron would point towards a CP symmetry violation in addition to possible P symmetry breaking [21]. An upper bound on the T (and therefore CP) violating neutron electric dipole moment is currently the best estimate we have for an observable effect of the Strong CP problem [22]. This will be the topic of discussion in Chapter 3.

Following Eq. (2.92), a CP transformation of the interaction Lagrangian Eq. (2.49) yields:

$$\mathcal{L}_{\Sigma^0-\Lambda} \xrightarrow{CP} -\frac{e\,c_B}{(m_\Sigma + m_\Lambda)}\overline{\Sigma}^0\gamma_5\sigma_{\mu\nu}\Lambda F^{\mu\nu} - \frac{e\,\bar{c}_B}{(m_\Sigma + m_\Lambda)}\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Sigma^0 F^{\mu\nu}. \tag{2.93}$$

Taken together with the hermiticity condition, this implies that for the Σ^0 decay to conserve CP, c_B must be purely real or $c_B = c_B^*$.

2.4 CKM Mechanism

As we stated earlier in this chapter, the theoretical framework needed to accommodate CP violation in the SM is described by the CKM mechanism of the weak theory [21]. We provide here a cursory review of the minimal implementation of CP violation in the weak interaction, and analyze whether the weak part of our radiative Σ^0 decay contributes towards a P or CP violation in the decay chain.

The SM incorporates one doublet (SU(2)) of the Higgs field. The Yukawa interaction, which describes the coupling of the quark field to the Higgs field, is given by [23]:

$$\mathcal{L}_Y = -G^U_{ij}\overline{Q}_{Li}\begin{pmatrix}\phi^0\\ -\phi^-\end{pmatrix}U_{Rj} - G^D_{ij}\overline{Q}_{Li}\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}D_{Rj} + h.c. \tag{2.94}$$

where Q_L are the left-handed fermion doublets, $Q_L = \begin{pmatrix}U_L\\ D_L\end{pmatrix}$; U_R and D_R are right-handed fermion singlets; with $U \equiv \{u, c, t\}$ and $D \equiv \{d, s, b\}$. Note that the chiral fermion fields are as defined in Eq. (2.50). Here $G^{U,D}$ are $n \times n$ matrices (with $n = 3$ in the Standard Model for the three quark families) which, due to spontaneous symmetry breaking, give us the mass terms [3]:

$$\mathcal{M}^U = vG^U, \qquad \mathcal{M}^D = vG^D \tag{2.95}$$

with v being the vacuum expectation value of the Higgs field, $\langle\phi^0\rangle \equiv v$. Because of the arbitrary nature of the Yukawa coupling, these mass matrices can in principle contain complex terms. It is precisely this complex Yukawa coupling that brings about an observable CP violating effect [23][3].

Before going further we must state that the appearance of this flavour-space coupling has an impact on the quark gauge interactions which are given by the charged current, weak neutral current and the electromagnetic current [23]:

$$\mathcal{J}^\mu_{CC} = \overline{U}_L\gamma^\mu D_L, \tag{2.96}$$

$$\mathcal{J}^\mu_{NC} = \frac{1}{2}(\overline{U}_L\gamma^\mu U_L - \overline{D}_L\gamma^\mu D_L) - \sin^2\theta_W\mathcal{J}^\mu_{EM}, \tag{2.97}$$

$$\mathcal{J}^\mu_{EM} = \frac{2}{3}\overline{U}\gamma^\mu U - \frac{1}{3}\overline{D}\gamma^\mu D. \tag{2.98}$$

By diagonalizing the mass matrix, we can write the Lagrangian in terms of the mass eigenstates of the quarks [3]. This can be done by adopting unitary matrices $T^U_{L/R}$ and $T^D_{L/R}$ which act as follows:

$$\mathcal{M}^U_{diag} = T^U_L \mathcal{M}^U T^{\dagger U}_R, \qquad \mathcal{M}^D_{diag} = T^D_L \mathcal{M}^D T^{\dagger D}_R \tag{2.99}$$

such that it changes the left and right-handed quark fields into their mass eigenstates [3]. This has no effect on the neutral currents, up to tree-level, which remain diagonal due to the unitarity of $T^{U,D}_{L/R}$ [23].

The charged weak current, on the other hand, transforms non-trivially [23]:

$$\mathcal{J}^\mu_{CC} = \overline{U}_L \gamma^\mu D_L = \overline{U}^m_L \gamma^\mu (T^U_L T^{D\dagger}_L) D^m_L \tag{2.100}$$

where U^m_L, D^m_L are fields with definite mass states due to the diagonalization process. We define a unitary matrix V, called the Cabibbo-Kobayashi-Masakawa (CKM) matrix, such that:

$$V \equiv T^U_L T^{D\dagger}_L. \tag{2.101}$$

It is this CKM matrix that parameterizes quark mixing in the flavour space so that flavour changing reactions are permissible in reactions governed by the weak interaction [23].

In the seminal paper by Kobayashi and Masakawa [6], they argued that it is precisely because of the nature of this V matrix that CP violation is observed in nature. Without going into the details, we can say that merely because V can contain complex phases it does not mean that CP becomes an observable consequence as we can always redefine the quark fields and their respective phases [3]. However, the question arises, when can one *not* rotate away a complex phase arising in V? Kobayashi and Masakawa noted that for three families of quarks, in addition to the three Euler mixing angles, one also gets a non-trivial complex phase that cannot be re-defined away [6]. This is the complex phase that lends itself to an observable CP violation [23][3]. There are several ways to construct V using three mixing angles and one non-trivial complex phase. The standard form is given by [23]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \tag{2.102}$$

with $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$ and δ_{CP} as the CP violating complex phase.

Now how does this non-trivial CP violating complex phase δ_{CP} arising in the weak charged current affect P and CP considerations in our radiative Σ^0 decay? Following Donoghue et al. [24], we state that theories which have CP violation tend to produce nEDMs close to the experimental bounds. Significantly, no electric dipole moment is produced at first order of the weak interaction. This is because at tree-level, due to the non-flavour changing nature of this order of the interaction, the CKM matrix combination $V_{ij}^* V_{ij}$ is real, and so there exists no CP inducing complex phase [24]. At second order in the weak interactions, there is a possibility of a dipole moment in the neutron being observed. This requires an additional gluon loop (see [24]) that could manifest itself as a distinct contribution to CP violation. However, such weak interaction contributions to the CP violating electric dipole moment are several orders of magnitude smaller than the current experimental upper bound of the nEDM [25].

2.5 Final State Interactions

The decay products of the radiative Σ^0 decay are not asymptotic states yet. Due to the strong/electromagnetic interaction(s) they undergo subsequent scattering before ceasing to interact [3]. The observable consequence of these strong/electromagnetic final state interactions (FSI) is the topic of discussion in this section.

FSI becomes important when considering CP violation in hyperon decays. In general, for a decay of hyperon M to a final state f, $M \to f$, that proceeds through two different elementary amplitudes A_1 and A_2, we can write the total transition amplitude as [23]:

$$A(M \to f) = |A_1|e^{i\phi_1}e^{i\delta_1} + |A_2|e^{i\phi_2}e^{i\delta_2}. \tag{2.103}$$

The phase ϕ_i changes sign for a CP conjugate decay (this is often called the *weak* phase) whereas the phase δ_i does not change sign (termed the *strong* phase or the *final state interaction* phase). The charge conjugate reaction has the following transition amplitude:

$$A(\overline{M} \to \overline{f}) = |A_1|e^{-i\phi_1}e^{i\delta_1} + |A_2|e^{-i\phi_2}e^{i\delta_2}. \tag{2.104}$$

The asymmetry between the two decays is parameterized using a decay asymmetry parameter, $\mathcal{A}_{CP}$, defined using partial decay rates as [23][3]:

$$\mathcal{A}_{CP} = \frac{\Gamma(M \to f) - \Gamma(\overline{M} \to \overline{f})}{\Gamma(M \to f) + \Gamma(\overline{M} \to \overline{f})}. \tag{2.105}$$

Utilizing Eq. (2.103) and Eq. (2.104), we have:

$$\mathcal{A}_{CP} = \frac{2|A_2/A_1|\sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2)}{1 + |A_2/A_1|^2 + 2|A_2/A_1|\cos(\phi_1 - \phi_2)\cos(\phi_1 - \phi_2)}. \tag{2.106}$$

Therefore, it is not the single phase of an amplitude that matters in discussions of CP violation (these can be redefined), but the *phase difference* between two amplitudes. That is, an interference between the amplitudes is necessary to have a CP violating effect [23]. Note also that for a CP violation to be observable in the partial decay width calculation, we require a non-zero weak phase difference ($\Delta\phi_W \equiv \phi_1 - \phi_2 \neq 0$) and a non-trivial phase shift due to FSI ($\Delta\delta_F \equiv \delta_1 - \delta_2 \neq 0$) as can be seen from Eq. (2.106) [3].

This is precisely what we expect happens in the radiative decay $\Sigma^0 \to \Lambda\gamma$ where the even (for example, an s wave) and odd (for example, a p wave) partial wave amplitudes of the final state interfere to produce a possible CP violation. We can define a new parameter, called the decay asymmetry, that shows an s and p wave interference:

$$\alpha = \frac{2\text{Re}(s^* p)}{|s|^2 + |p|^2} \tag{2.107}$$

with $s = |s|e^{i\delta_s}e^{i\phi_s}$ and $p = |p|e^{i\delta_p}e^{i\phi_p}$. Here δ_s and δ_p are the s and p wave phase shifts due to strong (or electromagnetic) FSI, while ϕ_s and ϕ_p are the weak phases of the s and p waves, respectively [26].

Since the $\Sigma^0 \to \Lambda\gamma$ decay proceeds largely through an electromagnetic channel, and because the weak part of this decay is suppressed, we consider here an FSI phase shift solely due to strong/electromagnetic interactions of the decay products. What remains to be seen is how a phase induced by FSI affects C and CP symmetry considerations in our Σ^0 decay.

Electromagnetic (and strong) FSI induces a phase shift δ_F so that we can define a new complex constant c as follows:

$$c = c_B e^{i\delta_F}, \qquad \bar{c} = \bar{c}_B e^{i\delta_F}. \tag{2.108}$$

The constants c_B, $\bar{c}_B$ emerge from the interaction Lagrangian in Eq. (2.49). Since c_B is complex, it can have a modulus and a phase δ_B such that:

$$c_B = |c_B|e^{i\delta_B}, \qquad \bar{c}_B = -|c_B|e^{-i\delta_B} = |c_B|e^{i(\pi-\delta_B)} \tag{2.109}$$

where in the second equation we have used the hermiticity condition $\bar{c}_B = -c_B^*$. The complex constant c can now be written as:

$$c = |c_B|e^{i(\delta_B+\delta_F)}, \qquad \bar{c} = |c_B|e^{i(\pi-(\delta_B-\delta_F))}. \tag{2.110}$$

Going back to our discussion in Section 2.2, a C symmetric decay implies $c_B = \bar{c}_B$ (i.e. c_B is purely imaginary) and so $c = \bar{c}$. Looking at Eq. (2.109), this means that $\delta_B = \pi - \delta_B$ (i.e. $\delta_B = \frac{\pi}{2}$) and the second possibility, $\delta_B = 3\pi - \delta_B$ (i.e. $\delta_B = \frac{3\pi}{2}$). As before, for a P violating Lagrangian, conservation of C implies that the decay violates CP. What about the case when CP is conserved? In Section 2.3 we stated that this is fulfilled when $c_B = -\bar{c}_B$ (i.e. c_B is purely real) and so $c = -\bar{c}$. Looking at Eq. (2.109), we see that $\delta_B = 0$

and $\delta_B = \pi$ are the solutions. For a P violating decay, conservation of CP implies C is violated. Note that for the case when P, C and CP are violated (i.e. c_B is neither purely real nor purely imaginary), δ_B is not a multiple of $\frac{\pi}{2}$.

Chapter 3

Strong CP Problem

Having discussed generally the discrete symmetries and their violation in the Standard Model, we will now study in detail their properties in the strong sector. In this chapter, we will fist touch upon the $U(1)_A$ problem, providing a brief review of this unrealized axial symmetry. This brings us to the QCD vacuum which is the topic of discussion in the subsequent section. The non-trivial structure of the QCD vacuum will lead us to the formulation of the Strong CP Problem. We then discuss the effective vacuum angle and present a possible solution to the $U(1)_A$ problem. The θ-vacuum term in the QCD Lagrangian gives rise to a CP violating neutron electric dipole moment which is also discussed in this section. In the last section, we make a slight digression from the Strong CP problem to discuss the relation between the neutron electric dipole moment and the electric dipole transition moment for the Σ^0-Λ transition relevant to the decay under study. This is possible due to the SU(3) flavour symmetry, and we illustrate this using the framework of Heavy Baryon Chiral Perturbation Theory.

3.1 The $U(1)_A$ Problem

The QCD Lagrangian in the limit of massless quarks for N flavours has a global $U(N)_V \times U(N)_A$ symmetry [14]. Since the up and down quarks have a significantly lower mass when compared to the masses of other flavoured quarks, one can safely say that the strong interaction respects an approximate $U(2)_V \times U(2)_A$ symmetry [27]. This can be further generalized if one accommodates the strange quark, together with the up and down quarks, in the massless limit. The strong interaction in this case is said to be approximately invariant under $U(3)_V \times U(3)_A$ transformations. Indeed, we find that experimentally the vector group $U(3)_V = SU(3)_V \times U(1)_B$ is a symmetry that is approximately respected in nature. The $SU(3)_V$ group manifests itself in the hadron spectrum as an approximate flavour symmetry among quarks since the strong interaction does not discriminate between quarks of differing flavours. Historically, this identification was made by Gell-Mann and his colleagues in what was termed the *Eightfold Way*, which represented this symmetry in the nucleon octet [21]. In fact, as we will see later in this chapter, it is this flavour symmetry that helps us relate the neutron electric dipole moment to the P violating electric dipole transition moment for the Σ^0-Λ transition. The $U(1)_B$ group, on the other hand, is an exact global symmetry which is realized in nature as the conservation of baryon number.

The axial symmetry group $U(3)_A = SU(3)_A \times U(1)_A$ is less straightforward. Dynamically, the formation of quark condensates $< \bar{u}u > = < \bar{d}d > \neq 0$ results in the spontaneous breakdown of this axial symmetry [27]. Following the Goldstone Theorem, we expect to find eight pseudo-Goldstone bosons due to the spontaneous breakdown of the $SU(3)_A$ group. In fact, this was shown to be the case when the octet of mesons $(\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \overline{K}^0, \eta)$ was identified as the corresponding pseudo-Goldstone bosons. This is also why one does not see mass degenerate parity doublets in the hadron spectrum. However, the absence of a ninth pseudo-scalar meson as a pseudo-Goldstone boson of the $U(1)_A$ group means that there is an excess $U(1)_A$ symmetry that is not realized in nature. The η' meson is too heavy to fulfill this role, with $m^2_{\eta'} \gg m^2_K$. This non-realization of the $U(1)$ axial symmetry in the strong interaction was labeled the $U(1)_A$ Problem by Weinberg [14][27]. As we will see in the next section, the $U(1)_A$ problem is connected to the topologically non-trivial structure of the QCD vacuum.

3.2 QCD Vacuum & the Strong CP Problem

QCD as a theory of strong interactions is a gauge theory. This means that local changes in the configurations of fields (gauge transformations) do not affect the energy of the fields. This property, coupled to the fact that gauge transformations in QCD are non-abelian, means that there are infinitely many minimas in the energy associated with these fields [28]. For a pure gauge field, i.e. a set of field configurations obtained by a gauge transformation on the null field, we can impose the boundary condition $A_a^\mu = 0$ at spatial infinity [27]. However, there are gauge transformations for which it is not possible to deform the field back to the null-field configuration via smooth transformations. We can define a topological quantity for a static field called the winding number as follows [28]:

$$n = \frac{1}{32\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}. \tag{3.1}$$

It is an integer for a pure gauge field. In slightly more formal terms, the winding number characterizes the homotopy class for the $S_3 \to SU(2)$ mapping, i.e. a mapping from the three dimensional Euclidean space to the $SU(2)$ space [27][21]. Because the gauge group $U(1)$ can be easily deformed into the null-field configuration for an $S_3 \to U(1)$ mapping, we say that there is no analogous definition of a winding number for Abelian gauge theories [21]. For a field which vanishes at spatial infinity, the winding number can be expressed in terms of a surface integral that is non-zero. This implies a non-zero vacuum-vacuum transition amplitude. Thus the true vacuum is a superposition of all these vacua (n in number) and is called the theta vacuum [27]:

$$|\theta\rangle = \sum_n e^{-in\theta} |n\rangle \tag{3.2}$$

where $|n\rangle$ denotes the pure gauge configurations and θ is called the *vacuum angle*. A gauge transformation that transforms the field configuration $|n\rangle \to |n+1\rangle$ has a well defined solution, and such a tunneling event is called an *instanton* in the literature [28]. The effect of these mutually distinct theta vacua is that the effective action gains an additional term. The path integral formulation of the vacuum to vacuum transition amplitude involves an effective action which is dependent on the vacuum angle θ [29][27]:

$$S_{eff}[A] = S_0[A] + \frac{\theta g^2}{32\pi^2} \int d^4x F_a^{\mu\nu} \widetilde{F}_{a\mu\nu} \tag{3.3}$$

with $S_0[A]$ being the usual QCD action and $\widetilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$. This means the QCD Lagrangian now has an addition θ-term:

$$\mathcal{L}_{QCD} = \mathcal{L}_0 + \mathcal{L}_\theta \tag{3.4}$$

where $\mathcal{L}_0$ is the usual QCD Lagrangian given by:

$$\mathcal{L}_0 = \frac{1}{2} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{q}(i\not{D} - M)q, \tag{3.5}$$

and

$$\mathcal{L}_\theta = \frac{\theta g^2}{32\pi^2} \widetilde{F}_{\mu\nu}^a F_a^{\mu\nu} \tag{3.6}$$

is the P and T violating term due to the structure of the QCD vacuum. Since the Lagrangian $\mathcal{L}_{QCD}$ conserves charge conjugation symmetry, this additional θ-term is a source of CP violation in the strong interaction. For QCD to remain a CP conserving theory, this term must be zero. However, as we will see in the next section, there is no reason why this should be the case. On the other hand, its value is derived from the neutron electric dipole moment, and is diminishingly small [27][29]. The fact that the θ angle has such a small value (and is not zero) is called the *Strong CP Problem*.

3.3 Effective Vacuum Angle

The $U(1)_A$ problem finds its resolution in the non-trivial structure of the QCD vacuum. Before discussing this possible solution however, we must note that the $U(1)_A$ problem is really a *chiral anomaly*, wherein the quantization procedure leads to a charge that is not conserved [21]. The current associated with nearly massless up, down and strange quarks is [29]:

$$J_5^\mu = \bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d + \bar{s}\gamma^\mu\gamma_5 s. \tag{3.7}$$

This current is conserved in the massless quark limit up to tree level. At first loop order, this current diverges and it is given as [21][27][29]:

$$\partial_\mu J_5^\mu = \frac{g^2 N}{16\pi^2} F_a^{\mu\nu} \widetilde{F}_{a\mu\nu}. \tag{3.8}$$

This is an exact result and $\partial_\mu J_5^\mu \neq 0$ in QCD [29]. As we can see, it holds true even in the massless quark limit. This $U(1)_A$ chiral transformation, $q_i \to e^{-i\alpha_f\gamma_5} q_i$, brings about a change in the Lagrangian [29]:

$$\delta\mathcal{L} = \alpha_f \frac{g^2 N}{16\pi^2} F_a^{\mu\nu} \widetilde{F}_{a\mu\nu} \tag{3.9}$$

If we now make the identification, $N = -\theta/2\alpha_f$, we notice that the effect of the theta vacua given in Eq. (3.6) is removed. Unhappily, things are not as simple. Since quarks are not massless (and if we were to also include weak interactions), we have a general mass term in the Lagrangian which can be written as:

$$\mathcal{L}_M = q_{iR} M_{ij} q_{jL} + h.c. \tag{3.10}$$

where M_{ij} represents the complex quark mass matrix. The $U(1)_A$ chiral transformation, $q_i \to e^{-i\alpha_f\gamma_5} q_i$, then leads to an additional phase when we diagonalize the mass matrix [29]:

$$m_f \to e^{-2i\alpha_f} m_f \tag{3.11}$$

where f and m_f denotes the flavour and mass of individual quarks, respectively. The chiral rotation does not leave the vacuum state invariant either:

$$\theta \to \bar{\theta} = \theta - \sum_f \alpha_f = \theta - \arg\det[M]. \tag{3.12}$$

This new vacuum term is called the Effective Vacuum Angle, $\bar{\theta}$, and it is non-zero. Put another way, it is this Effective Vacuum Angle that determines the CP violating neutron electric dipole moment and therefore it is more accurate to use $\bar{\theta}$ than θ-vacuum when speaking of the Strong CP Problem.

The chiral transformation of the quarks leads to a CP violating pion-nucleon interaction vertex (for details see [29], [21]):

$$\mathcal{L}_{\pi NN} = g_{\pi NN} \overline{N} \tau^a N \pi^a \tag{3.13}$$

with the coupling constant $g_{\pi NN}$ depending on the Effective Vacuum Angle $\bar{\theta}$ as follows:

$$g_{\pi NN} = -\bar{\theta} \frac{m_u m_d}{m_u + m_d} \frac{1}{F_\pi} \frac{m_\Xi - m_N}{2m_s - m_u - m_d}. \tag{3.14}$$

For the case where the nucleon is the neutron, the interaction vertex gives us a theoretical estimation of the neutron electric dipole moment (nEDM), $|d_n^{theo}| \sim 1.1 \times 10^{-16}\bar{\theta}$ e cm. The current experimental upper bound of the nEDM is $|d_n^{exp}| \leq 2.9 \times 10^{-26}$e cm. This yields a value of $\bar{\theta} \leq 2.5 \times 10^{-10}$ [10][29].

3.4 Heavy Baryon Chiral Perturbation Theory

As we briefly discussed in the introduction to this chapter, due to the $SU(3)$ flavour symmetry between quarks, a non-zero nEDM in an n-n transition translates to a non-trivial electric dipole transition moment for the Σ^0-Λ transition. This is precisely the transition that is of interest for our decay $\Sigma^0 \to \Lambda\gamma \to p\pi^-\gamma$. In order to extract the exact nature of this correspondence between the nEDM and the electric dipole transition moment, we make use of the framework of Heavy Baryon Chiral Perturbation Theory (HBChPT), which is the effective field theory used to describe interactions in the baryonic sector [30][31].

Following Ottnad et al. [10], we can write down the most general, relativistic effective Lagrangian comprising the baryon octet B, up to (and including) the second order in the derivative expansion. The effective vacuum angle is now treated as an external field which transforms under a $U(1)_A$ rotation as in Eq. (3.12). As can be seen from Eq. (3.12), the displacement of this source $\bar{\theta}$ compensates for the chiral rotation of the mass matrix arising from the Yukawa coupling. The Lagrangian in Eq. (3.4) is now invariant under an approximate $U(3)_R \times U(3)_L$ symmetry [31]. This $\bar{\theta}$ source is introduced in our effective Lagrangian as $\bar{\theta} = \theta - i \ln \det \widetilde{U}$ ($\widetilde{U}$ being a function of the matrix valued fields $U = \exp\left(\sqrt{\frac{2}{3}} \frac{i}{F_0} \eta_0 + \frac{2i}{F_\phi} \phi\right)$ [10]) along with the baryon octet B, and the octet mass $\mathring{m}$ in the chiral limit:

$$\begin{aligned}\mathcal{L}_{\phi B} = & \, i\mathrm{Tr}[\bar{B}\gamma^\mu [D_\mu, B]] - \mathring{m}\mathrm{Tr}[\bar{B}B] - \frac{D/F}{2}\mathrm{Tr}[\bar{B}\gamma^\mu \gamma_5 [u_\mu, B]_\pm] - \frac{\lambda}{2}\mathrm{Tr}[\bar{B}\gamma^\mu \gamma_5 B]\mathrm{Tr}[u_\mu] \\ & + b_{D/F}\mathrm{Tr}[\bar{B}[\chi_+ - i\mathcal{A}(U - U^\dagger), B]_\pm] + b_0 \mathrm{Tr}[\bar{B}B] Tr[\chi_+ - i\mathcal{A}(U - U^\dagger)] \\ & + 4\mathcal{A}\omega'_{10}\frac{\sqrt{6}}{F_0}\eta_0 \mathrm{Tr}[\bar{B}B] + i\left(\omega'_{13/14}\bar{\theta} + \omega_{13/14}\frac{\sqrt{6}}{F_0}\eta_0\right)\mathrm{Tr}[\bar{B}\sigma^{\mu\nu}\gamma_5 [F^+_{\mu\nu}, B]_\pm] \\ & + \omega_{16/17}\mathrm{Tr}[\bar{B}\sigma^{\mu\nu}[F^+_{\mu\nu}, B]_\pm]\end{aligned} \tag{3.15}$$

where D and F are axial vector couplings that can be determined from semileptonic hyperon decays, λ is the isosinglet axial coupling, $\mathcal{A}$ is a complicated function of $\bar{\theta}$ (see [31]), b_0 and $b_{D/F}$ are low energy constants (LECs) representing the leading explicitly symmetry breaking terms, while ω_i are related to the coupling of the baryon fields to the singlet field [10]. $\omega_{16/17}$ is the exception here, since these constants are nothing but the magnetic moment couplings. The baryon octet containing the hadrons relevant to our decay is given by the matrix [31]:

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \tag{3.16}$$

The covariant derivative of the baryon field is [10],[31]:

$$[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \tag{3.17}$$

with Γ^μ being the chiral connection, given as follows:

$$\Gamma_\mu = [u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]/2. \tag{3.18}$$

Note that r_μ and l_μ are conventional external sources that go into the definition of the field strength tensor, $F^+_{\mu\nu}$:

$$\begin{aligned} F^+_{\mu\nu} &= u^\dagger F^R_{\mu\nu} u + u F^L_{\mu\nu} u^\dagger \\ &= \partial_\mu(r_\nu + l_\nu) - \partial_\nu(r_\mu + l_\mu) - i([r_\mu, r_\nu] + [l_\mu, l_\nu]) \end{aligned} \tag{3.19}$$

where in the second step we have set $u = u^\dagger = \mathbb{1}$, and made the substitution $F^R_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$ and $F^L_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]$. If we now use the definition of these external sources, i.e. $r_\mu = v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$, and make the identification $a_\mu = 0$ and

$$v_\mu = eA_\mu \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}, \tag{3.20}$$

for electromagnetic interactions, we end up with a field strength tensor that has the following form:

$$F^+_{\mu\nu} = 2e \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} (\partial_\mu A_\nu - \partial_\nu A_\mu) = 2e \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} F_{\mu\nu}. \tag{3.21}$$

The term in Eq. (3.15) containing the constants $\omega'_{13/14}$ and $\omega_{13/14}$ are of particular interest to us, since it is this term in the Lagrangian that gives rise to a CP violation which manifests itself as the nEDM (for an n-n transition) and the electric dipole transition moment (for the Σ^0-Λ transition) [10][31]. In order to explicitly derive the interaction Lagrangian that is relevant to our decay, we isolate this Lagrangian term,

$$\mathcal{L}_I = i\Big(\omega'_{13/14}\bar{\theta} + \omega_{13/14}\frac{\sqrt{6}}{F_0}\eta_0\Big)\mathrm{Tr}[\bar{B}\sigma_{\mu\nu}\gamma_5[F^{+\mu\nu}, B]_\pm] \tag{3.22}$$

and utilize Eq. (3.21) and Eq. (3.16), while setting the fields $\Sigma^+, \Sigma^-, \Xi^+, \Xi^0,\ p \rightarrow 0$. We obtain:

$$\begin{aligned}\mathcal{L}_I = -\frac{2ie}{9}(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13})\Big[&6\bar{n}\gamma_5\sigma_{\mu\nu}nF^{\mu\nu} - 3\sqrt{3}\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Sigma^0F^{\mu\nu}\\ &- 3\sqrt{3}\overline{\Sigma}^0\gamma_5\sigma_{\mu\nu}\Lambda F^{\mu\nu} + 3\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Lambda F^{\mu\nu} - 3\overline{\Sigma}^0\gamma_5\sigma_{\mu\nu}\Sigma^0F^{\mu\nu}\Big].\end{aligned} \tag{3.23}$$

In the relativistic case, the nEDM is defined by the following interaction Lagrangian [31]:

$$\mathcal{L}_{nEDM} = \frac{1}{2}d_n^\gamma e\bar{n}i\sigma_{\mu\nu}\gamma_5 nF^{\mu\nu} \tag{3.24}$$

Comparing this to Eq. (3.23), we can determine the LEC coefficients common to all the interaction terms. Thus, we get an estimation of the electric dipole transition moment for the Σ^0-Λ transition from the current experimental upper bound of the nEDM. The details of this calculation, including the determination of a numerical value, is carried out in Chapter 6. However, we still require the most general matrix element for the current that characterizes this transition. This is the topic of the following chapter, where we discuss the most general current for the Σ^0-Λ transition using Loretnz invariance as a guiding constraint. We will derive functions called transition form factors which will play the crucial role of parameterizing the P violating electric dipole transition moment that we have just determined.

Chapter 4

Transition Form Factors

Form factors play an important role in determining the structure of baryons, their magnetic properties and charge distribution [13]. Of particular concern to our decay is the study of transition form factors, which has provided valuable information about the charge and magnetic properties of nucleons (also the Delta hadron) and mesons, as well as their underlying quark and gluon structure (see for instance [8],[9],[32],[33]). In this present work, we make use of transition form factors to parameterize the P (and possible CP) violating decay. The emergence of this possible P and CP violation was discussed in the previous chapter as part of the theta vacuum term in the QCD Lagrangian. This chapter is devoted to deriving the most general Lorentz invariant electromagnetic current, which will in turn help us identify the Lorentz invariant (electromagnetic) transition form factors. In the process, we will discuss properties such as current conservation and parity symmetry of this current. Lastly, we will eliminate terms in the current that are not relevant to our decay, and present the Lorentz invariant transition form factors for the Σ^0-Λ transition.

Lorentz Invariant Electromagnetic Current

We are interested in deriving the most general Lorentz invariant electromagnetic current which will enable us to pin down the transition form factors important to our radiative decay. The expectation value of a current $\mathcal{J}^\mu$ for a spin 1/2 baryon transition B-B' is generally given by [34],[35]:

$$\langle B'(p')| \mathcal{J}^\mu(x) |B(p)\rangle = e^{-i(p-p').x} \langle B'(p')| \mathcal{J}^\mu(0) |B(p)\rangle . \qquad (4.1)$$

where we have used the fact that under translation in space and time, the current transforms as:

$$\mathcal{J}^\mu(x) = e^{i\hat{P}x} \mathcal{J}^\mu(0) e^{-i\hat{P}x}$$

with $\hat{P}$ being the momentum operator. We can rewrite the above matrix element as [34]:

$$\langle B'(p')| \mathcal{J}^\mu(0) |B(p)\rangle = \bar{u}_{B'}(p')\Gamma^\mu(l,q)u_B(p) \qquad (4.2)$$

with $l^\mu = p'^\mu + p^\mu$ and $q^\mu = p^\mu - p'^\mu$. Note that $\Gamma^\mu(l,q)$ is the vertex function (a 4×4 matrix acting on spinors), which contains the Lorentz invariant transition form factors we are after.

If we now consider all possible structures (four-vectors, gamma matrices, tensors etc.) that can be contracted so that the current is manifestly Lorentz covariant, we are left with a matrix element that has the following form:

$$\begin{aligned}\langle B'(p')| \mathcal{J}^\mu(0) |B(p)\rangle = \bar{u}_{B'}(p')\Bigg(&\sum_i a_i(q^2) I A_i^\mu(l,q) + \sum_i b_i(q^2)\gamma_\nu B_i^{\mu\nu}(l,q) \\ &+ \sum_i c_i(q^2)\gamma_5 C_i^\mu(l,q) + \sum_i d_i(q^2)\gamma_\nu\gamma_5 D_i^{\mu\nu}(l,q) \\ &+ \sum_i e_i(q^2)\sigma_{\alpha\beta} E_i^{\mu\alpha\beta}(l,q)\Bigg)u_B(p).\end{aligned} \qquad (4.3)$$

Note that the functions a_i, b_i, c_i, d_i and e_i are Lorentz invariant as they depend only on q^2. We would now like to list, concretely, all structures that could accompany these functions, keeping in mind the condition of Lorentz invariance. For the first term in Eq. (4.3) with only one Lorentz index, we can safely conclude:

$$A_1^\mu = q^\mu, \qquad\qquad A_2^\mu = l^\mu. \tag{4.4}$$

Now, going back to Eq. (4.3), the tensor accompanying the second term has two Lorentz indices and can take on the following Lorentz invariant forms:

$$B_1^{\mu\nu} = g^{\mu\nu}, \qquad\qquad B_2^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} q_\alpha l_\beta. \tag{4.5}$$

The first of these two, $\gamma_\nu g^{\mu\nu}$, simply raises the index of the gamma matrix, which is an independent structure. The latter term, $\gamma_\nu B_2^{\mu\nu}$, can be further broken down, as we will show when analyzing a similar structure $\gamma_\nu \gamma_5 D_2^{\mu\nu}$ below [1]. Note that we haven't included terms like $B_i^{\mu\nu} = l^\mu l^\nu$ (or structures like $l^\mu q^\nu ; q^\mu l^\nu ; q^\mu q^\nu$) in Eq. (4.5), since they contract with the gamma matrix in the second term of Eq. (4.3) to yield structures like $l^\mu \not{l}$ and $l^\mu \not{q}$. These structures are redundant as they can be expressed in terms of Eq. (4.4) using the equation of motion for Dirac spinors.

Referring to Eq. (4.3), we now consider the third term in that expression. With only one Lorentz index available, we write down the same structures as we did in Eq. (4.4):

$$C_1^\mu = q^\mu, \qquad\qquad C_2^\mu = l^\mu. \tag{4.6}$$

C_1^μ when inserted back in Eq. (4.3) yields the independent structure $q^\mu \gamma_5$ which we would like to retain. The form of C_2^μ shown above gives us $l^\mu \gamma_5$.

We now analyze the fourth term in Eq. (4.3), which tells us by the same reasoning that D_1 and D_2 can have the same structures as $B_i^{\mu\nu}$ did in Eq. (4.5):

$$D_1^{\mu\nu} = g^{\mu\nu}, \qquad\qquad D_2^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} q_\alpha l_\beta. \tag{4.7}$$

Note that structures like $D_i^{\mu\nu} = l^\mu q^\nu$ (or structures like $l^\mu l^\nu ; q^\mu l^\nu ; q^\mu q^\nu$) in the fourth term of Eq. (4.3) are redundant as they can be expressed in terms of

[1] We use here the convention $\epsilon^{0123} = -1$ for the Levi-Civita tensor.

C_1^μ and C_2^μ by using the equation of motion for Dirac spinors. $D_1^{\mu\nu}$ contracts the gamma matrix to give us the independent term $\gamma^\mu\gamma_5$. $D_2^{\mu\nu}$ is a composite term, and this can be illustrated as follows:

$$\bar{u}_{B'}(p')i\gamma_\nu\gamma_5\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta u_B(p) = \bar{u}_{B'}(p')\gamma_\nu\frac{1}{4!}\epsilon^{\mu'\nu'\alpha'\beta'}\gamma_{\mu'}\gamma_{\nu'}\gamma_{\alpha'}\gamma_{\beta'}\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta u_B(p)$$
$$\sim \bar{u}_{B'}(p')\begin{vmatrix} g^{\mu'\mu} & g^{\nu'\mu} & g^{\alpha'\mu} & g^{\beta'\mu} \\ g^{\mu'\nu} & g^{\nu'\nu} & g^{\alpha'\nu} & g^{\beta'\nu} \\ g^{\mu'\alpha} & g^{\nu'\alpha} & g^{\alpha'\alpha} & g^{\beta'\alpha} \\ g^{\mu'\beta} & g^{\nu'\beta} & g^{\alpha'\beta} & g^{\beta'\beta} \end{vmatrix}\gamma_\nu\gamma_{\mu'}\gamma_{\nu'}\gamma_{\alpha'}\gamma_{\beta'}q_\alpha l_\beta u_B(p) \tag{4.8}$$

where we have used the formal definition of γ_5 matrix in the first step, $\gamma_5 = -\frac{i}{4!}\epsilon^{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta$ [36] and used the determinant of the metric tensor in place of the Levi-Civita tensor product in the second step. We see from the above expression that this is related to the terms $\bar{u}_{B'}l^\mu u_B$, $\bar{u}_{B'}q^\mu u_B$ and $\bar{u}_{B'}\gamma^\mu u_B$ and is therefore not independent. Using the same reasoning, we can also show that the term considered previously in Eq. (4.5), $\bar{u}_{B'}\gamma_\nu B_2^{\mu\nu}u_B$, can be shown to be composed of $\bar{u}_{B'}q^\mu\gamma_5 u_B$, $\bar{u}_{B'}l^\mu\gamma_5 u_B$ and $\bar{u}_{B'}\gamma^\mu\gamma_5 u_B$ by making use of the following trick and proceeding as before:

$$\bar{u}_{B'}(p')\gamma_\nu\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta u_B(p) = \bar{u}_{B'}(p')\gamma_\nu\gamma_5\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\gamma_5 u_B(p) \tag{4.9}$$

Finally, we come to the last term of Eq. (4.3), which contains $E_i^{\mu\alpha\beta}$. The structure $E_1^{\mu\alpha\beta} = g^{\mu\alpha}l^\beta$ is not independent and can be further decomposed. On contracting $E_1^{\mu\alpha\beta} = g^{\mu\alpha}l^\beta$ with the tensor $\sigma_{\alpha\beta}$, we obtain a structure of the form $\sigma^{\mu\beta}l_\beta$. We will now show that this structure can be simplified further. We use the following relation to aid our calculation:

$$i\sigma^{\mu\beta} = -\frac{1}{2}(\gamma^\mu\gamma^\beta - \gamma^\beta\gamma^\mu) = g^{\mu\beta} - \gamma^\mu\gamma^\beta \tag{4.10}$$

where we have used $\{\gamma^\mu, \gamma^\beta\} = 2g^{\mu\beta}$. Equivalently, the above relation can be written as:

$$i\sigma^{\mu\beta} = \gamma^\beta\gamma^\mu - g^{\mu\beta}. \tag{4.11}$$

Therefore,

$$\begin{aligned}
\bar{u}_{B'}(p')i\sigma^{\mu\beta}l_\beta u_B(p) &= \bar{u}_{B'}(p')i\sigma^{\mu\beta}(p'+p)_\beta u_B(p) \\
&= \bar{u}_{B'}(p')[(\gamma^\beta\gamma^\mu - g^{\mu\beta})p'_\beta + (g^{\mu\beta} - \gamma^\mu\gamma^\beta)p_\beta]u_B(p) \\
&= \bar{u}_{B'}(p')[\not{p}'\gamma^\mu - p'^\mu + p^\mu - \gamma^\mu\not{p}]u_B(p) \\
&= \bar{u}_{B'}(p')[(m_{B'} - m_B)\gamma^\mu + q^\mu]u_B(p)
\end{aligned} \tag{4.12}$$

where in the last step we made use of the equation of motion for the Dirac spinor, $(\not{p} - m)u(p)$ and its conjugate. Thus the structure $\sigma_{\alpha\beta}E_1^{\mu\alpha\beta}$ can be expressed in terms of γ^μ and q^μ.

Similarly, we can rewrite another possible structure $E_2^{\mu\alpha\beta} = g^{\mu\alpha}q^\beta$ in terms of a gamma matrix and four-momentum. On contracting $E_2^{\mu\alpha\beta}$ with $\sigma_{\alpha\beta}$, we obtain a structure of the form $\sigma^{\mu\beta}q_\beta$. As was done above, we derive the following property using Eq. (4.10) and Eq. (4.11):

$$\begin{aligned}
\bar{u}_{B'}(p')i\sigma^{\mu\beta}q_\beta u_B(p) &= \bar{u}_{B'}(p')i\sigma^{\mu\beta}(p-p')_\beta u_B(p) \\
&= \bar{u}_{B'}(p')[(g^{\mu\beta} - \gamma^\mu\gamma^\beta)p_\beta - (\gamma^\beta\gamma^\mu - g^{\mu\beta})p'_\beta]u_B(p) \\
&= \bar{u}_{B'}(p')[p^\mu - \gamma^\mu\not{p} - \not{p}'\gamma^\mu + p'^\mu]u_B(p) \\
&= \bar{u}_{B'}(p')[l^\mu - (m_B + m_{B'})\gamma^\mu]u_B(p)
\end{aligned} \tag{4.13}$$

We will henceforth express l^μ (recall that this is the structure A_2^μ) in terms of γ^μ and $\sigma^{\mu\beta}q_\beta$ as shown in Eq. (4.13).

Note that because $\sigma_{\alpha\beta}$ in the final term of Eq. (4.3) is an antisymmetric tensor, contracting it with $g^{\alpha\beta}l^\mu$ (or $g^{\alpha\beta}q^\mu$) gives us a null result since the product of a symmetric tensor like $g^{\alpha\beta}$ with an anti-symmetric one like $\sigma_{\alpha\beta}$ makes this term zero. The same reasoning elimantes structures of the form $E_i^{\mu\alpha\beta} = l^\mu l^\alpha l^\beta$ (or any such combination where both the indices of the $\sigma_{\alpha\beta}$ tensor is contracted by the same four-momentum). However, the structures of the following form are non-zero and allowed by Lorentz invariance:

$$E_3^{\mu\alpha\beta} = l^\mu l^\alpha q^\beta, \qquad E_4^{\mu\alpha\beta} = q^\mu l^\alpha q^\beta. \tag{4.14}$$

On expanding the $\sigma_{\alpha\beta}$ in terms of the gamma matrices (see Eq. (4.10)) and using the equation of motion for Dirac spinors, we note that the structures in Eq. (4.14) reduces to those in Eq. (4.4).

Another structure that satisfies the condition of Lorentz invariance is $\sigma_{\alpha\beta}E_5^{\mu\alpha\beta} = \sigma_{\alpha\beta}\epsilon^{\alpha\beta\mu\nu}q_\nu$. This term can be rewritten using the property [36]:

$$\sigma^{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon^{\alpha\beta\mu\nu}\sigma_{\alpha\beta} \tag{4.15}$$

so that the independent structure we would like to preserve in the last term of Eq. (4.3) is $\sigma_{\alpha\beta}E_5^{\mu\alpha\beta} \equiv i\sigma^{\mu\nu}\gamma_5 q_\nu$. The same method can be applied to the term $\sigma_{\alpha\beta}E_6^{\mu\alpha\beta} = \sigma_{\alpha\beta}\epsilon^{\mu\nu\alpha\beta}l_\nu$. If both the indices of $\sigma_{\alpha\beta}$ are contracted by the Levi-Civita tensor, we have the structures $\sigma_{\alpha\beta}E_7^{\mu\alpha\beta} = \sigma_{\alpha\beta}\epsilon^{\alpha\beta\rho\kappa}q_\rho l_\kappa q^\mu$ and $\sigma_{\alpha\beta}E_8^{\mu\alpha\beta} = \sigma_{\alpha\beta}\epsilon^{\alpha\beta\rho\kappa}l_\rho q_\kappa l^\mu$ which satisfy the condition of Lorentz invariance. These can be similarly shown to be redundant using the above property (Eq (4.15)) and following exactly what we did with the terms in Eq. (4.14).

If, on the other hand, only one index of $\sigma_{\alpha\beta}$ is contracted with the Levi-Civita tensor, then we can have the following structures:

$$\sigma_{\alpha\beta}E_9^{\mu\alpha\beta} = \sigma_{\alpha\beta}\epsilon^{\alpha\mu\kappa\rho}l_\kappa q_\rho l^\beta, \qquad \sigma_{\alpha\beta}E_{10}^{\mu\alpha\beta} = \sigma_{\alpha\beta}\epsilon^{\alpha\mu\kappa\rho}l_\kappa q_\rho q^\beta. \tag{4.16}$$

If we now use Eq. (4.12) and Eq. (4.13), we can reduce the above two structures to simpler ones that have been covered already.

Lastly, we use the following property to write $l^\mu\gamma_5$ (recall that this is $C_2^\mu\gamma_5$) in terms of $\sigma^{\mu\nu}\gamma_5$ (see Eq. (4.15)) and the gamma matrices:

$$\begin{aligned}
\bar{u}_{B'}(p')i\sigma^{\mu\nu}\gamma_5 q_\nu u_B(p) &= \bar{u}_{B'}(p')i\sigma^{\mu\nu}\gamma_5(p-p')_\nu u_B(p) \\
&= \bar{u}_{B'}(p')[(g^{\mu\nu}-\gamma^\mu\gamma^\nu)\gamma_5 p_\nu - (\gamma^\nu\gamma^\mu - g^{\mu\nu})\gamma_5 p'_\nu]u_B(p) \\
&= \bar{u}_{B'}(p')[\gamma_5 p^\mu + \gamma^\mu\gamma_5\not{p} - \not{p}'\gamma^\mu\gamma_5 + \gamma_5 p'^\mu]u_B(p) \\
&= \bar{u}_{B'}(p')[(m_B - m_{B'})\gamma^\mu\gamma_5 + l^\mu\gamma_5]u_B(p).
\end{aligned} \tag{4.17}$$

We will henceforth express $l^\mu\gamma_5$ in terms of $\gamma^\mu\gamma_5$ and $i\sigma^{\mu\nu}\gamma_5 q_\nu$. We therefore have eliminated the four-momentum l in favour of q in our Lorentz invariant current.

Based on our discussion above, the most general Lorentz invariant current containing only independent terms is:

$$\langle B'(p')| \mathcal{J}^\mu(0) |B(p)\rangle = \bar{u}_{B'}(p')\Big(a_1(q^2)q^\mu + b_1(q^2)\gamma^\mu + c_1(q^2)q^\mu\gamma_5 + d_1(q^2)\gamma^\mu\gamma_5 + a_2(q^2)\sigma^{\mu\nu}q_\nu + e_3(q^2)\sigma^{\mu\nu}\gamma_5 q_\nu\Big)u_B(p). \tag{4.18}$$

This is sometimes referred to in the literature as the weak-current form factor decomposition (see [35]). This is because we still have not imposed the constraint of current conservation on Eq. (4.1). This is analogous to imposing the condition of gauge invariance in QED which gives rise to massless gauge bosons (photons). Since the gauge invariance in the weak sector is spontaneously broken giving rise to massive gauge bosons, the weak current is not conserved [35]. Applying current conservation, we have:

$$q_\mu \langle B'(p')| \mathcal{J}^\mu(0) |B(p)\rangle = 0 \tag{4.19}$$

which results in the following demand when applied to Eq. (4.18):

$$a_1(q^2)q^2 + b_1(q^2)(m_B - m_{B'}) + c_1(q^2)q^2\gamma_5 - d_1(q^2)(m_B + m_{B'})\gamma_5 = 0. \tag{4.20}$$

This implies:

$$a_1(q^2) = b_1(q^2)\frac{(m_B - m_{B'})}{q^2}, \qquad \text{and} \qquad d_1(q^2) = \frac{c_1(q^2)q^2}{(m_B + m_{B'})}. \tag{4.21}$$

We are finally left with four independent transition form factors in our gauge invariant current for the B-B' transition which is conventionally expressed as [10],[35],[31]:

$$\langle B'(p')|\, \mathcal{J}^{\mu}_{em}(0)\, |B(p)\rangle = \bar{u}_{B'}(p')\Bigg(F_1(q^2)\Big(\gamma^{\mu} + \frac{(m_{B'} - m_B)}{q^2} q^{\mu}\Big) - \frac{i\sigma^{\mu\nu} q_{\nu}}{m_B + m_{B'}} F_2(q^2) + i\big(\gamma^{\mu} q^2 + (m_{B'} + m_B) q^{\mu}\big)\gamma_5 F_A(q^2) + \frac{\sigma^{\mu\nu} q_{\nu}\gamma_5}{m_B + m_{B'}} F_3(q^2)\Bigg) u_B(p) \tag{4.22}$$

where $q_\nu = (p - p')_\nu$. If B and B' have the same intrinsic parity then the functions $F_1(q^2)$ and $F_2(q^2)$ are the P conserving Dirac and Pauli transition form factors. Now, $F_A(q^2)$ and $F_3(q^2)$ are the P violating Lorentz invariant transition form factors and are termed the anapole form factor and the electric dipole form factor, respectively. At $q^2 = 0$, these form factors have specific normalizations [37]:

$$F_1(0) = Q_B, \qquad\qquad F_2(0) = \kappa$$

where Q_B is the electric charge and κ is the anomalous magnetic transition moment. The P violating terms at $q^2 = 0$ have values $F_A(0)$ and $F_3(0)$ which are the anapole transition moment and the electric dipole transition moment, respectively.

For the decay $\Sigma^0 \to \Lambda\gamma$ at the real photon point (i.e. $q^2 = 0$), we have a transition between two neutral particles and so $F_1(0) = 0$. The third term in Eq. (4.22) also disappears at $q^2 = 0$, since the contraction $q^\mu \varepsilon_\mu = 0$ with ε_μ being the photon polarization. In the end, we are left with a matrix element of the current for the Σ^0-Λ transition that takes the form:

$$\langle \Lambda|\, \mathcal{J}^{\mu}(0)\, |\Sigma^0\rangle = \bar{u}_{\Lambda}(p_{\Lambda}) e\Big(\frac{i\kappa\sigma^{\mu\nu} q_{\nu}}{m_{\Sigma} + m_{\Lambda}} - \frac{ic\sigma^{\mu\nu} q_{\nu}\gamma_5}{m_{\Sigma} + m_{\Lambda}}\Big) u_{\Sigma}(p_{\Sigma}) \tag{4.23}$$

where we have used $F_2(0) = \kappa$ and $iF_3(0) = c$. Note that q fulfills four-momentum conservation for a real photon, i.e. $q = (p_\Sigma - p_\Lambda)$. The term in the matrix element containing the electric dipole transition moment $F_3(0)$ thus parametrizes P violation in our decay. Given such a P violation, we expect to see an angular distribution between the final decay products. The kinematics of such a P violating three-body decay is discussed next.

Chapter 5

Neutral Sigma Decay

The Σ^0 hyperon decay is a two step process. The Σ^0 first decays into the Λ hyperon and a photon. In the next step, the Λ hyperon decays via the weak interaction to give a proton and a pion. The whole decay can then be characterized as a three-body decay given by $\Sigma^0 \rightarrow p\,\pi^-\gamma$. Following the idea presented by Dreitlein et al. [38], in this chapter we will show that for a parity non-conserving first decay, the P violation will manifest itself as an angular dependence between two of the final decay products in the Λ rest frame. We will first illustrate this using all possible spin configurations of the initial Σ^0 hyperon and its decay products, noting how a P violation affects these different possibilities. We will then analyze this more concretely by studying the decay rate for a three-body decay chain, i.e. a radiative decay followed by a weak decay. We will see how a parity conservation in the first decay implies a flat decay distribution, i.e. a differential decay rate that is constant with respect to the cosine of the angle between two of the final decay products.

5.1 Spin Configurations

The first decay, $\Sigma^0 \rightarrow \Lambda\gamma$, is dominated by the electromagnetic interaction and was thus thought to be parity conserving. In 1961, Dreitlein and Primakoff presented their seminal paper which established a method to calculate

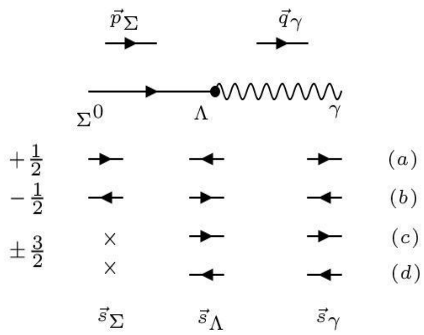

Figure 5.1: Illustrating the selection rules for the $\Sigma^0 \to \Lambda\gamma$ decay. Dreitlein, J. et al. [38]

the lifetime of the Σ^0 particle. In addition, in the same paper, they examined the effect of parity non-conservation in the Λ-Σ^0 transition. Starting from a P violating effective Lagrangian, they showed that the variation of the polarization of the Λ hyperon with the angle between the polarization of the Σ^0 particle and the line of flight of the Λ hyperon (in Σ^0 rest frame), is a suitable test for parity conservation in the reaction $\Lambda\gamma \to \Sigma^0$ [38]. Following the same logic, we will test the parity symmetry for our three-body decay of Σ^0, but with respect to the angle between the proton and the photon in the Λ rest frame.

Figure 5.1 illustrates the selection rules operational in the decay of the neutral Sigma particle in the Λ rest frame. $\vec{p}_\Sigma$ and $\vec{q}_\gamma$ indicate the three-momentum of the Σ^0 and the photon, respectively. The labels $\vec{s}_\Sigma$, $\vec{s}_\Lambda$ and $\vec{s}_\gamma$ denote the possible spin states of the Σ^0, Λ and photon. Note that a rightward arrow for spin configurations in Figure 5.1 indicates a positive value while a leftward arrow indicates negative spin values. The two-body decay reduces to a one-dimensional problem in the Λ rest frame. This means we do not have orbital angular momentum in this case. The spins (along the line of flight) of the Σ^0 and the photon just add up.

The radiative decay emits a real photon which therefore has two possible helicities of ± 1. Cases (c) and (d) are evidently forbidden due to angular momentum conservation since we are dealing with spin 1/2 hyperons. We are then left with cases (a) and (b) as the only two permissible spin configurations for the $\Sigma^0 \to \Lambda\gamma$ decay. Thus, if we know the spin of any of the three particles involved in this decay, then we know the spins of all particles. When parity is conserved, the cases (a) and (b) are identical. If we were to take a mirror image of case (a) and rotate it by 180° (a parity operation), we would end up with case (b). We will shortly explore what this means kinematically, in terms of the matrix element calculation of this decay. For now it is sufficient to note that P conservation results in the angular information of the Σ^0 hyperon carried by Λ being averaged out by the spin sums. If P is violated however, we can distinguish between cases (a) and (b), since this will result in different probabilities when calculating the decay width. If the spins of the initial (and final) states are not determined, P violation implies two distinct decay widths for cases (a) and (b). If the spins of the initial (or final) states are known, as is the case when considering the production process, then too we expect to find cases (a) and (b) to be distinguishable even if parity is conserved in the decay. Thus, for a radiative decay emitting a real photon, the initial helicity of the Σ^0 hyperon is sufficient to determine the helicities of Λ and the photon. This is the reason why this decay lends itself so nicely to the test of P violation via the method prescribed in Dreitlein et al. [38]. We will now look at how a P violation would manifest itself in the differential decay width calculation for the $\Sigma^0 \to p\pi\gamma$ decay.

5.2 P Violation & Angular Distribution

For a two-step decay, i.e. a radiative decay of a Σ^0 hyperon followed by a weak decay of Λ, the spin averaged differential decay rate is obtained by using:

$$d\Gamma \sim \sum_{s,r,r',g,p} \mathcal{M}_{s\to r,g}\mathcal{M}_{r\to p}\mathcal{M}^*_{s\to r',g}\mathcal{M}^*_{r'\to p} \tag{5.1}$$

where we have taken into account possible interferences between amplitudes. $\mathcal{M}_{s\to r,g}$ is the Lorentz invariant Feynman matrix element for the decay $\Sigma^0 \to$

$\Lambda\gamma$, and $\mathcal{M}_{r\to p}$ is the Lorentz invariant Feynman matrix element for the decay $\Lambda \to p\pi^-$. Now s denotes the spin orientation of the Σ^0 hyperon, while r and r' are the spin orientations of the Λ, g is the helicity of the photon and lastly, p refers to the spin orientation of the proton. Note that Eq. (5.1) does not show certain constant terms and phase space factors since these are not germane to the discussion at hand. The differential decay rate is therefore shown to be proportional to the quantities on the right-hand side.

Looking at the decay diagram in Figure 5.2, we see that due to momentum conservation, the direction of the Σ^0 hyperon and the photon are aligned in the Λ rest frame. Similarly, since the Λ hyperon is at rest, its decay products, the proton and the pion, are also aligned.

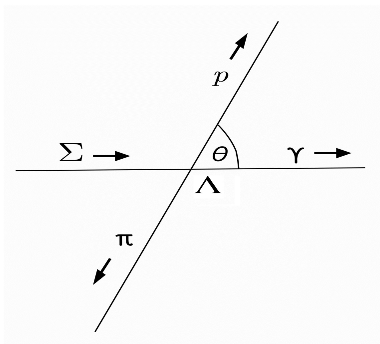

Figure 5.2: Σ^0 decay diagram in Λ rest frame.

As we saw in the previous section, due to angular momentum conservation, the only permissible spin states for this decay are the values $s = \pm 1/2$ with $r = \mp 1/2$ and $g = \pm 1$. Note that if the photon was virtual, then it could have three possible polarizations and the spin orientation s would not be sufficient to determine the values of r and g. Thus, for the decay with a real photon, Eq. (5.1) can be written as:

$$
\begin{aligned}
d\Gamma &\sim \sum_{s,p} |\mathcal{M}_{s\to r,g}|^2 |\mathcal{M}_{r\to p}|^2 \\
&= \sum_{p} (|\mathcal{M}_{+1/2\to -1/2,+1}|^2 |\mathcal{M}_{-1/2\to p}|^2 + |\mathcal{M}_{-1/2\to +1/2,-1}|^2 |\mathcal{M}_{+1/2\to p}|^2).
\end{aligned}
\tag{5.2}
$$

If our decay satisfies parity symmetry, then $|\mathcal{M}_{s\to r,g}| = |\mathcal{M}_{-s\to -r,-g}|$. Therefore, for a P conserving decay we see that:

$$
\begin{aligned}
d\Gamma &\sim |\mathcal{M}_{+1/2\to -1/2,+1}|^2 \times \sum_{p} (|\mathcal{M}_{-1/2\to p}|^2 + |\mathcal{M}_{+1/2\to p}|^2) \\
&= \frac{1}{2} \sum_{s,r,g} |\mathcal{M}_{s\to r,g}|^2 \sum_{r',p} |\mathcal{M}_{r'\to p}|^2.
\end{aligned}
\tag{5.3}
$$

Note that we end up with a product of two spin averaged quantities. These spin averaged quantities are constrained by Lorentz invariance, and so they can only depend on Lorentz invariant combinations of four-momenta. The matrix element $\mathcal{M}_{s\to r,g}$ in Eq. (5.3) can then be seen to be a function of Σ^0 momentum p_Σ^2 (nothing but the mass m_Σ^2 and thus constant), photon momentum q_γ^2 (0 for a real photon), and the scalar product $p_\Sigma . q_\gamma$. The momentum of Λ is fixed by energy-momentum conservation. Since the Λ hyperon is real, we note that the product $p_\Sigma . q_\gamma$ is also constant.

The second quantity in Eq. (5.3), $\mathcal{M}_{r'\to p}$, will depend on Lorentz invariant combinations of pion and proton four-momentum, i.e. p_π and l, with the momenta of Λ being fixed yet again by energy-momentum conservation. The possible combinations of four-momenta are p_π^2 ($= m_\pi^2$; constant), l^2 ($= m_p^2$; constant) and the scalar product $p_\pi . l$ (which for real Λ is also constant). Since the quantity $\mathcal{M}_{r'\to p}$ does not depend on q_γ, the photon momentum, the scalar product $q_\gamma . l$ (alternatively, $q_\gamma . p_\pi$) never arises. Thus the right-hand side of Eq. (5.3) cannot depend on the angle between the photon and the proton (or pion and the photon). Note that what we have said so far holds true even if we replace our Σ^0 hyperon with a spin 3/2 hyperon.

What the preceding observations mean is that our differential decay rate is flat and does not depend on the cosine of the angle between the pion and photon for a parity conserving radiative decay:

$$\frac{d\Gamma}{dm^2_{\gamma\pi}} = \text{const.} \tag{5.4}$$

with $m^2_{\gamma\pi} = (q_\gamma + p_\pi)^2 = m^2_\pi + 2E_\gamma E_\pi - 2|\vec{q}_\gamma||\vec{p}_\pi|\cos(\pi - \theta)$ evaluated in the rest frame of the Λ hyperon (See Figure 5.2). Equivalently, we could restate Eq. (5.4) with the decay rate differential in $m^2_{\gamma p}$ (angle between photon and proton). Conversely, if we obtain a decay rate for our decay chain $\Sigma^0 \to p\pi^-\gamma$ that is not flat, this would imply a parity non-conservation in this process. This is entirely independent of any parity violation that may take place in the second decay $\Lambda \to p\pi^-$. The explicit calculation of the angular dependence in the decay rate due to parity violation will now be carried out in the next chapter, Chapter 6.

The same analysis can also be carried out for the charge conjugated reaction $\overline{\Sigma}^0 \to \overline{p}\pi^+\gamma$. For a charge symmetric reaction, this should produce an identical differential decay width. We carry out this calculation explicitly in Chapter 6. Any possible CP violation would then produce differing decay asymmetries, an indication that the Strong CP effect has observable consequences.

Chapter 6

Calculations & Results

The aim of this thesis is to find possible P and CP violation in the decay of the Σ^0 hyperon. To this end we discussed in Chapter 5 how such a violation would manifest itself as an angular dependence between the proton and the photon in the decay rate of the Σ^0 hyperon. In Chapter 4 we discussed the parametrization of this CP violating term via the study of baryon transition form factors. Following Ottnad et al. [10], we identified the nEDM as the realization of this possible violation. Using the framework of HBChPT we analyzed, in Chapter 5, how we can provide an estimate of this angular distribution based on the current upper bound of the nEDM. This section provides the methods and calculations required to establish this estimate. First, we discuss the largely electromagnetic decay $\Sigma^0 \to \Lambda\gamma$ using the knowledge of Σ^0-Λ electromagnetic transition form factors. Next, we analyze the strangeness violating weak decay of the Λ hyperon, $\Lambda \to p\pi^-$. Finally, using our knowledge of these two decays, we study the combined three-body decay $\Sigma^0 \to p\pi^-\gamma$ for the case where the initial Sigma hyperon is unpolarized. We then determine the effects of the production process on the angular distribution of the final decay products and investigate whether such an effect can be disentangled from possible CP violation. Lastly, we perform the same calculations for the decay of the neutral anti-Sigma hyperon, with the goal to determine any discrepancy between this decay and that of the neutral Sigma particle.

6.1 Decay of the Neutral Sigma Hyperon

6.1.1 Electromagnetic decay $\Sigma^0 \to \Lambda\gamma$

In Chapter 4, we analyzed the electromagnetic transition form factors that play a crucial role in identifying the discrete symmetries respected by an interaction. The electromagnetic interaction is thought to conserve P and CP symmetry. Referring to Chapter 4, this would imply setting $F_3(q^2) = 0$ and $F_A(q^2) = 0$ in Eq. (4.22) which gives us the Lorentz covariant current for a purely electromagnetic transition. The matrix element for the current in terms of form factors is then given by [37]:

$$\langle B'(p')| \mathcal{J}_\mu |B(p)\rangle = e\bar{u}(p')\left(\gamma_\mu F_1^B(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_B + m_{B'}} F_2^B(q^2)\right)u(p) \qquad (6.1)$$

where $\mathcal{J}_\mu$ represents the quark vector current and q is the photon four-momentum, $q = p - p'$. $F_1^B(q^2)$ and $F_2^B(q^2)$ are the Dirac and Pauli form factors which contain information about the baryon structure at a given photon four-momentum. From our discussion of fermion bilinears in Chapter 2, we can see that this matrix element conserves both parity and CP symmetry. Following Kubis et al. [37], we state that in the low momentum limit $q^2 \to 0$ the form factors have the following normalization:

$$F_1^B(0) = Q_B, \qquad F_2^B(0) = \kappa_B \qquad (6.2)$$

where $F_1^B(0)$ is the electric charge Q_B and $F_2^B(0)$ is the anomalous magnetic moment κ_B of the probed baryon. We can also construct electric and magnetic Sachs form factors $G_{E/M}$ as linear combinations of the above two form factors. This is conventionally used by experimentalists while estimating the square radius of the charge or magnetic distribution, see for instance [37],[13].

For our specific decay $\Sigma^0 \to \Lambda\gamma$, the Σ^0-Λ transition form factors can be generalized as (see Eq. (4.22) and Eq. (4.23)):

$$\langle \Lambda(p_\Lambda)| \mathcal{J}_\mu |\Sigma^0(p_\Sigma)\rangle = e\bar{u}(p_\Lambda)\Big(\big(\gamma_\mu + \frac{m_\Lambda - m_\Sigma}{q^2} q_\mu\big) F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_\Lambda + m_\Sigma} F_2(q^2) \Big) u(p_\Sigma) \tag{6.3}$$

with $q = p_\Sigma - p_\Lambda$. The presence of the $\frac{1}{q^2}$ term in Eq. (6.3) enforces the vanishing of F_1 at the real photon point [9], i.e. $F_1(0) = 0$. The value of the anomalous transition magnetic moment, $F_2(0) = \kappa$, is determined by calculating the decay width for our electromagnetic decay.

In general, the matrix element for a vertex function Γ_μ corresponding to an electromagnetic decay is [13]:

$$i\mathcal{M} = \bar{u}(p')(ie\Gamma_\mu)u(p)\varepsilon^\mu. \tag{6.4}$$

In this particular case, the vertex function takes the form:

$$\Gamma_\mu = \frac{i\kappa\sigma_{\mu\nu}q^\nu}{m_\Lambda + m_\Sigma}. \tag{6.5}$$

Therefore the matrix element for the $\Sigma^0 \to \Lambda\gamma$ decay given by the Lorentz invariant transition form factors shown above is:

$$\mathcal{M}_{\Sigma^0\to\Lambda\gamma} = \bar{u}_\Lambda \frac{ei\sigma_{\mu\nu}q^\nu}{m_\Lambda + m_\Sigma} \kappa u_\Sigma \, \varepsilon^\mu. \tag{6.6}$$

For a two-body decay, the spin-averaged matrix element is constant because any Lorentz invariant combination of four-momenta is fully specified by the masses. The decay width is then given as [16]:

$$\Gamma_{\Sigma^0\to\Lambda\gamma} = \frac{1}{16\pi m_\Sigma^3}(m_\Sigma^2 - m_\Lambda^2)\langle|\mathcal{M}|^2\rangle \tag{6.7}$$

where $\langle|\mathcal{M}|^2\rangle$ is the spin-averaged squared matrix element for this decay. Squaring the matrix element given in Eq. (6.6) and averaging over initial spins and summing over final ones, we get:

$$\langle|\mathcal{M}|^2\rangle = \frac{e^2\kappa^2}{2(m_\Lambda + m_\Sigma)^2}\mathrm{Tr}[(\not{p}_\Lambda + m_\Lambda)\sigma_{\mu\nu}(\not{p}_\Sigma + m_\Sigma)\sigma_{\rho\alpha}]g^{\mu\rho}q^\nu q^\alpha. \quad (6.8)$$

Taking the trace, we see that the spin averaged matrix element reduces to:

$$\langle|\mathcal{M}|^2\rangle = \frac{2e^2\kappa^2}{(m_\Lambda + m_\Sigma)^2}(m_\Sigma^2 - m_\Lambda^2)^2. \quad (6.9)$$

Plugging this back into Eq. (6.7), we have the decay width:

$$\Gamma_{\Sigma^0\to\Lambda\gamma} = \frac{e^2\kappa^2(m_\Sigma^2 - m_\Lambda^2)^3}{8\pi m_\Sigma^3(m_\Lambda + m_\Sigma)^2} \quad (6.10)$$

obtained in [9]. Knowing the lifetime of the Σ^0 hyperon from the Particle Data Group compilation, we can establish the value of the anomalous magnetic moment, which turns out to be $\kappa \approx 1.98$ [9].

6.1.2 Weak decay $\Lambda \to p\pi^-$

The strangeness violating weak decay of the Λ hyperon constitutes the second decay that occurs in our reaction chain for $\Sigma \to p\pi^-\gamma$. Recall that in Chapter 5 we noted that without a possible P violation in the first decay, we will not see an angular dependence between the final decay products in our three-body decay width calculation, and that this holds true irrespective of the symmetries violated by the subsequent weak decay of the Λ. We follow here the discussion in [39] to obtain the decay width for this weak decay.

The matrix element for a non-leptonic hyperon weak decay such as this is given as [26][39]:

$$\mathcal{M}_{\Lambda\to p\pi^-} = \bar{u}_p(l)[A + B\gamma_5]u_\Lambda(p_\Lambda) \quad (6.11)$$

where A and B are complex numbers and l is the proton four-momentum. Before going on to calculate the spin averaged squared matrix element, we make use of the covariant Dirac spin projection operator for a general spin four-vector s^μ [40]:

$$\Sigma(s) = \frac{(1 + \gamma_5 \not{s})}{2}. \tag{6.12}$$

For a massive spin 1/2 particle with four-momentum $p^\mu = (E, \vec{p})$ and polarization $+\vec{n}$ in its rest frame, we have:

$$\begin{aligned} u(p, \vec{n})\bar{u}(p, \vec{n}) &= \Sigma(s) u(p, \vec{n})\bar{u}(p, \vec{n}) \\ &= \frac{1}{2}(1 + \gamma_5 \not{s})(\not{p} + m) \end{aligned} \tag{6.13}$$

such that $s.s = -1$ and $s.p = 0$. The spin four-vector takes the form:

$$s^\mu = \left(\frac{\vec{p}.\vec{n}}{m}, \ \vec{n} + \frac{\vec{p}(\vec{p}.\vec{n})}{m(m+E)} \right). \tag{6.14}$$

We can now utilize Eq. (6.13) for the calculation of the squared matrix element summed over the proton spins while keeping the Λ spin explicit:

$$\langle |\mathcal{M}|^2 \rangle_s = \mathrm{Tr}[(A^* - B^*\gamma_5)(\not{l} + m_p)(A + B\gamma_5)\frac{1}{2}(1 + \gamma_5 \not{s})(\not{p}_\Lambda + m_\Lambda)]. \tag{6.15}$$

Taking the trace, we obtain [39]:

$$\langle |\mathcal{M}|^2 \rangle_s = R_\Lambda + m_\Lambda S_\Lambda l.s \tag{6.16}$$

where,

$$R_\Lambda = |A|^2((m_\Lambda + m_p)^2 - m_\pi^2) + |B|^2((m_\Lambda - m_p)^2 - m_\pi^2), \tag{6.17}$$

$$S_\Lambda = 4\mathrm{Re}(A^* B). \tag{6.18}$$

When considering the rest frame of Λ we make the substitution $l.s = -\vec{l}.\vec{n}$, where we have used $\vec{s} = \vec{n}$ and $s^0 = 0$ which follows from Eq. (6.14) for $\vec{p} = 0$.

For the case of an unpolarized Λ, we see that the spin projection operator reduces to $\frac{1}{2}$ when averaged over initial spins. The matrix element squared is:

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{2}\sum_s \langle|\mathcal{M}|^2\rangle_s = \text{Tr}[\frac{1}{2}(A^* - B^*\gamma_5)(\not{l} + m_p)(A + B\gamma_5)(\not{p}_\Lambda + m_\Lambda)]. \tag{6.19}$$

Taking the trace, we have:

$$\langle|\mathcal{M}|^2\rangle = R_\Lambda. \tag{6.20}$$

Finally, the decay width for the two-body decay $\Lambda \to p\pi^-$ is [16]:

$$\Gamma_{\Lambda\to p\pi} = \frac{l_\Lambda}{8\pi m_\Lambda^2} R_\Lambda \tag{6.21}$$

where l_Λ is the modulus of the three-momentum of the proton in the rest frame of the Λ hyperon:

$$l_\Lambda = \frac{1}{2m_\Lambda}\left(\left((m_\Lambda + m_p)^2 - m_\pi^2\right)\left((m_\Lambda - m_p)^2 - m_\pi^2\right)\right)^{1/2}. \tag{6.22}$$

In terms of the Källén function this can be written as:

$$l_\Lambda = \frac{1}{2m_\Lambda}\lambda^{1/2}(m_\Lambda^2, m_p^2, m_\pi^2) \tag{6.23}$$

with the Källén function defined as $\lambda^{1/2}(a, b, c) \equiv a^2 + b^2 + c^2 - 2(ab + bc + ac)$.

6.1.3 Combined decay $\Sigma^0 \to p\pi^-\gamma$

We now consider the combined three-body decay of the neutral Σ hyperon. Before establishing the matrix element for this particular decay, we make use of a property of resonances to simplify our calculations. Because the Λ hyperon is relatively long-lived, enabling experimentalists to track the displaced vertex, we can use the width of the resonance peak (the decay rate) to simplify the S-matrix.

6.1.3.1 The Reduced Matrix Element

In non-relativistic quantum physics, an attractive potential leads to resonant scattering. The scattering amplitude for energies E around the resonance energy E_R is given by [13]:

$$f(E) \propto \frac{1}{E - E_R + \frac{i\Gamma}{2}}. \tag{6.24}$$

This is called the Breit-Wigner formula. In relativistic quantum mechanics, this formula can be generalized to calculate the transition amplitude for particles that combine to form unstable particles, which in turn decay. For an unstable particle of mass m and four-momentum p, the Lorentz invariant generalization of Eq. (6.24) leads to a propagator of the form [13]:

$$\frac{1}{p^2 - m^2 + im\Gamma} \approx \frac{1}{2E_p(p^0 - E_p + i\frac{m}{E_p}\frac{\Gamma}{2})} \tag{6.25}$$

with $E_p = \sqrt{|\vec{p}|^2 + m^2}$. The left hand side is manifestly Lorentz invariant and this is what we will use for our calculations. Note that a detailed derivation of this resonance condition involves using the optical theorem to pin down the imaginary part of the self energy, $\Pi(p^2)$. This self energy is nothing but the sum of one-particle irreducible contributions to the Feynman propagator. The imaginary part of the relevant loop diagrams then gives us the decay rate i.e. $\mathrm{Im}\Pi(p^2) \approx m\Gamma$ (see [4],[16]).

For our present case, we consider the matrix element given by:

$$\begin{aligned}\mathcal{M}_{\Sigma^0 \to p\pi^-\gamma} &= \bar{u}_p(l) V_2 \frac{\not{p}_\Lambda + m_\Lambda}{p_\Lambda^2 - m_\Lambda^2 + im_\Lambda\Gamma_\Lambda} V_{1\mu} u_\Sigma(p_\Sigma)\varepsilon^\mu \\ &= \mathcal{M}_R \frac{1}{p_\Lambda^2 - m_\Lambda^2 + im_\Lambda\Gamma_\Lambda},\end{aligned} \tag{6.26}$$

with

$$\mathcal{M}_R = \bar{u}_p(l) V_2 (\not{p}_\Lambda + m_\Lambda) V_{1\mu} u_\Sigma(p_\Sigma)\varepsilon^\mu. \tag{6.27}$$

Here we have labeled the contributions of the vertices for the processes $\Lambda \to p\pi^-$ and $\Sigma^0 \to \Lambda\gamma$ as V_2 and $V_{1\mu}$, respectively. For a three-body decay, we make use of the variables $m_{12}^2 = (q+l)^2$ and $m_{23}^2 = (l+p_\pi)^2$. The double differential decay rate is [16]:

$$\begin{aligned}\frac{d\Gamma}{dm_{12}^2 dm_{23}^2} &= \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\langle|\mathcal{M}|^2\rangle \\ &= \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\langle|\mathcal{M}_R|^2\rangle\frac{1}{(p_\Lambda^2-m_\Lambda^2)^2+m_\Lambda^2\Gamma_\Lambda^2} \\ &= \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\frac{\langle|\mathcal{M}_R|^2\rangle}{m_\Lambda\Gamma_\Lambda}\frac{m_\Lambda\Gamma_\Lambda}{(m_{23}^2-m_\Lambda^2)^2+m_\Lambda^2\Gamma_\Lambda^2}\end{aligned} \tag{6.28}$$

where we have used the relation $p_\Lambda^2 = (l+p_\pi)^2 = m_{23}^2$. Using the identity [41]:

$$\lim_{\epsilon\to 0}\frac{1}{\pi}\frac{\epsilon}{x^2+\epsilon^2} = \delta(x) \tag{6.29}$$

we see that as $m_\Lambda\Gamma_\Lambda \to 0$, Eq. (6.28) reads:

$$\frac{d\Gamma}{dm_{12}^2 dm_{23}^2} = \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\frac{\langle|\mathcal{M}_R|^2\rangle}{m_\Lambda\Gamma_\Lambda}\pi\delta(m_{23}^2-m_\Lambda^2). \tag{6.30}$$

The delta function enforces the on-shell condition $p_\Lambda^2 = m_{23}^2 = m_\Lambda^2$. On integrating once, we get:

$$\begin{aligned}\frac{d\Gamma}{dm_{12}^2} &= \int dm_{23}^2\frac{d\Gamma}{dm_{12}^2 dm_{23}^2} \\ &= \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\int dm_{23}^2\frac{\langle|\mathcal{M}_R|^2\rangle}{m_\Lambda\Gamma_\Lambda}\pi\delta(m_{23}^2-m_\Lambda^2) \\ &= \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\frac{\pi}{m_\Lambda\Gamma_\Lambda}\langle|\mathcal{M}_R|^2\rangle.\end{aligned} \tag{6.31}$$

Thus we see that the general form of the differential decay rate is greatly simplified, and the reduced matrix element is sufficient to understand the dynamics of this decay.

6.1.3.2 Calculation of the Differential Decay Rate

As was the case for the electromagnetic decay, the baryon transition form factors are given by:

$$\langle B'(p')|\, \mathcal{J}^{\mu}\, |B(p)\rangle = e\bar{u}_{B'}(p')\Gamma^{\mu}(q^2)u_B(p) \tag{6.32}$$

where $\mathcal{J}^{\mu}$ is the current. Following our discussion in Chapter 4, we are interested in all possible Lorentz invariant form factors irrespective of their discrete symmetry properties. We were able to derive a general expression for the vertex function Γ^{ν} (see Eq. (4.22)) which takes the form [10]:

$$\begin{aligned}\Gamma^{\mu}(q^2) = &(\gamma^{\mu} + \frac{m_{\Lambda} - m_{\Sigma}}{q^2} q^{\mu})F_1(q^2) - \frac{i}{m_{\Sigma} + m_{\Lambda}}\sigma^{\mu\nu}q_{\nu}F_2(q^2) \\ &+ i(\gamma^{\mu}q^2 + (m_{\Sigma} + m_{\Lambda})q^{\mu})\gamma_5 F_A(q^2) \\ &+ \frac{1}{m_{\Sigma} + m_{\Lambda}}\sigma^{\mu\nu}q_{\nu}\gamma_5 F_3(q^2)\end{aligned} \tag{6.33}$$

for $q = p - p'$. As before, F_1 and F_2 are the P conserving Dirac and Pauli form factors. F_A and F_3 constitute the P violating anapole form factor and electric dipole form factor, respectively. We are interested in the low momentum ($q^2 \to 0$) properties of these form factors and use the following normalization:

$$F_1(0) = 0, \qquad F_2(0) = \kappa. \tag{6.34}$$

In our subsequent calculations we do not need the anapole form factor F_A since it drops out for $q^2 = 0$ (i.e. $q_{\mu}\varepsilon^{\mu} = 0$). Instead, we focus on the form factor F_3 which yields us the transition electric dipole moment at $q^2 = 0$. The normalization of F_3 is obtained as a result of the definition of the neutron dipole moment, which is given by [10]:

$$d_n^{\gamma} = \frac{e\, F_{3,n}(0)}{2m_n}. \tag{6.35}$$

Following Eq. (6.4), the reduced matrix element corresponding to Eq. (6.33) is:

$$\mathcal{M}_R = \bar{u}_p(l)(A + B\gamma_5)(\not{p}_\Lambda + m_\Lambda)\left(\frac{ie\kappa\sigma_{\mu\nu}q^\nu}{m_\Lambda + m_\Sigma} - \frac{iec\sigma_{\mu\nu}q^\nu\gamma_5}{m_\Lambda + m_\Sigma}\right)u_\Sigma(p_\Sigma)\varepsilon^\mu \tag{6.36}$$

where we have assigned a complex number $c = iF_3(0)$ for our particular decay. Since we are only sensitive to the relative phase between the P conserving and P violating terms in the matrix element, we can define κ to be a positive real number, $\kappa \in \mathbb{R}^+$. Now, taking the average of initial spins and summing over final spins, we obtain the squared reduced matrix element:

$$\begin{aligned}\langle|\mathcal{M}_R|^2\rangle =& \frac{-e^2}{2(m_\Lambda + m_\Sigma)^2}\,\mathrm{Tr}[(\not{l} + m_p)(A + B\gamma_5)(\not{p}_\Lambda + m_\Lambda)(\kappa\sigma_{\mu\nu} - c\sigma_{\mu\nu}\gamma_5)\\ &(\not{p}_\Sigma + m_\Sigma)(\kappa\sigma_{\rho\alpha} + c^*\sigma_{\rho\alpha}\gamma_5)(\not{p}_\Lambda + m_\Lambda)(A^* - B^*\gamma_5)]q^\nu g^{\mu\rho}q^\alpha.\end{aligned} \tag{6.37}$$

Taking the trace:

$$\begin{aligned}\langle|\mathcal{M}_R|^2\rangle =& \frac{-2e^2}{(m_\Lambda + m_\Sigma)^2}(m_\Sigma^2 - m_\Lambda^2)\Big[4\kappa m_\Lambda^2(c + c^*)(AB^* + A^*B)l.q\\ &- (m_\Sigma^2 - m_\Lambda^2)\Big(AA^*(cc^* + \kappa^2)((m_\Lambda + m_p)^2 - m_\pi^2) + AB^*\kappa(m_\Lambda^2 + m_p^2 - m_\pi^2)\\ &+ BB^*(cc^* + \kappa^2)((m_\Lambda - m_p)^2 - m_\pi^2) + A^*B\kappa(c + c^*)(m_\Lambda^2 + m_p^2 - m_\pi^2)\Big)\Big].\end{aligned} \tag{6.38}$$

Finally, using the variables defined in the weak decay, R_Λ (Eq. (6.17)) and S_Λ (Eq. (6.18)), we can re-write the above equation as:

$$\begin{aligned}\langle|\mathcal{M}_R|^2\rangle =& \frac{2e^2}{(m_\Lambda + m_\Sigma)^2}(m_\Sigma^2 - m_\Lambda^2)\Big[(m_\Sigma^2 - m_\Lambda^2)\Big((cc^* + \kappa^2)R_\Lambda\\ &+ \frac{\kappa}{2}(c + c^*)(m_\Lambda^2 + m_p^2 - m_\pi^2)S_\Lambda\Big) - 2\kappa m_\Lambda^2(c + c^*)S_\Lambda l.q\Big].\end{aligned} \tag{6.39}$$

Setting $c = c^* = 0$, we notice that the angular dependence between the proton and the photon vanishes:

$$\langle |\mathcal{M}_R|^2 \rangle = \frac{2e^2\kappa^2}{(m_\Lambda + m_\Sigma)^2}(m_\Sigma^2 - m_\Lambda^2)^2 R_\Lambda. \tag{6.40}$$

This is to be expected. When the P and CP violating form factors are set to zero, the resulting decay does not carry any information regarding the angular distribution. The dot product $l.q$ can also be written in terms of the angle between the proton and the photon. In the rest frame of the Λ hyperon, we have:

$$\begin{aligned} l.q &= E_p E_\gamma - \vec{l}.\vec{q} \\ &= E_p E_\gamma - |\vec{l}||\vec{q}| \cos\theta_{p\gamma} \\ &= E_p E_\gamma \left(1 - \frac{l_\Lambda}{E_p} \cos\theta_{p\gamma}\right) \end{aligned} \tag{6.41}$$

where in the last step we have used the fact that $|\vec{q}| = E_\gamma$ for an on-shell photon, and the equality $|\vec{l}| = l_\Lambda$ in the Λ rest frame, with l_Λ given in Eq. (6.23). Note that in the Λ rest frame, the photon energy E_γ and the proton energy E_p are not free variables. Instead, E_γ can be expressed in terms of the rest masses of the particles involved in the first decay:

$$E_\gamma = \frac{q.p_\Lambda}{m_\Lambda} = \frac{1}{2}\frac{m_\Sigma^2 - m_\Lambda^2}{m_\Lambda}. \tag{6.42}$$

while E_p takes the form:

$$E_p = \frac{m_\Lambda^2 + m_p^2 - m_\pi^2}{2m_\Lambda}. \tag{6.43}$$

Thus, Eq. (6.41) tells us that the S_Λ featuring in the last term of Eq. (6.39) is not accompanied by a constant but the cosine of the angle between the photon and the proton. Likewise, we see that the factor accompanying R_Λ in Eq. (6.39) is indeed a constant.

Now, since the right-hand side of Eq. (6.31) is a function of $\cos\theta_{p\gamma}$, we would like to obtain the differential decay rate, i.e. the left-hand side of Eq. (6.31), in terms of the differential angle $dcos\theta_{p\gamma}$. To this end, we rewrite m_{12}^2:

$$m_{12}^2 = (q+l)^2 = m_p^2 + 2E_pE_\gamma - 2|\vec{q}||\vec{l}|\cos\theta_{p\gamma}. \tag{6.44}$$

Taking the differential of the above equation, we obtain:

$$\begin{aligned} dm_{12}^2 &= -2|\vec{q}||\vec{l}|\, dcos\theta_{p\gamma} \\ &= -2E_\gamma l_\Lambda\ dcos\theta_{p\gamma}. \end{aligned} \tag{6.45}$$

Inserting this in Eq. (6.31) gives us:

$$\frac{d\Gamma}{d\cos\theta_{p\gamma}} = -2E_\gamma l_\Lambda \frac{1}{(2\pi)^3}\frac{1}{32m_\Sigma^3}\frac{\pi}{m_\Lambda\Gamma_\Lambda}\langle|\mathcal{M}_R|^2\rangle. \tag{6.46}$$

The right-hand side of the above equation is a function of $\cos\theta_{p\gamma}$. For pedagogical reasons, let us call this function $F(\cos\theta_{p\gamma})$ for the moment. Further let us consider that this function takes the form:

$$F(\cos\theta_{p\gamma}) = \xi + \xi^{'}\cos\theta_{p\gamma} \tag{6.47}$$

where ξ and $\xi^{'}$ are constants. Eq. (6.46) can now be written as:

$$\frac{d\Gamma}{d\cos\theta_{p\gamma}} = F(\cos\theta_{p\gamma}) = \xi(1 + \frac{\xi^{'}}{\xi}\cos\theta_{p\gamma}). \tag{6.48}$$

It is this ratio of ξ' over ξ that is of importance to us. Barring a normalization factor, this ratio gives us the slope when plotting the number of events against the angular dependence, say $\cos\theta_{p\gamma}$. Experimentalists can obtain this ratio by considering the average and the weighted average of this function

$F(\cos\theta_{p\gamma})$ over the entire range of $\cos\theta_{p\gamma}$:

$$\mathcal{I}_1 = \int_{-1}^{1} d(\cos\theta_{p\gamma}) F(\cos\theta_{p\gamma}) = 2\xi, \tag{6.49}$$

$$\mathcal{I}_2 = \int_{-1}^{1} d(\cos\theta_{p\gamma}) \cos\theta_{p\gamma} F(\cos\theta_{p\gamma}) = \frac{2}{3}\xi'. \tag{6.50}$$

And this gives us the ratio in terms of the quantities $\mathcal{I}_1$ and $\mathcal{I}_2$:

$$\frac{\xi'}{\xi} = \frac{3\mathcal{I}_2}{\mathcal{I}_1}. \tag{6.51}$$

Performing the integration and substituting it in the above equation, we obtain:

$$\frac{\xi'}{\xi} = \frac{4\kappa m_\Lambda^2 E_\gamma (c + c^*)\, l_\Lambda\, S_\Lambda}{2(m_\Sigma^2 - m_\Lambda^2) R_\Lambda (\kappa^2 + cc^*)}. \tag{6.52}$$

Using Eq. (6.42) for E_γ in the Λ rest frame, the slope is:

$$\frac{\xi'}{\xi} = \frac{\kappa(c + c^*) m_\Lambda\, l_\Lambda\, S_\Lambda}{(\kappa^2 + cc^*)\, R_\Lambda} \tag{6.53}$$

In the above expression, we note that κ is the only constant that we have determined so far. In order to get an estimate of the slope, we also need to figure out the value of the constants R_Λ, S_Λ and c.

6.1.3.3 Determination of R_Λ and S_Λ

In order to determine the value of R_Λ we make use of the result obtained in Eq. (6.21):

$$R_\Lambda = \frac{8\pi m_\Lambda^2}{l_\Lambda}\, \Gamma_{\Lambda\to p\pi^-}. \tag{6.54}$$

Since the decay $\Lambda \to p\pi^-$ takes place with a probability of about 63.9% of the total observed decays, we can determine the value of the decay width from the lifetime of the Λ hyperon [42]:

$$\Gamma_{\Lambda \to p\pi^-} = 0.639\,\Gamma_{\text{total}} = \frac{0.639}{\tau}. \tag{6.55}$$

This gives us $R_\Lambda = 4.97(073) \times 10^{-7}(\text{MeV})^2$. Having established the value of R_Λ, we now proceed to find the value of S_Λ as defined in Eq. (6.18). For this purpose we make use of the baryonic decay parameters for a non-leptonic decay. The amplitude of any spin 1/2 hyperon decaying weakly into a spin 1/2 baryon and a spin 0 meson can be written as [26]:

$$\mathcal{M} = G_F m_\pi^2 B_f(p')(\mathcal{A} - \mathcal{B}\gamma_5)B_i(p) \tag{6.56}$$

where $\mathcal{A}$ and $\mathcal{B}$ are constants. One of the decay parameters, γ, is defined as:

$$\gamma = \frac{|s|^2 - |p|^2}{|s|^2 + |p|^2} \tag{6.57}$$

with $s = \mathcal{A}$ and $p = \eta\mathcal{B}$ where η is given by:

$$\eta = \frac{|p_f|}{E_f + m_f}, \tag{6.58}$$

$|p_f|$ and E_f being the magnitude of three-momentum and energy of the final baryon in the rest frame of the decaying hyperon. In the Λ rest frame, we make the identification:

$$|p_f| = l_\Lambda, \tag{6.59}$$

$$m_f = m_p, \tag{6.60}$$

$$E_f = E_p. \tag{6.61}$$

Using Eq. (6.23), Eq. (6.43) and the above three relations in Eq. (6.58), we obtain:

$$\eta = \frac{\lambda^{1/2}(m_\Lambda^2, m_p^2, m_\pi^2)}{(m_\Lambda + m_p)^2 - m_\pi^2}. \tag{6.62}$$

For our weak decay (given in Eq. (6.11)), we have:

$$s = \frac{A}{G_F m_\pi^2}, \qquad p = -\frac{\eta B}{G_F m_\pi^2}, \tag{6.63}$$

We can now re-write Eq. (6.57) to get the ratio:

$$\frac{|A|^2}{|B|^2} = \eta^2 \frac{(1+\gamma)}{(1-\gamma)}. \tag{6.64}$$

Using this relation and that of R_Λ given in Eq. (6.17), we can establish the constants $|A|$ and $|B|$. Another decay parameter is required to determine S_Λ and it is given by [26]:

$$\alpha_\Lambda = \frac{2\text{Re}(s^* p)}{|s|^2 + |p|^2}. \tag{6.65}$$

Re-writing this for our case, we get:

$$\alpha_\Lambda = \frac{-2\text{Re}(A^* \eta B)}{|A|^2 + \eta^2 |B|^2}. \tag{6.66}$$

Since $S_\Lambda = 4\text{Re}(A^* B)$, we finally have:

$$S_\Lambda = -\frac{2\alpha_\Lambda}{\eta}(|A|^2 + \eta^2 |B|^2) \tag{6.67}$$

which gives us $S_\Lambda = -2.84(382) \times 10^{-12}$. We also note that our expression for S_Λ can be simplified further on using the relation Eq. (6.62):

$$S_\Lambda = \frac{-2\alpha_\Lambda}{\lambda^{1/2}(m_\Lambda^2, m_p^2, m_\pi^2)} R_\Lambda. \tag{6.68}$$

Inserting this in Eq. (6.53), the calculated slope becomes:

$$\frac{\xi'}{\xi} = -\alpha_\Lambda \frac{\kappa(c + c^*)}{\kappa^2 + cc^*} \tag{6.69}$$

where we have used the fact that the modulus of the proton three-momentum l_Λ in the Λ rest frame is as given in Eq. (6.23).

In sequential weak decays, one finds expressions for angular distributions of the type (see for instance [43],[44]):

$$\frac{dN}{d\cos\theta} = \frac{N}{2}(1 + \alpha_1\alpha_2\cos\theta)$$

where N is the number of events while α_1 and α_2 are decay asymmetry parameters of the first and second decay, respectively. We can now define a decay asymmetry for the first decay similar to the α_Λ parameter in Eq. (6.66) for the weak decay. By analogy, we define [26]:

$$\alpha_{\Sigma^0} \equiv \frac{2\mathrm{Re}(c^*\kappa)}{|\kappa|^2 + |c|^2} \tag{6.70}$$

Inserting this in Eq. (6.39) gives us the reduced matrix element squared as:

$$\langle|\mathcal{M}_R|^2\rangle = 2e^2(m_\Sigma - m_\Lambda)^2(cc^* + \kappa^2)R_\Lambda\Big[1 - \alpha_{\Sigma^0}\alpha_\Lambda\cos\theta_{p\gamma}\Big]. \tag{6.71}$$

Consequently, the slope is then seen to be:

$$\frac{\xi'}{\xi} = -\alpha_{\Sigma^0}\,\alpha_\Lambda. \tag{6.72}$$

6.1.3.4 Determination of the Complex Number c

Referring to our discussion in Chapter 3, we make use of the upper bound of the nEDM in the framework of HBChPT in order to make an estimate of the complex number c. We will first determine a 'bare' quantity c_B based on HBChPT. As a second step, we will introduce a phase due to final state interactions. The interaction Lagrangian for our decay in question which fits to Eq. (6.36) is given as:

$$\mathcal{L}_{\Sigma^0\Lambda} = \frac{e\,c_B}{2(m_\Sigma + m_\Lambda)}\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Sigma^0 F^{\mu\nu} + h.c. \tag{6.73}$$

In Chapter 3, we also saw that the interaction Lagrangian which emerges from the effective Lagrangian of HBChPT has the general form (see Eq. (3.23)):

$$\begin{aligned}\mathcal{L}_{\text{int}} = &-\frac{2ie}{9}(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13})\Big[6\bar{n}\gamma_5\sigma_{\mu\nu}nF^{\mu\nu} - 3\sqrt{3}\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Sigma^0F^{\mu\nu} \\ &- 3\sqrt{3}\overline{\Sigma}^0\gamma_5\sigma_{\mu\nu}\Lambda F^{\mu\nu} + 3\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Lambda F^{\mu\nu} - 3\overline{\Sigma}^0\gamma_5\sigma_{\mu\nu}\Sigma^0F^{\mu\nu}\Big].\end{aligned} \tag{6.74}$$

From this we can extract the interaction terms of the Lagrangian that is relevant to our decay:

$$\mathcal{L}_{\Sigma^0\Lambda} = \frac{2\sqrt{3}}{3}ie\big(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13}\big)\overline{\Lambda}\gamma_5\sigma_{\mu\nu}\Sigma^0F^{\mu\nu}. \tag{6.75}$$

Comparing Eq. (6.73) and Eq. (6.75), we are led to the conclusion:

$$c_B = \frac{4\sqrt{3}}{3}i(m_\Sigma + m_\Lambda)\big(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13}\big). \tag{6.76}$$

Now, consider the interaction term for the neutron that emerges from Eq. (6.74):

$$\mathcal{L}_n = -\frac{4ie}{3}\big(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13}\big)\bar{n}\gamma_5\sigma_{\mu\nu}nF^{\mu\nu}. \tag{6.77}$$

Recall Eq. (6.35), where we provided a normalization for the form factor at $q^2 = 0$, $F_3(0)$, based on the value of the nEDM. Comparing that to Eq. (6.77), we obtain (up to tree-level of the nEDM):

$$\frac{e\,F_{3,n}(0)}{2m_n} = d_n^{\text{tree}} = -\frac{8e}{3}\big(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13}\big). \tag{6.78}$$

Re-writing this, we have:

$$\big(\omega'_{13}\bar{\theta}_0 + \frac{\sqrt{6}}{F_0}\eta_0\omega_{13}\big) = \frac{-3\,d_n^{\text{tree}}}{8e}. \tag{6.79}$$

Lastly, we plug this into Eq. (6.76) to get a value of the c_B in terms of the tree level contribution to the nEDM:

$$c_B = -\frac{\sqrt{3}\,i}{2e}(m_\Sigma + m_\Lambda)\, d_n^{\text{tree}}. \tag{6.80}$$

6.1.3.5 Final State Interactions

In Chapter 2 we discussed how final state interactions are indispensable in making CP violation effects observable in terms of the decay width. In our Σ^0 particle decay calculations, we note that our complex constant c can be expressed in terms of the bare (complex) constant in the interaction Lagrangian (Eq. (6.73)) as:

$$c = c_B e^{i\delta_F} \tag{6.81}$$

where δ_F is the phase induced by the electromagnetic final state interactions between Λ and the photon in the first decay. Further, we noted in Chapter 2 that the condition:

$$c_B = \bar{c}_B = -c_B^* \tag{6.82}$$

implies a C conservation in our decay. For a P violating decay this implies CP is violated. This is the breaking pattern caused by the theta vacuum angle.

We saw in Eq. (6.69) that the slope of the particle decay was proportional to the real part of c:

$$\frac{\xi'}{\xi} \sim \text{Re}(c) = \text{Re}(c_B e^{i\delta_F}) \tag{6.83}$$

Since we are interested in the case where CP is violated, we have a purely imaginary c_B as shown in Eq. (6.80):

$$\text{Re}(c) = \pm|c_B|\sin\delta_F \tag{6.84}$$

where $|c_B|$ is the absolute value of the complex number c_B. We can now calculate the slope by plugging in Eq. (6.80) and Eq. (6.84) into the slope that was calculated in Eq. (6.69), with the reasonable assumption that $|\kappa| \gg |c|$:

$$\frac{\xi'}{\xi} = -\alpha_\Lambda \frac{(c + c^*)}{\kappa} = -\alpha_\Lambda \frac{2\mathrm{Re}(c)}{\kappa}. \tag{6.85}$$

Following Eq. (6.84), we have:

$$\left|\frac{\xi'}{\xi}\right| = \frac{|\alpha_\Lambda|}{\kappa} 2|c_B||\sin\delta_F| = \frac{|\alpha_\Lambda|}{\kappa}\frac{\sqrt{3}(m_\Sigma + m_\Lambda)}{e}|d_n^{\mathrm{tree}}||\sin\delta_F|. \tag{6.86}$$

Since we are making an order of magnitude estimate, we utilize the current experimental upper bound of the nEDM [10], $|d_n^{\mathrm{tree}}| \leq 2.9 \times 10^{-26} e\,\mathrm{cm}$ with $|\sin\delta_F| \leq 1$. The value of the decay asymmetry for $\Lambda \to p\pi^-$ has been determined and it is $\alpha_\Lambda = 0.642$ [42]. This gives us a slope $|\frac{\xi'}{\xi}| \leq 1.902 \times 10^{-12}$.

We can also provide an upper bound for the decay asymmetry α_{Σ^0}. Utilizing Eq. (6.80), this yields an upper bound of $|\alpha_{\Sigma^0}| \leq 2.962 \times 10^{-12}$ in the limit $|\kappa| \gg |c|$.

We now turn our attention to the production process of the neutral Sigma hyperon.

6.2 Production Process

Often hyperons (denoted here as Y) are produced by parity conserving strong or electromagnetic processes, for instance $e^+e^- \to Y\overline{Y}$ [45] or $p\bar{p} \to Y\overline{Y}$ [46]. The production process for hyperons gives important insights concerning the strangeness and charm production in QCD [45][46]. It serves as a probe in the region between the perturbative and non-perturbative regime, i.e. in the confinement region, of QCD. In our case, the production of hyperons can also serve as a test for CP violation through the study of spin observables.

In quantum mechanical systems, the spin density matrix ρ contains the information we need to analyze that system [45]. The spin density matrix for a particle with spin j can be decomposed as:

$$\rho = \frac{1}{2j+1}\mathcal{I} + \sum_{L=1}^{2j} \frac{2j}{2j+1} \sum_{M=-L}^{L} Q^L_M r^L_M \tag{6.87}$$

where Q^L_M are hermitian matrices [45]. The first term of the above equation is related to the unpolarized differential cross section of the particle. The second term contains the polarized part, with r^L_M denoting the polarization vectors for a given angular momentum L and its projection, M [45]. For a spin 1/2 particle, the Q^L_M matrices take the form of Pauli matrices τ^a. For vector polarizations P_l, P_m and P_n, the spin density matrix for a spin 1/2 particle takes the form [47]:

$$\rho(1/2) = \frac{1}{2}(\mathcal{I} + \vec{P}.\vec{\tau})^T = \frac{1}{2}\begin{bmatrix} 1+P_l & P_m+iP_n \\ P_m-iP_n & 1-P_l \end{bmatrix}. \tag{6.88}$$

We now have to define the directions $\hat{l}$, $\hat{m}$ and $\hat{n}$. For a production process such as $A + B \to \Sigma^0 + X$ viewed in the center-of-mass system, we have two directions given by $\vec{p}_A$ and $\vec{p}_\Sigma$. We define $\hat{n}$ to be the direction normal to the plane formed by $\vec{p}_A$ and $\vec{p}_\Sigma$. The unit vectors $\hat{l}$ and $\hat{m}$ then lie in the $\vec{p}_A$–$\vec{p}_\Sigma$ plane, with $\hat{l}$ aligned with $\vec{p}_\Sigma$ and $\hat{m}$ being orthogonal to it.

If the production process is parity conserving, then the only polarization vector that remains due to the symmetry of the spin density matrix is P_n, the component normal to the production plane [47]:

$$\rho(1/2) = \frac{1}{2}\begin{bmatrix} 1 & iP_n \\ -iP_n & 1 \end{bmatrix}. \tag{6.89}$$

The spin-averaged total matrix element for the production and decay process is then given by [48]:

$$\sum |\mathcal{M}_{\text{total}}|^2 = \text{Tr}(\rho^P \rho^D) = \rho^P_{\lambda\lambda'}\rho^D_{\lambda'\lambda} \tag{6.90}$$

where λ and λ' are the helicity labels of the particle with eigenvalues $\pm\frac{1}{2}$.

We must now establish a frame of reference to perform our calculations.

For a massive fermion with four-momentum p and energy E, we define three orthonormal four-vectors s_a^μ, with $a = 1, 2, 3$, such that:

$$s_1^\mu = (0, \cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta), \tag{6.91}$$

$$s_2^\mu = (0, -\sin\phi, \cos\phi, 0), \tag{6.92}$$

$$s_3^\mu = \left(\frac{|\vec{p}|}{m}, \frac{E}{m}\hat{p}\right) \tag{6.93}$$

in a coordinate system where $\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. As a matter of convenience we choose the frame where $\theta = \phi = 0$. This leads us to a frame where $\hat{p} = (0, 0, 1)$ and the three four-vectors reduce to:

$$s_1^\mu = (0, 1, 0, 0), \tag{6.94}$$

$$s_2^\mu = (0, 0, 1, 0), \tag{6.95}$$

$$s_3^\mu = (\frac{|\vec{p}|}{m}, 0, 0, \frac{E}{m}). \tag{6.96}$$

The helicity spinor satisfies [48]:

$$\gamma_5 \not{s}^a u(p, \lambda') = \tau^a_{\lambda\lambda'} u(p, \lambda) \tag{6.97}$$

where τ^a are the Pauli matrices. From the above relation we obtain the Bouchiat-Michel formula for massive spin 1/2 fermions:

$$u(p, \lambda')\bar{u}(p, \lambda) = \frac{1}{2}[\delta_{\lambda\lambda'} + \gamma_5 \not{s}_a \tau^a_{\lambda\lambda'}](\not{p} + m). \tag{6.98}$$

For a complete derivation of this see, for instance, [48].

The decay spin density matrix takes on the familiar form of the spin-averaged reduced matrix element encountered previously:

$$\rho^D_{\lambda'\lambda} = \langle|\mathcal{M}_R|^2\rangle_{\lambda'\lambda} = \sum \mathcal{M}_{\lambda'}\mathcal{M}^*_\lambda. \tag{6.99}$$

Following Eq. (6.37) and using the Bouchiat-Michel formula, Eq. (6.98), we have:

$$\sum \mathcal{M}_{\lambda'}\mathcal{M}^*_{\lambda} = \frac{-e^2}{(m_\Lambda + m_\Sigma)^2}\Bigg((\not{l} + m_p)(A + B\gamma_5)(\not{p}_\Lambda + m_\Lambda)(\kappa\sigma_{\mu\nu} - c\sigma_{\mu\nu}\gamma_5) \\ \frac{1}{2}[\delta_{\lambda\lambda'} + \gamma_5 \not{s}_a \tau^a_{\lambda\lambda'}](\not{p}_\Sigma + m_\Sigma)(\kappa\sigma_{\rho\alpha} + c^*\sigma_{\rho\alpha}\gamma_5) \\ (\not{p}_\Lambda + m_\Lambda)(A^* - B^*\gamma_5)\Bigg) q^\nu q^\alpha g^{\mu\rho}. \tag{6.100}$$

Eq. (6.90) now takes the form:

$$\sum |\mathcal{M}_{\text{total}}|^2 = \rho^P_{\lambda\lambda'} \text{Tr}\Big(\sum \mathcal{M}_{\lambda'}\mathcal{M}^*_{\lambda}\Big). \tag{6.101}$$

Since for each eigenvalue λ and λ', the spin density matrix for the production process Eq. (6.89) picks out a number, we can re-write the above equation as:

$$\sum |\mathcal{M}_{\text{total}}|^2 = \text{Tr}\Big(\rho^P_{\frac{1}{2}\frac{1}{2}} \sum \mathcal{M}_{\frac{1}{2}}\mathcal{M}^*_{\frac{1}{2}} + \rho^P_{\frac{1}{2}-\frac{1}{2}} \sum \mathcal{M}_{-\frac{1}{2}}\mathcal{M}^*_{\frac{1}{2}} \\ + \rho^P_{-\frac{1}{2}\frac{1}{2}} \sum \mathcal{M}_{\frac{1}{2}}\mathcal{M}^*_{-\frac{1}{2}} + \rho^P_{-\frac{1}{2}-\frac{1}{2}} \sum \mathcal{M}_{-\frac{1}{2}}\mathcal{M}^*_{-\frac{1}{2}}\Big) \tag{6.102}$$

with

$$\rho^P_{\frac{1}{2}\frac{1}{2}} = \rho^P_{-\frac{1}{2}-\frac{1}{2}} = \frac{1}{2},$$

and the off-diagonal elements

$$\rho^P_{\frac{1}{2}-\frac{1}{2}} = \frac{iP_n}{2}, \qquad \rho^P_{-\frac{1}{2}\frac{1}{2}} = -\frac{iP_n}{2}.$$

Taking the trace, we have:

$$\sum |\mathcal{M}_{\text{total}}|^2 = \frac{2e^2(m_\Sigma^2 - m_\Lambda^2)^2}{(m_\Lambda + m_\Sigma)^2} R_\Lambda (cc^* + \kappa^2) \Big(1 + \frac{2m_\Sigma}{(m_\Sigma^2 - m_\Lambda^2)} P_n \, q.s_2 \, \alpha_{\Sigma^0} - \alpha_\Lambda \alpha_{\Sigma^0} \cos\theta_{p\gamma} - \alpha_\Lambda \cos\theta_{p\gamma} \frac{2m_\Sigma}{(m_\Sigma^2 - m_\Lambda^2)} P_n \, q.s_2 \Big) \tag{6.103}$$

where we have substituted the decay asymmetry α_Λ, α_{Σ^0} according to Eq. (6.66) and Eq. (6.70). We have also expanded $l.q$ to include terms containing the angle between the photon and proton line of flight in the Λ rest frame, as in Eq. (6.41) (see Figure 5.2). Thus, we see that the effects of a possible P violation can be distinguished from the effects of the production process.

We note that on setting $P_n = 0$ in Eq. (6.103) the total squared matrix element is (barring a normalization constant):

$$\sum |\mathcal{M}_{\text{total}}|^2 \sim 1 - \alpha_\Lambda \alpha_{\Sigma^0} \cos\theta_{p\gamma}. \tag{6.104}$$

Calculating the slope for the above expression, we obtain the slope derived for the unpolarized case (Eq. (6.72)), as expected. Further, the spin density matrix for the production process, given in Eq. (6.89), holds true both in the center of mass frame for the production and when boosted to the Σ^0 rest frame (since the normal vector P_n is unaffected by this boost). Therefore, the product $q.s_2$ can be expressed in the Σ^0 rest frame as follows:

$$q.s_2 = -q_y = -E_\gamma \cos\phi_2 = -\frac{(m_\Sigma^2 - m_\Lambda^2)}{2m_\Sigma} \cos\phi_2 \tag{6.105}$$

where ϕ_2 is the angle between $\hat{n}$ and the photon line of flight. Substituting this in Eq. (6.103), we have:

$$\sum |\mathcal{M}_{\text{total}}|^2 = \frac{2e^2}{(m_\Lambda + m_\Sigma)^2} (m_\Sigma^2 - m_\Lambda^2)^2 R_\Lambda (cc^* + \kappa^2) \Big(1 - \alpha_{\Sigma^0} P_n \cos\phi_2 - \alpha_\Lambda \alpha_{\Sigma^0} \cos\theta_{\gamma p} + \alpha_\Lambda P_n \cos\theta_{\gamma p} \cos\phi_2 \Big). \tag{6.106}$$

When $\alpha_{\Sigma^0} = 0$, i.e. the case where $c = c^* = 0$, we obtain (up to a normalization factor):

$$\sum |\mathcal{M}_{\text{total}}|^2 \sim 1 + \alpha_\Lambda P_n \cos\theta_{\gamma p} \cos\phi_2. \tag{6.107}$$

That is, even when there is no P violation in the $\Sigma^0 \to \Lambda\gamma$ decay, we get an angular dependence between the final decay products due to the production process and the P violation of $\Lambda \to p\pi^-$. The slope calculated for this particular case is:

$$\frac{\xi'}{\xi} = \alpha_\Lambda\, P_n \cos\phi_2. \tag{6.108}$$

This vanishes when averaged over the angle ϕ_2, while the P violating term in Eq. (6.103) ($\sim \alpha_\Lambda \alpha_{\Sigma^0} \cos\theta_{\gamma p}$) remains.

6.3 Decay of the Neutral Anti-Sigma Hyperon

We will now analyze the decay of an unpolarized neutral anti-Sigma hyperon, analogous to the treatment of the particle decay in Section 6.1. We will focus here only on those aspects of the decay that differ from the particle case, with an aim to determine the impact of charge conjugation symmetry on our final slope estimation.

We will first make some general remarks about the conditions necessary for a strong CP violation, as was done in Chapter 2. Following our discussion in Chapter 2, we can introduce a modulus and a phase for our complex number c_B:

$$c_B = |c_B| e^{i\delta_B} \tag{6.109}$$

for the particle decay, with $|c_B|$ being the magnitude of the bare constant in the interaction Lagrangian, and:

$$\bar{c}_B = |c_B| e^{i\bar{\delta}_B} \tag{6.110}$$

for the antiparticle decay. Eq. (2.109) shows us that hermiticity constrains this new phase $\bar{\delta}_B$ such that:

$$\bar{\delta}_B = \pi - \delta_B. \tag{6.111}$$

Since we are concerned here with the impact of electromagnetic final state interactions on our slope, we bring in an additional phase δ_F (see Eq. (2.108)) in our estimation of the decay asymmetry α_{Σ^0}. Following Eq. (6.70) for the particle decay, we observe:

$$\alpha_{\Sigma^0} \sim \mathrm{Re}(c) = \mathrm{Re}(c_B e^{i\delta_F}) = \mathrm{Re}(|c_B| e^{i\delta_B} e^{i\delta_F}) = |c_B| \cos(\delta_B + \delta_F). \tag{6.112}$$

Similarly, for the antiparticle case we can state:

$$\alpha_{\overline{\Sigma}^0} \sim \mathrm{Re}(\bar{c}) = \mathrm{Re}(\bar{c}_B e^{i\delta_F}) = \mathrm{Re}(|c_B| e^{i(\pi-\delta_B)} e^{i\delta_F}) = -|c_B| \cos(\delta_B - \delta_F) \tag{6.113}$$

where we have used the relation Eq. (6.111).

For a decay which conserves charge conjugation symmetry but violates parity, CP is also violated. Charge conjugation symmetry enforces the conditions $\delta_B = \frac{\pi}{2}$ or $\delta_B = \frac{3\pi}{2}$ so that:

$$\bar{c}_B = c_B, \tag{6.114}$$

and therefore $\bar{c} = c$ when we include final state interactions. Note that Eq. (6.112) above is the more general formulation of which the strong CP violation condition given in Eq. (6.84) is a particular case. That is, for $\delta_B = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, Eq. (6.112) is identical to Eq. (6.84).

For the antiparticle slope, if C symmetry is conserved, we have:

$$\alpha_{\Sigma^0} = -|c_B| \sin \delta_F = \alpha_{\overline{\Sigma}^0}. \tag{6.115}$$

with $\delta_B = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ as a consequence. If CP is conserved in this decay, then we have $\alpha_{\Sigma^0} = -\alpha_{\overline{\Sigma}^0}$ and $\delta_B = 0$ or π. Now if both CP and C symmetries are violated then δ_B is not a multiple of $\frac{\pi}{2}$. In our subsequent calculations we deal with the case where CP is violated, i.e. $\delta_B = \bar{\delta}_B = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

We would now like to calculate a concrete expression for the slope of the $\overline{\Sigma}^0$ hyperon decay. To this end, we note that the matrix element for the weak decay of the $\overline{\Lambda}$ hyperon takes the form [39]:

$$\mathcal{M}_{\overline{\Lambda}\to\bar{p}\pi^+} = \bar{v}_\Lambda(p_\Lambda)[A' + B'\gamma_5]v_p(l). \tag{6.116}$$

Just as in the Λ decay, this gives us a spin-averaged squared matrix element:

$$\langle|\mathcal{M}|^2\rangle = \overline{R}_\Lambda \tag{6.117}$$

with $\overline{R}_\Lambda$ defined as:

$$\overline{R}_\Lambda = |A'|^2((m_\Lambda + m_p)^2 - m_\pi^2) + |B'|^2((m_\Lambda - m_p)^2 - m_\pi^2) \tag{6.118}$$

If CP invariance holds for this particular decay, then $A' = A$ and $B' = -B$ (note that $\overline{R}_\Lambda = R_\Lambda$ and $\overline{S}_\Lambda = -S_\Lambda$). This in turn implies that the decay asymmetry defined in Eq. (6.66) for the Λ particle, now satisfies $\alpha_{\overline{\Lambda}} = -\alpha_\Lambda$. Experimentally, no deviation from this relation has been observed [42]. In the subsequent calculations we implicitly assume this condition to be true.

The combined three-body decay $\overline{\Sigma}^0 \to \bar{p}\pi^+\gamma$ will now have a reduced matrix element (analogous to the particle decay matrix element derived using transition form factors):

$$\mathcal{M}_{\overline{\Sigma}^0\to\bar{p}\pi^+\gamma} = \bar{v}_{\overline{\Sigma}}(p_\Sigma)\left(\frac{ie\kappa'\sigma_{\mu\nu}q^\nu}{m_\Sigma + m_\Lambda} - \frac{ie\bar{c}\sigma_{\mu\nu}\gamma_5 q^\nu}{m_\Sigma + m_\Lambda}\right)(\not{p}_{\overline{\Lambda}} - m_\Lambda)(A - B\gamma_5)v_{\bar{p}}(l)\varepsilon^\mu. \tag{6.119}$$

Averaging over initial spins and summing over final spins gives us the squared reduced matrix element:

$$\begin{aligned}\langle|\mathcal{M}_R|^2\rangle =& \frac{-e^2}{2(m_\Lambda + m_\Sigma)^2}\,\mathrm{Tr}\,[(\not{p}_\Sigma - m_\Sigma)(\kappa'\sigma_{\mu\nu} - \bar{c}\sigma_{\mu\nu}\gamma_5)(\not{p}_\Lambda - m_\Lambda)(A - B\gamma_5) \\ & (\not{l} - m_p)(A^* + B^*\gamma_5)(\not{p}_\Lambda - m_\Lambda)(\kappa'\sigma_{\rho\alpha} + \bar{c}^*\sigma_{\rho\alpha}\gamma_5)]q^\nu g^{\mu\rho}q^\alpha.\end{aligned} \tag{6.120}$$

Taking the trace, we obtain:

$$\langle|\mathcal{M}_R|^2\rangle = \frac{2e^2}{(m_\Sigma + m_\Lambda)^2}\Bigg((m_\Sigma^2 - m_\Lambda^2)\Big[R_\Lambda(\bar{c}^2 + \kappa'^2) - \frac{S_\Lambda}{2}\kappa'(\bar{c} + \bar{c}^*)(m_\Lambda^2 + m_p^2 - m_\pi^2)\Big] + 2\kappa' m_\Lambda^2(\bar{c} + \bar{c}^*) l.q S_\Lambda\Bigg). \tag{6.121}$$

That is,

$$\langle|\mathcal{M}_R|^2\rangle = 2e^2(m_\Sigma - m_\Lambda)^2(\bar{c}^2 + \kappa'^2)R_\Lambda\Big[1 + \alpha_{\overline{\Sigma}^0}\alpha_\Lambda \cos\theta_{p\gamma}\Big] \tag{6.122}$$

where we have used

$$\alpha_{\overline{\Sigma}^0} = \frac{\mathrm{Re}(\bar{c}^*\kappa')}{|\kappa'|^2 + |\bar{c}|^2} = \frac{\kappa'(\bar{c} + \bar{c}^*)}{(\kappa'^2 + \bar{c}^2)}. \tag{6.123}$$

Following the steps in Section 7.1, we can determine the slope for this decay. In the $\overline{\Lambda}$ rest frame this turns out to be:

$$\begin{aligned}\frac{\xi'}{\xi} &= +\alpha_\Lambda \frac{\kappa'(\bar{c} + \bar{c}^*)}{(\kappa'^2 + \bar{c}^2)} \\ &= +\alpha_\Lambda \alpha_{\overline{\Sigma}^0},\end{aligned} \tag{6.124}$$

For a C conserving process, we noted that $\alpha_{\overline{\Sigma}^0} = \alpha_{\Sigma^0}$. Therefore, the slope for the decay of the anti-Sigma hyperon becomes:

$$\frac{\xi'}{\xi} = +\alpha_\Lambda \alpha_{\Sigma^0}. \tag{6.125}$$

This bears the opposite sign to that of the particle-decay slope calculated in Eq. (6.72), which is what we expect from a decay that violates CP.

It is useful to define a decay asymmetry parameter (see Eq. (2.106)) in terms of the decay asymmetries, which serves as an indicator of CP violation in a decay [3]. This parameter, $\mathcal{Q}_{CP}$, is defined as [43]:

$$\mathcal{Q}_{CP} = \frac{\alpha_{\Sigma^0}\alpha_{\Lambda} - \alpha_{\overline{\Sigma}^0}\alpha_{\overline{\Lambda}}}{\alpha_{\Sigma^0}\alpha_{\Lambda} + \alpha_{\overline{\Sigma}^0}\alpha_{\overline{\Lambda}}}. \tag{6.126}$$

As we stated earlier, if we consider CP to be conserved in the weak decay $\Lambda \to p\pi^-$, we have $\alpha_{\overline{\Lambda}} = -\alpha_{\Lambda}$. This reduces the decay asymmetry parameter $\mathcal{Q}_{CP}$ to:

$$\mathcal{Q}_{CP} = \frac{\alpha_{\Sigma^0} + \alpha_{\overline{\Sigma}^0}}{\alpha_{\Sigma^0} - \alpha_{\overline{\Sigma}^0}} \tag{6.127}$$

If CP is conserved in our decay $\Sigma^0 \to p\pi^-\gamma$, we have $\alpha_{\Sigma^0} = -\alpha_{\overline{\Sigma}^0}$. Therefore, from Eq. (6.127), we see that the decay asymmetry parameter vanishes, i.e. $\mathcal{Q}_{CP} = 0$. On the other hand, if C is conserved in our decay, we have $\alpha_{\Sigma^0} = \alpha_{\overline{\Sigma}^0}$. From Eq. (6.127), we see that for this case the quantity $\mathcal{Q}_{CP}^{-1} = 0$. Note that we have implicitly assumed that the weak interaction, which violates C, does not appreciably contribute to our $\Sigma^0 \to \Lambda\gamma$ decay. This need not be true, and a study of C violation due to the weak interaction in the Σ^0 decay could reveal the extent of such a contribution. The general formalism laid out here can also be applied to such a study. However, since this thesis deals primarily with the possibility of CP violation in the decay due to the non-trivial nature of the QCD vacuum, for the sake of simplicity we have ignored the effects of C violation due to the weak interaction.

Chapter 7

Conclusions & Summary

Through this study on CP violation in Σ^0 decay, we have made an order of magnitude estimate of the angular dependence which serves as a test for P and CP violation in this decay. The estimate, a slope of about 10^{-12}, implies that any possible P and CP violation that may arise due to Strong CP effects is negligibly small. That is, despite considering beyond Standard Model effects in the form of a non-trivial QCD vacuum, there seems to be no observable CP (or P) violation in the decay of the Σ^0 hyperon. Conversely, restricting ourselves to the strong sector, if a slope is observed orders of magnitude larger than our estimate, this would imply not only physics beyond the Standard Model but also physics beyond Strong CP.

The decay asymmetry, α_{Σ^0}, has yet to be measured experimentally. It is interesting to compare our study to other radiative hyperon decays, in particular, the CERN NA48/1 experiment which analyzed the radiative decay $\Xi^0 \to \Lambda\gamma$ for an unpolarized Ξ^0 hyperon [44]. Of course, unlike our case where the first decay channel is dominated by the electromagnetic interaction, this decay proceeds largely through the weak interaction. Thus one can expect a decay asymmetry on the order 1 from the two interfering partial waves, since they both come from the weak interaction. This is clearly different from our Σ^0 decay. What is the same is the weak hadronic decay $\Lambda \to p\pi^-$ which serves as an analyzer with the angular distribution measured in the Λ rest frame [44]. The form of the matrix element obtained for our unpolarized Σ^0

decay is familiar to that found by NA48/1. The experiment yielded a decay asymmetry $\alpha_{\Xi^0} = -0.704 \pm 0.019 \pm 0.064$. As expected, this is many orders of magnitude larger than our estimate of α_{Σ^0}. However, no evidence of CP violation was found with $\bar{\alpha}_{\overline{\Xi}^0} = -0.798 \pm 0.064$ [44]. Our analysis of the Σ^0 however, predicts a negligibly small decay asymmetry, α_{Σ^0}. The measurement of α_{Σ^0} for this decay is thus a crucial experiment. Such an experiment can be performed if a large number of Σ^0 is available. The upcoming PANDA experiment will serve as a hyperon factory, and is therefore equipped for such a study (see also [9]).

Additionally, in this study we analyzed the production process of the Σ^0 hyperon. We obtained a general form of the matrix element including the production process which gives us the liberty to choose the angular distribution we would like to examine. That is, we can either integrate out the angular dependence between two of the final decay products leaving us with a slope independent of this factor, or alternatively, we can choose to integrate over the angle between the photon line of flight and the polarization of the Σ^0, thereby giving us a slope dependent on the angle between two of the decay products. This study extends the works done by Dreitlein et al. [38] wherein the matrix element was simply a function of the Σ^0 polarization angle with respect to the photon direction. Note that we can, if required, further generalize our results if we do not assume parity conservation in the production process. This give us two additional polarization directions for our Σ^0 particle which would in turn add more angular distributions that can be analyzed experimentally.

We have so far established that the weak part of our radiative decay has a negligible contribution towards CP violation in the decay chain. As a possible extension to this study, one can analyze the extent of C violation that could, in principle, take place due to the weak interaction. This is of course beyond the considerations of CP violation due to Strong CP effects which was undertaken here, but it would nonetheless be interesting to probe the exact nature of C violation in the Σ^0 decay.

To summarize, in this thesis we examined the possibility of P and CP violation in the neutral Sigma hyperon decay. In the process, we first gave a general overview of discrete symmetries in quantum field theory, aligning our discussion with the specifics of the decay at hand. We briefly described the minimal implementation of CP violation in the Standard Model via the CKM mechanism. We noted that the contribution of the weak interaction towards CP violation in our decay is negligible. We then touched on final state interactions and the crucial role played by them in making CP violation effects observable. We examined the Strong CP problem in some detail before moving on to the framework of Heayy Baryon Chiral Perturbation Theory which made an estimation of the P violation via the neutron electric dipole moment possible. We then derived the most general Lorentz invariant electromagnetic current which led us to the Lorentz invariant transition form factors and their role in determining a possible P and CP violation in our decay. The kinematics of the decay was then examined, and the connection between the angular distribution of the final decay products and P violation was made. We then performed the necessary calculations in order to establish the value of angular dependence arising due to P violation in the initial decay. This yielded a small slope of about 10^{-12} in our estimation. The production process for the Σ^0 hyperon was also analyzed and general form of the matrix element was found. The slope for the charge conjugated decay was also calculated.

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