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Patent Licensing and Vertical Integration in Complementary Markets

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Résumé en français

Licences de Brevets et Intégration Verticale dans les Marchés Complémentaires

Le secteur des TIC est caractérisé par de nombreux arrangements stratégiques pour les transferts de technologies tels que les licences et les regroupements de brevets. Par ailleurs, les produits et services ont souvent de fortes relations de complémentarité dans ce secteur. Il est important pour les consommateurs de pouvoir bénéficier d'un écosystème de produits compatibles. Afin de garantir un niveau satisfaisant d'interopérabilité aux utilisateurs, les producteurs de biens complémentaires doivent échanger des informations techniques. Cette thèse cherche à prendre en compte ces deux dimensions de l'industrie des nouvelles technologies et à produire de nouveaux éclairages sur les cas de politique de concurrence impliquant des marchés complémentaires (e.g Intel/McAfee, Google/Motorola). Nous étendons la littérature sur les licences de brevets en modélisant des marchés avals différenciés et complémentaires. En utilisant les méthodes de l'économie industrielle, nous caractérisons les stratégies optimales pour un innovateur en situation de monopole concernant le nombre de licences, les instruments tarifaires ainsi que l'intégration verticale et conglomérale.

En 2011, Intel, entreprise dominante sur le marché de la conception et de la production de processeurs utilisés dans les ordinateurs (environ 80% de part de marché en 2016) a acquis l'entreprise McAfee qui faisait partie des trois principales firmes développant des logiciels de sécurité. Après une analyse approfondie, la commission européenne qui a soulevé de nombreuses inquiétudes concurrentielles a finalement autorisé cette fusion. Intel était notamment suspecté de vouloir favoriser McAfee sur le marché des logiciels en garantissant un accès privilégié aux informations techniques concernant les technologies utilisées dans les processeurs. Intel serait en mesure d'utiliser sa position dominante pour influencer la manière dont fonctionnent les marchés avals complémentaires de processeurs et de logiciels de sécurité. En effet, les produits de la plateforme Intel-Windows, dominante sur le marché des ordinateurs, requièrent l'utilisation d'un logiciel de sécurité.

L'analyse menée dans cette thèse ne permet pas de montrer qu'il serait dans l'intérêt d'Intel de freiner la diffusion de sa technologie, même lorsque les deux composants sont fabriqués en interne. Nous modélisons explicitement la dimension verticale, d'échange de propriété intellectuelle du secteur des TIC, également caractérisé par de fortes relations de complémentarités entre les produits. En effet, rendre effective la complémentarité entre deux produits en atteignant un niveau élevé d'interopérabilité requière la distribution d'informations technologiques complexes et sensibles. Ces travaux permettent d'étudier l'impact de la complémentarité des marchés finals sur les stratégies optimales pour un monopole amont en comparaison avec le cas d'un marché aval isolé. Dans le cas de la tarification fixe, il est préférable pour le monopole de distribuer une licence exclusive sur le marché final alors que dans le cas de marchés complémentaires, nous montrons que le monopole ne choisit pas nécessairement d'offrir une licence exclusive. Dans ce travail, nous analysons les conditions et les manières dont les technologies permettant la production de biens complémentaires dans les secteurs des TIC sont diffusées et valorisées. Nous étudions l'impact de trois déterminants sur les stratégies de distribution de brevets :

- le degré de différenciation des produits
- la structure verticale du secteur
- le type de tarification disponible.

Nous cherchons également à explorer les éventuelles incitations à la forclusion verticale et à éclairer la régulation des fusions verticales et conglomérales dans ce secteur particulièrement dynamique et stratégique. De plus, ces travaux peuvent permettre de répondre à certaines questions sur la régulation des contrats de licences de technologies quant à la forme des contrats utilisés ainsi que sur les conditions dans lesquelles il peut être nécessaire d'imposer, aux propriétaires de certains brevets essentiels, la mise en place de termes FRAND (raisonnables et non discriminatoires).

Cadre d'analyse général

Dans le secteur des TIC, les coûts fixes d'investissement pour le développement d'une nouvelle technologie et les effets de réseaux tendent à générer de fortes positions dominantes sur la technologie de composants essentiels utilisés en combinaison avec d'autres produits. Nous analysons la dimension stratégique des interactions entre les acteurs présents dans ce type de structure de marché grâce à la théorie des jeux.

Nous introduisons ici notre modèle principal servant de référence dans la majeure partie de cette analyse. Nous supposons qu'une entreprise exerce un pouvoir de monopole sur le marché des idées. Sa technologie est nécessaire à la production de deux produits qui sont parfaitement complémentaires. Les consommateurs peuvent obtenir une satisfaction strictement positive lorsqu'ils achètent une unité de chaque bien (e.g processeur et système d'exploitation ou logiciel de sécurité). Nous considérons le cas d'une innovation drastique sans laquelle il est impossible pour les firmes avals de participer au marché et de réaliser un profit positif. Le coût marginal de production d'une unité de composant est constant, fini et positif pour les entreprises qui ont accès à la technologie du monopole. Nous présentons le modèle dans lequel l'un des deux marchés est différencié horizontalement (i.e modèle de la ville circulaire de Salop , 1979) et le nombre de firmes potentiellement actives dans chaque marché de composant est limité à deux. Les différentes structures de l'industrie que le monopole amont peut choisir d'implémenter sont donc :

- le double monopole
- le monopole différencié (monopole sur le marché différencié)
- la structure asymétrique (monopole sur le marché homogène)
- le double duopole.

Lors de la première étape du jeu, le monopole amont détermine le nombre de licences de technologie qu'il souhaite distribuer sur chaque marché final ainsi que le tarif auquel la technologie sera transférée. Dans un second temps, les firmes ayant accès à la technologie choisissent leurs prix qui déterminent les quantités vendues et les profits réalisés à l'équilibre. Nous résolvons le modèle à rebours en commençant par la dernière étape du jeu ce qui nous permet de déterminer les équilibres de Nash parfait en sous-jeux.

En fonction du niveau de valorisation des consommateurs pour le bien système, du degré de sensibilité des consommateurs à la différenciation des produits et du niveau des prix des composants fixés par les producteurs, la fonction de demande peut prendre plusieurs formes comme cela a été démontré par Salop (1979) dans son article sur la ville circulaire :

- non-couvert, lorsque la valorisation résiduelle (i.e valorisation moins les prix des composants) est trop faible pour assurer la participation des consommateurs les plus éloignés des producteurs du bien différencié. Ces derniers sont alors dans une situation de monopole local ;
- coudé, lorsque l'ensemble des consommateurs participent au marché mais qu'il n'y a pas de concurrence effective entre les producteurs ;
- compétitif, lorsque la valorisation résiduelle est suffisamment élevée (i.e la sensibilité à la différenciation des produits suffisamment faible) pour que les producteurs tentent d'attirer les consommateurs les plus indécis.

Tarification fixe des licences de technologie

Dans le chapitre un et deux, nous étudions le comportement d'un monopole qui a la possibilité de transférer sa technologie par l'intermédiaire d'un contrat à tarification fixe dont le montant est choisi en fonction du nombre de licences distribuées. Le pouvoir de négociation est concentré en amont avec des offres du monopole à prendre ou à laisser. Dans le cas d'un monopole séparé, nous trouvons qu'une licence exclusive n'est délivrée que lorsque la valorisation des consommateurs pour le bien système est suffisamment élevée. Dans le cas contraire, la structure de double duopole est préférée par le monopole. Ce résultat diffère du cas de référence où la technologie est utilisée dans un seul marché aval pour lequel le monopole préfère utiliser une licence exclusive (monopoles successifs) et extraire tout le profit de monopole aval par la tarification fixe. La prise en compte de la relation de complémentarité entre les deux composants importe pour la stratégie d'équilibre de distribution de licence et la structure de l'industrie.

Nous pouvons expliquer la préférence de l'innovateur pour la structure de double duopole dans les marchés de niche (i.e faible niveau de valorisation pour le système ou haut niveau de sensibilité à la différenciation des produits) par le fait que cette structure élimine la double marginalisation horizontale entre producteurs de biens complémentaires par le biais d'une intense concurrence sur le marché homogène. En comparaison, la structure asymétrique implique la présence d'un pouvoir de monopole sur chacun des marchés de composants (i.e monopole sur le marché homogène et monopoles locaux sur le différencié) induisant un prix excessif et une demande insuffisante pour le bien final. Les marchés de niche sont en effet caractérisés par une élasticité négative de la demande totale puisque le nombre de consommateurs ne pouvant pas participer au marché augmente avec le prix du bien système.

En revanche, lorsque la technologie est utilisée dans un marché de masse (valorisation importante pour le bien système), la double marginalisation à l'œoeuvre dans la structure asymétrique n'a pas d'effets néfastes puisque l'ensemble des consommateurs sont alors désireux d'acquérir le bien. Le niveau des prix d'équilibre permet de maintenir le niveau de demande sans laisser inutilement de surplus aux consommateurs. Au contraire, le double duopole générerait une intense concurrence, des marges et un prix pour le bien final excessivement bas aux yeux du propriétaire de brevets.

Ce modèle montre que le nombre de licences sur le marché homogène augmente avec le degré de différenciation des composants ce qui est en contradiction avec le travail de Arora et Fosfuri (2003). Cette relation positive entre le degré de différenciation et le nombre de licences délivrées est également présente dans l'article de Doganoglu et Inceoglu (2014).

En mettant de côté les effets sur l'étape auquel le profit du monopole est réalisé, la stratégie de distribution de licence est identique lorsque le détenteur de la technologie est intégré verticalement avec l'une des firmes avals. L'effet de la complémentarité des marchés avals sur cette stratégie persiste avec l'intégration verticale simple puisqu'elle n'est pas systématiquement identique dans des marchés complémentaires et dans un marché unique intégré. Nous étudions à la suite de Sandonis et Fauli-Oller (2006) et Rey et Salant (2012), l'impact de l'intégration verticale sur la stratégie de licence de brevets ainsi que sa profitabilité. Cette dernière n'a que peu d'effets sur le fonctionnement des marchés de composants et n'est jamais profitable pour le monopole amont.

Néanmoins, lorsque le propriétaire est présent sur chacun des deux marchés (i.e double intégration verticale), il choisit la structure de double duopole pour un intervalle plus large de valorisations du bien système. Cela s'explique par la diminution de l'efficacité de la structure asymétrique provoquée par la double intégration verticale alors que le profit du propriétaire de la technologie dans la structure de double duopole n'est pas affecté. Même lorsque le niveau intense de concurrence commence à générer un prix excessivement bas à l'équilibre compétitif, le double duopole reste préférable lorsque la valorisation pour le système prend des valeurs intermédiaires. En effet, dans la structure de double duopole, la concurrence est tellement intense sur le marché homogène que la firme intégrée ne peut pas fixer librement le prix de ce composant. Dans la structure asymétrique, la firme intégrée est tentée d'augmenter son volume de ventes (et le profit en résultant) en diminuant le prix de son alternative sur le marché différencié tout en augmentant le prix de l'unique composant homogène. Ceci résulte en une diminution de la marge du producteur indépendant qui lui permet seulement de contenir la baisse de ses ventes. Les parts de marchés sont donc asymétriques, ce qui génère une inefficacité dans l'allocation des consommateurs sur le marché différencié. En effet, certains consommateurs localisés un peu plus près du producteur indépendant choisiront à l'équilibre, de se fournir auprès de la firme intégrée. L'augmentation des coûts de transports supportés par les consommateurs (en particulier par le consommateur marginal et indifférent) en résultant implique une diminution du surplus social, du profit de l'industrie et celui de la firme intégrée qui est égal à la somme des revenus des ventes de produits et de la licence de brevet.

Dans les deux premiers chapitres, nous montrons que la relation de complémentarité entre les biens importe pour la détermination de la stratégie d'équilibre de distribution de brevet du monopole détenteur de la technologie. Lorsque la tarification fixe est utilisée, la distribution d'une licence exclusive ne demeure la stratégie choisie par le monopole que lorsque la valorisation pour le bien final est suffisamment élevée (i.e dans les marchés de masse). Cet intervalle sur lequel la politique de distribution de licence sur le marché homogène est identique que l'on considère le marché complémentaire ou non, se réduit lorsque le détenteur des brevets est actif dans chacun des deux marchés.

Tarification Binôme

Dans le chapitre trois, nous considérons le cas d'un contrat de licence public binôme qui inclut une composante fixe et une composante variable appelée royalty dont le montant dépend du volume de vente du bien final produit par la firme aval signataire d'un accord de licence. Pour chaque unité de bien vendu, l'entreprise doit verser au monopole amont le montant fixé par ce dernier. Il s'agit d'une royalty unitaire dont l'introduction implique l'endogénéisation du coût marginal constant de production d'un composant compatible avec la technologie. Celui-ci inclut désormais la royalty reversée au propriétaire. Cet instrument permet au monopole amont de contrôler précisément le prix d'équilibre du composant homogène et de rendre la structure de double duopole efficace dans les marchés de niche. Le montant de royalties est ainsi fixé de sorte que le surplus laissé aux consommateurs soit juste suffisant pour assurer la couverture totale du marché. Ceci permet de maintenir les prix d'équilibre à un niveau suffisamment élevé, malgré l'intensité de la concurrence et le haut niveau de valorisation pour le bien final. En conséquence, l'innovateur préfère faiblement la structure de double duopole pour tout niveau de valorisation pour le système.

Lorsque la technologie est utilisée sur un seul marché homogène et que les tarifs binômes sont disponibles, le monopole amont est indifférent entre l'usage d'un contrat de licence exclusive (i.e monopole) et la mise en place d'une structure compétitive (i.e duopole). Lorsque l'on considère des marchés parfaitement complémentaires, on ne retrouve cette indifférence que lorsque la technologie est utilisée dans un marché de masse. Autrement, le monopole préfère utiliser une tarification fixe pour mettre en place la structure de double duopole qui permet d'éviter la double marginalisation horizontale. On note que l'usage de la royalty est profitable pour le monopole uniquement lorsqu'il considère le degré de concurrence comme étant excessif (i.e équilibre compétitif de double duopole).

L'intégration verticale entre le monopole amont et l'une des firmes avals n'a pas d'impact significatif sur la stratégie de distribution de licence. Celle-ci n'a d'effets que sur les prix d'équilibres. La structure de double duopole reste faiblement dominante. En revanche lorsque l'innovateur est actif dans les deux marchés de composants, celui-ci préfère strictement la structure de double duopole pour tout niveau de valorisation du bien système. L'indifférence entre le monopole et le duopole, observée lorsqu'un seul marché aval est considéré, disparaît complètement dans la structure de l'industrie résultant d'une intégration verticale et conglomérale. Ce résultat s'explique par le fait que l'efficacité de la structure asymétrique est réduite lorsque l'innovateur détermine librement les prix des deux composants. Le propriétaire de la technologie rencontre un problème d'engagement qui le pousse à privilégier les revenus issus des ventes au détriment de celui généré par le transfert de la technologie au producteur indépendant. Cette incitation résulte en une allocation asymétrique et inefficace des ventes sur le marché différencié qui détériore le surplus et le profit de la firme doublement intégrée. Par ailleurs, le tarif binôme permet à l'innovateur d'augmenter le profit généré par la structure de double duopole. Celle-ci est donc strictement préférée dans les marchés de niche mais aussi de masse. La complémentarité et la double intégration verticale importent pour la stratégie de distribution de licence de technologie dans le cas de contrats à tarification fixe et binôme.

Tarification trinôme, demande incertaine et aversion au risque

Nous introduisons dans le chapitre quatre un troisième instrument tarifaire. Il s'agit d'une royalty ad valorem dont la base de calcul est le revenu généré par les ventes de produits finals qui utilisent la technologie de l'innovateur. La profitabilité et l'optimalité de l'usage de ce type d'instrument dans les contrats de licences a notamment été étudié par Llobet et Padilla (2016). Bousquet et al. (1998) montrent que l'incertitude sur la performance de la technologie peut expliquer la préférence des innovateurs pour cette forme de royalty. Dans ce chapitre, nous examinons l'impact de la relation de complémentarité entre les biens finals sur la forme du contrat utilisé par un innovateur neutre au risque lorsque la demande pour le bien système est incertaine et les firmes avals sont averses au risque. Nous étudions cette question dans le cadre de la structure de double duopole. En particulier, nous considérons deux types de marchés et d'équilibres distincts qui sont d'un côté, le marché de niche dans lequel la couverture du marché est nécessairement incomplète, et de l'autre, le marché de masse concurrentiel dans lequel il y a une concurrence effective entre les producteurs du composant différencié.

Nous trouvons que l'innovateur choisit d'utiliser un tarif binôme incluant une royalty ad valorem dans les deux types d'équilibres que nous explorons. Ceci reste vrai lorsque le détenteur de la technologie est actif dans l'un des marchés de composants. Ce résultat s'explique par le fait que la composante variable de la tarification permet de contourner l'aversion au risque des firmes avals, en prélevant le surplus directement sur les ventes quand le niveau de demande est de connaissance commune (i.e lors de la seconde étape). Par ailleurs, l'usage de la royalty ad valorem est bénéfique pour l'innovateur dans l'équilibre compétitif, car il génère un pass-through inférieur à la royalty unitaire (et une marge des firmes avals inférieure). Ceci permet à l'innovateur d'augmenter davantage le montant de la royalty tout en préservant le volume des ventes. Dans l'équilibre non-couvert correspondant aux marchés de niche, la royalty ad valorem est préférée car le montant du transfert qu'elle induit est calculé sur le revenu des ventes qui dépend du volume mais aussi de la marge sur coût marginal. Ceci permet de prendre en compte l'effort consenti par les producteurs de composants pour contenir la baisse des ventes (générée par l'augmentation de la royalty et donc du coût marginal) en réduisant leur marge. La baisse du revenu issu des ventes implique une relative diminution (ou une moindre augmentation) du transfert monétaire vers l'innovateur. Cette préférence des firmes avals pour la royalty ad valorem se transfère à l'innovateur puisque cela lui permet d'augmenter davantage la composante variable du contrat tout en préservant la participation des producteurs avals.

Le résultat principal de Bousquet et al. (1998) sur la forme du contrat profitable de l'innovateur est préservé lorsque sa technologie est utilisée dans des marchés complémentaires. De plus, nous montrons que l'intégration verticale sur le marché différencié est bénéfique pour l'innovateur dont la technologie est utilisée dans un marché de niche puisqu'elle permet d'internaliser les royalties perçus sur les deux marchés sans générer d'inefficacité de transport.

Conclusion

Cette thèse montre que la modélisation de la complémentarité entre les biens finals permet de mettre en évidence des différences dans les stratégies profitables de distribution de licences de brevets selon la structure du marché aval. La stratégie employée varie quand on prend en compte la relation de complémentarité entre deux produits fréquemment observée dans les marchés de l'ordinateur et des téléphones intelligents. De manière générale et en comparaison au cas où les brevets sont distribués à un unique marché aval, l'innovateur dont la technologie est utilisée pour la production de biens complémentaires a tendance à davantage privilégier la stratégie qui consiste à encourager la diversité de l'offre de composants et un certain degré de concurrence sur les marchés avals. Dans le cas d'une tarification fixe, la distribution d'une licence exclusive est seulement strictement préférable dans les marchés de masse. Cette stratégie qui consiste à monopoliser l'un des deux marchés avals n'est jamais dominante lorsqu'une tarification binôme est utilisée. On note également que les royalties unitaires ne sont utilisées que dans les marchés de masse afin de réguler l'intensité de la concurrence.

L'intégration verticale n'est en général pas profitable pour l'innovateur qui préfère donc rester en dehors des marchés de produits à l'exception du cas où une seule firme est prête à produire le composant homogène (i.e structure asymétrique).

Enfin nous montrons que l'incertitude sur la demande finale peut expliquer l'utilisation de royalties ad valorem dans les marchés complémentaires et que l'intégration verticale est profitable dans les marchés de niche afin de pallier aux difficultés d'extraction de la rente dues à l'aversion au risque des firmes avals.

Nos résultats montrent que la relation de complémentarité entre les marchés finals influe sur la manière dont sont transférées les technologies et doit être considérée pour l'analyse des transferts de technologie. De plus, les fusions verticales et conglomérales ne semblent pas générer de comportements de forclusion d'inputs technologiques.

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Contents

General Introduction

The demand for compatible goods is one of the fundamental characteristics of technological markets. Most of the technological devices we own can only be useful if they can be connected and work properly with other products. High technology markets are often characterized by a strong valuation of consumers for complementary goods. System markets represent an important share of ICT products. Computers are made of various components that are only valuable to OEMs (Original Equipment Manufacturers) and in turn to consumers if they can function perfectly with each others. No matter whether end products are assembled by consumers or integrating firms, component producers must ensure full effective complementarity with other elements of the final good.

On the other hand, the growing complexity of system products makes effective complementarity more difficult to attain. The production process of each component involves a massive amount of patents and intellectual property as well as human capital. It is a challenge for the members of a technological firm to successfully take the production process to an end which makes it virtually impossible for complementary good producers to independently produce a readily compatible component. The tremendous amount of technological knowledge embedded in components used in ICT industries makes the production of complementary goods challenging (e.g in 2012 a defensive Patent aggregator estimated that around 250 000 active patents impact the smartphone market). Reverse engineering becomes more and more difficult and expensive.

As a consequence, complementarity becomes a strategic and essential characteristic of a component. It is both required by consumers and resource consuming to supply. In effect, the only reasonable way to satisfy its consumers and ensure a sufficient degree of interoperability with other key elements of a technological system is to obtain information from complementary good producers. This technical information is an essential input for the production of a demanded component. Such technical information is very sensitive and strategic in technological system markets.

Antitrust authorities have been recently paying careful attention to the exchange of information allowing the production of interoperable products. Incentives to supply such an essential input have been studied in depth in various recent competition policy cases. The merger between Intel and McAfee has only been allowed by the European competition authority under the condition of strict commitments. One of the main competition concerns raised by the authority was the incentive for the new entity to strategically manipulate the supply of technical information to the rivals of McAfee in the Security Software Solution (SSS) market. Intel had to commit to provide under Fair Reasonable And Non Discriminatory (FRAND) terms the information necessary to allow for alternative interoperable complementary goods. The quality and the timing of the delivery of the information concerning new products or updates were to be controlled by an independent trustee. We can also cite the Google/Motorola or Microsoft/Nokia as well as the Cisco/Tandberg mergers and the IBM mainframes maintenance services case that raised similar issues of interoperability and technical information licensing.

In sum, complementarity is required by consumers of technological products and the supply of a compatible component is only made possible by the access to an essential input that is quality, up to date, technical information about new versions of complementary goods. A monopoly producer of a component using a specific technology is thus in a situation to decide whether or not it is desirable to supply the technology enabling the production of complementary goods. In essence, the importance given to complementarity in technological system markets generates a strategic vertical interaction between complementary good producers. The analysis of this particular strategic interaction is the aim of this thesis. Component producers and technology developers indeed face a licensing policy problem raising many questions. Is it more profitable to license and remain outside the downstream market? How many license to issue, what kind of contract to use and how much to charge for this technology in order to maximize the profit of the innovator? How efficient is the structure of the industry resulting from these trade-offs?

These questions have been extensively studied in the licensing policy literature in various strategic environments. This literature analyzes the impact of downstream and upstream market structures on the optimal licensing policy of an inventor. We only refer to some of the closest work among this vast literature. Arora and Fosfuri (2003) combine the licensing with the vertically related markets literature. They study the consequences of the introduction of upstream competition on the incentives to license to potential competitors. Upstream differentiated innovators also choose whether they want to compete in the downstream product market or not. The authors characterize an oligopolistic supply side of the market for ideas where technology is exchanged through patent licensing. Gambardella and Giarratana (2013) consider the case where a technology could be used in several product markets. The relationship between the upstream market for ideas and the product market is more general in their model. Licensing policy indeed vary with the degree of generality of the technology as well as the degree of fragmentation of downstream markets. In their terminology, a market is said to be fragmented when downstream assets cannot be used in other markets. They argue that licensing is encouraged by both factors.

The vertical structure of the industry has also been shown to matter to licensing in the work of Chen et al. (2013). An inventor may license its technology to a downstream firm in order to induce a strategic supplier to favorably change its behavior. Rey and Salant (2012) consider the impact of the optimal licensing policy of an upstream firm on the degree of downstream competition and product variety using the circular city model of product differentiation. The number of licenses can either be excessive or insufficient depending on the valuation given to variety. The effect of product differentiation on the profit-maximizing licensing contract of both drastic and non-drastic innovations is also analyzed by Poddar and Sinha (2004) in the context of a spatial differentiation model. They show that the shape of the optimal licensing contract of a patentee is influenced by spatial differentiation. The choice of the number of licensee and subsequent degree of downstream competition has also been considered by Doganoglu and Inceoglu (2014) in a logit demand model. They show that an inside innovator prefers not to compete in the downstream market. Moreover, the profit-maximizing number of licenses increases in the relative performance of the innovation. The question of integration of subsystem is also closely related to our present work. Erat et al. (2013), concludes that the integration of an additional functionality into a subsystem can be undesirable for a licensor because it prevents licensees to sufficiently differentiate in the downstream market. All these various issues have been studied for drastic or minor cost-reducing innovations as well as product innovation. Moreover, the optimal licensing policy has been shown to differ for an inside and outside innovator.

In chapters one and two, we contribute to the literature on the optimal number of licenses of a monopoly innovator and the effect of vertical integration in considering the case of downstream complementary markets. To our knowledge, the fact that industries where technology licensing is decisive are very often characterized by strong complementary relations between products has not been explicitly analyzed. This thesis aims at studying the impact of downstream complementarity on the profit-maximizing licensing policy (i.e number of licenses, integration and contracts) of a monopoly innovator. We study the extreme case of perfect complementarity and system goods. We analyze the optimal licensing problem of an upstream monopoly licensor that is given the power to shape downstream competition in two component markets.

The analysis of this industry structure is motivated by the fact that it is effectively observed. The Intel/McAfee merger gives an example of how a component producer can be in a position of an upstream monopolist having to choose how to license its technology to firms active in different component markets. In our terminology, Intel is an upstream monopolist (on the x86 Central Processing Unit architecture) active in R&D. Its technology is required for the manufacturing of Intel designed CPU as well as for the development of fully interoperable security software solutions (our two component markets). Consumers strongly value fully compatible components that ensure the best system performance. Intel is vertically integrated in the sense that it manufactures its own chips. In contrast, ARM who successfully design CPU for portable devices is a good example of a research lab, not active in one of the downstream component markets and making profit in licensing its technology to various component producers such as chips manufacturers, soft and middleware developers and system integrators.

Our work belongs to the vast literature on patent licensing initiated by the seminal article written by Arrow (1962). In this paper, the author argued that incentives to innovate are higher in a perfectly competitive industry than in the presence of a monopoly. Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985) analyze patent licensing in oligopolistic industries. These pioneers show that auction and fixed fee are more profitable than royalties. For example, Kamien and Tauman (1986) show that fixed fee licensing is superior to per unit royalty for both the outside innovator and consumers. This literature mainly focused on the case of homogeneous Cournot competition.

These results are in sharp contradiction with empirical evidence (e.g Taylor et al., 1973, Rostoker, 1984, Macho-Stadler et al., 1996, Degnan and Horton, 1997, Bousquet et al., 1998) suggesting that royalty is the mostly used pricing instrument in licensing agreements. Numerous explanations for royalties have been pushed forward by theoretical work such as innovator incumbency (e.g Kamien and Tauman, 2002), risk aversion (Bousquet et al., 1998), product differentiation (e.g Muto, 1993, Wang and Yang, 1999, Poddar and Sinha, 2004, Erkal, 2005) and the integer number of licenses (Sen, 2005). Gallini and Wright (1990) take into consideration the informational asymmetry and show that high-value innovations are licensed through a royalty contract. Under asymmetric information, royalties based on output are found to be profitably used as a separating tool (e.g Macho-Stadler and Perez-Castrillo, 1991, Beggs, 1992).

An important conclusion of this literature is that the profit-maximizing licensing mechanism depends on the type of downstream competition, the degree of product differentiation and integration (i.e outside or inside innovator). For example, Sen and Tauman (2007) find that the profit-maximizing contract involves a positive royalty when the size of the industry is not too large and the non-drastic innovation is significant. They use a general class of demand functions and show that royalties perform better than fee or auction for an innovation of standard magnitude when the industry is sufficiently large. Two-part tariffs always include a positive royalty rate and the technology is largely diffused if it represents a relatively significant innovation. In chapter three, we characterize the optimal two-part tariff licensing contract of the patentee and analyze the effects and the scope for vertical and conglomerate mergers and contribute to the explanation for the use of royalty rates.

Recently the literature on licensing has also been studying the optimal royalty base. Royalty payments are mainly computed on the basis of the volume of sales (i.e the level of the royalty times the number of end products sold) or sales revenue generated by the final product (i.e a share of the sales revenue defined by the ad valorem royalty rate). Bousquet et al. (1998) shows that uncertainty on the demand for the final good and risk aversion of the licensee, make the ad valorem royalty more profitable for the patentee. The authors also show that ad valorem royalty rates are the most frequently used licensing price instrument in the French telecommunication industry.

In deterministic frameworks and under Cournot competition, ad valorem royalties are found to constitute a better commitment mechanism allowing downstream prices to rise more easily than per unit royalties (San Martín and Saracho, 2010). The opposite is found to be true under Bertrand competition (Colombo and Filippini, 2015). In their article, Colombo and Filippini show that a per unit royalty contract is more profitable for an inside innovator of a non-drastic cost-reducing technology. The per unit royalty performs better because it represents a more effective commitment tool to preserve downstream profits. The strategic effect of royalties which reduces the intensity of downstream competition and increases aggregate industry profit is stronger with per unit than ad valorem royalty rates which explains the profitability of the former. We contribute to this literature in chapter four by introducing demand uncertainty and ad valorem royalty rates in a differentiated Bertrand model of downstream complementary markets.

The work presented in this dissertation is also related to the more general literature on vertical relations, integration and foreclosure. In their seminal article, Rey and Tirole (2007) contribute to the literature on vertical relations and vertical foreclosure and the impact of the nature and observability of contracts (i.e public or private contracts). One important conclusion of this literature with public contracts (e.g Mathewson and

Winter, 1984 and Perry and Porter, 1989) is that the monopolist is able to reach industry profit maximization. Under secret contracts (i.e with private renegotiation) however, an upstream monopolist would fail from maximizing the profit of the industry and attain the integrated monopoly outcome. In this case, there is an incentive for the upstream firm and one of the retailer to deviate from the integrated outcome and maximize their joint profit. Under passive beliefs, the equilibrium is the Cournot outcome which results in the profitability of vertical integration and foreclosure in order to restore monopoly profit (e.g Hart et al., 1990, O'Brien and Shaffer, 1992 and McAfee and Schwartz, 1994). Considering the case of public contracts with only one downstream market where the technology of the patentee is used, the profit-maximizing licensing policy with fixed fee contracts would be to issue an exclusive license.

In chapter one, we explore the profitability of this strategy when the technology is used in two different complementary markets. We find that an exclusive license can be profitably issued under some circumstances (e.g unlimited number of potential licensees in a spatially differentiated component market). In chapter two, we study the impact of vertical integration on profit maximizing strategies in downstream complementary markets and look for profitable vertical mergers that may not exist with a single downstream market. Finally, under two-part tariff public contracts, an upstream monopolist selling to one market would be indifferent between a monopolistic and oligopolistic downstream market, using the per unit royalty rate to regulate the degree of downstream competition. The upstream monopolist is able to reach the monopoly outcome under public contracts and there is no room for profitable vertical integration in such a context. We see in chapter three that there is also little room for single vertical integration when we consider downstream complementary markets. However, the upstream monopolist does not generally remain indifferent between downstream monopoly and oligopoly structures.

In this dissertation, we take into account the fact that complementarity is an important feature of IT industries and explicitly model the relationship between downstream complementarity markets. This allows us to explore whether or not the prediction on profit-maximizing vertical contracts in a model with a single downstream market remains valid. We characterize the conditions in which the patentee is willing to license an exclusive license, merge and foreclose. Our analysis of these issues is presented in four chapters. Fixed fee licensing is studied in chapter one. Chapter two deals with various forms of integration. Two-part tariffs are included in the analysis in chapter three while ad valorem royalties and demand uncertainty is considered in chapter four. In our framework, the inside patentee (i.e active in one of the component markets) has the opportunity to practice input foreclosure and leave some potential component producers inactive when choosing how many licensees will have access to its technology. We see if it is possible to find a rationale for vertical foreclosure under public contracts in complementary markets.

This thesis is also closely related to the literature on system markets, compatibility and tying (i.e horizontal foreclosure Rey and Tirole, 2007). Matutes and Regibeau (1988, 1992) and Economides (1989) found that in the absence of network externalities, standardization and full compatibility is both socially and privately desirable. This was shown to be true even in the case of conglomerate mergers. More recently, Kim and Choi (2015) study a more general model of product differentiation in system markets allowing for more than two varieties per component. They show that in general, there is a conflict between the interests of consumers and producers regarding the issue of compatibility. Church and Gandal (2000) show that profitable conglomerate merger and foreclosure can occur when the availability of complementary goods brings valuation to the system. Foreclosure does not arise when both components are symmetrically differentiated because it would make it unprofitable or would induce retaliation. Heeb (2003) finds that integration of a monopoly producer of an essential component with a complementary good producer would be socially desirable unless it induces the high quality component producer to exit the market. An integrated monopolist would implement an extremely aggressive pricing strategy (i.e zero price) of the complementary good in a mass market (i.e when all consumers highly value the complementary good) and charge a more standard price (i.e positive margin) in a niche market.

We characterize in chapters two and three, the impact of a conglomerate merger on the profit-maximizing licensing policy, the scope for profitable vertical foreclosure and the number of component varieties available to end users. Following a conglomerate merger between the inside innovator and one of the complementary good producers, the upstream monopolist would be producing both components in-house. Excluding one of its rivals from a licensing deal would make it impossible for these to produce a compatible component and would result in their exclusion from the market. In our model, incompatibility can be reached through the exclusion from a patent licensing deal which amounts to vertical foreclosure.

Chapter one presents the general framework of our model and focuses on characterizing the optimal number of licenses offered by an outside innovator using fixed fee contracts. Chapter two explores the profitability of vertical and conglomerate mergers and their impacts on licensing policy. Chapter three generalizes the two first chapters to the case of two-part tariff licensing contracts. Chapter four explores the impact of the introduction of ad valorem royalty rate and demand uncertainty on the shape of the profit-maximizing licensing contract and the profitability of vertical merger.

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Chapter 1

Optimal fixed fee licensing in complementary markets

1.1 Introduction

In this chapter we show that the profit-maximizing number of licenses delivered by an outside innovator in technological system markets vary with the structure of downstream competition. In particular, we study the impact of demand elasticity as well as product differentiation on the licensing policy of the innovator. Following Rey and Salant (2012) and Poddar and Sinha (2004), we use the spatial differentiation framework developed by Salop (1979). We characterize the optimal number of licenses sold by an upstream innovator active in the R&D of a technology used in two different component markets. We observe that its licensing policy depends on downstream market characteristics such as demand elasticity and component differentiation. Nevertheless, the asymmetric licensing across component markets (i.e when the number of licenses differs from one downstream market to another) tends to dominate in our models. It appears that perfect complementarity pushes for asymmetric licensing. However, we show that a symmetric licensing structure is preferred when the number of potential licensees is limited (i.e high fixed entry costs) and one of the component is produced in a niche market (i.e consumers are highly sensitive to the distance with their ideal variety). As a result, the profit-maximizing contract in general differs in complementary markets and in a single homogeneous market. In addition, we find that the strategy consisting in transferring all the rent to a monopolized component market does not always dominate. Product differentiation may encourage a symmetric distribution of profits across downstream complementary markets.

The literature on patent licensing has addressed the issue of the desirable number of licenses issued in a single differentiated downstream market. Most of these existing articles use two-part tariffs contracts. There is no clear agreement on the effect of the degree of product differentiation on the number of licenses. Doganoglu and Inceoglu (2014) find that the profit-maximizing number of licenses increases with the valuation for the final good and decreases in their degree of substitutability. In line with Doganoglu and Inceoglu (2014) we find that, when the number of firms likely to acquire the technology is unlimited, the profitable number of licenses to be issued by the innovator is increasing with the valuation for the technology. We obtain the opposite result when the number of potential licensees is capped. Moreover, the profitable number of licenses is in this case found to be increasing with the degree of product differentiation. Arora and Fosfuri (2003) obtain the opposite result in a model with multiple innovators. In their article product differentiation simply lessens the intensity of downstream competition which reduces the desirable number of active producers. In our framework and in Doganoglu and Inceoglu (2014), the number of varieties however does increase consumer satisfaction.

Rey and Salant (2012) do not allow for two-part tariffs and use a spatial differentiation framework. The authors consider fixed cost of entry in the downstream market and discuss the efficiency of the licensing policy. In the case of downstream competition between independent licensees, depending on the number of active downstream firms n, the equilibrium price can be uncovered (i.e with local monopolists when n is low enough), competitive (i.e when n is sufficiently high) or kinked (i.e market segmentation when the value of n is intermediate). The effect of an increase in the number of licensees varies across these intervals (i.e downstream equilibria). The monopoly innovator finds it profitable to remain on the kinked equilibrium interval and capture the entire profit of the industry. When the sensitivity to product differentiation is low, the equilibrium number of licensees is profitably set to a low level in order to avoid profit dissipation and maintain the downstream market in the kinked equilibrium. To the contrary, there is an excessive number of licensees issued by the upstream monopolist when the valuation given to variety is high as entry then allows downstream firms to charge much higher prices (because entry makes their marginal consumers located closer to the firms). Our model under perfect information and fixed fee licensing is close to this model when the cost of entry is equal to zero.

The following two sections present two simple benchmarks of our model. These are useful to understand the mechanisms at work in the spatially differentiated model. Section two describes the homogeneous inelastic model and the third introduces elasticity in the demand for the system good. We consider product differentiation in section four and characterize the corresponding profit-maximizing licensing policy in section five. Section six concludes.

1.2 Homogeneous inelastic demand benchmark

1.2.1 Framework

In this section, we assume that the products are homogeneous. The players are:

- an upstream monopolist U, active in the R&D who offers licenses for its technological information
- an exogenous number of undifferentiated downstream firms in the component market A, M_A
- an exogenous number of undifferentiated downstream firms in the complementary market B, M_B

- consumers of systems made of one unit of each complementary goods

Technological information is required for the downstream firms to be active in their respective markets. We choose to represent the decisive role of technological information through compatibility costs. Consumers require components to be made compatible with the technology developed by the upstream monopolist. This can only be achieved by obtaining a license from the innovator. This information allows downstream firms to reduce the cost of making their components compatible with its technology. For each component to be produced, firms who obtained a license must pay a finite compatibility cost c. Non-licensed firms face an infinite compatibility costs and are consequently inactive. The innovation produced by the upstream monopolist is drastic. The innovator is free to determine the number of licenses (N_A, N_B) it wishes to supply, making (N_A, N_B) firms active in their respective markets.

We assume a simplistic vertical contracting model where the upstream monopolist is able to capture the entire downstream profit through a perfect auction mechanism. The desirability of auctions has been studied in the licensing literature (e.g Kamien, 1992 and Kamien and Tauman, 2002) and can vary across types of innovation and structures of the industry. We will focus here on the determination of the profitable number of license to be issued by the upstream monopolist. Because we study the licensing of a drastic innovation, the outside option of potential licensees does not depend on the type of contract offered by the patentee. Without being granted access to the technology, downstream firms are inactive, cannot participate to the market and get zero profit. In these circumstances, fixed fee licensing is equivalent to a first price sealed bid auction licensing. In this chapter, we will present the model with an auction process. Each component market is associated with a specific technological license and auction. The innovator is able to determine how many firms will be active in each component market as well as a reservation price for each kind of license. Downstream entry decisions are taken as given. Downstream component producers have already paid fixed costs of entry. They must choose whether or not to be active in their respective component market in deciding to bid for a technological information license.

Downstream firms are not allowed to resell the information contained in the license. This can be ensured through intellectual property legislation or technological barriers imposed on the information product sold by the upstream monopolist. Inactive firms make zero profit and active ones must choose the price they charge consumers for a unit of their component. Consumers make purchasing decisions. In the inelastic case, we have a mass of homogeneous consumers who derive a utility v from the use of a system that they assemble from one unit of each component. Consumers maximize their net utility equal to $v - p$ where p is the sum of the lowest prices available for each component. They purchase a unit of system if p is equal to or below their valuation v. For a given industrial structure and a given number of downstream firms, the timing is as follows.

1. The upstream stage in which the innovator chooses the number of licenses (N_A and N_B) to issue in each component market as well as the respective reservation prices.

2. The auctioning game where downstream firms observe the number of licenses to be auctioned and choose how much they want to bid for a license in order to be able to produce demanded goods. Their choice depends on the number of licenses the innovator delivered in each component market. This indeed shapes downstream industrial structure and competition, determines downstream profits and their willingness to pay for an information license.

3. The downstream stage where downstream active firms simultaneously set their component price. Consumers make their purchasing decisions. Profits and utilities are derived.

We consider the case of perfect information where firms know the valuation consumers give to a system. We solve this game using backward induction. We look for Nash equilibria of the downstream stage for different number of firms active in each market. We consider the case where unit costs are equal to compatibility costs (i.e marginal costs of production are constant and normalized to zero). Moreover, the mass of homogeneous consumers is normalized to one. There is no bypass technology. The only way to produce a demanded good is to obtain a license from the technological information monopolist. This captures the fact that for instance, a chip manufacturer cannot profitably operate without being technologically able to supply consumers with an Intel designed chip fully compatible with Intel based Security Software Solutions. The market analyzed here is the market for readily compatible components. We define different subgames corresponding to different number of firms present in each downstream market (M_A and M_B). These are the maximum numbers of licenses and competitors in each market. The benefit procured by the technology depends on the number of license released in each market. We assume that the two downstream markets are clearly separated. Downstream firms have previously borne fixed costs of entry that are specific to their component market. They are unable to switch from one component production to another. We assume that the innovator is able to identify the market in which a downstream firm previously entered. In our model, technology licensing is more easily reversible than market specific investments. The lifetime of the technology is shorter than the one of the assets needed for the production of a given component. License auctions allow the upstream monopolist to credibly commit on a number of license released in each market and on respective reservation prices.

1.2.2 Downstream stage

We first study symmetric subgames where there is the same number of active firms in each component market (i.e $N_A = N_B$). In the double monopoly subgame ($N_A = N_B = 1$), none of the surplus is left to consumers. They purchase a unit of each component if the sum of component prices is lower than their valuation for a system and do not purchase any unit otherwise. It is worth noting that a component producer relies on another firm to produce the other component that is required by consumers and is necessary for the firm to make positive gross profits. Given the demand function and the price of the other component p_j, each monopolist optimally charges $p_i = v - p_j$. This is true if and only if this price allows the component producer to cover the cost c of making a unit compatible with the technology (i.e $p_j < v - c$). Otherwise, the monopolist charges any price p_i greater or equal to c. Best responses being symmetric, we have multiple equilibria in the interval of prices between c and $v - c$. Deviations are unprofitable. In particular, if one firm raises its price, its demand and gross profit fall to zero. We also have equilibria where both firms charge very high prices and do not produce (i.e when prices are both over $v - c$). We focus on the first set of equilibria where positive surplus is created through production and sales of components.

In this model, we consider a symmetric framework where each component is essential to one another. We can thus assume that bargaining power is balanced and that monopolists share social surplus equally. Hence, the balanced equilibrium where each monopolist set a price equal to half of the valuation of the consumers stands out from the other equilibria. It results in equilibrium profits equal to $\frac{v}{2} - c$. This equilibrium is sustained by all system valuations such that the production of the good is socially desirable (i.e $c < \frac{v}{2}$) so that firms are willing to produce at the equilibrium price $\frac{v}{2}$.

As soon as there is more than one producer in one of the component market, homogeneous Bertrand competition leads to marginal cost pricing. Competition within the market is so intense that the price of the complementary good does not affect the determination of the price equilibrium. In effect, for any given price of the other component such that there is positive surplus to share between rivals, they all have an incentive to undercut in order to capture the whole component demand. When there is competition in both downstream markets, consumers retain full social surplus.

We now consider the asymmetric subgame when there is a monopoly in one component market and competition in the other. Knowing that price competition leads to marginal cost pricing the downstream monopolist captures the whole social surplus in setting its price to $v - c$. Profits in the competitive component market are equal to zero. We find that a monopolist of an essential facility (i.e one of the component producers) benefits from intense competition in the complementary segment. This result echoes with the Chicago school critique of the leverage theory and the so called single monopoly profit theory of complementary goods. In our model, we indeed have a facility essential for all uses of the other. A component monopoly is thus able to extract surplus from the competitive complementary market. There is consequently no need to monopolize the complementary market using conglomerate mergers or tying strategies (e.g Whinston, 1990).

1.2.3 Auctioning game

In each component market, players bid for the number of license auctioned by the upstream monopolist. Given the value of the lowest winning bid y_j (i.e we focus on the auction of the most accessible license), players have three strategies available.

1. $y_i > y_j$, if player i follows this strategy, its optimal bid would be $y_i = y_j + \epsilon$, where ϵ is arbitrarily small. This would ensure the player obtains a license and makes associated profits with probability one.
2. y_i=y_j, then player i gets the license with probability one half.
3. $y_i < y_j$, and player i makes zero profit.

We assume that the lowest winning bid is different from the other winning bids which implies that the probability of winning the auction using strategy two is equal to one half. We first consider the case where there are more bidders than licenses in each auction (i.e $M_A > N_A$ and $M_B > N_B$). We derive best response functions in comparing the profitability of each of these strategies given the value of y_j. We find that strategy one is preferred as long as $y_j < \frac{v}{2} - c$, that strategy 2 is optimal if $y_j = \frac{v}{2} - c$ and strategy 3 otherwise. We can note that the valuation given to a license V is equal to the profit it generates in the subsequent competition stage. This results in a unique equilibrium where players bid their valuation for the license. As long as there is positive surplus to be made if winning the bid, players are willing to bid epsilon more to make sure they earn this surplus. This leads to an equilibrium where the upstream firm captures the entire surplus generated by its license. We describe the equilibrium of the auctioning game in each subgame:

- when $N_A = N_B = 1$ and $(M_A, M_B) > 1$, $y_A = V_A = \frac{v}{2} - c$,

- when $N_A = 1$, $N_B = 2$ and $M_A > 1$, $M_B > 2$, $y_A = V_A = y_B = V_B = v - 2c$ and $y_B = V_B = 0$ and symmetrically,
- when $M_A = M_B = 1$, $y_A = V_A = y_B = V_B = 0$, because there is at least as many licenses as bidders in each market, thus the probability of winning the auction is equal to one for any non negative bid,
- when $M_A = 1$, $M_B > 1$ and $N_A = N_B = 1$, $y_A = 0$ and $y_B = V_B = \frac{v}{2} - c$,
- when $M_A = 1$, $M_B = 2$ and $N_A = 1$, $N_B = 2$, $y_A = 0$ and $y_B = 0$,
- when $M_A = 1$, $M_B > 2$ and $N_A = 1$, $N_B = 2$, $y_A = 0$ and $y_B = 0$.

When there are as many licenses as bidders in a given market, the equilibrium bid is equal to zero. On the other hand, when there are more players than licenses, the equilibrium bid is equal to the maximum bid and valuation for the license. Nevertheless, we assume that the upstream monopolist is able to set a public reservation price under which each license cannot be sold. This reservation price is in essence very similar to a fixed fee. Even in the non favorable subgames with a low number of potential buyers, the innovator is able to capture downstream profits through the auction process. We use this result in the remaining of this chapter and only consider the downstream competition stage as well as the choice of the upstream monopolist on the number of licenses to issue given the fact that it is able to capture downstream profits.

1.2.4 Upstream stage

We now turn to the first stage of the game. The upstream monopolist chooses how many licenses to auction in each downstream market given the number of firms that have previously entered (M_A, M_B).

- When $M_A = M_B = 1$, the reservation price R is such that $R = V$. The innovator does not have any choice to make as the only structure of the industry available is the double monopoly. The upstream monopolist extracts downstream profits through the reservation price.

- Otherwise, the monopolist is indifferent between the asymmetric and double monopoly structures as it captures full social surplus in both cases. Upstream profit is either equal to: $2 \times y = 2 \times V = v - 2c$ under the double monopoly, or to: $1 \times y_A = V_A = v - 2c$ under the asymmetric structure with an exclusive license. The double duopoly structure (i.e $N_A = N_B = 2$) is unprofitable as it leads to marginal cost pricing and dissipation of the profit of the industry.

Lemma 1.1. *As long as both structures are available, the upstream monopolist is indifferent between the double monopoly and the asymmetric structure in the homogeneous inelastic model.*

1.3 Homogeneous elastic demand benchmark

1.3.1 Downstream competition

We introduce heterogeneity in the valuation that consumers give to a system. We have a large number of heterogeneous consumers who choose to purchase one system if the sum of the component prices is lower than their valuation. This rational behavior generates a smooth, elastic demand function for systems that is common knowledge for all firms in the industry. We maintain assumptions of pure system valuations (i.e no stand alone value for a component), fixed proportions (i.e the system is made of one unit of each component) and single unit system valuation (i.e the second unit of the system is worthless). We assume that the aggregate system demand function takes a very simple linear form: $d(p) = q = u - p$, where q is the number of systems (and components) purchased, and p is such that $p = \underline{a} + \underline{b}$ (i.e with $\underline{a}$ and $\underline{b}$ respectively denoting the lowest price level of components A and B). Downstream firms in each component market simultaneously maximize their profits. We first consider the double monopoly subgame where $N_A = N_B = 1$ and thus $p = a + b$ (i.e with a and b respectively denoting the price level of components A and B). There is no competition within component markets. This is a surplus sharing game leading to a coordination problem because each producer has to make sure that the other producer is willing to supply its essential component. On the other hand, each producer is eager to take a bigger part of the social surplus. Moreover, because of the demand elasticity, component prices determine the size of the social surplus to be shared. We take the price of the other component as given and derive symmetric best response functions leading to the following Nash equilibrium:

$$BR_A = a = \frac{u-b+c}{2} \tag{1.1a}$$

$$BR_B = b = \frac{u-a+c}{2} \tag{1.1b}$$

$$a = b = \frac{u+c}{3} \tag{1.1c}$$

$$\Pi_A = \Pi_B = \left(\frac{u-2c}{3}\right)^2. \tag{1.1d}$$

When $(N_A, N_B) > 1$, there is Bertrand competition in each component market leading to marginal cost pricing. Downstream profits are equal to zero. We now consider the asymmetric subgame where $N_A = 1$ and $N_B > 1$ (or conversely). Price competition drives profits to zero in the market for component B as the equilibrium price is equal to marginal cost. Given the price, b paid by consumers, the monopoly producer of component A fully determines the price of the system and the associated demand: $p = c + a$, and $d(p) = u - c - a$. Profit maximization leads to:

$$a = \frac{u}{2} \tag{1.2a}$$

$$p = \frac{u}{2} + c \tag{1.2b}$$

$$\Pi_A = \left(\frac{u}{2} - c\right)^2. \tag{1.2c}$$

This is equivalent to the pricing behavior of a monopolist facing the modified demand function: $d(a) = \hat{u} - a$, where $\hat{u} = u - c$ is the residual highest valuation. Profit maximization with respect to a leads to $p_A^m = \frac{\hat{u}+c}{2} = \frac{u}{2}$. Moreover, the system price equilibrium is equal to the integrated monopoly solution. The integrated monopolist faces the cost of making both components compatible and system demand function $d(p)$. The maximization of the integrated profit function with respect to p leads to a price system equal to: $p = \frac{u}{2} + c$. As a consequence, the asymmetric structure enables the maximization of the profit of the industry. Comparing equilibrium system prices under asymmetric and symmetric structures, we find from equations 1.1c and 1.2b that the double monopoly induces a higher system price for all valuations of the system (i.e $u > 2c$). This is a well known result in complementary goods pricing theory owed to Cournot. We observe that horizontal double marginalization leads to excessive inefficient pricing. License valuations V is equal to downstream profits:

- when $(N_A, N_B) > 1$, $V = 0$
- when $N_A = N_B = 1$, $V = (\frac{u-2c}{3})^2$
- when $N_A = 1$ and $N_B > 1$, $V_A = (\frac{u}{2} - c)^2$ and $V_B = 0$

1.3.2 Upstream stage

We previously found that the upstream monopolist was able to capture the entire valuation of downstream firms. We look for the number of license (N_A, N_B) that maximizes the profit of the upstream monopolist. We find that the asymmetric structure is preferred whenever it is available (i.e when $(M_A, M_B) > 1$):

$$\Pi_U^S = 2 \times V = 2 \times \left(\frac{u-2c}{3}\right)^2 < \Pi_A^A = V_A = \left(\frac{u}{2} - c\right)^2, \text{ if: } u > 2c. \tag{1.3}$$

Thus, the asymmetric structure is strictly preferred whenever the good is socially desirable. In effect, Cournot showed that the double monopoly structure leads to excessive prices and suboptimal industry profit. On the other hand, we know that the asymmetric structure maximizes downstream profits because it avoids the double marginalization problem. Given that the upstream monopolist is able to capture the profit of the industry, it aims at maximizing it. Therefore, its licensing policy is efficient with respect to the total profit of the industry. The innovator issues at least two licenses in one of the component markets in order to implement an intense price competition. Homogeneous Bertrand competition leads to marginal cost pricing which avoids the horizontal double marginalization problem pointed out by Cournot. All the rent is transferred to the complementary market and is captured by the use of an exclusive license. That is why the upstream monopolist chooses the asymmetric structure whenever it is available.

Lemma 1.2. *In the homogeneous elastic model, horizontal double marginalization makes the asymmetric structure more profitable than the double monopoly structure for the upstream monopolist.*

1.4 Differentiated component downstream stage

1.4.1 Differentiation framework

We now turn to a new version of the downstream competition stage where we introduce differentiation between products. We will consider models of spatial differentiation following the work of Salop (1979). Our model features a circular city and quadratic costs of transportation. Only one of the components is differentiated. This framework can be justified by the fact that in many system markets, consumers are only sensitive to the variety of some of the components constituting one unit of the system. Some components are indeed invisible or are not in direct interaction with the user of the system. It seems reasonable to assume that consumers do not express preferences over variants of a component they do not actually perceive. As a consequence, for a given level of quality, consumers are indifferent between two different varieties of such a product. These imperceivable components are then homogeneous.

For instance, in a system made of one Central Processing Unit (CPU) and one Operating System (OS), consumers can hardly perceive a difference in the variety of the CPU whereas they directly and frequently interact with the OS. The latter could, for instance be differentiated with respect to the extent to which the OS is user friendly or customizable. The products in each market are in essence, identical. The preferences of consumers are heterogeneous leading them to perceive each product differently. Following Lancaster (1975), we consider a space of characteristics in which each product is defined. We assume that preferences differ in only one dimension of these characteristics. We can then restrict the space of characteristics in which products are located to be one dimensional. Each consumer has an ideal version of the product which defines its position in the characteristic space. From this location, each product looks differently depending on where these products are situated in the characteristic space. Consumers are uniformly distributed along a circle of perimeter one. Products being in essence identical, consumers choose to get their product from the firm offering the lowest full price that includes individual transportation costs.

We focus on the price determination process taking the location of firms as exogenous. For any given number of firms N active in the market, the distance between each of them is equal to $\frac{1}{N}$. Product differentiation (i.e the distance between firms) is maximized. The assumption of quadratic transportation costs ensures that firms are indeed willing to locate as far as possible from any given rival as shown by d'Aspremont et al. (1983) in the linear city model. It implies that assumed locations are optimal for the differentiated producers. Differentiation between any two firms in the market is symmetric.

We need to keep in mind that in our model, consumers only have valuations for a system combining two components produced in two distinct markets. We will study two different structures of the industry varying in the number of firms active in the homogeneous downstream market (N_A):

1. the monopoly subgame with a monopolist in market A and spatial product differentiation in market B
2. the duopoly subgame with homogeneous Bertrand competition in market A and spatial product differentiation in market B.

1.4.2 Homogeneous Monopoly subgame

1.4.2.1 Market B equilibria

We first explore the price setting behavior of component B producers for any given price a charged by the homogeneous monopoly producer of component A. In this section, we present the framework of spatial product differentiation developed by Salop (1979) in the context of our model of complementary goods. We look for the price b charged in equilibrium by component B producers for each given level of price of component A. We focus on a representative segment of the circular city where firms B_1 and B_2 are active. The consumer located at a distance x from firm B_1 purchases B_1's component if two conditions are satisfied:

1. consumer x derives a higher net utility if consuming component B_1 than B_2 (i.e the full price of component B_1 is lower than B_2's)

2. consumer x derives a non negative net utility if consuming component B_1.

We express the two respective values of x (i.e the distance from B_1) such that above conditions are binding. The indifferent consumer $\hat{x}$ (binding condition 1) is located at a distance $\hat{x}$ from B_1:

$$\hat{x} = \frac{1}{2N} + \frac{N}{2t} \times (d - b), \tag{1.4}$$

where t represents the degree of sensitivity of consumers to the distance with their ideal variety, d and b the price of components B respectively charged by B_1 and B_2. The marginal consumer $\bar{x}$ (binding condition 2) of B_1 is characterized by:

$$\bar{x} = \sqrt{\frac{v - a - d}{t}}. \tag{1.5}$$

We define the potential market as the interval of locations in the representative segment where consumers derive a non negative net utility from consuming the system made out of the component B_1. The potential market of firm B_1 is given by the location of the marginal consumer. The price and potential market of the neighbor B_2 is taken as given. The marginal consumer of B_2 is located at a distance $\bar{x}_2$ from B_2 and at $x = \frac{1}{N} - \bar{x}_2$ from B_1. When d is very high, the second constraint is binding for a low value of x and the marginal consumer of B_1 is located closer to B_1 than the marginal consumer of B_2 (i.e $\bar{x}_1 < \frac{1}{N} - \bar{x}_2$). Then, there is no effective competition in the representative segment, the market is uncovered as long as potential markets of the two firms do not meet and demand functions addressed to B_1 and B_2 respectively are:

$$D_{B_1} = \bar{x}_1 = \sqrt{\frac{v-a-d}{t}} \tag{1.6a}$$

$$D_{B_2} = \bar{x}_2 = \sqrt{\frac{v-a-b}{t}}. \tag{1.6b}$$

As soon as potential markets cross (i.e $\bar{x}_1 \geq \frac{1}{N} - \bar{x}_2$), the market is covered and respective demand functions are given by the location of the indifferent consumer:

$$D_{B_1} = \frac{1}{2N} + \frac{N}{2t} \times (b-d) \tag{1.7a}$$

$$D_{B_2} = \frac{1}{2N} + \frac{N}{2t} \times (d-b). \tag{1.7b}$$

Profits made by component B producers in the representative segment are symmetric and equal to:

$$\Pi_{B_1} = \begin{cases} (d-c) \times \sqrt{\frac{v-a-d}{t}} & (1.8) \\ (d-c) \times \left(\frac{1}{2N} + \frac{N}{2t} \times (b-d)\right). & (1.9) \end{cases}$$

We derive equilibria of market B for any given value of the price charged by the homogeneous monopolist A. We first consider the case where a is sufficiently high to make the market uncovered (i.e $\bar{x}_1 < \frac{1}{N} - \bar{x}_2$). Then both firms act like local monopolists facing an elastic demand. They independently and symmetrically maximize their profit given the demand consistency. Uncovered price equilibrium is entirely determined by the strategic interaction between complementary good producers. Best response functions of component B producers are such that:

$$d = b = \frac{2v + c - 2a}{3}. \tag{1.10}$$

Given these best response functions, we derive the condition on a for the market B to be consistently uncovered:

$$a > v - c - \frac{3t}{4N^2}. \tag{1.11}$$

We now consider values of a such that the market B is covered and neighboring firms compete with each others. We look for the corresponding best response functions and subgame equilibrium price of component B. We know that demand functions are determined by the location of the indifferent consumer. Profit functions in the competitive case depend on the prices charged by the two component B producers active in the representative segment. They do not depend on a because the market is only competitive when all consumers enjoy a strictly positive net surplus from consumption of the system. The residual valuation $v - a$ is indeed sufficiently large to make it unnecessary for consumers to take into account the price of the complementary good as their participation in the market is guaranteed (i.e for consistent values of the parameters). Competition within market B is sufficiently intense to make complementarity irrelevant to the determination process of the component B price (when the price of the complementary good a is in the sustaining interval). The best response function of any firm active in the market B takes the following form:

$$d = \frac{b + c + \frac{t}{N^2}}{2}. \tag{1.12}$$

Firms being symmetric, we obtain the following competitive component B price equilibrium candidate:

$$b = d = c + \frac{t}{N^2}. \tag{1.13}$$

The component B market will indeed be competitive if the indifferent consumer (i.e $\hat{x} = \frac{1}{2N}$ in equilibrium) participates (i.e if the location of the marginal consumer $\bar{x}_1$ is such that $\bar{x}_1 > \frac{1}{2N}$). This yields to the following consistency condition on a:

$$a < v - c - \frac{5t}{4N^2}. \tag{1.14}$$

We now turn to the third kind of equilibrium price candidate in which the market B would be covered but competition between neighbors would actually not take place. This is the case when the indifferent and the marginal consumer lie in the exact same place. This consumer is indifferent between buying component B_1, buying component B_2 and consuming the outside good. We refer to this situation of component market B as being kinked, that is when potential markets of both firms just meet: $\bar{x}_1 = \frac{1}{N} - \bar{x}_2$ implying under symmetric prices that $\bar{x}_1 = \hat{x} = \frac{1}{2N}$. This is the case if:

$$b = d = v - a - \frac{t}{4N^2}. \tag{1.15}$$

Given a, we verify that this kinked price of market B is the best response of B component producers and a price equilibrium candidate. Assuming the rival charges the price b given by equation 1.15, we explore the conditions in which B_1 finds it optimal to charge the same price and when it is willing to deviate. If B_1 chooses to set a higher price, the market would become uncovered and its profit-maximizing price would be given by equation 1.10. Feasible profitable deviations making the market uncovered are only found to be existing if:

$$v - c - \frac{3t}{4N^2} < a \leq v - c[1]. \tag{1.16}$$

As a consequence, the kinked price of component B is profitably charge as long as this condition is not satisfied (i.e when the price of component A is sufficiently low). If B_1 deviates from the kinked equilibrium candidate with a lower price, the market becomes covered and demand addressed to B_1 is given by the location of the indifferent consumer.

[1]More details are available in the appendix.

We find that such a deviation is profitable and feasible only if:

$$a < v - c - \frac{5t}{4N^2}. \tag{1.17}$$

Combining the two conditions for unprofitable deviations, we characterize the following kinked equilibrium candidate of the market B:

$$b = d = v - a - \frac{t}{4N^2} \tag{1.18a}$$
$$\text{if: } v - c - \frac{5t}{4N^2} < a < v - c - \frac{3t}{4N^2}. \tag{1.18b}$$

Lemma 1.3. *The market B price equilibrium candidate and associated conditions on the price of component A are:*

$$d = b = \frac{2v + c - 2a}{3}, \textit{ if: } a > v - c - \frac{3t}{4N^2} \tag{1.19a}$$
$$b = d = v - a - \frac{t}{4N^2}, \textit{ if: } v - c - \frac{5t}{4N^2} < a < v - c - \frac{3t}{4N^2} \tag{1.19b}$$
$$b = d = c + \frac{t}{N^2}, \textit{ if: } a < v - c - \frac{5t}{4N^2}. \tag{1.19c}$$

Proof. Details of the proofs are available in the appendix and in Salop (1979). □

1.4.2.2 Best response function of the homogeneous monopoly A

We characterized market B equilibrium candidates for any given value of a. We now turn to the analysis of the optimal behavior of the homogeneous monopoly producer of component A. We characterize its best response function for any given symmetric price equilibrium b. We consider the case where a is sufficiently high to make the market uncovered and B producers act like independent local monopolists (i.e $\sqrt{\frac{v-a-b}{t}} < \frac{1}{2N}$). The demand addressed to the monopolist A is given by the sum of all consumers served by local monopolists in market B. Therefore, its profit function takes the following form:

$$\Pi_A^U = (a - c) \times 2N \times \sqrt{\frac{v - a - b}{t}}. \tag{1.20}$$

Maximizing it with respect to a leads to the following best response function:

$$a = \frac{2v + c - 2b}{3} \tag{1.21a}$$

$$\text{if: } b > v - c - \frac{3t}{4N^2}. \tag{1.21b}$$

In effect, when the price charged by the complementary good producer is too high, the residual valuation for the other component is too low to make it profitable for its producer to cover the market. Otherwise, A would prefer to cover the market and charge the highest price consistent with the covered market condition:

$$a = v - b - \frac{t}{4N^2} \tag{1.22a}$$

$$\text{if: } b \leq v - c - \frac{3t}{4N^2}. \tag{1.22b}$$

If $b = v - c - \frac{3t}{4N^2}$, the uncovered and covered best response expressions give the same value for the price of the homogeneous component:

$$a = c + \frac{t}{2N^2}. \tag{1.23}$$

1.4.2.3 Downstream equilibrium characterization

We first characterize the uncovered market equilibrium. This equilibrium must satisfy the three following conditions:

1. $b_i = BR_{B_i}(a)$
2. $a = BR_A(b)$
3. market B is uncovered.

Combining uncovered best response functions derived in the two previous sections, we obtain the uncovered Nash equilibrium:

$$a = b = \frac{2v + c}{5}, \tag{1.24a}$$

$$\text{if: } v < 2c + \frac{5t}{4N^2}. \tag{1.24b}$$

This is the condition for the market to be consistently uncovered in equilibrium (i.e such that the location of the marginal consumer is $\bar{x} < \frac{1}{2N}$). This uncovered market structure features N_B independent local system markets in which two monopolists produce complementary goods (A and B_i) facing an elastic demand. Horizontal double marginalization leads to an equilibrium where prices are excessive. The profit of the industry fails to be maximized. We turn to the characterization of the competitive covered equilibrium which must satisfy the following conditions:

1. $b_i = BR_{B_i}\left(b_j\right)$
2. $a = BR_A(b)$
3. market B is covered (i.e $\bar{x} > \frac{1}{2N}$).

Combining the competitive best response functions, we obtain the following competitive covered equilibrium characterization:

$$a = v - c - \frac{5t}{4N^2} \tag{1.25a}$$

$$b = c + \frac{t}{N^2} \tag{1.25b}$$

$$\text{if: } v > 2c + \frac{7t}{4N^2}. \tag{1.25c}$$

Finally, we characterize the kinked equilibrium. It must satisfy the following conditions:

1. $b_i = BR_{B_i}\left(b_j, p_A\right)$
2. $a = BR_A(b)$

3. market B is just covered (i.e $\bar{x} = \frac{1}{2N}$).

Using the respective expressions of best response functions and conditions, we find that there exists a kinked equilibrium if and only if: $v > 2c + \frac{5t}{4N^2}$. We determine an interval on the equilibrium prices (a, b) which are necessarily such that:

$$(a, b) \in \left[c + \frac{t}{2N^2}, v - c - \frac{3t}{4N^2}\right], \tag{1.26a}$$

$$\text{if: } 2c + \frac{5t}{4N^2} < v < 2c + \frac{7t}{4N^2}; \tag{1.26b}$$

$$\text{and } a \in \left[v - c - \frac{5t}{4N^2}, v - c - \frac{3t}{4N^2}\right], \tag{1.26c}$$

$$b \in \left[c + \frac{t}{2N^2}, c + \frac{t}{N^2}\right] \tag{1.26d}$$

$$\text{if: } v > 2c + \frac{7t}{4N^2}. \tag{1.26e}$$

When v is large (i.e when equation 1.26e is satisfied), the constraint on a ensuring that deviations towards the competitive demand (i.e charging a lower price than the kinked equilibrium candidate charged by its neighbor) are unprofitable for B producers tightens as the valuation for the system increases. The minimum level of a required for a kinked equilibrium to exist is expressed in equation 1.26c and indeed depends positively on v. On the other hand, component prices are upper bounded because otherwise, there would be some feasible profitable deviations (making the market uncovered) consisting in charging a higher price than the kinked equilibrium candidate. The upper bound on price a is also positively related to v making the interval expressed in equation 1.26c translate upward with the level of system valuation. When a takes the highest value in this interval consistent with a kinked equilibrium, B charges the minimum consistent value of b (i.e lower bound of the interval in equation 1.26d).

We now describe three particular equilibria selected among all possible kinked equilibria. We consider the two extremely unbalanced equilibria as well as the balanced equilibrium where $a = b$. The following equations characterize the kinked equilibrium that is the least (most) favorable to A (B) where the price of the homogeneous component is at its lowest sustaining level:

$$a = v - c - \frac{5t}{4N^2} \quad (1.27a)$$

$$b = c + \frac{t}{N^2} \quad (1.27b)$$

$$\text{if: } v > 2c + \frac{7t}{4N^2}. \quad (1.27c)$$

The kinked equilibrium that is the most favorable (unfavorable) to A (B) is conversely characterized:

$$a = v - c - \frac{3t}{4N^2} \quad (1.28a)$$

$$b = c + \frac{t}{2N^2} \quad (1.28b)$$

$$\text{if: } v \geq 2c + \frac{5t}{4N^2}. \quad (1.28c)$$

When A captures a high share of the surplus (i.e as in equation 1.28a), the price b is relatively low and then the constraint on v such that it is optimal for A to cover the market relaxes. To the contrary, when A captures a low share of the surplus (i.e as in equation 1.27a), b is relatively high and equal to the competitive equilibrium of market B. Thus, the constraint on v ensuring that A does find it optimal to serve the entire market tightens and the minimum level of v is higher in equation 1.28c than in equation eq: chaponeonediffmostfavorabletoBkinkedequilibriumthresholdonv. As a consequence, the kinked equilibrium that is the most unfavorable to A is the most difficult to sustain (i.e more restrictive sustaining values of v). For instance, when the valuation for the system is equal to its lowest sustaining value, there is a unique price equilibrium such that the price of component B is equal to the level of its lower bound (i.e the most favorable to A and balanced equilibrium):

$$a = b = c + \frac{t}{2N^2} \tag{1.29a}$$

$$\text{when: } v = 2c + \frac{5t}{4N^2}. \tag{1.29b}$$

In this case, both kinked and uncovered best response sub-functions lead to the same price level and both components are equally priced. The remaining kinked equilibria are not sustained by the lower bound value of v. We now determine existence conditions of the balanced kinked equilibrium where $a = b$:

$$b = a = \left(v - \frac{t}{4N^2}\right) \times \frac{1}{2} \tag{1.30a}$$

$$\text{if: } 2c + \frac{5t}{4N^2} < v < 2c + \frac{9t}{4N^2}. \tag{1.30b}$$

We obtain an interval on v, for this particular kinked equilibrium, because a is required to be sufficiently high compared to v in order to ensure that deviations of B towards a lower price are unprofitable. The balanced surplus sharing condition implies that for a sufficiently high value of v, a would cease to satisfy this condition. Thus, v should not be too large to sustain the balanced kinked equilibrium. On the other hand, for sufficiently large (low) system valuations (sensitivity to the distance t), the equilibrium the most favorable to B producers making the price of component B equal to the competitive price equilibrium obtained by Salop (1979) also exists (see equation 1.27c).

If a particular kinked equilibrium had to be selected, we could argue that in an asymmetric structure with a differentiated oligopoly, the homogeneous monopoly could have a stronger bargaining power which would lead to the selection of the kinked equilibrium the most favorable to A. On the other hand, in the double monopoly structure, it is reasonable to think that the balanced equilibrium would be selected as long as it is supported by the value of v. We summarize our findings on the characterization of downstream equilibria when there is a monopoly on the homogeneous component market A in the following lemma.

Lemma 1.4. *We characterize the following Nash equilibria of the downstream competition stage of the subgame in which there is an exclusive producer of component A:*

- *the uncovered equilibrium if:* $v < 2c + \frac{5t}{4N^2}$
- *kinked equilibria if:* $v > 2c + \frac{5t}{4N^2}$
- *the competitive equilibrium if:* $v > 2c + \frac{7t}{4N^2}$.

It is worth noting that for a sufficiently large v, the competitive equilibrium coexists with kinked equilibria and might not be selected.

There is a multiplicity of equilibria as soon as v is sufficiently large (i.e such that condition 1.28c is satisfied). This differs from the outcome of a standard circular city model. Considering the equilibrium that is the most favorable to A, we observe that the monopolist A is able to maintain market B in the kinked equilibrium by adjusting its price a, no matter the value of v or the distance between firms $\frac{1}{N}$. This effect does not appear in the standard model in which the market necessarily becomes competitive as v increases. It is not in the interest of monopolist A to charge a lower price so that B producers would be allowed to charge the competitive price. There is a multiplicity of equilibria because there are various ways for A and B producers to share the extra surplus generated by an increase in v. We find the same strategic behavior and the same multiplicity of equilibria than in our model with inelastic demand (i.e homogeneous consumers). Given that the market needs to be just covered (i.e $\bar{x} = \frac{1}{2N}$), the net valuation of the good (i.e the system valuation taking into account the full participation constraint) is then equal to: $v - \frac{t}{4N^2}$. Each firm aims at capturing the residual net valuation: $p_i = v - \frac{t}{4N^2} - p_j$.

1.4.3 Homogeneous Duopoly subgame

We now study an industry structure where there are two homogeneous producers of component A. Homogeneous Bertrand competition leads to marginal cost pricing on market A. The market B remains differentiated and the number of active firms is equal to N_B. In section 1.4.2, we characterized the equilibrium of the market B for any given price a charged by the monopoly producer of the homogeneous component. The homogeneous duopoly structure corresponds to the case where a is equal to marginal cost. Replacing a by c in the characterization of market B equilibria (i.e lemma 1.3) results in the following conditions for the various equilibria of the duopoly structure.

Lemma 1.5. *When there are two producers of the homogeneous component (i.e A), the pricing stage results in one of the following price equilibria depending on the number of firms, N active in the differentiated market (i.e B):*

$$a = c \tag{1.31a}$$

$$d = b = \frac{2v - c}{3} \tag{1.31b}$$

$$v < 2c + \frac{3t}{4N^2} \tag{1.31c}$$

$$a = c \tag{1.32a}$$

$$d = b = v - c - \frac{t}{4N^2} \tag{1.32b}$$

$$2c + \frac{3t}{4N^2} < v < 2c + \frac{5t}{4N^2} \tag{1.32c}$$

$$a = c \tag{1.33a}$$

$$d = b = c + \frac{t}{N^2} \tag{1.33b}$$

$$v > 2c + \frac{5t}{4N^2}. \tag{1.33c}$$

When the market is uncovered (i.e $\bar{x}_1 < \frac{1}{N} - \bar{x}_2$), both firms act like local monopolists facing an elastic demand. They independently and symmetrically maximize their profit given the demand consistency condition which results in the price expressed in equation 1.31b. This is an equilibrium of the market B when the market is indeed uncovered (i.e when the condition 1.31c on system valuation is satisfied). Assuming that the market B is competitive, the corresponding profit and best response sub-functions lead to the following price level: $b = c + \frac{t}{N^2}$. This is indeed a competitive covered equilibrium of the market B if the indifferent consumer ($\hat{x} = \frac{1}{2N}$, in a symmetric equilibrium) participates (i.e if the location of the marginal consumer $\bar{x}_i$ is such that $\bar{x}_i \geq \frac{1}{2N}$). The kinked equilibrium in which market B is covered but competition between neighbors does not actually take place arises when potential markets of both firms just touch (i.e $\bar{x}_1 = \frac{1}{N} - \bar{x}_2 = \frac{1}{2N}$). Replacing a by c, in the characterization of the equilibrium price of market B, we obtain the following expression under the condition 1.32c: $b = v - c - \frac{t}{4N^2}$. We see that there is no multiplicity of equilibria in the homogeneous duopoly structure just as in the model of Salop (1979). Moreover, we do not have any horizontal double marginalization and the profit of the upstream monopolist differs across covered equilibria (i.e between the kinked and competitive covered equilibria).

1.5 Optimal licensing policy

1.5.1 Unlimited number of component producers

We now turn to the profit-maximizing problem of the upstream firm (i.e the upstream stage). Let us remind the expression of the profit of the upstream monopolist where N stands for N_B and superscript C for covered, K for kinked and U for uncovered equilibria. When $N_A = 1$, we have:

$$\Pi_U = \Pi_A + N_B \times \Pi_B \tag{1.34a}$$

$$\Pi_U^C = (a^C - c) + N_B \times (b^C - c) \tag{1.34b}$$

$$\Pi_U^K = (a^K - c) + N_B \times (b^K - c) \tag{1.34c}$$

$$\Pi_U^U = (a^U - c) \times 2\bar{x} \times N_B + N_B \times (b^U - c) \times 2\bar{x}. \tag{1.34d}$$

In particular, the profit of the upstream monopolist takes the following form for each type of equilibrium:

$$\Pi_U^U = \frac{8N}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}} \tag{1.35a}$$

$$\Pi_U^C = v - 2c - \frac{t}{4N^2} \tag{1.35b}$$

$$\Pi_U^K = v - 2c - \frac{t}{4N^2}, \tag{1.35c}$$

$$\tag{1.35d}$$

When $N_A = 2$, homogeneous Bertrand competition drives profit to zero and the entire profit of the innovator is made in the differentiated market B:

$$\Pi_U = N_B \times \Pi_B \tag{1.36a}$$

$$\Pi_U^C = N_B \times (p_B^C - c) \tag{1.36b}$$

$$\Pi_U^K = N_B \times (p_B^K - c) \tag{1.36c}$$

$$\Pi_U^U = N_B \times (p_B^U - c) \times 2\bar{x}. \tag{1.36d}$$

In particular, the profit of the upstream monopolist takes the following form for each sort of equilibrium:

$$\Pi_U^C = \frac{t}{N^2} \tag{1.37a}$$

$$\Pi_U^U = \frac{4N}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}} \tag{1.37b}$$

$$\Pi_U^K = v - 2c - \frac{t}{4N^2} \tag{1.37c}$$

Everything else being equal, social surplus and profit of the innovator are increasing with the number of active firms in the differentiated markets. That is because entry in the differentiated market generates a decrease in transportation costs which makes it less costly for downstream firms to cover the market. This unambiguously increases social surplus as entry costs are assumed to be sunk.

Most importantly, we observe in equations 1.35b and 1.35c that when the homogeneous component is monopolized, the upstream monopolist is indifferent between all covered equilibria including the competitive equilibrium. This is the case because there is no unnecessary surplus left to consumers and the aggregate level of sales remains constant.

It is worth noting that there is no horizontal double marginalization at work in the case of homogeneous duopoly. When the market is uncovered, homogeneous Bertrand competition has a positive effect on the profit of the upstream monopolist. Comparing uncovered equilibrium profits, we indeed observe in equations 1.35a and 1.37b that it is higher with a homogeneous duopoly than with a monopoly because,

$$\frac{4N}{3\sqrt{3t}} > \frac{8N}{5\sqrt{5t}}, \tag{1.38}$$

which evaluates the negative impact of the horizontal double marginalization on the uncovered profit of the innovator.

Considering the case where the number of active firms is unlimited, the upstream monopolist will be able to capture the entire social surplus. It will transfer all the rent to the homogeneous monopoly market and capture it through an exclusive license. This is feasible by issuing an infinite number of licenses in the differentiated product market making it perfectly competitive and efficient (as entry costs are sunk). The downstream market is characterized by the following covered monopoly equilibrium:

$$b = c \tag{1.39a}$$

$$a = v - c \tag{1.39b}$$

$$\Pi_U^* = v - 2c. \tag{1.39c}$$

Proposition 1.5.1. *The optimal licensing policy of an upstream monopoly innovator whose technology is used in complementary markets and when the number of potential licensees is unlimited consists in implementing perfect competition in the differentiated component market by freely delivering its technology to an infinite number of firms and* *to issue an exclusive license for the production of the complementary good in exchange for the monopoly profit (i.e full social surplus $v - 2c$).*

This is the optimal licensing policy since it enables the upstream monopolist to maximize and to capture the entire social surplus. Therefore, the profit-maximizing strategy is in this case efficient with respect to the total profit of the industry and total social surplus. Let us assume a sufficiently high system valuation making all feasible structures covered in equilibrium (i.e $v > 2c + \frac{5t}{4}$) including the double monopoly structure (i.e $N_A = N_B = 1$). Then, the demand is inelastic as the market is covered in any case. We recall that inefficient horizontal double marginalization explains the preference of the upstream monopolist for the asymmetric structure (i.e over the double monopoly) in the homogeneous model with elastic demand. Even in the absence of demand elasticity, the introduction of product differentiation breaks the indifference found in our homogeneous inelastic model (between the asymmetric and the double monopoly structure) in favor of the asymmetric structure. This is due to the fact that an increase in the number of active firms in the differentiated markets decreases (increases) transportation costs (social surplus). In equilibrium, the marginal consumer $\bar{x}$ that is the most likely to abstain from consumption is the indifferent consumer $\hat{x}$ located at a distance $\frac{1}{2N}$ from closest producers. This is the case because $\hat{x}$ needs to travel the longest distance in the representative segment. The net valuation (i.e $v - \frac{t}{4N^2}$ the valuation of the indifferent consumer) of the good increases with the number of active firms, which makes it less costly for A (and the innovator) to serve the entire market. Monopolist A takes the participation constraint of the indifferent consumer into account. As the number of firms increases, market segments become shorter, distance between each active firm decreases and so does the distance and transportation costs borne by the indifferent consumer. The net valuation to be extracted from consumers is increasing with the number of component varieties. Following the increase in the number of active firms in the differentiated market, social surplus increases, and the innovator captures it through the homogeneous monopoly producer of component A. As a consequence, the upstream monopolist favors the asymmetric structure in the case of zero demand elasticity (i.e covered market assumptions for all industry structures) because of the efficiency effect of the increase in varieties. In the case of unlimited potential downstream firms, the efficiency effect of the increase in licensees implies that there must be a number of component B producers such that the differentiated market becomes covered.

We do not discuss here the general efficiency of the increase in varieties because we consider that all potentially active firms have already entered one of the component market (i.e fixed costs of entry are already paid and sunk). It is worth noting that no matter which covered equilibrium is selected, the innovator continues to favor the asymmetric structure when the number of active firms is unlimited. For any given number of firms active in the market B, the upstream monopolist is indifferent between the distribution of the social surplus between component A and B producers as long as the market is covered. It is indeed able to capture downstream profits in both markets anyway. However, as we stated earlier, social surplus is increasing with the number of B producers as the transportation cost of the indifferent consumer decreases. As a consequence, the innovator will optimally deliver an infinite number of licenses in the differentiated market and an exclusive license in the homogeneous market. This is in contradiction with the result found by Rey and Salant (2012). In our model, there is no trade-off between the positive effect of increased number of varieties and the negative effect of an intensified downstream competition. The surplus generated by competition is indeed captured by the exclusive complementary good producer and in turn by the innovator.

1.5.2 Limited number of component producers

We now consider the situation where the number of active firms is limited to a low value (i.e two firms in each market). This constraint on the number of active firms is determined by the number of firms that have previously entered downstream markets. The upstream monopolist can only issue as many licenses as there are firms able to produce a given component. The fact that the number of active firms is upper bounded seems to be particularly realistic in high technology industries. In effect, the industries where complementarity and technology licensing play a big role are typically characterized by high fixed costs and barriers to entry. For instance, a large amount of financial and technological resources is required in order to enter the computing industry. An entrant also needs to construct an ecosystem made of a network of consumers and cooperative firms selling complementary goods. These characteristics tend to constrain the number of firms present in each component market. As a consequence, there could be a limited number of firms potentially interested in the license of the upstream monopolist. We are looking for the constrained profit-maximizing licensing policy for the upstream monopolist depending on the value of v. Considering the case where the maximal number of active firms is equal to two in each market, we first describe the mechanisms at work in each of the structures available to the upstream monopolist. The innovator can choose between: the double monopoly, the differentiated monopoly (i.e homogeneous duopoly with a differentiated monopolist), the asymmetric structure (i.e homogeneous monopoly with differentiated duopoly) and the double duopoly structure.

We first describe how the model behaves in the cases where there is only one firm active in the differentiated component market (i.e $N_B = 1$). In general, we observe that when the valuation for the system is sufficiently high, a competitive covered equilibrium exists. This is not true in the differentiated monopoly case. It is indeed impossible to get competition in a differentiated market with only one active firm. We however find the same uncovered and kinked equilibria than in the unconstrained model.

Lemma 1.6. *We derive the following equilibria of the game in the double monopoly case ($N_A = N_B = 1$). The uncovered Nash equilibrium price:*

$$a = b = \frac{2v + c}{5} \tag{1.40a}$$

$$\text{if: } v < 2c + \frac{5t}{4}. \tag{1.40b}$$

And the kinked equilibrium:

$$a = v - b - \frac{t}{4} \tag{1.41a}$$

$$b = v - a - \frac{t}{4} \tag{1.41b}$$

$$\text{if: } v > 2c + \frac{5t}{4}. \tag{1.41c}$$

Kinked equilibrium prices, a and b are necessarily included in the following interval:

$$(a, b) \in \left[c + \frac{t}{2}, v - c - \frac{3t}{4}\right]. \tag{1.42}$$

Since the competitive equilibrium does not exist in the differentiated monopolist case, there is no profitable deviation towards the competitive demand implying no lower bound on the price a consistent with the market to be just covered (i.e kinked component B market). We find that the double monopoly structure features horizontal double marginalization when the demand is elastic (i.e in the uncovered case). This is also the case in the basic elastic model. When the market is covered, we have multiple equilibria varying in the way the social surplus is shared between the two monopolists. Moreover, as there is no competitor in market B, the competitive equilibrium does not exist.

In the differentiated monopoly structure ($N_A = 2, N_B = 1$), we know that marginal cost pricing of component A implies that there is no multiplicity of equilibria. There is no coordination problem between A and B producers because the behavior of firms active in the market A is entirely determined by the competition within their market. We indeed consistently observe in our framework that complementarity only matters when competition within a market is not too intense. On the other hand, we have a monopoly on the differentiated market implying that the competitive equilibrium does not exist because there is no rival to compete with. As a consequence, as we showed in the double monopoly structure, there is no constraint on the minimum level of a in order to maintain a kinked equilibrium.

Lemma 1.7. *The only covered equilibrium of the differentiated monopoly structure is such that:*

$$a = c \tag{1.43a}$$

$$b = v - c - \frac{t}{4} \tag{1.43b}$$

$$v > 2c + \frac{3t}{4} \tag{1.43c}$$

$$b = \frac{2v - c}{3}. \tag{1.43d}$$

Given that the upstream monopolist is able to capture downstream profits through auctions and reservation prices, we derive the following expression of the profit of the patentee across various licensing policies. We now describe the profit function of the upstream monopolist in each different case of each available structure.

1. In the double monopoly structure (i.e $N_A = N_B = 1$):
   (a) Multiplicity of kinked equilibria and no competitive equilibrium.
   (b) Profit function of the innovator the innovator in covered cases:
   $$\Pi_U^K = v - 2c - \frac{t}{4}. \tag{1.44}$$
   (c) Both firms charge the covered valuation minus the price of the complementary good.
   (d) Profit function of the innovator in the uncovered case:
   $$\Pi_U^U = \frac{8}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}}. \tag{1.45}$$
   (e) Horizontal double marginalization in the uncovered case.
   (f) Uncovered market condition: $v < 2c + \frac{5t}{4}$.
2. In the differentiated monopoly structure (i.e $N_A = 2, N_B = 1$):
   (a) No multiplicity of kinked equilibria and no competitive equilibrium.
   (b) Profit function of the innovator in the covered case:
   $$\Pi_U^K = v - 2c - \frac{t}{4}. \tag{1.46}$$
   (c) Profit function of the innovator in the uncovered case:
   $$\Pi_U^U = \frac{4}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}}. \tag{1.47}$$
   (d) No double marginalization.

(e) Uncovered market condition: $v < 2c + \frac{3t}{4}$.

3. In the asymmetric structure (i.e $N_A = 1, N_B = 2$):

   (a) Multiplicity of kinked equilibria and existence of the competitive equilibrium. The innovator is indifferent between various social surplus sharing. A does not leave extra surplus to consumers. Lower transportation costs and higher social surplus.

   (b) Profit function of the innovator in covered cases:

$$\Pi_U^C = \Pi_U^K = v - 2c - \frac{t}{16}. \tag{1.48}$$

   (c) Profit function of the innovator in the uncovered case:

$$\Pi_U^U = \frac{16}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}}. \tag{1.49}$$

   (d) Horizontal double marginalization in the uncovered case.

   (e) Uncovered market condition: $v < 2c + \frac{5t}{16}$.

4. In the double duopoly structure (i.e $N_A = N_B = 2$):

   (a) No multiplicity of kinked equilibria and existence of the competitive equilibrium.

   (b) Profit of the innovator in the competitive covered case:

$$\Pi_U^C = \frac{t}{4}. \tag{1.50}$$

   (c) Competitive covered equilibrium condition: $v > 2c + \frac{5t}{16}$.

   (d) Profit of the innovator in the uncovered case:

$$\Pi_U^U = \frac{8}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}}. \tag{1.51}$$

   (e) Horizontal no double marginalization in the uncovered case.

(f) Uncovered market condition: $v < 2c + \frac{3t}{16}$.

(g) Profit of the innovator in the kinked case:

$$\Pi_U^K = v - 2c - \frac{t}{16}. \tag{1.52}$$

(h) Kinked equilibrium condition: $2c + \frac{3t}{16} < v < 2c + \frac{5t}{16}$.

Proposition 1.5.2. *When the number of potential downstream firms in each component market is limited to $N_B = 2$, the monopoly innovator, holder of a drastic innovation finds it profitable to deliver as many licenses as possible on the differentiated component market (i.e $N_B = 2$).*

On the other hand it profitably releases an exclusive license for the production of the homogeneous component (i.e $N_A = 1$ and $N_B = 2$) when the valuation for the system is high (i.e when it is a mass market):

$$v > 2c + \frac{5t}{16}. \tag{1.53}$$

Otherwise, when the valuation for the system is sufficiently low (i.e when it is a niche market), the patentee prefers to implement perfect competition on the homogeneous component market and to extract surplus through the differentiated local monopolists (i.e $N_A = 2$ and $N_B = 2$).

Proof. Details of the proof are available in the appendix. □

In our model of spatial product differentiation, it is always profitable for the patentee to deliver two licenses on the differentiated market. The asymmetric structure always dominates both the double monopoly and the differentiated monopoly. Varieties increase (decrease) consumer satisfaction (transportation costs) and social surplus which is captured by the monopoly innovator. The patentee must then choose between the asymmetric structure and the double duopoly structure. The asymmetric structure is the most favorable for the upstream monopolist as soon as the valuation for the system is sufficiently high (i.e when $v > 2c + \frac{5t}{16}$). For these values of the parameters (i.e in covered equilibria), the efficiency of the asymmetric structure does not suffer from the horizontal double marginalization and the asymmetric structure allows the innovator to avoid the dissipation of profits generated by the double duopoly structure.

When the valuation for the system is sufficiently low (i.e when $v < 2c + \frac{5t}{16}$), the double duopoly structure is preferred. We found that when consumers are highly sensitive to the distance with their ideal variety (i.e $t > (v - 2c) \times \frac{16}{5}$), which characterizes a niche component market, the profit-maximizing licensing policy is symmetric (i.e double duopoly policy). For these values of the parameters, the double duopoly structure does not generate intense competition and avoids the horizontal double marginalization of the asymmetric structure. Despite the symmetry of the profit-maximizing licensing policy, profits are only generated in one of the component markets. All the rent is indeed transferred to one downstream market and extracted by two local monopolists. Given the constraint on the number of active firms, the licensing strategy is in this case optimal with respect to the profit of the innovator, the industry and social surplus.

The asymmetry in the degree of competition is indeed to be distinguished from the asymmetry in the number of active firms. When the number of potential licensees is unlimited, the asymmetric licensing policy is preferred in general. Nevertheless, the degree of competition is quite symmetric. There is indeed no effective competition in the differentiated market. In the asymmetric structure, there is multiple kinked equilibria varying in the way the surplus is shared between the two component markets. This shows that despite the asymmetric licensing policy, profits are not necessarily asymmetric across complementary markets. On the other hand, when there is a limited number of potential licensees and a niche component market, the symmetric licensing policy is shown to be the most profitable strategy. However, the degree of competition is very asymmetric and all the rent is transferred to the differentiated market where local monopolists are active. In the context of limited number of licenses, competition in the homogeneous market is more effective to eliminate the rent. It is indeed enough to issue two licenses in the homogeneous market to transfer all the rent to the other component market. The maximal number of licenses may also not suffice to cover the niche component market which can explain the preference for the double duopoly structure.

The strategy consisting in transferring all the rent to one market where an exclusive license is sold dominates in the unlimited differentiated model as well as in the limited niche market case. We showed that the asymmetric licensing policy is in fact symmetric in terms of profits generated across component markets when the number of licenses is limited. The generality of the preference of the innovator for such a policy is questionable. The fact that there is no inefficient double marginalization (i.e in all covered equilibria) is only due to the fixed size of the market in the circular model. The optimality of the asymmetric licensing policy in the case of a limited number of active firms may be very model dependent. Nevertheless, we can argue that for a sufficiently low degree of distance sensitivity t, inefficient horizontal double marginalization at work in the asymmetric structure is preferable to the intense competition implemented in the double duopoly structure. The asymmetric structure allows the innovator to avoid the competitive equilibrium of the double duopoly structure where some unnecessary surplus is left to consumers. To summarize, we show that the asymmetry in the market structure does not imply asymmetry in the degree of competition and in the subsequent surplus sharing. Moreover, we find that a symmetric licensing policy is optimal for the innovator in the case of a differentiated niche component market. Finally, the rent transferring and capturing strategy is not always privately desirable in our differentiated model.

1.5.3 Illustrations

We illustrate our results with some calibrations and examples. The sensitivity of consumers to the distance with their ideal variety of the differentiated component is set to be equal to one. Moreover, the cost for a licensee of making components compatible is normalized to zero. We will run some calibrations statics taking different values of the valuation for the system. The number of active firms in the differentiated markets remains limited to two.

We use the relationship between the number of active firms in the homogeneous market N_A and the price of the component A to illustrate the dominance of the double duopoly over the asymmetric structure in niche markets. The price of component A

indeed decreases when the number of licensees in this market increases. The double duopoly structure (i.e $N_A = 2$) implies perfect Bertrand competition and marginal cost pricing. Considering the price of component A as the strategic variable of the innovator, we associate the choice of the double duopoly structure with a price of component A equal to marginal cost (i.e $a = c = 0$).

We then compute the resulting profit of the innovator which determines the intercept point of the profit function in figures 1.1 and 1.2. In order to set a strictly positive price a, the innovator must choose the asymmetric structure (i.e $N_A = 1$). For a given level of system valuation, the price of component A determines which available equilibrium of the asymmetric structure is selected and what level of profit the innovator is able to reach. When the valuation is high, all three kind of equilibrium are available (i.e the competitive, kinked and uncovered equilibria) but as v decreases, the competitive equilibrium and then the kinked equilibria are no more sustained by positive values of the price of component A. As the price of component A increases, the asymmetric structure moves towards the uncovered equilibrium and eventually reaches zero level of sales and profits.[2]

We first consider a system valuation equal to one quarter for which the asymmetric structure (i.e $N_A = 1$ and $N_B = 2$) is at the uncovered equilibrium (i.e $a = b = \frac{1}{10}$) resulting in the following profit of the innovator: $\Pi_U = 0,178$. Figure 1.1 depicts the profit of the innovator as a function of a. As we can see, the equilibrium profit of the upstream monopolist is higher under the double duopoly structure (i.e when $N_A = 2$ and $a = c = 0$) than under the asymmetric structure (i.e when $N_A = 1$ and $a = \frac{1}{10}$). This illustrates the fact that the upstream monopolist favors the double duopoly when the valuation of the good is sufficiently low (i.e when $v < \frac{5}{16}$). To the contrary when v is sufficiently high (e.g $v = \frac{6}{16}$), the asymmetric structure is at a kinked equilibrium such that:

$$(a, p_B) \in \left[\frac{1}{8}, \frac{3}{16}\right] \text{and } \Pi_U = \frac{5}{16}. \tag{1.54}$$

[2]The value of a such that the profit of the innovator is equal to zero determines the intercept with the horizontal axis.

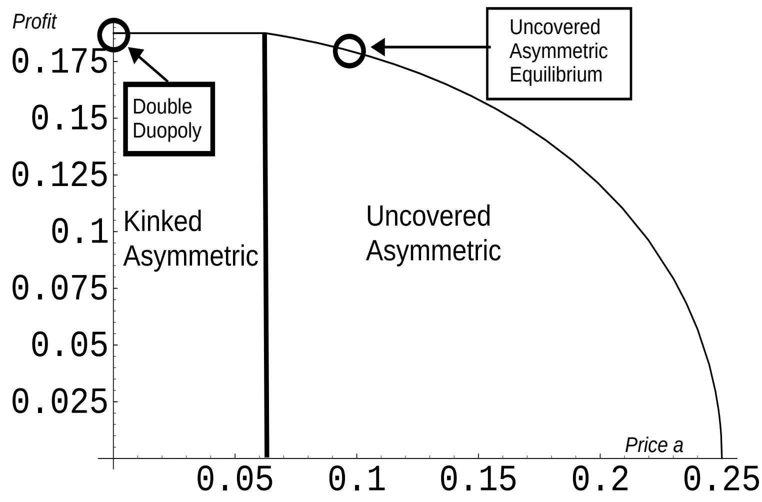

FIGURE 1.1: Profit of the innovator as a function of a when: $N_B = 2$ and $v = \frac{1}{4}$

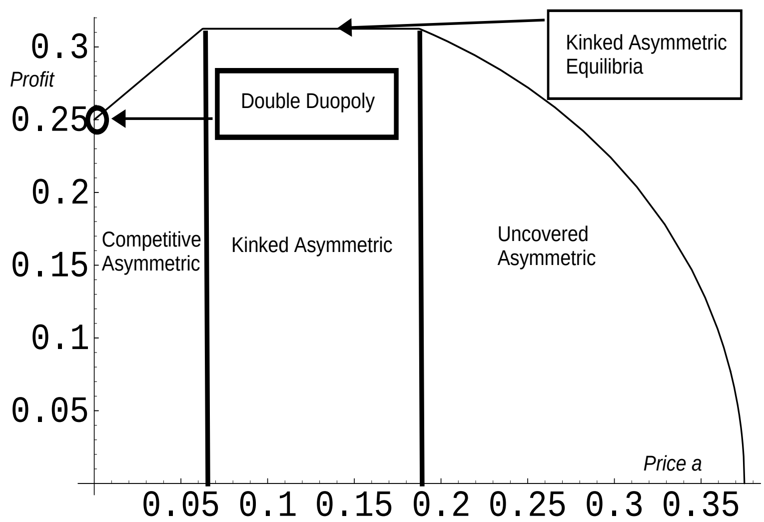

FIGURE 1.2: Profit of the innovator as a function of a when: $N_B = 2$ and $v = \frac{6}{16}$

The valuation is high enough for the asymmetric structure (i.e $N_A = 1$) to be at a covered equilibrium. The double duopoly structure would imply a loss for the upstream monopolist because a would be too low and some unnecessary surplus would be left to consumers. As a consequence, the asymmetric structure would be preferred to the double duopoly one. This can be observed in figure 1.2. The profit of the innovator is higher when $a \in \left[\frac{1}{8}, \frac{3}{16}\right]$ than when $a = 0$.

We now apply the same reasoning to illustrate how the double monopoly and the differentiated monopoly structures compare. In both structures, there is only one producer of the component B (i.e $N_B = 1$). When $v = 1$, both structures are in the uncovered equilibrium but the differentiated monopoly structure eliminates the double marginalization problem. We can see on figure 1.3 that the profit of the innovator is higher in the differentiated monopoly structure (i.e when $a = 0$) than in the double monopoly structure (i.e when $a = \frac{2}{5}$). On the other hand, when $v = \frac{6}{4}$, both structures are in a covered equilibrium. The double monopoly structure is characterized by multiple covered equilibria such that $a \in \left[\frac{1}{2}, \frac{3}{4}\right]$. The innovator is indifferent between both structures because both are covered in equilibrium (see figure 1.4). There is no inefficient double marginalization in the double monopoly structure anymore and both structures lead to full social surplus extraction. This illustrates an interesting feature of the model which is that an homogeneous monopoly is not more effective than a differentiated monopolist to capture consumer surplus. We indeed observe that when the competitive equilibrium exists the move towards homogeneous Bertrand competition implies a loss in the ability of the upstream monopolist to extract the available social surplus. However, in the absence of effective competition, differentiated firms have the same ability as an homogeneous monopolist to extract surplus from consumers. This can also be shown by the fact that when the double duopoly structure is at a kinked equilibrium point (i.e no effective competition), the upstream monopolist is able to capture the entire net social surplus. The introduction of an additional producer in the homogeneous component market does not necessarily affect the ability of the upstream monopolist to extract system valuation.

1.6 Double circular city model

In this section, we aim at developing a model of spatially differentiated complementary goods where both components are differentiated. We consider a double circular city model in which each component product space is represented by a unit circle.

We assume that the distribution of consumers is identical in both markets (i.e both circles). A consumer located at a distance x of producer B_1 needs to travel the same

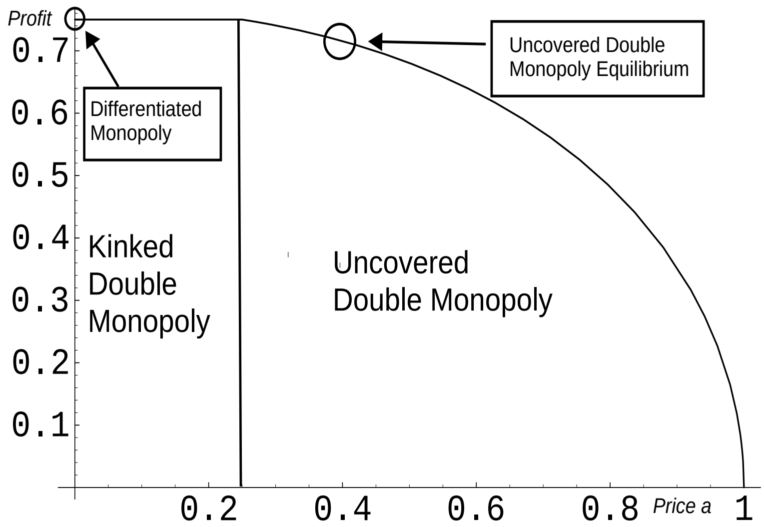

FIGURE 1.3: Profit of the innovator as a function of a when: $N_B = 1$ and $v = 1$

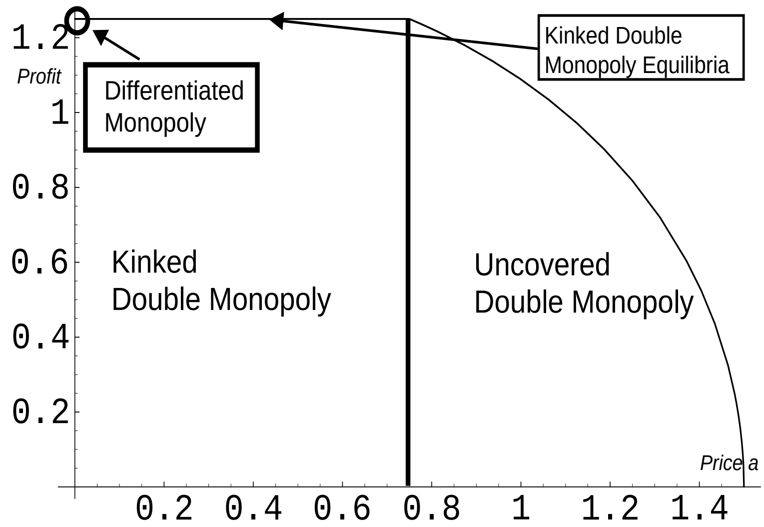

FIGURE 1.4: Profit of the innovator as a function of a when: $N_B = 1$ and $v = \frac{6}{4}$

distance between its ideal variety and the product produced by A_1. This essentially amounts to assuming that products are differentiated in only one dimension that is common to both components. The difference with the model with one differentiated market is that consumers are here sensitive to this characteristic for both products. They suffer from the disutility of distance in both downstream markets. We first restrict our attention to symmetric structures where there is an equal number of active firms in both markets ($N_A = N_B$). The locations of downstream firms are thus symmetric implying that consumer x, located at distance x_i of firm A_i is also situated at a distance x_i of the corresponding firm B_i (though the two are different suppliers). The disutility given to the distance traveled t is also the same for both products. This perfectly symmetric framework allows us to focus on a representative segment in each market.

In the covered market case, demand addressed to producers is given by the location of the indifferent consumer. It is derived from the comparison of full prices between the two available varieties of a component. Each consumer selects its favorite variety of each component independently. The location of the indifferent consumer depends only on prices charged by the two firms active in the representative segment. Under the assumption that the market is covered, all potential consumers that have selected a given producer as their favorite supplier actually purchase a system. Therefore, profit functions are the same as in the standard Salop (1979) model. Symmetric price equilibrium is consequently:

$$b = c + \frac{t_B}{N_B^2} \tag{1.55a}$$

$$a = c + \frac{t_A}{N_A^2}. \tag{1.55b}$$

The condition for the indifferent consumer to participate is the following:

$$v > 2c + \frac{5(t_A + t_B)}{4N^2}. \tag{1.56}$$

If the market is uncovered, each firm is a local monopolist and sets its price independently of its neighbors. However, the demand addressed to each firm depends on the price charged by the symmetrically corresponding producer of the complementary good.

The perfect symmetry of the model allows us to focus on one representative segment in each component market where only one local monopolist is active. There are in fact N independent monopolized local markets. Assuming the market is uncovered, the only strategic interaction at work is the one between complementary good monopoly producers. The profit function of each local monopolist takes the following form:

$$(a - c) \times 2\sqrt{\frac{v - a - b}{t_A + t_B}}. \tag{1.57}$$

Profit maximization leads to symmetric best response functions for each local monopolist in both complementary markets:

$$a = \frac{2v - 2b + c}{3}. \tag{1.58}$$

Using the symmetry between and within markets, we find the following uncovered equilibrium price:

$$a = b = \frac{2v + c}{5}. \tag{1.59}$$

The market is indeed consistently uncovered in equilibrium under the following condition:

$$v < 2c + \frac{5(t_A + t_B)}{4N^2}. \tag{1.60}$$

We will now allow for asymmetric number of downstream firms and characterize the optimal licensing policy of the monopoly innovator in the double circular city model. In the following section, we restrict our attention to fully covered downstream markets.

1.6.1 Double circular city model under the covered market assumption

1.6.1.1 Covered market condition

We are now trying to generalize the previous model to asymmetric licensing policies. We will however restrict the analysis to the covered market case. We derive a sufficient condition for both complementary markets to be covered. We will study the case in which the market is the most likely to be uncovered. This is the case when there is only one firm active in each differentiated market (i.e the double monopoly structure). Each downstream market is characterized by a circular city of perimeter one. Consumers are uniformly and symmetrically distributed along each circle. Each consumer is characterized by its location on each circle $x_A = x_B = x$. The two monopolists are symmetrically located in their respective circle so that the consumer that is the most likely to exit the market and use its outside option is the consumer characterized by $x_A = x_B = \frac{1}{2}$. Given the price charged by the other firm, the potential market of a monopolist is a decreasing function of its own price. Transportation costs are assumed to be quadratic so that the expression of the location of the marginal consumer $\bar{x}$, given the prices charged by both monopolists (a, b) is: $\bar{x} = \sqrt{\frac{v-a-b}{t_A+t_B}}$. We focus on the representative segment such that x belongs to the interval $[0, \frac{1}{2}]$. Knowing the expression of the location of the marginal consumer, we can define the demand function addressed to each monopolist that is equal to the number of consumers purchasing a system in the representative segment which is given by the value of $\bar{x}$. This is only true when $\bar{x}$ belongs to the interval $[0, \frac{1}{2}]$, that is when the market is uncovered. Otherwise, the demand is equal to one. We can now define profit functions that depend on a and b.

We derive best response functions in maximizing the profit of each downstream firm given the behavior of the other monopolist and obtain Nash equilibria of this subgame. The best response function of the producer of component A (and B symmetrically) is:

$$a = \begin{cases} \dfrac{2v - 2b + c}{3}, \text{ if: } v - c > b > v - c - \dfrac{3}{4(t_A + t_B)} & (1.61) \\ v - b - \dfrac{t_A + t_B}{4}, \text{ if: } v - c - \dfrac{3}{4(t_A + t_B)} > b > c. & (1.62) \end{cases}$$

These best response functions lead to the following price equilibria:

- the uncovered price equilibrium:

$$a = b = \frac{2v + c}{5} \tag{1.63}$$

- the covered price equilibria: (a, b) belongs to the following interval:

$$\left[c + \frac{t_A + t_B}{2}, v - c - \frac{3}{4(t_A + t_B)}\right]. \tag{1.64}$$

We select a particular covered equilibrium, where surplus is shared equally between the two monopolists. The balanced covered equilibrium price is such that:

$$a = b = \frac{v}{2} - \frac{t_A + t_B}{8}. \tag{1.65}$$

We should keep in mind that consumers value the system as a whole and have no intrinsic valuation for individual component. Consumers decide whether or not they want to purchase the whole system. This is why our demand functions in this game are symmetric. As a consequence, when one of the component markets is covered, the other is necessarily covered as well. From the expression of the equilibrium prices, we can derive the following sufficient condition on parameters v in order for the system market to be covered. The two monopolists optimally serve all potential consumers if:

$$v > 2c + \frac{5}{4(t_A + t_B)}. \tag{1.66}$$

If this condition is respected, both differentiated complementary markets are covered in the extreme case of symmetric monopolies and in all other structures of the industry where the number of active firms is higher or equal to one. Any additional entrant would reduce the maximum transportation distance and potentially increase the degree of competition causing the constraint on the participation of the marginal consumer to relax.

1.6.1.2 Downstream competitive equilibrium

We will now assume that the above covered market condition is satisfied and we turn to the analysis of asymmetric differentiated complementary markets under the covered market assumption. We need to derive demand functions which are based on the location of the indifferent consumer in each of the component market. Consumers compare the two options that possibly best meet their needs (i.e transportation costs are such that the only reasonable alternatives are the closest from any consumer x) in each of the component markets independently. We focus on a representative segment lying between any two firms. Let it be reminded that locations are automatically defined by maximal differentiation so that the distance between any two firms in the market i is equal to $\frac{1}{N_i}$. The indifferent consumer $\hat{x}$ located in the representative segment, is the one for which full prices of its two alternatives are equal. Solving this equation leads to the following expression of the indifferent consumer:

$$\hat{x} = \frac{x_{A_1} + x_{A_2}}{2} - \frac{a_1 - a_2}{2t_A(x_{A_2} - x_{A_1})}, \tag{1.67}$$

with x_{A_i} denoting the location of the firm. The expression of the location of the indifferent consumer is symmetric for the component market B.

Knowing that both markets are covered under the sufficient covered market condition expressed in equation 1.66, the location of the indifferent consumer in each of the segments where the representative firm is active defines the demand function addressed to each downstream firm depending on the prices charged by its two neighbors. We obtain the following expression of the total demand function addressed to any downstream firm:

$$\frac{1}{N_A} + \frac{N_A}{2t_A} \times ((a_{i-1} - a_i) - (a_i - a_{i+1})). \tag{1.68}$$

This demand function is only valid when there are at least two firms in the downstream market. Otherwise, the expression of the indifferent consumer does not make sense and its location is undefined. If there is a monopoly on both markets we fall back into the double monopoly case. On the other hand, if only one of the component markets is monopolized, the industry is characterized by a differentiated asymmetric structure. Then the covered market condition remains satisfied implying the demand for the monopolized good is equal to one and the demand for the competitive component is determined by the indifferent consumer. We first derive the equilibrium of the game in the competitive cases when there are at least two firms in each markets (i.e $N_A > 1$, $N_B > 1$). From the expression of the competitive total demand, we obtain the profit function of the representative firm i that we maximize with respect to its own price given the prices of both neighbors. We obtain the following best response function:

$$b_i = \frac{1}{4} \times \left(2c + b_{i+1} + b_{i-1} + \frac{2t_B}{N_B^2} \right). \tag{1.69}$$

Given the symmetry of the model, we know that any downstream firm active in a component market has symmetric profit and best response functions. Two identical firms cannot be maximizing their profit using different strategies so that in equilibrium, the price of the representative firm must be equal to the price charged by both its neighbors. We obtain the following symmetric Nash equilibrium price and profits:

$$b = c + \frac{t_B}{N_B^2} \tag{1.70a}$$

$$a = c + \frac{t_A}{N_A^2} \tag{1.70b}$$

$$\Pi_B = \frac{t_B}{N_B^3} \tag{1.70c}$$

$$\Pi_A = \frac{t_A}{N_A^3}. \tag{1.70d}$$

1.6.1.3 Optimal licensing policy

We now turn to the profit-maximizing problem of the upstream firm (i.e the upstream stage) and look for the profit-maximizing structure of the industry. Under our assumptions on the vertical contracting stage, we know that the upstream monopolist is able to capture the entire downstream profit. It is indeed the equilibrium of the auction game for the downstream firms to bid their valuation for the license that is equal to their operating profit as long as there are more potential buyers than there are licenses to be purchased. Moreover, when the number of potential buyers is low, the upstream monopolist sets a reservation price which acts like a fixed fee. The valuation of downstream firms for the license depends on the degree of downstream competition generated by the number of licenses issued. The profit of the upstream monopolist is equal to the number of licenses issued multiplied by the valuation of potential buyers in each component markets:

$$\Pi_U = \frac{t_B}{N_B^2} + \frac{t_A}{N_A^2}. \tag{1.71}$$

We observe that the profit function of the upstream monopolist is decreasing in the number of licenses issued. Profit maximization implies minimization of the number of active downstream firm. Demand and profit functions previously derived are valid as long as there are at least two firms active in each market. Otherwise we fall into the double monopoly case that we studied in section 1.6.1.1. Maintaining the assumption of full market coverage, upstream profit in the double monopoly structure is equal to the sum of the downstream monopoly profits. In the case of the balanced covered equilibrium, the profit of the upstream monopolist takes the following value:

$$\Pi_U = v - \frac{t_A + t_B}{4} - 2c. \tag{1.72}$$

Under the covered market assumption, the profit of the upstream monopolist in the double monopoly case is higher than under the double duopoly structure (i.e minimum consistent level of (N_A, N_B)). This is the case because the horizontal double marginalization is costless to the upstream monopolist with inelastic demand (i.e covered market). Profit maximization of the upstream monopolist consequently leads to the double monopoly structure to be preferred. We must now compare the double monopoly with an asymmetric differentiated structure with one monopoly and a duopoly on the other market. We now turn to the monopolized structures where there is an exclusive component producer in one of the downstream markets. Since market is assumed to be covered, the price equilibrium of the competitive market is equal to the standard competitive circular city price equilibrium. This allows the monopolist active in the complementary market to capture the whole remaining surplus under the covered market constraint. This equilibrium is very similar to the fully unbalanced covered equilibrium of the double monopoly model where one of the monopolists only gets the necessary surplus to ensure full participation while the other monopoly captures the entire residual surplus.

In the differentiated asymmetric structure however, the presence of an extra B component producer decreases transportation costs borne by consumers making it less costly to ensure full participation than under the symmetric double monopoly structure. The equilibrium prices and profits of the asymmetric structure are the following:

$$a = v - c - \frac{t_A}{4} - \frac{t_B}{N_B^2} \tag{1.73a}$$

$$b = c + \frac{t_B}{N_B^2} \tag{1.73b}$$

$$\Pi_A = v - 2c - \frac{t_A}{4} - \frac{t_B}{N_B^2} \tag{1.73c}$$

$$N_B \times \Pi_{B_i} = \frac{t_B}{N_B^2} \tag{1.73d}$$

$$\Pi_U = v - 2c - \frac{t_A}{4}. \tag{1.73e}$$

With quadratic transportation costs, the marginal consumer (i.e the consumer facing the highest total transportation costs over the two markets and the most likely to exit the market) is located at $\bar{x} = \frac{1}{2}$ as long as the unit transportation cost in the monopolized good is not much lower than in the complementary market (i.e $t_B < 3t_A$). The very high transportation cost faced to get component A from the monopoly producer more than outweigh its perfect location in market B.

As a long as the upstream monopolist is able to capture downstream profits, it prefers the differentiated asymmetric structure. The upstream monopolist captures the full social surplus net of the maximum transportation cost borne by the marginal consumer of the monopolized market (i.e $\bar{x} = \frac{1}{2}$). The innovator favors the asymmetric structure to the double monopoly because it decreases the maximum transportation cost borne by the marginal consumer and thus the cost of maintaining the market fully covered. In the asymmetric structure, the marginal consumer does not pay any transportation cost to obtain its unit of component B whereas in the double monopoly it has to travel a distance equal to one half. This positive effect of entry in the competitive market does not persist when there are more than two active firms. The location of the marginal consumer indeed remains the same as long as there is only one variety of the component A. Further entry in market B would make the market more competitive, decrease the price of component B and the average transportation cost but would not allow the monopoly producer of component A to charge a higher price without making the marginal consumer exit the market. As a consequence, the upstream monopolist prefers the asymmetric to the double monopoly structure. It is indifferent between all asymmetric structures as long as one of the markets is monopolized. More intense competition would decrease the profit of each component B producer along with total profit of the market B industry. This negative effect on the profit of the innovator is fully compensated by the decrease in component B price which allows the monopoly producer of component A to increase its price while maintaining the market fully covered. This extra surplus is in turn captured by the patentee through the delivering of the exclusive license on market A. The equilibrium profit function of the innovator does not depend on N_B:

$$\Pi_U = v - 2c - \frac{t_A}{4}. \tag{1.74}$$

The number of active firms in market B only influences the surplus sharing between B component producers and the monopolist A. It does not influence the size of the surplus but only its distribution. Since the patentee is able to capture the entire downstream profit, it is indifferent between its various distributions.

Proposition 1.6.1. *In the double circular city model of complementary goods and under the market covered condition (i.e equation 1.66), a monopoly patentee profitably issues an exclusive license on one of the component market and at least two licenses on the other.*

Proof. More details are available in the appendix. □

Since we have abstracted from any vertical contracting issues such as commitment problems, downstream firms bid their valuation and the upstream monopolist capture the whole downstream profit. The profit of the industry is maximized because under our assumption of full market coverage, all potential consumers are served in equilibrium and the demand function is inelastic (i.e the aggregate demand is bounded to one).

There is consequently no double marginalization problem. The decrease in the degree of competition in downstream markets has indeed no quantity effect. That explains why delivering an exclusive license is profitable for the innovator.

In our model with inelastic demand and homogeneous goods, the upstream monopolist is indifferent between symmetric and asymmetric structures of the industry. In the symmetric structure, the upstream monopolist issues an exclusive license in each component market (i.e double monopoly structure). Maintaining inelastic demand and introducing horizontal differentiation breaks this indifference. Because the quantity effect is absent with inelastic demand, the double monopoly structure does not suffer from inefficient horizontal double marginalization. Moreover, the decrease in transportation costs associated with the entry of a second producer in one of the markets increases the social surplus. This is captured by the exclusive component producer and in turn by the upstream monopolist.

In this section, we find the same result as in the single differentiated market model. In a model where only one component market is differentiated, the most favorable industry structure for the upstream monopolist is the asymmetric one. It sells an exclusive license to a homogeneous monopoly and sells as many licenses as possible on the differentiated market. If there is no upper bound on the number of firms able to enter the market and willing to purchase a license, the upstream monopolist can replicate the homogeneous structure by making the differentiated market perfectly competitive in issuing an infinite number of licenses. When the number of potential licensees is capped and the system valuation is sufficiently high to make the market covered, the asymmetric structure is also found to be optimal for the patentee in both the single and double differentiated complementary markets. This can be explained by product differentiation and the fact that entry generates additional social surplus.

We have shown in this section that the asymmetric structure is preferred to the symmetric double monopoly when the market is fully covered both in the single and double differentiation circular city framework. More competitive structures are dominated by both types of structures involving at least one exclusive license. It appears that the decrease in transportation costs and the associated welfare gains induced by symmetric competition do not compensate the decreased ability of downstream firms and in turn of the upstream monopolist to capture the social surplus.

1.6.2 Uncovered double circular city model

We studied the differentiated double monopoly structure from which the covered market condition we used to analyze the double circular city model is derived. We aim now at generalizing this analysis to uncovered market structures. We focus on cases where the number of potential licensees is low. We simply consider the model from the previous section and relax the assumption on full market coverage. We restrict the analysis to situations where the number of active firms in each downstream market is lower than or equal to two. We consider three different structures namely the double monopoly, the asymmetric ($N_A = 1$, $N_B = 2$) and the double duopoly ($N_A = N_B = 2$).

The upstream monopolist determines the downstream industry structure in a first stage. We solve the model backward and start to derive the downstream equilibrium in each subgame. This equilibrium can either be covered or uncovered. We already characterized the equilibrium of the game in which the system valuation and the transportation cost are such that the market is covered. We now study intermediate situations where some structures are covered and others are not. We rank structures in order of increasing likelihood of full market coverage (i.e decreasing maximal transportation cost and increasing number of active firms): the double monopoly, the asymmetric structure and the double duopoly. The double monopoly market can indeed be uncovered whereas the asymmetric and double duopoly structures are covered. The double duopoly can also be the only covered structure. Finally, there are values of v and (t_A, t_B) for which none of the structures studied here are covered. When v is sufficiently high, all structures are covered and as v decreases some structures become uncovered. These are the possible intermediate situations for specific values of v:

- all structures are covered
- only the double monopoly is uncovered

- asymmetric structure is partially uncovered and the double duopoly is covered
- asymmetric structure is partially uncovered and the double duopoly is uncovered
- asymmetric structure is fully uncovered and the double duopoly is uncovered (i.e all structures are uncovered)

The asymmetric structure is said to be partially uncovered when the second producer of B component makes some positive sales. Comparing the two offers, each consumer chooses which firm it wishes to patronize (i.e the one with the lowest full component price). The location of the indifferent consumer (i.e $\hat{x}$) determines the range of consumers preferring one firm to the other. This location would also define in a fully covered market, the demand addressed to each producer of the component B. In a partially uncovered market however, the consumers facing the highest total transportation cost (i.e those located away from the monopoly producer of component A and beyond the marginal consumer $\bar{x}_2$) do not participate in the market. The demand addressed to the second producer of component B is then defined by the spread between its marginal consumer and the indifferent consumer (i.e $\bar{x}_2 - \hat{x}$).

On the other hand, the asymmetric structure is fully uncovered when B_2 is unable to attract any consumer and none of its patronizing consumers participate. In this case, B_1 is in a situation of local monopoly and it does not serve all patronizing (i.e dedicated) consumers (i.e its marginal consumer is located closer to B_1 than the indifferent consumer). As we showed before, the move from the double monopoly to the asymmetric structure decreases the maximum transportation cost and loosens the full participation constraint. That is why the double monopoly can be uncovered while the asymmetric structure is not. Nevertheless, the consumer facing the highest transportation cost remains in the double circular city, the one located at a distance $x = \frac{1}{2}$ of the monopoly producer of one of the two components (i.e A). That is why there is no symmetry between the two producers of the complementary good (i.e B) in the asymmetric structure. We recall that in the asymmetric structure, pricing behavior in the duopoly market does not depend on the price of the complementary good as long as the duopoly market is competitive. This result is similar to the one obtained by Cheng and Nahm (2007) on horizontal double marginalization under asymmetric complementarity.

We now characterize downstream equilibria of the specific market outcomes that are the partially and fully uncovered asymmetric equilibria. We indeed previously analyzed the double monopoly as well as the double duopoly structure which is a special case of the symmetric differentiated model. We have also characterized the covered case of the asymmetric structure.

1.6.2.1 Downstream equilibria

We have two differentiated component markets with a monopoly on one side and a duopoly on the other. We know that the market is not fully covered. We want to explore the existence of an equilibrium where both systems available are sold (i.e all firms are effectively active). We have two circular cities representing the two downstream complementary markets. There is one firm in each market situated at a given point x (named A and B_1). We arbitrarily set this point to be equal to zero. In market B, the extra active firm (B_2) is situated at distance equal to one half from B_1. Under quadratic transportation costs and reasonably low spread between unit transportation costs in each market (i.e between t_A and t_B), the consumer facing the highest transportation cost is located at $x = \frac{1}{2}$. This is the consumer that is the most likely to be unwilling to participate in the market. The market is covered when this consumer purchases a system good. We have previously analyzed this model under the covered market assumption derived in the double monopoly model. We found that the asymmetric structure is preferred by the upstream monopolist and the competitive price equilibrium is:

$$b = c + \frac{t_B}{N_B^2} \tag{1.75a}$$

$$a = v - c - \frac{(t_A + t_B)}{4}. \tag{1.75b}$$

We now derive the condition for this asymmetric structure to be fully uncovered. If this is the case, the uncovered equilibria of the asymmetric and double monopoly structures are identical:

$$b = a = \frac{2v + c}{5}. \tag{1.76}$$

We now look for conditions ensuring the existence of this monopoly equilibrium in the asymmetric structure. B_2 is ready to set its price to marginal cost in order to avoid the fully uncovered situation. As soon as the market is fully uncovered (i.e location of the marginal consumer $\bar{x}_1$ of the system $A - B_1$ is lower than the location of the indifferent consumer), there is indeed no positive demand addressed to B_2. Unlike in the symmetric case, B_2 is here ready to give up margin to make the market competitive and convince some of its patronizing consumers participate. B_2 does not enjoy effective local monopoly power as the elasticity of demand in its local market is much higher (i.e lower residual valuation for the system). As a consequence, the monopoly equilibrium exists if and only if marginal cost pricing of B_2 does not suffice to convince some consumers to refuse the local monopoly price charged by B_1. We derive the fully uncovered equilibrium condition on v for the marginal consumer of B_1 to be inferior to the indifferent consumer when the producers located at $x = 0$ charge uncovered prices (i.e $a = b = \frac{2v+c}{5}$) and B_1 is setting its price equal to marginal cost ($d = c$):

$$2c < v < \frac{16c(t_A + t_B) + 5t_B(2t_B + t_A - \sqrt{t_B(3t_B + 2t_A)})}{(8(t_B + t_A))}. \tag{1.77}$$

We now turn to the case of partially uncovered equilibrium. There is some positive demand addressed to B_2 but some patronizing consumers do not participate. System valuation is too high for the fully uncovered equilibrium to exist at least the consumer located at $x = \frac{1}{2}$ does not participate. We first determine the location of the indifferent consumer in market B (i.e who is characterized by equal full prices of B components). The expression of the indifferent consumer corresponds to the one derived in the covered model in the case where the number of active firms is equal to two. Assuming that there is some positive demand for B_2 implies that all consumers favoring product B_1 participate in the market. Demand for product B_1 is thus determined by the location of the indifferent consumer:

$$D_{AB_1} = \hat{x} = \frac{1}{4} - \frac{d - b}{t_B}. \tag{1.78}$$

Remaining consumers situated beyond the indifferent consumer prefer product B_2. However, there is only a fraction of these potential B_2 consumers who actually purchase system AB_2. Those with the highest value of x (i.e located the furthest away from A) would derive negative surplus from system consumption and thus abstain from participation in the market. We derive the expression of the location of the marginal consumer of a system AB_2 in equalizing net surplus to zero. The number of effective consumers of this system is given by the distance between the marginal consumer and the indifferent consumer:

$$D_{AB_2} = \frac{t_B + \sqrt{4(t_A + t_B) \times (v - a - b) - t_A \times t_B}}{2(t_A + t_B)} - \frac{1}{4} + \frac{(d - b)}{t_B}. \tag{1.79}$$

The demand function addressed to A is given by the sum of the two above demand functions that is the sum of all consumers deriving non negative net surplus from system consumption (i.e the distance between zero and the marginal consumer of system AB_2):

$$D_A = \bar{x}_2 = \frac{t_B + \sqrt{4(t_A + t_B) \times (v - a - b) - t_A \times t_B}}{2(t_A + t_B)}. \tag{1.80}$$

Once we obtain demand functions, we can derive profit and best response functions for A and B_1. It is worth noting that the best response function of B_1 is identical to the one in the standard circular model of Salop (1979):

$$d = \frac{4c + t_B + 4b}{8}. \tag{1.81}$$

The expression of the best response function of B_2 is much more complicated and made it impossible for us to characterize a partially uncovered equilibrium despite simplifications and calibrations.

We observe in equation 1.80 that the demand addressed to the monopoly producer of the homogeneous component A depends on its own price a and the price of the producer B_2 (i.e b). Similarly, the demand and profit functions of B_1 depend on two price variables b and d (see in equation 1.78). On the other hand, we can see in equation 1.79 that the profit function of B_2 depends on all three prices (i.e a, b and d). Even when we set a to a given value exogenously determined, the best response function of B_2 remains highly complicated. We observe that unlike in the standard circular city model, B_2 is here willing to charge a price equal to marginal cost in order to generate some positive demand. The structure we are studying here is more competitive than the standard circular city model. B_2 has effectively no dedicated consumers (i.e no local market power). Indeed, consumers situated closer to B_2 are the first to abstain from consumption because they are the furthest away from A and thus face the highest transportation cost. As a consequence, the only consumers B_2 can serve are the one that are situated closer to B_1 and those who can switch more easily to the alternative product. This implies that B_2 is ready to reduce its margin in order to avoid the uncovered outcome (i.e when some B_1 potential buyers do not participate) because if it were the case, B_2 would face no positive demand. Marginal cost pricing can be optimal for B_2. The behavior of the firm B_1 is exactly the same as in the standard competitive model of a circular city. Profit function of the monopoly producer of component A does not depend on the behavior of firm B_1 within the interval consistent with the partially uncovered equilibrium. B_2 determines jointly with A the location of the marginal consumer which generates the demand addressed to the monopolist A.

1.6.2.2 Optimal licensing policy

Despite calibrations and simplifying assumptions we were unable to characterize a partially uncovered equilibrium. Even though we cannot predict the outcome of the asymmetric structure in the partially uncovered case, we know that the asymmetric structure is weakly preferred to the double monopoly. The fully uncovered equilibrium reproduces the double monopoly solution for very low values of v. We know that B_1 behaves as in the covered case, and that the behavior of B_2 converges towards the standard competitive behavior as the partially uncovered case comes closer to the covered case. The profit functions of B_2 and B_1 look alike except that the demand function faced by B_2 is elastic in two dimensions. B_2 can attract new consumers by making the indifferent consumers move further away from it (i.e getting consumers to switch producer) but can also extend its market segment in increasing the number of potential consumers (i.e making B_2 marginal consumer come closer to its location). This extra dimension in the elasticity of the demand function of B_2 provides it with an additional incentive to decrease its price. It appears that B_2 would be at least as much induced to decrease its price in the partially uncovered than in the covered equilibrium. Thus, the partially uncovered structure could be more competitive than the covered one. Nevertheless, the extra dimension of demand elasticity is bounded by the covered case as the marginal consumer cannot lie beyond the location of B_2 (i.e $x < \frac{1}{2}$). When we observe the general best response function when the market is covered (expressed in equation 1.69), we notice that given the prices practiced by its rival, the number of active firms in the market impacts negatively the price charged by a downstream firm. The length of the market segment where a firm is active is given by the inverse of the number of active firms. That is that the length of the market segment has other things being equal a positive effect on price. Given that B_2 is very similar to B_1 in its objective function, the fact that it is active in a smaller market segment should induce B_2 to practice a lower price compared to B_1 and to the price charged in the covered market case. We can thus speculate that no matter whether the asymmetric structure is covered, partially or fully uncovered, it ensures at least equal level of participation in the market and equal level of total industry profits than in the double monopoly structure. The fact that B_2 behaves more aggressively in the asymmetric structure leading to potentially lower component B prices, benefits monopolist A and in turn the upstream innovator.

In order to find out what would be the optimal licensing policy of the innovator in the uncovered limited number of active firms framework, we should study the possibility for the patentee to choose a symmetric structure. The upstream monopolist could do so in order to increase the number of sales. This benefit would be earned at a cost of reducing downstream profits margin. A necessary condition for this to happen would be that the innovator prefers the double duopoly structure to the double monopoly one. The asymmetric structure being an intermediate situation in terms of market coverage, it would require a more restrictive condition for the innovator to favor the double duopoly to the asymmetric structure.

We now leave aside the asymmetric structure and compare the profit of the innovator in the double monopoly and double duopoly structures. When the double duopoly structure is uncovered (i.e $v < 2c + \frac{5}{16(t_A+t_B)}$, this structure is clearly preferred because it enables the patentee to reproduce the double monopoly outcome on each side of the circle. There are indeed two independent double monopoly markets. The only downside of this structure from the view point of the innovator is that this structure does not solve the double marginalization problem. Nevertheless, the fact that the monopoly outcome is replicated reduces considerably the cost of horizontal double marginalization as the number of consumers abstaining from consumption can be very low. When the double duopoly structure is covered however, the double duopoly structure generates a more competitive outcome with lower margin and higher sales. We found that the profit of the upstream monopolist is higher under the double duopoly structure when the valuation is sufficiently low (i.e if $v < 2c + \frac{5(t_A+t_B)}{8\times 2^{\frac{1}{3}}}$ approximately equivalent to $v < 2c + \frac{(t_A+t_B)}{2}$). Otherwise, the double duopoly structure generates a very intense competition that dissipates the profits of the industry. In this case, the double monopoly is preferred to the double duopoly structure which is dominated by the asymmetric structure.

We know that when the valuation is low enough for the asymmetric structure to be fully uncovered, the double duopoly structure generates more upstream profits. When the asymmetric structure is fully uncovered it implies that the double duopoly structure is also uncovered (i.e condition on v for the asymmetric structure to be fully uncovered is more restrictive than the uncovered double duopoly condition). Then the monopoly equilibrium can be replicated on both sides of the market and the double duopoly structure dominates both the double monopoly and the asymmetric structures. There might also be a range of intermediate values of v such that the asymmetric structure is partially uncovered but v is too low for the asymmetric structure to ensure enough participation. The upstream monopolist may then also prefer the double duopoly structure, even outside of the fully uncovered asymmetric situation. Despite being unable to precisely determine the value of the threshold on system valuation v from which the asymmetric dominates the double duopoly structure, we find that our qualitative results on the profitable licensing policy in a differentiated system market carries over to the double circular city model. The patentee indeed chooses the asymmetric structure in mass markets in order to avoid intense downstream competition, and the double duopoly structure in niche markets to attenuate the effect of the horizontal double marginalization problem.

Proposition 1.6.2. *In the double circular city model (i.e allowing for both covered and uncovered market structures) with a limited number of potential firms, the monopoly innovator finds it weakly profitable to issue an exclusive license on one of the component market and two licenses on the other (i.e asymmetric structure) when the valuation for the system is sufficiently high.*

Otherwise (i.e in a niche market), the patentee prefers to deliver two licenses in each component market (i.e double duopoly structure).

We cannot directly compare the models with either one or two differentiated component markets because, for given values of v and location of a consumer x, disutility of transportation will be much higher in the double circular city. But assuming $t_A + t_B = t$, we believe that the asymmetric structure performs relatively better in the double circular city model which is likely to lower the threshold on system valuation for the domination of the asymmetric structure.

1.7 Conclusion

In this chapter we attempt to extend the literature on optimal licensing policy by considering an essential characteristic of technological goods that is product complementarity. In other words, our model aims at taking into account the vertical dimension of technological system markets (i.e the licensing stage). We characterized the optimal number of licenses sold by an upstream innovator active in the R&D whose technology is used in two different component markets. We observe that the profit-maximizing licensing policy depends on downstream market characteristics such as demand elasticity and component differentiation. Nevertheless, in the models we presented in this chapter, the asymmetric licensing policy tends to dominate. It seems like perfect complementarity pushes for asymmetric licensing. However, we show that a symmetric licensing (i.e double duopoly) structure can be optimal for the innovator when the number of potential licensees is limited and one of the component is produced in a niche market (i.e consumers are highly sensitive to the distance with their ideal variety).

This result is robust to the extension of our model to the double circular city where both component markets are differentiated. When the number of licensees is capped, the double duopoly structure remains to be optimal for the innovator in niche component markets. Otherwise the asymmetric structure is weakly dominating. On the other hand, when the number of downstream firms is unlimited (and the market covered), the asymmetric structure always dominates. In addition, we show that the strategy consisting in transferring all the rent to a monopolized component market does not always dominate. Product differentiation may encourage a symmetric distribution of profits across downstream complementary markets. We can also note that we find a multiplicity of covered equilibria of the downstream competition stage. The competitive covered equilibrium of the standard circular city model of Salop (1979) does not stand out in this differentiated system good framework.

Rey and Salant (2012) also analyze fixed fee patent licensing in a spatial differentiation framework. As mentioned earlier, the authors consider the level of the cost of entry in the downstream market and discuss the efficiency of the licensing policy with respect to the socially desirable number of firms (i.e depending on the valuation for the system and the fixed cost of entry). When the sensitivity to product differentiation is low (i.e low level of t or high level of v), the equilibrium number of licensees is profitably set to an excessively low level in order to avoid profit dissipation and maintain the downstream market in the kinked covered equilibrium. To the contrary, there is an excessive number of licensees issued by the upstream monopolist when the value of variety is high (i.e high t) because entry allows downstream firms to charge higher prices (as entry makes their marginal consumers located closer to the firms). We obtained consistent results on the effect of the sensitivity to product differentiation t on the optimal licensing policy of the patentee when the number of potential licensees is limited (i.e $(N_A, N_B) \leq 2$). We indeed found that the monopoly innovator profitably offers more licenses on the homogeneous component market when the differentiated component market is a niche (high level of sensitivity t or low valuation for the good v) rather than a mass market. This is in line with Doganoglu and Inceoglu (2014) who find that the optimal number of licenses decreases with the degree of substitutability between final goods (i.e with a lower t).

In contrast Arora and Fosfuri (2003) show in a model with multiple innovators and no taste for variety that the number of licenses decreases with the level of product differentiation. Doganoglu and Inceoglu (2014) also argue that the optimal number of licenses increases with the valuation for the final good. We find the opposite in our model assuming a limited number of licensees. Nevertheless, we obtain similar results in the case where there is an unlimited number of potential downstream firms. The number of profitable licenses offered by the monopoly innovator increases with the valuation for the system good and decreases with the sensitivity to distance in the product space. We also show in this chapter that our qualitative results on the optimal licensing of a monopoly innovator in differentiated complementary markets hold in the double circular city model where both component markets are differentiated.

One of the limitations of our model as pointed out by Reisinger and Schnitzer (2012) is the fact that under the covered market assumption, the aggregate demand is constant in the (single and double) circular city. Moreover, in the uncovered asymmetric structure of the double circular city model, the assumption of maximal differentiation locations is unprofitable for the second producer (e.g B_2) located on the opposite side from both the rival and the complementary good producer. In this framework, it implies a lower degree of competition but most importantly a lower consumer valuation for the system. This is an important limitation of this model because it assumes that the second producer makes a non profitable choice of location in equilibrium or that the locations are exogenously given and suboptimal from the point of view of both the licensees and patentee.

The literature on patent licensing extensively discusses the profitability and desirability of vertical integration between the licensor and a downstream licensee. It appears that the structure of the industry and the shape of the licensing contract matter for the desirability of vertical integration in the context of patent licensing contracts. That is why in the second chapter, we study various integration regimes and their impacts on downstream equilibria as well as the number of licenses profitably offered by the patentee active in complementary markets.

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Chapter 2

Vertical integration and fixed fee licensing in complementary markets

2.1 Introduction

In IT industries, we commonly observe vertically integrated firms producing both, the technology at the core of an ecosystem, as well as one of the component of the final good. Intel for instance develops the technology and manufactures its chips in-house. Strategic moves from one market to a complementary one is also frequently observed through integration or in-house development of products. Google used both strategies to move from the search engine market to the web browser, operating system, smartphone and computer markets. We can also mention the following conglomerate mergers in IT markets: Google/Motorola or Microsoft/Nokia as well as the Cisco/Tandberg mergers. In the previous chapter, we characterized the profit-maximizing policy of an innovator-monopolist whose technology is used in a perfectly complementary market (i.e system market). Considering the number of licenses issued, the monopolist finds it more profitable to capture downstream profits through an asymmetric licensing policy (implying an exclusive license on the homogeneous downstream market) when the technology is used in a mass component market (i.e when the sensitivity of consumers to product differentiation in the differentiated component market is low). On the other hand, when the technology is used in a niche component market (i.e when consumers have a high sensitivity to the distance with their ideal variety), the outside innovator chooses a symmetric licensing policy.

In this chapter, we aim at exploring the impact of vertical and conglomerate mergers on the licensing policy of an innovator-monopolist in complementary markets. In the following section, we consider the case in which the monopolist produces one of the downstream components. Then we will study in section three a double vertical integration making the patentee produce both complementary goods.

2.2 Vertical integration and licensing in complementary markets

2.2.1 Introduction

We aim at characterizing the profit-maximizing licensing policy of an inside innovator producing one of the downstream component. Our analysis refers to the large literature on the licensing of a monopoly technology by an inside innovator. Sandonis and Fauli-Oller (2006) find that a monopoly innovator of a cost-reducing innovation would prefer to remain outside the product market when the innovation generates a large cost reduction. Rey and Salant (2012) show in a model of spatial differentiation that vertical integration does not matter for the licensor of a drastic innovation. We will see if this remains true when the technology is used in complementary markets.

In the first section, we study the situation where the upstream innovator competes in the homogeneous downstream component market. The second and third sections analyze the profit-maximizing licensing policy of an inside innovator producing one variety of the differentiated component. We finally conclude on how our results differ from the one obtained in a standard model with one downstream market.

2.2.2 Vertical integration into the homogeneous market

2.2.2.1 Framework

We study the case where the upstream monopoly innovator is an inside innovator active in the production of the homogeneous downstream good. Following our work presented in the chapter one, we build a model with a monopoly patent holder whose technology is required for the production of two perfectly complementary goods. One of the downstream component is homogeneous as it is considered to be the basis of the final good (i.e the CPU, hidden in the device). For a given equal quality of this component, consumers have no preferences on the identity of its producer. On the other hand, the various varieties of the second component are spatially differentiated because consumers directly interact with the final good through this component (i.e an interface, software or operating system). In our framework, the monopoly innovator is able to capture full downstream profits with the use of a first price sealed auction for each downstream market (or with fixed fee take-it-or-leave-it offers). In this setting, the licensing policy consists in the choice of the number of licenses to issue in each of the downstream markets.

In the separated case, we found that when the number of potential firms in the component markets is unlimited, the asymmetric licensing policy with an exclusive license in the homogeneous market is the most profitable structure for the patentee. When only two firms in each downstream market are ready to purchase a license in order to be active, we showed that the outside innovator favors the asymmetric licensing policy as long as the consumers are not too sensitive to the distance with their ideal variety. To the contrary, the double monopoly structure is chosen as soon as the consumers are sufficiently sensitive to product differentiation.

The vertically integrated structure differs in two dimensions from the separated model. The first is that the monopoly innovator can now directly set the price of the component it sells while it was bound to influence it through its licensing policy (i.e the number of licenses) in the separated case. The monopoly has indeed an additional optimizing tool. The second is that the timing of the game and specifically the timing of the earning of the profit of the innovating firm has changed. We can now write the total final profit of the inside innovator as the sum of the profits made in each stage of the game:

$$\Pi_U = \Pi_A + N_B \times \Pi_B \tag{2.1}$$

In the last stage of the game, the downstream department of the integrated firm only maximizes its sales profit with respect to the price of its component given the prices of the other active firms. The profit of the upstream division is indeed maximized and earned in the first stage of the game. There is a perfect separability between the licensing and the sales profits. This is due to the fact that in our framework, licenses are distributed through auctions which is equivalent to public fixed fee licensing contracts in our drastic innovation case. It is worth noting that the optimization problem of the downstream division of the integrated firm is exactly the same as any separated homogeneous good producer.

$$\Pi_A = \begin{cases} (a-c) \times 2N \times \sqrt{\dfrac{v-a-b}{t}} \text{if: } \sqrt{\dfrac{v-a-b}{t}} < \dfrac{1}{2N}, & (2.2) \\ (a-c) \times 1 \text{ otherwise.} & (2.3) \end{cases}$$

Anticipating the behavior of its manufacturing arm (identical to a separated firm) and the impact of its licensing policy on downstream equilibrium, the innovator maximizes the sum of the present licensing and future sales revenues with respect to the number of licenses issued in each component market. In most ICT industries, it appears that a significant amount of time runs between the acquirement of the necessary technology and the sales of the final goods (i.e development stage). We believe that it might be interesting to introduce an extra parameter representing the degree of the preference for present revenues of the integrated firm.

$$\Pi_U = \delta\Pi_A + N_B \times \Pi_B \tag{2.4}$$

We will now characterize the profit-maximizing licensing policy of the inside innovator depending on the values of parameter δ. We focus on the degree of patience of the innovator and the impact it has on the profitability of its licensing and vertical integration strategies. We do not analyze the effect of the sensitivity of potential downstream firms on the strategies of the upstream monopolist. Potential licensees are assumed to be infinitely patient as they do not bear any cost for having to pay the fixed fee up front.

2.2.2.2 Licensing policy under polar degrees of patience

Assuming the innovator is extremely patient (i.e $\delta = 1$), it would then maximize sales profits made in the second stage. If this is the case, the profit-maximizing licensing policy is identical for inside and outside innovators. The objective function of the patentee indeed remains the same because revenues made on the homogeneous component market are either equal to zero (when there are at least two active producers) or equal to the downstream homogeneous monopoly profit which is unaffected by vertical integration. In the end, vertical integration only affects the time at which the rent on the homogeneous component market is captured by the innovator. In our framework, vertical integration delays the earning of this rent whenever it is positive (i.e when an exclusive license is issued on the homogeneous market). Assuming that the innovator is perfectly patient, its total profit is unaffected by vertical integration.

Proposition 2.2.1. *The licensing policy is unaffected by vertical integration when the inside innovator is extremely patient (i.e $\delta = 1$). The profit-maximizing licensing policy of an extremely patient innovator is identical for outside and inside innovators (i.e the asymmetric structure for a mass component market and the double duopoly otherwise).*

Proof. The proposition directly results from the above reasoning. □

Taking now the other extreme value for the degree of patience of the inside innovator, we show that the profit-maximizing licensing policy is now affected by vertical integration. An extremely impatient inside innovator only maximizes the licensing revenue earned in the first stage that corresponds to the total downstream profit made on the differentiated component market. The innovator would thus be more tempted to eliminate the rent on the homogeneous market and transfer it onto the differentiated market. As soon as the innovator is imperfectly impatient, sales revenues are discounted giving an incentive to the inside innovator to transfer rent away from the market where it is active by delivering a license to a potential rival on component market A. Considering the case where there are only two potential firms, the innovator would then have to choose between the differentiated monopoly and the double duopoly structures. We find that the outcome of this trade-off relies on the relative value of parameters v and t, that is the relative sensitivity of consumers to the distance between their ideal and available varieties.

Proposition 2.2.2. *The double duopoly is the most profitable structure for an extremely impatient inside innovator when the differentiated component market is a niche market:*

$$v < 2c + \frac{3t}{4 \times 2^{\frac{2}{3}}}. \tag{2.5}$$

Otherwise, the differentiated monopoly is preferred.

Proof. See in the appendix. □

The double duopoly is the most profitable structure whenever the sensitivity to the distance is sufficiently high making the level of sales in the differentiated monopoly structure sufficiently low. Otherwise, the differentiated monopoly is the optimal structure for an extremely impatient inside innovator. This structure is desirable because it avoids the dissipation of downstream profits due to intense competition. Comparing the profit-maximizing licensing policy across integration regimes, we find that the double duopoly structure is chosen for a wider range of system valuations under vertical integration (i.e the threshold on v is higher in equation 2.5 than in equation 1.53 making the condition on t or v less restrictive). We can also notice that the differentiated monopoly is preferred to the asymmetric structure by the extremely impatient inside innovator. The differentiated monopoly structure enables the elimination of the rent on the homogeneous market (i.e discounted sales profit) persisting under both the double monopoly and asymmetric structures.

We compared the profit-maximizing licensing policy in the two polar cases regarding the degree of preference for the present. We showed that vertical integration matters for the licensing policy when the sensitivity to the distance is sufficiently high. The additional direct price instrument does not matter in this setting since it only determines the profit earned in the second stage which is identical to the profit function of an independent producer.

2.2.2.3 Profit-maximizing licensing policy for intermediate degrees of patience

We now attempt to derive the licensing policy in a more general setting where the preference for the present of the inside innovator can take any value between zero and one. For each range of the relative values of v and t, we compare the structures chosen by the patentee in the two polar cases and look for the value of the discounting factor for which the integrated and separated licensing policy becomes identical (i.e the innovator is sufficiently patient).

Lemma 2.1. *When the inside innovator is active in the homogeneous component and the valuation for the good is very high so that the differentiated monopoly structure is covered (i.e $v \geq 2c + \frac{3t}{4}$), the double monopoly structure is dominated by the differentiated monopoly .*

Proof. This lemma directly result from our results under polar values of δ. In both extreme cases, the double monopoly is dominated by the differentiated monopoly. □

Comparing the asymmetric and the differentiated monopoly structures, we find that the asymmetric structure is more profitable for the innovator when the discount rate is sufficiently high. The kinked equilibrium which is the most unfavorable to B producers is the unique equilibrium sustained by all values of v consistent with kinked equilibrium existence. It is also unfavorable to an imperfectly patient inside innovator UA because it results in more surplus being left to the downstream division and to the last (i.e discounted) period of the game.

The profit of the innovator under the asymmetric structure is increasing with the degree of patience whereas it is independent of δ under the double duopoly structure. From propositions 2.2.1 and 2.2.2, we know that for the niche component market range of parameters, the double duopoly structure is preferred even by an extremely patient patentee. It must then also be the case when the inside innovator is not as patient (i.e decreasing δ and decreasing profit under the asymmetric structure).

We present our results on the profit-maximizing licensing policy of the inside innovator active in the homogeneous component market assuming the unfavorable equilibrium is always selected. We find that for sufficiently high level of δ, it is profitable for the innovator to follow the optimal licensing policy of the perfectly patient innovator. We know from the previous section that the condition on v for which the double duopoly structure is the most profitable is less restrictive for the impatient innovator (i.e higher value of $\bar{v}$). As a consequence, there is an interval on v for which sufficiently impatient innovators will choose the double duopoly structure whereas patient innovators will prefer the asymmetric structure. Figure 2.1 illustrates the following proposition. When the value of the degree of patience of the inside innovator corresponds to a value located above the frontier, the licensing policy is the one of a perfectly patient patentee (and an outside innovator). There is no disagreement in niche component markets (i.e when the valuation for the system is sufficiently low) as all types of innovators (i.e with all values of δ) find it preferable to implement the double duopoly structure.

Proposition 2.2.3. *The profit-maximizing licensing policy of the inside innovator active in the homogeneous component market is:*

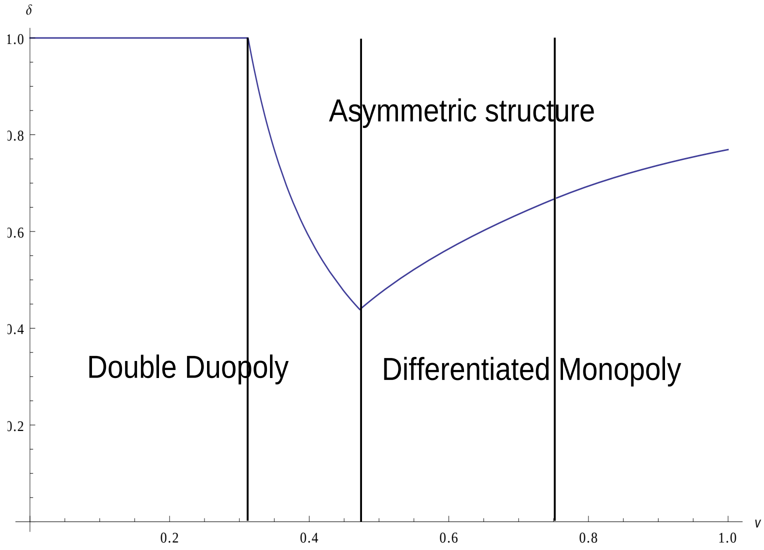

FIGURE 2.1: Licensing policy for various values of δ and v when $c = 0$ and $t = 1$

- *when* $v \geq 2c + \frac{3t}{4}$, *the kinked covered asymmetric structure dominates the covered differentiated monopoly structure and is the most profitable if:*

$$\delta \geq 1 - \frac{\frac{3t}{16}}{v - 2c - \frac{3t}{16}}; \tag{2.6}$$

- *when* $2c + \frac{3t}{4\times 2^{\frac{3}{2}}} \leq v \leq 2c + \frac{3t}{4}$, *the kinked covered asymmetric structure dominates the uncovered differentiated monopoly structure and is the most profitable if:*

$$\delta \geq \frac{2}{9} \times \left(\frac{9t}{32c + 3t - 16v} + 32\sqrt{3}\sqrt{\frac{(v - 2c)^3}{t(32c + 3t - 16v)^2}} \right); \tag{2.7}$$

- *when* $2c + \frac{3t}{4\times 2^{\frac{2}{3}}} \geq v \geq 2c + \frac{5t}{16}$, *the kinked covered asymmetric licensing policy dominates the competitive double duopoly structure and is the most profitable if:*

$$\delta \geq \frac{\frac{2t}{16}}{v - 2c - \frac{3t}{16}}; \tag{2.8}$$

- *when $v \leq 2c + \frac{5t}{16}$ (i.e niche component market), the double duopoly structure dominates the asymmetric structure and is the most profitable whatever the value of δ.*

Proof. These threshold values of δ directly result from comparisons of consistent profit functions. See in the appendix for explicit expressions. □

Taking $v = 2c + \frac{3t}{4}$, the asymmetric licensing policy is chosen by the patentee if $\delta \geq \frac{2}{3}$. Otherwise, the differentiated monopoly dominates. The value of $\bar{\delta}$ is increasing with v so that the range of values of δ for which the asymmetric structure is preferred is very narrow when the technology is used in a mass component market. We find that integration between the upstream innovator and one of the producers of the homogeneous component increases the range for which the double duopoly structure is the most profitable. Moreover the asymmetric structure is found to be dominated for sufficiently large levels of v. As a result vertical integration can, for some range of the parameters induce the inside innovator to deliver more licenses than an outside innovator. In the next section, we will study the case of an inside innovator active in the differentiated component market.

2.2.3 Vertical integration into the differentiated market

We now consider a different structure of the industry where the innovator is active in the differentiated component market. We study the incentives for such an inside differentiated innovator to license its technology to both competitors and complementors. As in the previous section, vertical integration mainly results in a delay of the earning of some downstream profit by the monopoly innovator. The fixed fee licensing agreement allows the innovator to capture expected downstream profit in the first period, before the development of the two final components. Vertical integration implies that the downstream component is developed and produced in-house and that the corresponding downstream profit is only made once it is actually sold. Since the amount of time between the two periods can be significant in the ICT industries to which our model mainly applies, we introduce a discount factor δ for the profit made in the second period. The profit of the inside innovator is written:

$$\Pi_U = \delta\Pi_A + N_B \times \Pi_B \tag{2.9}$$

Profits are earned and fully determined in each of the periods independently, meaning that the optimization problem of each firm remains the same as in the separated structure. The downstream division of the integrated firm simply maximizes the downstream profit with respect to its component price as any other downstream active firm. The downstream division does not internalize the effect of its pricing decision on the upstream profit because the latter is fully determined and earned in the first period through fixed fee or auction licensing. The profit of the industry is captured through the auctioning of licenses and through the downstream division of the integrated firm. In the end, vertical integration results in the innovator to discount (if imperfectly patient) its sales profit. In this section, the innovator is active in the differentiated market which makes it feasible to extract positive licensing revenues from that market. The discounted profit made in the second period evolves less dramatically with the number of other active firms in the same component market (i.e the number of licenses delivered in that specific market) compared to the case of vertical integration into the homogeneous market. We notice that if the inside innovator is perfectly patient (i.e $\delta = 1$), its objective function and licensing policy are identical to those of an outside innovator. In this case, we showed that the profit-maximizing licensing policy is to implement the asymmetric structure when v is sufficiently large and the double duopoly structure when the technology is used in a niche component market. We also proved that the asymmetric structure always dominates the double monopoly and the differentiated monopoly structures. Lemma 2.2 shows that it is also the case for an inside differentiated innovator.

Lemma 2.2. *When the inside innovator is active in the differentiated component market, the asymmetric structure dominates the double monopoly and differentiated monopoly structures for any $\delta \in (0, 1)$ and any level of system valuation (i.e $v \geq 2c$), no matter the selected kinked equilibrium.*

Proof. See in the appendix. □

Now that it is clear that the asymmetric structure dominates both the double monopoly and the differentiated monopoly structures, we study the trade-off between the two remaining structures. We expect that vertical integration into the differentiated market will increase the range for the asymmetric structure private desirability. This should be the case because a lower share of the industry profit would be made on the differentiated market in the asymmetric than in the double duopoly structure. The share of discounted profit in the total profit would be reduced. In our setting, vertical integration provides an incentive to transfer the rent away from the market where the integrated firm is active. This favorable effect of the asymmetric structure is to be traded-off against the increase in efficiency provided by the covered double duopoly structure while the asymmetric structure is uncovered. As a result and in contrast with the separated case, we find that the asymmetric structure can be favored when it is uncovered. In particular, when $\delta = 0$, we find that the asymmetric structure is the most profitable structure for any level of system valuation (i.e $v > 2c$ such that the good is demanded). In the following proposition, we derive the profit-maximizing licensing policy depending on the values of δ and v.

Proposition 2.2.4. *The profit-maximizing licensing policy of the inside innovator active in the differentiated component market is to implement the asymmetric structure:*

- *when its technology is used in a mass component market (i.e $v \geq 2c + \frac{5t}{16}$) whatever the value of δ.*
- *when the technology of the inside innovator is used in a niche component market (i.e when both structures are uncovered, $v \leq 2c + \frac{3t}{16}$) and if the inside innovator is sufficiently impatient:*
$$0 \leq \delta \leq \bar{\delta}_1 = 0,737. \tag{2.10}$$
- *for intermediate values of v (i.e $2c + \frac{3t}{16} \leq v \leq 2c + \frac{5t}{16}$) when the inside innovator is sufficiently impatient:*
$$\delta \leq \bar{\delta}_2{}^{1}. \tag{2.11}$$

Otherwise, the innovator chooses the double duopoly structure.

[1]The precise expression of $\bar{\delta}_2$ is presented in the appendix.

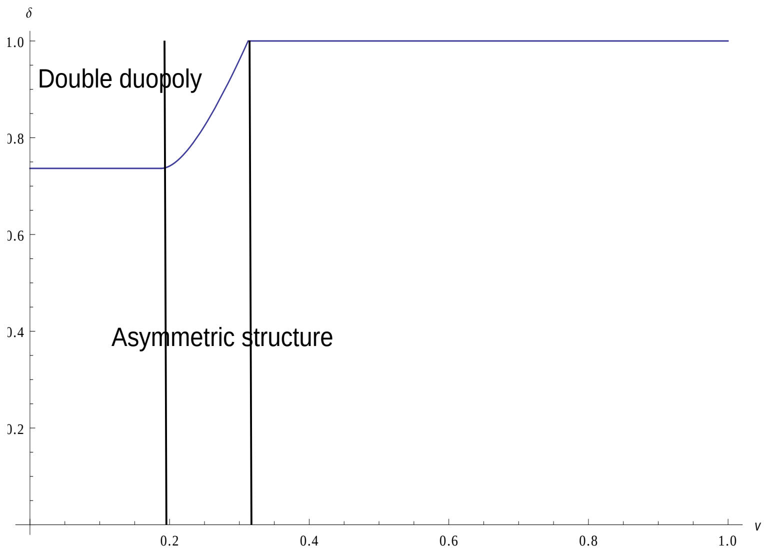

FIGURE 2.2: Optimal structures for various values of δ and v when $c = 0$ and $t = 1$

Proof. See in the appendix. □

Figure 2.2 illustrates this proposition. When the value of the degree of patience of the inside innovator corresponds to a value located above the frontier, the profit-maximizing licensing is the one of a perfectly patient patentee (and of an outside innovator). There is no disagreement in mass component markets (i.e when the valuation for the system is sufficiently low) as all types of innovators (i.e with all values of δ) find it preferable to implement the asymmetric structure.

When v is such that $v \leq 2c + \frac{3t}{16}$, a sufficiently impatient inside innovator chooses to earn a higher profit in the first period and to disregard the increase in social surplus (and total profit of the industry) due to the elimination of double marginalization provided by the double duopoly structure. Otherwise (i.e when the inside innovator is sufficiently patient), the double duopoly structure is preferred when all structures are uncovered. The above result can be explained by the difference in the way costs and benefits of the double duopoly structure behave. On the one hand, the gain associated with the double duopoly structure is independent of δ. The elimination of the horizontal double marginalization increases the share of the potential social surplus $v-2c$ that is effectively realized. This positive effect is constant for a given value of the gross social surplus. On the other hand, the loss associated with the discounting of a share of the differentiated downstream profit decreases with the discounting rate δ. As a consequence, there exists a sufficiently low discounting rate such that the costs overcome the constant benefit of the double duopoly structure.[2]

When v is such that $2c + \frac{3t}{16} \geq v \geq 2c + \frac{5t}{16}$, the value of $\bar{\delta}_2$ tends to 1 as v goes to the higher bound of the interval. The asymmetric structure is indeed chosen for any δ when it is covered. Similarly, $\bar{\delta}_2$ tends to 0.737 as v goes to the lower bound of the interval which is consistent with 2.2.4. In between, the loss in sales and in efficiency increases as the asymmetric structure becomes more and more uncovered (i.e the level of sales decreases with the level of v) while the double duopoly structure remains just covered. The range for which the asymmetric structure is preferred shrinks as v decreases and the loss in sales rises (i.e the partial derivative of $\bar{\delta}_2$ with respect to v is positive). When v takes its lowest value in the interval, $\bar{\delta}_2$ reaches its minimum level. Thus when the double duopoly becomes uncovered, the loss in sales increases in the same way for both structures so that the value of $\bar{\delta}_1$ remains constant as shown in proposition 2.2.4.

2.2.4 Alternative variety in-house development

In this section, we allow the upstream monopoly innovator to develop an additional alternative of the differentiated downstream component. The number of potentially active firms initially remains equal to 2 in both downstream markets. Instead of considering the case where the upstream monopolist is integrated with one of these potentially active firms, we now analyze the situation where the fact of being an inside innovator brings an additional benefit that is the increase in the number of potentially active firms. It implies that new structures of the downstream industry are now available to the innovator:

- the very asymmetric structure (i.e $N_A = 1, N_B = 3$)

[2]Equilibrium selection only matters in situations where the differentiated market is at a kinked equilibrium that is when the asymmetric structure is covered. We find that the asymmetric also dominates the double duopoly structure when the favorable and balanced equilibrium are selected (when existing). Proposition 2.2.4 remains valid for these kinked equilibrium selection processes.

- the differentiated oligopoly structure (i.e $N_A = 2$, $N_B = 3$)

These structures seem to be more privately and socially desirable since they enable a decrease in the maximal transportation cost faced by consumers that are located the furthest away from firms selling component B. The market covered constraint loosens as it is less costly to serve the entire market. We find that the structure of the licensing policy remains the same as in the case of integration into the differentiated market. A perfectly patient innovator prefers the double duopoly structure in a niche market and the very asymmetric structure in a mass market. When the innovator is sufficiently impatient, the very asymmetric structure is found to be the most profitable for any kind of downstream markets (i.e any levels of v and t).

Proposition 2.2.5. *The very asymmetric structure is the most profitable structure for a sufficiently impatient patentee (i.e low value of δ):*

- *an innovator developing an additional variety of the the differentiated component in-house finds it preferable to implement the Very Asymmetric Structure (i.e $N_A = 1, N_B = 3$) when its technology is used in a mass component market (i.e when $v \geq 2c + \frac{5t}{36}$ and both structures are covered) whatever the value of δ;*
- *the Very Asymmetric Structure is preferred when the technology of the inside innovator is used in a niche component market (i.e when $v \leq 2c + \frac{3t}{36}$ and both structures are uncovered) and if the inside innovator is sufficiently impatient: $0 \leq \delta \leq \bar{\delta}_3 = 0,605$;*
- *the Very Asymmetric Structure is preferable for intermediate values of v (i.e when $2c + \frac{3t}{16} \leq v \leq 2c + \frac{5t}{16}$ and only the differentiated oligopoly structure is uncovered) and if the inside innovator is sufficiently impatient: $\delta \leq \bar{\delta}_4$.*[3]

Otherwise, the innovator chooses the differentiated oligopoly structure.

Proof. See the appendix. □

[3] The precise expression of $\bar{\delta}_4$ is presented in the appendix.

The structure of the proof remains the same as in the previous section. We extend lemma 2.2 to the very asymmetric structure and show that it dominates both the asymmetric and the double duopoly structures as well as the double monopoly and differentiated monopoly structures. Then we compare the very asymmetric and the differentiated oligopoly structures. The same intuition as in the previous section drives our results. Starting from the value of v for which the very asymmetric structure is kinked covered (i.e just covered), the loss in sales due to the uncovered very asymmetric structure increases as v decreases (i.e positive partial derivative of $\bar{\delta}_4$ with respect to v). The inside innovator is required to be more and more impatient in order to maintain the private desirability of the very asymmetric structure as the valuation v decreases and the spread in sales with the differentiated oligopoly structure increases. Once v is low enough to make both structures uncovered, the difference in sales remains constant (i.e the value of $\bar{\delta}_3$ remains equal to 0.605). We find that $\bar{\delta}$ takes lower values when an additional variety of component B is produced in-house as compared to the differentiated integration case.[4] The interval on parameter δ for which an exclusive license is sold on the homogeneous market is reduced.

2.2.5 Conclusion

We can now compare the effects on the profit-maximizing licensing policy of a monopoly innovator in a system market of the two forms of vertical integration we have analyzed so far. Assuming that the time dimension matters for the innovator, both forms induce the inside innovator to transfer the rent away from the market where it is active. In the separated case, the innovator indeed captures profit ex ante through fixed fee licensing. A vertically integrated firm has to wait for the final component to be developed in-house to earn the profit generated by its technology for that particular product. Vertical integration delays the earning of the profit of the production unit to the time when sales are effectively realized. An imperfectly patient inside innovator will mitigate this effect by reducing the share of the total industry profit affected (i.e transferring profits to the complementary market).

[4] We can see in propositions 2.2.4 and 2.2.5 that $\bar{\delta}_3 \leq \bar{\delta}_1$.

The in-house production of the differentiated component by a sufficiently impatient innovator results in a strong dominance of the asymmetric structure. The double duopoly structure is preferable for a narrow range of parameters. The exclusive license on the complementary good market reduces the share of the social surplus available on the differentiated market. In this case, integration results in a less competitive complementary market for some range of the parameters (i.e v, t). On the other hand, integration into the homogeneous component market makes the differentiated monopoly and the double duopoly structures (i.e $N_A = 2$) more attractive (to the expense of the asymmetric one). The inside innovator is willing to increase the number of varieties of the complementary good for a wider range of parameters than an outside innovator. Integration with a producer of the homogeneous component generates more competition in the complementary market. Both forms of vertical integration tend to affect the structure of the complementary market but not the one where the patentee is active.

We can compare these results with the literature on vertical integration of a monopoly innovator whose technology is used in a single downstream market. Arora and Fosfuri (2003) find in a model with multiple innovators and product differentiation that the profit-maximizing number of two-part tariff licenses is lower when an innovator is active in the product market. In our model of licensing in complementary markets we do not observe this effect. We show that vertical integration matters for the degree of competition of the complementary market. Depending on the form of integration, it results in an increase or a decrease in the intensity of competition in the other component market.

Rey and Salant (2012) show in a model of spatial differentiation that vertical integration does not matter for the licensor of a drastic innovation. Doganoglu and Inceoglu (2014) find that it is optimal for an innovator of a drastic innovation using per unit royalty two-part tariffs to remain outside the final market when there is downstream product differentiation. These authors generalize the result of Sandonis and Fauli-Oller (2006) showing that the outside innovator is able to achieve the multiproduct monopoly outcome. Ignoring the difference in the time of the earning between licensing and sales revenues, we find that single vertical integration had no effects on downstream competition nor on the optimal licensing policy of the innovator. As a result, vertical integration is not found to be strictly profitable for the innovator. This is consistent with the results of Rey and Salant (2012) and tends to show that the effect of vertical integration on licensing does not change in complementary markets. In the following section, we show that this is not the case when the innovator is active in the production of both components. The double vertical integration has effects on both downstream price equilibria and the licensing policy of the patentee.

2.3 Conglomerate merger and Optimal Licensing

2.3.1 Introduction

This section extends our previous work on technology licensing used in downstream perfectly complementary goods to the case of an inside innovator producing both downstream goods. We characterize the profit-maximizing number of licenses issued by an upstream monopolist active in the production of both downstream components. We show that the structure of the licensing policy chosen by the patentee remains the same as in other ownership structures. The double duopoly is more profitable in niche component markets whereas the asymmetric structure is preferred when the valuation for the system is high (i.e in mass component markets). Nevertheless, in the covered asymmetric structure, the inside innovator producing both components faces a commitment problem with respect to the use of its additional pricing instruments leading to a degraded licensing (i.e first stage) revenue, partially compensated by an increase in sales (i.e second stage) revenue. As a consequence, when the valuation for the system is high, the innovator is doing worse when it is active in both complementary markets. As long as there are enough potential downstream firms for the innovator to freely determine the structure of the industry through the number of licenses it delivers, we find that it would rather not be active in both complementary markets. However, when the double duopoly is not feasible, we show that in the uncovered asymmetric structure (i.e in niche component markets), it is profitable for the upstream monopolist to vertically integrate into both complementary good markets.

The analysis of this vertical structure is motivated by the existence of significant conglomerate firms in the IT industry in which we find a tendency for high technology firms to move onto complementary markets either by developing their own solution or by merging with one complementor. We can for example mention some merger cases such as Intel/McAfee, Google/Motorola, Microsoft/Nokia and Cisco/Tandberg. We can also argue that Google is strategically moving from one complementary good to another (e.g Chrome web browser and operating system, Nexus smartphones, Youtube...).

In the previous section of this chapter, we explored the implications of single vertical integration of the upstream monopolist into one of the downstream component markets. Ignoring the difference in the time of the earning between licensing and sales revenues (i.e ignoring the effect of the required time for the development of products based on the technology), we found that single vertical integration had no effects on downstream competition nor on the equilibrium licensing policy of the innovator. When we consider double vertical integration (i.e conglomerate merger), it turns out that it might affect downstream equilibria and thus the its profit-maximizing licensing policy. The asymmetric structure is particularly impacted by the conglomerate merger between the inside innovator and one of the complementary good producers. The merged entity is now able to increase its sales revenue at the expense of the independent component B producer. In mass component markets, the licensing revenue (equal to the equilibrium profit of the independent producer) of the merged entity decreases because of its inability to commit not to use its additional pricing instruments. As a consequence, from the view point of the monopoly innovator, the conglomerate merger makes the asymmetric structure less attractive as it is made less efficient with respect to the total profit of the industry which is fully captured by the patentee. The condition for which the asymmetric structure is chosen tightens as a result of the merger. We use the framework developed in chapter one to analyze the situation where the monopoly innovator directly produces both downstream complementary goods. The monopolist thus directly set component prices and earns the corresponding sales revenue. The first section deals with the characterization of downstream equilibria in the post merger structure. Section two derives the implications of the merger in terms of licensing policy and section three concludes on how our results differ from the literature on conglomerate mergers and patent licensing.

2.3.2 Downstream equilibria

Given the number of licenses issued by the upstream innovator, we describe the downstream equilibria for each structure of the industry. The conglomerate merger matters only for the double monopoly and the asymmetric structures. This is due to the fact that in both structures, the price of component A is not fully determined by perfect Bertrand competition. This implies that the merging firms effectively control both pricing tools. Otherwise, it only sets the price of component B and the conglomerate case is equivalent to the inside B component innovator case. As we mentioned earlier, single vertical integration does not matter in our framework (assuming the innovator is perfectly patient or assuming away the time dimension) due to profit separability resulting from fixed fee or auction licensing. Then downstream equilibria of the differentiated monopoly and double duopoly remain the same as in the separated case. We focus now on the description of structures for which the conglomerate merger matters.

2.3.2.1 Double monopoly

In the conglomerate double monopoly case, the industry is fully integrated. There is no competitor and the price externalities across complementors are internalized. For the given number of varieties of the differentiated product (i.e equal to one), the profit of the industry is maximized. The price of the system is equal to the integrated monopoly price (see equations 2.14 2.15). The double monopoly structure is made more efficient with the conglomerate merger as the horizontal double marginalization problem disappears. It is also worth noting that the conglomerate double monopoly and the differentiated monopoly structure equally perform. In the differentiated monopoly structure, horizontal double marginalization is however eliminated with a different tool that is perfect

Bertrand competition. It leads to the same total profit of the integrated inside innovator:

$$\Pi_{UAB_2} = \begin{cases} -\dfrac{4(2c-v)\sqrt{\frac{v-2c}{t}}}{3\sqrt{3}}, \text{ when the market is uncovered,} & (2.12) \\ v - 2c - \dfrac{t}{4}, \text{ when the market is covered.} & (2.13) \end{cases}$$

First order conditions for profit maximization lead to the following system price:

$$p^M = a + b = \begin{cases} \dfrac{2(v+c)}{3}, \text{ when the market is uncovered,} & (2.14) \\ v - c - \dfrac{t}{4}, \text{ when the market is covered.} & (2.15) \end{cases}$$

2.3.2.2 Asymmetric structure

In this section, we show that there is no symmetric equilibrium of the asymmetric structure. This results in an efficiency loss when the valuation for the system is high. In this structure of the industry, there are only two firms active in the production of components: the merged entity that produces both components and the independent firm producing one variety of the differentiated component B. Given the prices set by its competitor, the independent firm behaves here exactly in the same way as in the separated cases. Its profit function indeed takes the same form:

$$\Pi_{UAB_2} = \begin{cases} (d-c)\sqrt{-\dfrac{a+d-v}{t}}, \text{ when the market is uncovered,} & (2.16) \\ (d-c)\left(\dfrac{b-d}{t} + \dfrac{1}{4}\right), \text{ when the market is covered.} & (2.17) \end{cases}$$

When the price of the integrated system $a+b$ is sufficiently large to make the independent firm unwilling to cover the market, it behaves as a local monopolist and charges the following price d (i.e a function of price a):

$$d = \frac{1}{3}(-2a + c + 2v), \text{ if:} \tag{2.18a}$$

$$a \geq v - c - \frac{3 \times t}{16}. \tag{2.18b}$$

Otherwise, the independent firm is willing to steal business from the merged entity and charges the following price d (i.e a function of price b):

$$d = \frac{1}{8}(4b + 4c + t). \tag{2.19}$$

On the other hand, the integrated firm is maximizing its downstream profit with respect to its two pricing instruments given the value of the price charged by the independent firm:

$$\Pi_{UAB_2} = \begin{cases} \dfrac{a-c}{2} + (b-c)\left(\dfrac{1}{4} + \dfrac{b-d}{t}\right), \text{ when the } B \text{ market is competitive} & (2.20) \\ (a-c)\left(\sqrt{-\dfrac{a+b-v}{t}} + \sqrt{-\dfrac{a+d-v}{t}}\right) + (b-c)\sqrt{-\dfrac{a+b-v}{t}}, & (2.21) \end{cases}$$

when $\frac{-16b^2+32bd-8bt-16d^2-8dt-t^2+16tv}{16t} \leq a \leq v - d$. For given values of v and d, we find that it is never profitable for the integrated firm to set its system price such that the market B is purely competitive. The merged entity can always do better by increasing its price a till a kinked covered equilibrium is reached. It leaves just enough surplus to consumers to get the maximum level of sales. This rational pricing behavior is also found in the separated asymmetric structure. Maximizing the uncovered profit function of the new entity with respect to both price instruments, we obtain the following uncovered best response sub-function, only valid when the price d is sufficiently high (with respect to

v) making it unprofitable to fully cover the market:

$$a = \frac{1}{3}(c - 2d + 2v) \tag{2.22a}$$

$$b = \frac{(c + 2d)}{3}, \text{ if and only if:} \tag{2.22b}$$

$$d > \sqrt{3tv - 6ct} + \frac{1}{4}(4c - 3t). \tag{2.22c}$$

It is worth noting that the expression of the price of component A in equation 2.22a is the same as in the separated case when the market is profitably uncovered. The expression of b however differs in the conglomerate case (see equation 2.22b). It is lower than in the separated case which results in the reduction of the inefficiency due to double marginalization and in the increase of system sales. This subsidization behavior carries over to the market covered case in which d is low (with respect to v) leaving sufficiently high surplus to consumers and making the merged entity willing to cover the market. Given the fact that the integrated firm is unwilling to leave unnecessary surplus to consumers, it charges the level of price a determined by the following kinked market condition:

$$a = -\frac{(c - d)^2}{9t} - \frac{c}{6} - \frac{5d}{6} - \frac{t}{16} + v. \tag{2.23}$$

Under this condition on a, the integrated firm maximizes its profit with respect to b leading to its profit-maximizing level:

$$b = \frac{1}{3}(c + 2d). \tag{2.24}$$

The component B market is subsidized by the merged entity. This low price b allows an increase in a while continuing to serve the entire market (see equations 2.23 and 2.24). For consistent values of v, the price of component A increases with the conglomerate merger. These best response functions lead to a unique uncovered equilibrium and multiple asymmetric kinked equilibria. There is no symmetric equilibrium allocation of component B sales because of the price charged by the integrated firm for the component B. In the uncovered equilibrium, the subsidy to the component B market allows an increase in sales and a reduction in the inefficiency caused by the horizontal double marginalization problem. The price of system 1 (i.e $a + d$ expressed in equation 2.25d) remains excessively high and identical to its separated level while system 2 (i.e $a + b$ expressed in equation 2.25e) now maximizes the joint profit:

$$a = \frac{1}{5}(c + 2v) \tag{2.25a}$$

$$b = \frac{1}{15}(7c + 4v) \tag{2.25b}$$

$$d = \frac{1}{5}(c + 2v) \tag{2.25c}$$

$$a + d = \frac{2}{5}(c + 2v) \tag{2.25d}$$

$$a + b = \frac{2(c + v)}{3}. \tag{2.25e}$$

This is the equilibrium of the asymmetric structure if and only if:

$$2c + \frac{15t}{8}(4 - \sqrt{15}) \leq v. \tag{2.26}$$

We also characterize a particular kinked equilibrium in which the uncovered best response sub-function of the independent producer results in a covered market. We assume that the two downstream firms coordinate on this kinked equilibrium that we name the strategic kinked equilibrium. It differs from the kinked equilibria of the asymmetric structure in the separated case because component B prices are asymmetric and price a is higher:

$$\begin{cases} b = c - \frac{1}{2}\sqrt{15}\sqrt{t^2} + 2t & (2.27) \\ a = -c + \frac{9}{8}\sqrt{15}\sqrt{t^2} - \frac{9t}{2} + v & (2.28) \\ d = c - \frac{3}{4}\sqrt{15}\sqrt{t^2} + 3t. & (2.29) \end{cases}$$

We find that the merged entity increases its price a inducing the independent firm to decrease its price d because of the strategic substitutability between the prices of complementary goods in the uncovered and kinked markets. This results in a decrease in the margin of the independent firm benefiting the merged entity. On the other hand, the price of its component B is set to just cover the market. Equivalently we can consider that the merged entity decreases its price b forcing the independent firm to charge a lower price in order to avoid further loss in sales. The decrease in the prices of component B allows the integrated firm to increase its price a (determined by the kinked condition) without suffering any loss in sales.

Proposition 2.3.1. *Given that the market is in the asymmetric structure, the conglomerate merger (A - B_2) is in our framework always profitable (i.e in uncovered and covered equilibria) for the merging firms because of the resulting reduction in the profit margin of the independent producer.*

Proof. In a niche market, the conglomerate merger between downstream divisions is profitable because it eliminates the inefficient horizontal double marginalization. Sales increase and the joint profit is maximized. In the strategic kinked equilibrium, the merger results in higher market shares and allows the merged entity to extract more from the surplus generated by the sales of system 1. This is due to the reduction of the profit margin of the independent producer. Overall, the conglomerate merger between the downstream divisions producing complementary goods is profitable. □

Proposition 2.3.2. *Given that the market is in the asymmetric structure, the conglomerate merger increases the total profit of the industry and the welfare when the market of component B is a niche market (i.e when $v \leq 2c + \frac{15}{8}\left(4 - \sqrt{15}\right) \times t$) because of the elimination of the horizontal double marginalization on the integrated system.*

Proof. In the uncovered equilibrium of the separated asymmetric structure, there are two distinct local markets in which monopolists sell the components of the system to consumers. There is horizontal double marginalization resulting in excessively high prices and low sales. The double vertical integration ensures efficient pricing of the integrated system and maintains the price of the alternative system (see equations 2.25d and 2.25e). As a result, the total profit of the industry and social surplus necessarily increase with the merger in niche component markets. □

In the kinked equilibrium, the asymmetric pricing of component B in the conglomerate case is socially costly (compared to a symmetrically covered B market) as some consumers will choose in equilibrium a product that is further away from their ideal variety. The resulting additional disutility constitutes a loss for society. As a consequence, the conglomerate merger decreases social welfare and total industry profit (i.e total profit of the innovator) when the valuation for the system is sufficiently high.

Knowing from proposition 2.3.2 that the total profit of the industry increases for some range of the parameters (i.e in niche markets), the inside innovator benefits from directly producing both complementary goods in the asymmetric structure. Taking the licensing policy as being exogenously fixed (e.g in the short run or when there are not enough potential licensees), the double vertical merger (merging with one downstream producer of each component) is profitable for the innovator when the component B market is a niche market. This is true because the upstream innovator is able to capture the entire profit of the downstream industry which is increased by the merger.

Taking the licensing policy and the structure of the industry as given and focusing on the outcome of the downstream market, we find that a conglomerate merger between the monopoly producer of the homogeneous component and one of the two differentiated producers of the complementary good is profitable and welfare enhancing when the valuation for the system v is sufficiently low with respect to the sensitivity to product differentiation t. This is the standard Cournot result applied to spatially differentiated complementary markets. Cournot showed that an industry is more efficient with one monopoly than two monopolies in complementary markets when the demand function for the system is elastic. In the uncovered asymmetric structure, the demand is elastic and differentiated producers act as local monopolists on two segmented markets. The conglomerate merger reduces the number of monopolists on one of the local complementary markets and eliminate the double marginalization problem on that market. Finally, we show that in the asymmetric structure, the innovator benefits from directly producing both complementary goods when the valuation for the system is low whereas the opposite is true when it is sufficiently high (i.e in mass markets that are asymmetrically covered). Given the outcome of downstream competition when the patentee produces both components, we look for the profit-maximizing number of licenses. It is important to keep in mind that in this case, there is no symmetric price equilibrium of the asymmetric structure.

2.3.3 Optimal licensing policy

We know that in our framework, there is no specific vertical effects of the conglomerate merger and that the upstream firm is able to capture the full industry profit in any integration regime.[5] As a consequence, the profit-maximizing licensing policy of the patentee results in the maximization of the downstream industry profit. In the previous section, we found that the conglomerate merger does not impact downstream equilibria of the structures where two homogeneous producers are active. Namely the differentiated monopoly and the double duopoly structures. We also found that the double monopoly is made more efficient by the conglomerate merger and is now doing as well as the differentiated monopoly. However, we know from our analysis of the separated case that both structures are undesirable because of the higher transportation costs implied by the production of a unique variety of component B. These structures are more likely to be uncovered and are more privately and socially costly. The optimal licensing policy of the integrated inside innovator that maximizes the total profit of the downstream industry can only consist in the choice of the asymmetric or the double duopoly structure. We now compare the total downstream profit in each structure for each interval of system valuation v.

When both structures are uncovered, the double duopoly is more efficient as it enables the full elimination of the double marginalization problem (i.e $a = c$) whereas the price of system 1 is still excessively high in the uncovered equilibrium of the asymmetric structure. The profit of the integrated inside innovator in the uncovered case (i.e

[5] Both forms of vertical integration do not have any effect on the ability of the innovator to extract downstream profits.

$v < 2c + \frac{3t}{16}$) is equal to:

$$\Pi_{UAB} = \frac{-4}{225}\left(25\sqrt{3} + 18\sqrt{5}\right) \times t^{\frac{1}{2}}(v-2c)^{\frac{3}{2}}, \text{ in the asymmetric structure,} \tag{2.30a}$$

$$\Pi_{UAB} = \frac{8}{3\sqrt{3}} \times t^{\frac{1}{2}}(v-2c)^{\frac{3}{2}}, \text{ in the double duopoly.} \tag{2.30b}$$

We observe in equations 2.30a and 2.30b that the expression $t^{1/2}(v-2c)^{3/2}$ is multiplied by a higher factor in the double duopoly structure as $\frac{8}{3\sqrt{3}}$ which can be approximated by 1.5396 is greater than $\frac{4}{225}\left(25\sqrt{3} + 18\sqrt{5}\right)$ which is approximately equal to 1.48534. Thus, when $v < 2c + \frac{3t}{16}$, the innovator chooses the double duopoly structure because it yields to a higher profit of the industry. We get the same result when v increases making the double duopoly structure just covered. When all structures are uncovered except the double duopoly (i.e $2c + \frac{3t}{16} < v < 2c + \frac{15}{8}\left(4 - \sqrt{15}\right)t$), the latter still generates a higher profit:

$$\Pi^{2D}_{UAB} = v - 2c - \frac{t}{16} \geq \Pi^{A}_{UAB} = \frac{1}{225}(-4)\left(25\sqrt{3} + 18\sqrt{5}\right)(2c - v)\sqrt{\frac{v-2c}{t}}. \tag{2.31}$$

We know from the previous section that for $v < 2c + \frac{15}{8}\left(4 - \sqrt{15}\right)t$, the efficiency of the asymmetric structure is increased by the conglomerate merger but the innovator can still do better with the use of the double duopoly structure which fully eliminates the horizontal double marginalization problem. There is an interval on v (i.e $2c + \frac{15}{8}\left(4 - \sqrt{15}\right)t < v < 2c + \frac{5t}{16}$) such that both structures are at a kinked equilibrium. In this case the double duopoly structure still dominates because in the asymmetric structure, full coverage of the market is reached through an asymmetric pricing of component B leading to higher transportation costs, lower social surplus as well as a lower price a and total profit of the innovator:

$$\Pi^{2D}_{UAB} = v - 2c - \frac{t}{16} \geq \Pi^{A}_{UAB} = v - 2c + \frac{1}{8}\left(12\sqrt{15} - 47\right)t, \tag{2.32a}$$

$$\text{because: } -\frac{1}{16} = -0.0625 \geq \frac{1}{8}\left(12\sqrt{15} - 47\right) = -0.065525. \tag{2.32b}$$

Turning now to the case where the asymmetric structure is at the strategic kinked equilibrium while the double duopoly structure is at the competitive equilibrium (i.e $2c+\frac{3t}{4} > v > 2c + \frac{5t}{16}$). The double duopoly is not perfectly efficient because the competition between the two producers of the differentiated good is too intense from the view point of the innovator. Unnecessary amount of surplus is left to consumers and the ability of the industry to capture social surplus is degraded. As opposed to the outcome in the separated case, the asymmetric structure is here imperfectly efficient as well. In the separated case, the asymmetric structure is covered and at kinked equilibrium when $v > 2c + \frac{5t}{16}$. Since both component B producers are independent, the equilibrium price is symmetric and variety choices of consumers are efficient. In the conglomerate case however, the downstream division of the merged entity maximizes its second period (i.e sales) profit by stealing some business from the independent component B producer. Leading to asymmetric B component pricing, efficiency losses and decreased downstream industry profit. We indeed found in the previous section that when the market is covered, the conglomerate merger is profitable for downstream divisions of the merging parties but it decreases the profit of the independent firm (i.e the licensing, first period revenue of the merged entity). As a consequence, there is an interval on v for which the loss due to competition in the differentiated market (i.e under the double duopoly structure) is lower than the loss generated by inefficient allocation of sales (i.e under the asymmetric structure). The values of v for which the innovator prefers the competitive double duopoly to the kinked asymmetric structure are:

$$2c + \frac{5t}{16} < v < 2c + \frac{1}{8}\left(49 - 12\sqrt{15}\right)t, \tag{2.33}$$

where $\frac{1}{8}\left(49 - 12\sqrt{15}\right) = 0.315525$ and $\frac{5}{16} = 0.3125$ meaning that this situation arises for only a very small portion of the consistent interval. As the value of v is increasing with respect to t, the cost of inefficient allocation of sales decreases and competition intensifies. A lower value of t implies a lower social loss caused by the inefficient asymmetric allocation of sales (i.e decreased sensitivity to the distance of consumers) as well as a more intense competition between independent differentiated producers (and an associated rise in the surplus unnecessarily left to consumers in the double duopoly).

Proposition 2.3.3. *The asymmetric structure is preferred by the innovator active in both complementary markets to the double duopoly when the sensitivity to the distance is sufficiently low (i.e the valuation for the system is sufficiently large) making the loss due to inefficient allocation of sales (i.e in the asymmetric structure) lower than the one caused by intense competition (i.e in the double duopoly structure) which is when:*

$$v > 2c + \frac{1}{8}\left(49 - 12\sqrt{15}\right)t. \tag{2.34}$$

Otherwise, the double duopoly structure is more profitable for the innovator and the industry.

We observe that the structure of the profit-maximizing licensing policy is very similar across integration regimes. The double duopoly structure is still privately desirable when it does not result in a competitive outcome. Nevertheless, there is a small range of parameters for which the double duopoly remains the most profitable structure despite effective competition in the differentiated market. As a consequence, the double vertical integration (or vertical and conglomerate merger) results in a slight increase in the range of parameters for which the double duopoly is chosen. It is worth noting that the conglomerate merger increases the downstream profit made under the asymmetric structure but results in a decrease in the total profit of the integrated firm (due to a decrease in the first period licensing revenue) when the market is covered. Such a conglomerate merger is thus strictly unprofitable in the covered asymmetric structure because of the lack of a commitment tool for the merged entity not to steal business from the independent B component producer. Given the constraint on the number of potentially active firms (i.e $N_A = N_B = 2$), the equilibrium licensing policy maximizes the social surplus because each firm has access to the technology when the valuation for the system is low. Otherwise, the market is fully covered and all gains from trade are realized in both the asymmetric and double duopoly structures.

2.4 Conclusion

In this chapter, we analyzed the impact of vertical integration on the licensing strategy used by a monopoly innovator whose technology is used in complementary markets. Putting aside the time dimension of the development process of downstream products, we find in line with Rey and Salant (2012) that single vertical integration does not change the profitable patent licensing strategy. The double vertical integration will however have more impacts on downstream equilibria and licensing policy.

Taking the structure of the industry as given ($N_A = 1$, $N_B = 2$) and focusing on the downstream profit of the merged entity (i.e sales revenues), we find that the conglomerate merger between downstream divisions is profitable. Moreover, in the uncovered equilibrium of the asymmetric structure, an inside innovator would find it profitable to vertically integrate with one producer of each component. This is in contradiction with Doganoglu and Inceoglu (2014) and Sandonis and Fauli-Oller (2006) showing that the innovator prefers to stay outside the downstream market.

We show that an integrated inside innovator active in each of the downstream markets always licenses to an independent producer of component B. In addition it licenses to an independent producer of the homogeneous component A for a greater range of parameters (i.e lower threshold on v for the profitability of the double duopoly structure) than an outside innovator does. As a result, the tendency for the innovator to license is higher for an integrated inside innovator than for an outside innovator. In contrast with Rey and Salant (2012), we conclude that integration can matter for the licensing of a drastic innovation in complementary markets. Our findings are also in contradiction with the work of Arora and Fosfuri (2003) showing in a model with multiple innovators that the profit-maximizing number of licenses is lower when an innovator is active in the product market.

We find that vertical and conglomerate mergers do not provide incentives for vertical foreclosure. In the following chapter, we will introduce two-part tariffs in our model of technology licensing in complementary markets. Empirical evidence (e.g Taylor et al., 1973, Rostoker, 1984, Macho-Stadler et al., 1996, Bousquet et al., 1998) suggest that royalty rates are predominant in patent licensing agreements. We will study the impact of the availability of per unit royalty rates on downstream price equilibria and the profit-maximizing licensing policy of a monopoly innovator across integration regimes.

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Chapter 3

Two-part tariff licensing and vertical integration in complementary markets

3.1 Introduction

The empirical literature shows the prevalence of royalties in practice (e.g Taylor et al., 1973, Rostoker, 1984, Macho-Stadler et al., 1996, Degnan and Horton, 1997, Bousquet et al., 1998) whereas early theoretical models tend to conclude on the dominance of fixed fees over royalties. The fact that the patentee is active in the final good production also appears to be commonly observed. Bessy and Brousseau (2000) show on French self reported data, that ex ante pure lump sump payments are very rarely used in practice. The authors argue that it is due to the difficulty of evaluating the value of the technology given the uncertainty of the market outcome. Royalties are found to be used in 90% of the observed licensing agreements making the licensor bear the technical and commercial risk of its licensee and requiring the monitoring of its market performance. Two-part tariff contracts used in 50% of the agreements in this data base may be seen as a way to share the risk burden between patentee and licensee. Exclusivity provisions are used in three out of four contracts. Half of the agreements involve territorial exclusivity. Restrictions on the use of the technology (i.e to a geographical area or a specific field) are also very common. Granting back clauses reduce the moral hazard problem and the likelihood for the licensee to be able to develop an alternative technology and become a competitor in the upstream market. Renegotiations provisions are used in 63% of the time. One third of the contracts introduce governance mechanisms to renegotiate the level of royalties. In this dissertation, we will ignore this stylized fact in assuming that contracts are public and that the innovator is fully able to commit to its licensing strategy.

In the present chapter, we study the role played by royalties and integration in the distribution of licenses in perfectly complementary markets. We focus on the case where royalties take the form of a transfer (from the licensee to the owner of the technology) of a given amount of money for each unit of the final good sold. As we have assumed constant marginal cost of production, such royalties can be modeled as an increase in the marginal cost of licensees. The use of royalties is found to restore the profitability of downstream competition when the sensitivity of consumers to the distance from their ideal variety is low (i.e mass market) which makes it a weakly dominating strategy for both outside and inside monopoly innovator (see the summarizing diagram C.1 page 230).

There is a vast theoretical literature characterizing the optimal patent licensing strategy for a technology used in a single differentiated downstream market. One of the main conclusion of this literature is that product differentiation can explain the use of per unit royalties in licensing contracts (e.g Muto, 1993, Wang and Yang, 1999, Poddar and Sinha, 2004, Sandonis and Fauli-Oller, 2006, Stamatopoulos and Tauman, 2008). Wang and Yang (1999) are one of the firsts to analyze patent licensing in price competition framework. They compare per unit royalty and fixed fee licensing of a cost-reducing innovation by an inside innovator and find that per unit royalties may be profitable for both drastic and non-drastic innovation. This article is corrected by Colombo and Filippini (2015) in taking into account appropriately the effect of royalty licensing on the profit of the inside innovator. Royalty revenue indeed depends positively on the volume of sales made by its rival (i.e licensee). This correction results in the cost-reducing innovation to be always profitably licensed to the rival. Per unit royalty licensing is preferred to fixed fee licensing when the degree of substitutability between alternatives is sufficiently high. This can be explained by the fact that the per unit royalty rate dampens downstream competition.

Lu and Poddar (2014) show that two-part tariff licensing are very robustly the optimal licensing contract in spatial differentiation models of an inside innovator of a cost-reducing technology even in the presence of cost asymmetry between the licensee and the patentee.

Sandonis and Fauli-Oller (2006); Sandonís and Faulí-Oller (2008) also contribute to this literature of patent licensing with product differentiation and integration. In their article published in 2006, they explore the profitability of a vertical merger between an outside innovator, holder of a cost-reducing technology and a downstream differentiated producer. Since the technology is non-drastic, the outside option of potential licensees depends on the licensing contracts being offered (if accepted) to the rival. This effect makes it more difficult for the innovator to extract profit from the downstream market. Another feature of their model is that an outside innovator is allowed to publicly offer a different contract to each potential licensee which provides an effective commitment tool for the upstream monopolist to restrict total output. However, a vertically integrated firm would not be able to credibly commit. The inside innovator only has one tariff instrument (i.e one licensing contract) available to influence the total level of production. On the other hand, vertical integration enables the innovator to directly benefit from its innovation through the product market in eliminating one participation constraint. Because in their model, a small innovation generates high outside options, it makes it costly for the innovator to ensure the participation of the licensees. The outside innovator will then find it profitable to merge with one of the two downstream producers in order to directly earn some profit from its innovation. To the contrary, when the new technology induces a great shift in efficiency, it is easy for the innovator to capture downstream profits as an outsider (i.e because of the low outside option of potential licensees) while keeping its ability to commit to the profitable level of output. From the social welfare point of view, the desirability of such a vertical merger is reversed. An inside innovator will indeed profitably raise the cost of production of its rival through the per unit royalty rate but will be unable to commit to output restriction. The second effect dominates thus making vertical integration socially desirable (privately undesirable) for large cost-reducing innovations. There is no socially desirable and profitable vertical merger in this model.

In the following section we look for the profit-maximizing two-part tariff licensing strategy of the separated monopoly innovator. The innovator is able in this framework to capture the full profit of the industry which makes it willing to maximize the total profit of the industry. Thus, the profit-maximizing licensing contract of the innovator generates an optimal structure of the industry in the sense that its equilibrium maximizes its total profit. We also study the impact of vertical integration into the homogeneous and differentiated market in the third and fourth sections respectively. In the fifth, we characterize the optimal licensing policy of an inside innovator producing both components (i.e in a post conglomerate merger industry). The sixth section concludes this chapter.

3.2 Separated model

3.2.1 Framework

Following our previous work, we build a model with a monopoly patent holder whose technology is required for the production of two perfectly complementary goods. One of the downstream component is homogeneous as it is considered to be the basis of the final good (e.g the CPU, hidden in the device). On the other hand, the various varieties of the second component are spatially differentiated (i.e circular city model)

In this chapter, we consider that two-part tariff licensing contracts are available to the monopoly innovator. Fixed fees are paid by licensees upfront and royalties are based on the sales of the end products using the technology of the innovator. Royalties are taking here the form of a price charged by the upstream monopolist on each unit of the end product sold by its licensees (i.e per unit royalty rates). These royalties increase the marginal cost of production of downstream producers. There is perfect symmetric information and no commitment problem. The innovator is able to monitor sales and to commit to a given number of licenses (i.e contracts are assumed to be public). In this setting, the licensing policy consists in the choice of the number of licensees (N_A, N_B) in each of the downstream complementary markets as well as the level of their marginal cost (k, g). These will determine downstream equilibrium prices and profits which are captured by the upstream innovator through the fixed fees. In the case of a pure fixed fee contract, we show in chapter two that when only two firms in each downstream market are ready to purchase a license in order to be active, the asymmetric licensing policy (with an exclusive license on the homogeneous market) remains optimal as long as consumers are not too sensitive to the distance from their ideal variety (i.e sufficiently low unit transportation cost t). To the contrary, the double monopoly structure becomes optimal as soon as consumers are sufficiently sensitive to product differentiation. In this analysis of optimal two-part tariff licensing, we maintain the assumption that due to very high sunk costs, the number of total producers of each downstream component is either one or two. The available structures for the patentee are:

- the double monopoly (i.e $N_A = 1$ and $N_B = 1$)
- the differentiated monopoly (i.e $N_A = 2$ and $N_B = 1$)
- the asymmetric structure (i.e $N_A = 1$ and $N_B = 2$)
- the double duopoly (i.e $N_A = 2$ and $N_B = 2$).

The timing of the game is the following. In the first stage of the game, the outside innovator chooses the number of licenses (N_A, N_B) to deliver in each component market as well as the two-part tariff licensing contracts offered to potential licensees. In the second stage of the game, downstream competition takes place between the (N_A, N_B) number of firms who accepted the licensing contract. Downstream firms pay royalties, utilities and profits are realized.

In the present chapter, we show that under royalty licensing contracts, the double duopoly weakly dominates the asymmetric structure for all values of the parameters.

When the valuation for the system is sufficiently low, the double duopoly structure is indeed strictly more profitable for the outside innovator. This is in contradiction with the case of a single homogeneous downstream market in which the upstream firm is indifferent between dealing with a downstream monopoly or a duopoly under two-part tariff public contracts. Taking into account the differentiated complementary market, we find that this indifference remains valid in mass component market only.[1]

In the separated two-part tariff licensing model, we find that an outside innovator is indifferent to the level of royalties in the kinked equilibria. Royalties do not influence system sales in these cases. Losses in fixed fee profits are indeed exactly compensated by royalty revenues. On the other hand, the use of royalties is found to be unprofitable and inefficient when the valuation for the good is low enough so that the market is uncovered. Higher marginal costs of production enhance the double marginalization effects. In contrast, per unit royalty rates enable the innovator to increase its profit in the case of a competitive equilibrium in the double duopoly structure. It is now able to dampen competition by increasing royalty rates earned on component markets. Marginal costs of production can be fine-tuned in order to capture the full residual surplus of the marginal consumer (leaving to consumers just enough surplus to ensure full participation). We solve the model backward and first characterize the downstream pricing equilibria in each structure of the industry.

3.2.2 Downstream equilibria

We derive downstream equilibria in each of the available structures of the industry. Given the number of active firms, we express the new profit functions for all values of the parameters. The difference with the model used in previous chapters is that the per unit royalty rate is added to the cost of producing a compatible component c. Total marginal cost is now denoted g in the differentiated market and k in the homogeneous market. We maximize each profit function and derive best response functions allowing us to characterize price equilibria. This enables the determination of the effects of

[1]This non-equivalence result between the licensing policy with a single downstream market and with complementary markets will remain true throughout this chapter.

per unit royalty rates on downstream competition. Royalties increase marginal costs of production which affect equilibrium price levels and associated sustaining values of parameters. We find that the pass-through is lower than one in the uncovered equilibrium, equal to zero in kinked equilibria and equal to one in the competitive equilibrium. We first illustrate these effects in characterizing downstream equilibria in the double monopoly structure.

3.2.2.1 Double monopoly

We first present the respective profit functions of active downstream firms when $N_A = N_B = 1$:

$$\Pi_A = \begin{cases} (a-k)\sqrt{\dfrac{-a-d+v}{t}}, \text{ if: } \sqrt{\dfrac{-a-d+v}{t}} < \dfrac{1}{2}, & (3.1) \\ \dfrac{a-k}{2}, \text{ otherwise.} & (3.2) \end{cases}$$

$$\Pi_B = \begin{cases} (d-g)\sqrt{\dfrac{-a-d+v}{t}}, \text{ if: } \sqrt{\dfrac{-a-d+v}{t}} < \dfrac{1}{2}, & (3.3) \\ \dfrac{d-g}{2}, \text{ otherwise.} & (3.4) \end{cases}$$

We maximize each profit function and obtain the following best response functions:

$$BR_A = a = \begin{cases} -d - \dfrac{t}{4} + v, \text{ if: } d < \dfrac{1}{4}(-4k - 3t + 4v), & (3.5) \\ \dfrac{1}{3}(-2d + k + 2v), \text{ if: } \dfrac{1}{4}(-4k - 3t + 4v) < d \leq v - k. & (3.6) \end{cases}$$

$$BR_B = b = \begin{cases} -a - \dfrac{t}{4} + v, \text{ if: } a < \dfrac{1}{4}(-4g - 3t + 4v), & (3.7) \\ \dfrac{1}{3}(-2a + g + 2v), \text{ if: } \dfrac{1}{4}(-4g - 3t + 4v) < a \leq v - r. & (3.8) \end{cases}$$

Using these best response functions, we characterize the following Nash equilibria of the double monopoly subgame:

- Uncovered equilibrium:

$$a = \frac{3k}{5} - \frac{2g}{5} + \frac{2v}{5} \tag{3.9a}$$

$$d = -\frac{2k}{5} + \frac{3g}{5} + \frac{2v}{5} \tag{3.9b}$$

$$p = a + d = \frac{4v + k + g}{5} \tag{3.9c}$$

$$\text{if: } v < \frac{1}{4}(4k + 4g + 5t), \tag{3.9d}$$

- Kinked equilibria:

$$a = -d - \frac{t}{4} + v \tag{3.10a}$$

$$d = -a - \frac{t}{4} + v \tag{3.10b}$$

$$p = a + d = v - \frac{t}{4} \tag{3.10c}$$

$$\text{if: } v > \frac{1}{4}(4k + 4g + 5t). \tag{3.10d}$$

In the double monopoly structure, there is inefficient double marginalization resulting in excessively high system price when the market is uncovered (i.e low levels of v or high levels of t and high royalty rates). When the market is covered there is a multiplicity of equilibria of the surplus sharing game between producers of complementary products facing an inelastic system demand. The competitive equilibrium does not exist because there is only one differentiated producer.

We find that per unit royalties may have an effect on downstream equilibrium prices through the marginal cost of production faced by licensees. In the covered equilibrium (see equations 3.10a and 3.10b), marginal cost increase does not directly result in higher prices. The covered equilibrium price fully depends on the net surplus of the marginal consumer (i.e $\bar{x} = \frac{1}{2}$) and hence on the price of the complementary good. On the other hand, producers transfer some of their marginal cost increase to consumers in the uncovered equilibrium (see in equations 3.9a and 3.9b that the pass-through is lower than one). Moreover, per unit royalty rates influence the value of the threshold on system valuation v such that the market is covered or uncovered in equilibrium. A higher marginal cost implies an increase in the minimum level of system valuation consistent with the covered equilibrium (see equation 3.10d). This can be explained by the rise in the uncovered equilibrium price generating, everything else being equal, a decrease in the number of participating consumers. In order to reach the full coverage of the market, the level of valuation for the system must be higher. We now turn to the case where an additional license is delivered on the homogeneous market.

3.2.2.2 Differentiated monopoly

In the differentiated monopoly subgame, there is an additional producer active in the homogeneous component market (i.e $N_A = 2$ and $N_B = 1$) generating perfect competition and marginal cost pricing (i.e $a = k$). There is no horizontal double marginalization problem. On the other hand, the competitive equilibrium does not exist as there is only one differentiated producer. Perfect competition prevents multiplicity of covered equilibria. The behavior of the monopoly component B producer is the same as in the double monopoly structure. Nash equilibria are derived from the best response functions in the double monopoly in which we replace the price of the homogeneous component by its total marginal cost of production (i.e $a = k$).

- Uncovered equilibrium:

$$a = k \tag{3.11a}$$

$$d = \frac{1}{3}(-2k + g + 2v) \tag{3.11b}$$

$$p = a + d = \frac{1}{3}(k + g + 2v) \tag{3.11c}$$

$$\text{if: } v < \frac{1}{4}(4k + 4g + 3t). \tag{3.11d}$$

- Kinked equilibrium:

$$a = k \tag{3.12a}$$

$$d = -k - \frac{t}{4} + v \tag{3.12b}$$

$$p = a + d = -\frac{t}{4} + v \tag{3.12c}$$

$$\text{if: } v > \frac{1}{4}(4k + 4g + 3t). \tag{3.12d}$$

Perfect competition on the homogeneous component market and associated marginal cost pricing makes the pass-through of the component A price equal to one for all values of the parameters v and t. The uncovered and kinked prices of component B remains characterized by the same pass-through as in the double monopoly (i.e is lower than one and zero in the uncovered and kinked equilibrium respectively). However, considering the price of the system made of the combination of the two components, we observe in comparing equations 3.11c and 3.9c that the increase in the uncovered system price (i.e $a+d$) following a rise in marginal cost is higher in the differentiated monopoly structure.

3.2.2.3 Double duopoly

In the double duopoly subgame ($N_A = N_B = 2$), the price of the homogeneous component A is still equal to the marginal cost because of perfect competition. There are now two differentiated component B producers as well as both kinked and competitive covered equilibria. This competitive equilibrium price follows the standard circular city expression with the per unit royalty rate being incorporated into the total marginal cost of component B (i.e g). On the other hand, the expression of the uncovered price does not change compared to the differentiated monopoly subgame. The unique kinked equilibrium price level is impacted by the decrease in transportation costs faced by marginal consumers generated by the entry of the second differentiated producer. This also affects the values of system valuation sustaining these equilibria. The best response functions of the B component producers are symmetric and lead to the following equilibria:

- Uncovered equilibrium:

$$a = k \tag{3.13a}$$

$$d = b = \frac{1}{3}(g - 2k + 2v) \tag{3.13b}$$

$$p = a + d = \frac{1}{3}(k + g + 2v) \tag{3.13c}$$

$$\text{if: } v < \frac{1}{16}(16g + 16k + 3t). \tag{3.13d}$$

- Kinked equilibrium:

$$a = k \tag{3.14a}$$

$$d = b = -k - \frac{t}{16} + v \tag{3.14b}$$

$$p = a + d = -\frac{t}{16} + v \tag{3.14c}$$

$$\text{if: } \frac{1}{16}(16g + 16k + 3t) < v < \frac{1}{16}(16g + 16k + 5t). \tag{3.14d}$$

- Competitive equilibrium:

$$a = k \tag{3.15a}$$

$$d = b = g + \frac{t}{4} \tag{3.15b}$$

$$p = a + d = k + g + \frac{t}{4} \tag{3.15c}$$

$$\text{if: } v > \frac{1}{16}(16g + 16k + 5t). \tag{3.15d}$$

We observe in equations 3.15b and 3.15a that in the competitive equilibrium, the rise in marginal cost due to the introduction of per unit royalty rates is fully transmitted to consumers (i.e pass-through equal to one). Moreover, all sustaining values of parameter v move upward as royalty rate increases (see equations 3.15d, 3.13d and 3.14d). We now present downstream equilibria of the last available structure of the industry which is the asymmetric structure.

3.2.2.4 Asymmetric structure

In the asymmetric structure (i.e $N_A = 1$ and $N_B = 2$), an inefficient double marginalization is at work when the market is uncovered just like in the double monopoly structure. There are also kinked and competitive equilibria as in the differentiated monopoly and double duopoly structures. Profit and best response functions of the monopoly producer of the component A are similar to the one observed in the double monopoly structure. When the market is kinked, complementary good producers play a surplus sharing game generating multiple kinked equilibria.

$$\Pi_A = \begin{cases} (a-k)\left(\sqrt{\frac{-a-d+v}{t}} + \sqrt{\frac{-a-b+v}{t}}\right), \text{ if: } \sqrt{\frac{-a-d+v}{t}} < \frac{1}{2} & (3.16) \\ \frac{a-k}{2}, \text{ if: } \sqrt{\frac{-a-d+v}{t}} + \sqrt{\frac{-a-b+v}{t}} = \frac{1}{2} & (3.17) \end{cases}$$

We derive the following best response function under the assumption that the price of component B is symmetric. This is true because the two component B players have symmetric objective functions.

$$a = \begin{cases} \frac{1}{3}(-2b+k+2v), \text{ if: } b > \frac{1}{16}(-16k-3t+16v) & (3.18) \\ -b - \frac{t}{16} - t + v, \text{ if: } b < \frac{1}{16}(-16k-3t+16v) & (3.19) \end{cases}$$

The behavior of component B producers remain the same as in the previous structure. Combining these best response functions, we characterize the following equilibria:

- the uncovered equilibrium:

$$b = d = \frac{3g}{5} - \frac{2k}{5} + \frac{2v}{5} \quad (3.20a)$$
$$a = -\frac{2g}{5} + \frac{3k}{5} + \frac{2v}{5} \quad (3.20b)$$
$$a + b = \frac{g}{5} + \frac{k}{5} + \frac{4v}{5} \quad (3.20c)$$
$$\text{when: } v < \frac{1}{16}(16g + 16k + 5t), \quad (3.20d)$$

- the most favorable to A kinked equilibrium:

$$a = -b - \frac{t}{16} + v = \frac{1}{16}(-16g - 3t + 16v) \tag{3.21a}$$

$$b = d = -a - \frac{t}{16} + v = g + \frac{t}{8} \tag{3.21b}$$

$$a + b = \frac{1}{16}(-t + 16v) \tag{3.21c}$$

$$\text{when: } v > \frac{1}{16}(16g + 16k + 5t). \tag{3.21d}$$

- the competitive equilibrium equilibrium:

$$a = -g - \frac{5t}{16} + v \tag{3.22a}$$

$$d = b = g + \frac{t}{4} \tag{3.22b}$$

$$a + b = \frac{1}{16}(-t + 16v) \tag{3.22c}$$

$$\text{when: } v > \frac{1}{16}(16g + 16k + 7t). \tag{3.22d}$$

Overall, we find that per unit royalty rates have an influence on the levels of equilibrium prices or on their sustaining values of v but do not change the strategic interactions between downstream firms. We show that equilibrium prices are in general unaffected in kinked equilibria (i.e zero pass-through) whereas the entire increase in marginal cost is passed onto consumers in competitive equilibrium. In the asymmetric structure, the system price remains unchanged in all covered equilibria. However, in the double duopoly structure, the competitive system price increases with the level of royalties. In uncovered equilibrium, the pass-through is lower than one for both component prices resulting in an increase in system price and the associated drop in sales. Positive royalty rates also require higher system valuations to ensure the full coverage of the market.

3.2.3 Optimal licensing policy

3.2.3.1 Royalties

Royalties have a negative effect on sales and thus on uncovered profit of the upstream monopolist. They indeed amplify the effect of double marginalization when the demand for the system is elastic (i.e uncovered). The uncovered profit function of the patentee is strictly decreasing in the level of royalties. As a consequence, it profitably sets royalty rates to zero. The kinked profit function of the innovator does not depend on the level of royalties because they do not have an influence on the kinked price equilibrium. It is indifferent between all values of the royalty rates consistent with kinked equilibria. In order to simplify the exposition, we will assume that the patentee only chooses a positive royalty rates when it is strictly profitable. Otherwise, the royalty rate will be equal to zero (i.e $g = k = c$). Per unit royalty rates however have a positive impact in the double duopoly structure when the market is at the competitive equilibrium. The innovator charges the highest level of royalties consistent with the full coverage of the market (i.e $g + k = \frac{1}{16}(16v - 5t)$). The upstream monopolist is indifferent to the structure given to royalties (i.e the burden sharing across the two component markets). Taking the case of symmetric royalties, we have: $g = k = \frac{1}{2}(v - \frac{5t}{16})$.

Lemma 3.1. *Under the assumption that the innovator charges a positive royalty rate only when it is strictly profitable, optimal royalty rates in the asymmetric subgame ($N_A = 1, N_B = 2$) are such that $g = k = c$ (i.e zero royalty rates) as long as the good is demanded (i.e $v > 2c$). In the double duopoly structure, optimal royalty rates are such that:*

- $g = k = c$ *(i.e zero royalty rates), when the market is uncovered or kinked (i.e $2c + \frac{5t}{16} > v > 2c$) and*
- $g = k = \frac{1}{2}(v - \frac{5t}{16})$, *in the competitive equilibrium (i.e $v > 2c + \frac{5t}{16}$).*

Proof. More details are available in the appendix. □

3.2.3.2 Number of licenses

In our framework, it is always optimal (no matter the shape of the licensing contract) for the upstream innovator to issue the highest feasible number of licenses on the differentiated market. The number of varieties increases the level of social surplus in reducing the transportation cost of the marginal consumer. This surplus is in turn captured by the upstream monopolist. The double monopoly and the differentiated monopoly are always dominated.

Thanks to the two-part tariff contracts, the double duopoly structure is never dominated. In the fixed fee model, the asymmetric structure is indeed preferred when the double duopoly structure is at the competitive equilibrium. The competition in the differentiated downstream market is too intense leading to excessively low system prices from the point of view of the innovator. This result does not hold under two-part tariff licensing because royalties allow the upstream monopolist to control the intensity of downstream competition through the level of marginal costs. Equilibrium system prices are such that just enough surplus is left to consumers so that the competitive equilibrium is sustained. The asymmetric structure remains inefficient and dominated by the double duopoly when the market is uncovered (i.e $v < \frac{1}{16}(16g + 16k + 5t)$). Double marginalization problems are aggravated by positive royalty levels. Otherwise the monopoly innovator is indifferent between the asymmetric and double duopoly structure.

Proposition 3.2.1. *The double duopoly structure (i.e $N_A = N_B = 2$) using a pure fixed fee licensing contract (i.e $g = k = c$) is strictly preferred to the uncovered asymmetric structure when $v < \frac{1}{16}(2c + 5t)$.*

Otherwise (i.e when $v > \frac{1}{16}(2c+5t)$), the outside innovator is indifferent between the kinked asymmetric and the competitive double duopoly structure. Its profit-maximizing two-part tariff licensing policy can either be:

- *the double duopoly structure (i.e $N_A = N_B = 2$) with a two-part tariff licensing contract such that $g + k = v - \frac{5t}{16}$, or*

- *the asymmetric structure (i.e $N_A = 1$ and $N_B = 2$) with a pure fixed fee licensing contract (i.e $g = k = c$).*

Proof. See in the appendix. □

In the separated integration regime, we find that the result obtained in chapter one showing that the licensing policy differs in complementary markets is robust to the introduction of two-part tariff licensing contracts. The upstream monopolist is not always indifferent between a downstream monopoly and a duopoly on the homogeneous market.[2] The fact that the patentee delivers as many licenses as possible in a niche market remains true. However, under two-part tariff licensing contracts, it is never strictly optimal for an outside innovator to issue an exclusive license as the double duopoly structure weakly dominates (see the summarizing diagram C.1 page 230).[3] We will now study the case where the patentee is producing the homogeneous component and characterize its profitable licensing strategy.

3.3 Homogeneous integration

In the present section, we show that the profitable number of licenses when the inside innovator is active in the homogeneous component market remains under two-part tariff, identical to the one of an outside innovator.[4] In the case of an homogeneous vertical integration, we find that the innovator and monopoly producer of component A (i.e when $N_A = 1$) is now neutral to the level of royalties when the market is uncovered (instead of being adverse to positive royalties). The profit function of the inside innovator does not depend on royalty rates. Royalty revenue made on the market B is now internalized by

[2] Contrary to the case of a single homogeneous downstream market.

[3] The outside innovator is indifferent between the double duopoly and the asymmetric structure when the latter is at a kinked equilibrium.

[4] Ignoring the time dimension associated with the development of downstream goods, we find the same result in chapter 2 under pure fixed fee licensing (see the summarizing diagram C.1 page 230).

the monopoly producer of component A who is also facing a lower marginal cost than in the separated case (i.e there is no royalty within the integrated firm).[5]

3.3.1 Downstream equilibria

We will not present the inefficient outcomes of the double monopoly and differentiated monopoly structures (i.e $N_B = 1$). These are suboptimal because they generate higher transportation costs for consumers and lower social surplus with a single variety of the differentiated component. We focus on the trade-off between the two potentially optimal structures (i.e $N_B = 2$) and on the main effect of vertical integration in that framework which is to make the innovator indifferent to royalties when the market is uncovered.

3.3.1.1 Double duopoly

Integration into the homogeneous component market does not matter when $N_A = 2$. Vertical integration indeed makes both downstream producers of component A asymmetric in their marginal cost. The independent producer faces a higher marginal cost equal to k whereas the marginal cost of the integrated firm is equal to c. Perfect Bertrand competition leads to: $a = k$, at which the integrated firm serves the entire market and makes a profit equal to: $\frac{k-c}{2}$. This is equal to the royalty revenue made by the innovator on the component A in the separated case. Equilibrium price of the component A are not affected nor the behavior of the component B producers. System prices and profits remain unchanged.

3.3.1.2 Asymmetric structure

When $N_A = 1$, vertical integration has an impact on the way royalties affect the social surplus in the uncovered equilibrium. The profit-maximizing behavior of the integrated

[5]The negative effect of royalties found in the separated case is exactly compensated by integration so that the double marginalization problem is just as severe as with a pure fixed fee contract.

monopoly producer of the component A is now characterized by the following best response function:

- Uncovered

$$a = \frac{1}{3}(2c - 2b - g + 2v) \tag{3.23a}$$

$$b > \frac{1}{16}(-32c + 16g - 3t + 16v) \tag{3.23b}$$

- Covered

$$a = -b - \frac{t}{16} + v \tag{3.24a}$$

$$b < \frac{1}{16}(-32c + 16g - 3t + 16v). \tag{3.24b}$$

Comparing the uncovered expression of this best response function with the one of the separated case, we see that the marginal cost of an independent producer k is replaced by: $c - (g - c) = 2c - g$. This is due to the internalization of the royalty revenue made on market B (i.e $g - c$) and the decrease in marginal cost of production of component A from k to c. As a result, when royalties are strictly positive, the inside monopoly innovator charges a lower price of component A (for a given level of price of the complementary good B) than an independent producer would.

On the other hand, the covered best response sub-function is unchanged because there is no demand elasticity and the monopoly producer charges the highest price compatible with the market to be fully covered. The behavior of component B producers remains unchanged so that the kinked and competitive covered equilibria are unaltered by this vertical integration. Combining best response functions we derive the following uncovered equilibrium prices:

$$a = \frac{1}{5}(6c - 5g + 2v), \tag{3.25a}$$

$$b = -\frac{4c}{5} + g + \frac{2v}{5}, \tag{3.25b}$$

$$d = -\frac{4c}{5} + g + \frac{2v}{5} \tag{3.25c}$$

$$a + b = a + d = \frac{2}{5}(c + 2v). \tag{3.25d}$$

The negative impact of royalties on sales and surplus that we observe in the separated two-part tariff situation does not arise here because the downstream division of U_A fully internalizes costs and benefits related to royalties.[6] In the integrated two-part tariff situation, royalties do not amplify the negative effect of horizontal double marginalization on sales because the inside innovator takes into account its royalty revenue. Thus as we can see in equation 3.25d, system price and thus upstream profits do not depend on g. The other equilibria remain unchanged by the integration that we consider in this section.

3.3.2 Optimal licensing policy

3.3.2.1 Royalties

We now look at the effect of royalty rates on the profit of the inside innovator. We focus on the outcome of the uncovered equilibrium of the asymmetric structure which is altered by vertical integration into the homogeneous market. Considering the associate total uncovered equilibrium profit of the innovator, we observe in the following equation

[6] It charges a price a so that the uncovered equilibrium system price is the same as in the fixed fee case.

that royalties do not have any influence:

$$\Pi_U = -\frac{8(2c-v)\sqrt{\frac{v-2c}{t}}}{5\sqrt{5}}{}^{7}. \tag{3.26}$$

Since integration into the homogeneous market does not have any effect on kinked equilibria of the asymmetric structure, the inside innovator is now indifferent to the level of per unit royalty rates for all valuations of the system in the asymmetric structure (i.e in both uncovered and covered equilibria). The level of profitable royalty rates remain unchanged in other equilibria of the asymmetric structure as well as in the double duopoly structure.

3.3.2.2 Number of licenses

The optimal licensing policy is unchanged compared to the separated two-part tariff case. The upstream innovator remains indifferent between the asymmetric and the double duopoly when the latter is in the competitive equilibrium. Otherwise, the asymmetric structure is uncovered and is strictly dominated by the double duopoly just as in the separated two-part tariff and fixed fee models (see the summarizing diagram C.1 page 230). The fact that the innovator is now indifferent to the level of royalties in the uncovered equilibrium of the asymmetric structure does not change the profit of the innovator because in equilibrium royalty rates are equal to zero in the separated structure.

Proposition 3.3.1. *Under two-part tariff licensing contracts, the optimal number of licenses delivered by an inside innovator active in the homogeneous downstream component market is identical to the one of an outside innovator.*

Proof. This directly results from the fact that integration on the homogeneous market does not affect the double duopoly structure nor the kinked equilibria of the asymmetric structure. □

[7] The uncovered equilibrium profit of the inside innovator in the asymmetric structure is the same as in the fixed fee licensing case.

We now turn to the analysis of the case where the innovator is active in the differentiated component market.

3.4 Differentiated integration

We now analyze the case where the upstream monopolist merges with one of the component B producers (i.e B_2). We focus on possibly optimal structures namely the asymmetric and the double duopoly and show that the latter remains weakly preferred for all values of parameters[8]. The number of licenses delivered by the inside innovator active in the differentiated market is found to be the same as in the cases of vertical homogeneous integration and separation. Moreover, we find that strictly positive royalty rates can only be charged for high valuation of the system if, despite the indifference between the asymmetric and double duopoly structures, the latter is chosen by the innovator.

In this section, we indeed show that the positive effect of royalties remains at work in the double duopoly structure. When the valuation for the good is high enough to make the double duopoly structure competitive, the royalty rate charged on the sales of component B_1 are enough to fine-tune the competitive market B equilibrium so that, no unnecessary surplus is left to consumers. On the other hand, when the valuation for the system is low enough to make the market uncovered, we know that there is no inefficient horizontal double marginalization due to perfect competition on component A market. Royalty rates are undesirable in the uncovered equilibrium of the double duopoly because they increase marginal costs and the negative vertical double marginalization effect. For intermediate values of system valuation, an efficient kinked equilibrium is reached when royalties are equal to zero.

Despite the internalization of the royalty revenue perceived on the sales of the complementary good A, the vertical integration into the differentiated market does not eliminate the horizontal double marginalization effect at work in the uncovered equilibrium of the asymmetric structure. This equilibrium is sustained by a larger range of system

[8]See the summarizing diagram C.1 page 230.

valuations than under the double duopoly structure.[9] The innovator avoids the decrease in sales and profits due to the excessive uncovered equilibrium prices of the asymmetric structure by choosing to implement the double duopoly structure. The latter strictly dominates as long as the asymmetric structure is uncovered.

We will now formally characterize the profitable licensing policy of the inside innovator active in the differentiated component market. This structure of the industry generates asymmetry in the behaviors of differentiated producers making the expression of demand functions more complicated. The characterization of general kinked equilibria with strictly positive royalty rates becomes very challenging. This is particularly true in the asymmetric structure. However, we show that it is never strictly profitable for the innovator to use royalty rates in the asymmetric structure which allows us to focus on the outcome of the pure fixed fee licensing.[10] We will first present downstream equilibria in both the double duopoly and asymmetric structures and then the resulting profit-maximizing licensing policy of the inside innovator.

3.4.1 Downstream equilibria

3.4.1.1 Double duopoly structure

We start by characterizing the downstream equilibria of the double duopoly structure (i.e $N_A = 2$ and $N_B = 2$). Since the component A market is homogeneous, perfect competition leads to marginal cost pricing (i.e $a = k$). The fact that the price of component A is fully determined simplifies the analysis. In particular, it allows the full characterization of kinked equilibria. The profit function of the inside innovator includes component B

[9] In this section, we show that the threshold on system valuation under which the asymmetric structure is uncovered is strictly increasing with the levels of royalty rates. It converges towards the pure fixed fee level as royalty rates tend to zero. In order to avoid exacerbating horizontal double marginalization effects, the innovator will never charge strictly positive royalty rates in the asymmetric structure.

[10] When the market is covered, we find in this section that the use of a pure fixed fee contracts in the asymmetric structure would ensure a symmetric, efficient and profitable allocation of sales in kinked equilibria.

sales revenues as well as royalty revenues. Profit functions of the differentiated producers take the following expressions depending on the structure of the market:

$$\Pi_{B_1} = \begin{cases} (d-g)\left(\dfrac{b-d}{t}+\dfrac{1}{4}\right), \text{ if: } \bar{x}_1+\bar{x}_2 > \dfrac{1}{2} & (3.27) \\ (d-g)\sqrt{-\dfrac{d+k-v}{t}}, \text{ if: } \bar{x}_1+\bar{x}_2 = \dfrac{1}{2} & (3.28) \\ (d-g)\sqrt{-\dfrac{d+k-v}{t}}, \text{ if: } \bar{x}_1+\bar{x}_2 < \dfrac{1}{2}. & (3.29) \end{cases}$$

$$\Pi_{UB_2} = \begin{cases} (g-c)\left(\dfrac{b-d}{t}+\dfrac{1}{4}\right)+(b-c)\left(\dfrac{d-b}{t}+\dfrac{1}{4}\right)+\dfrac{k-c}{2}, \text{ if: } \bar{x}_2+\bar{x}_1 > \dfrac{1}{2} & (3.30) \\ (b-c)\sqrt{-\dfrac{b+k-v}{t}}+(k-c)\sqrt{-\dfrac{b+k-v}{t}} & \\ +(g-c)\sqrt{-\dfrac{d+k-v}{t}}+(k-c)\sqrt{-\dfrac{d+k-v}{t}}, \text{ if: } \bar{x}_1+\bar{x}_2 < \dfrac{1}{2}. & (3.31) \end{cases}$$

When the market is kinked, $\bar{x}_2 + \bar{x}_1 = \sqrt{-\frac{b+k-v}{t}} + \sqrt{-\frac{d+k-v}{t}} = \frac{1}{2}$, the profit function of the integrated differentiated producer can be rewritten in the following way:

$$\Pi_{UB_2} = (b-g)\sqrt{-\frac{b+k-v}{t}} + \frac{g-c}{2} + \frac{k-c}{2}. \tag{3.32}$$

Profit maximization results in the standard best response function for the independent B_1 producer. It only differs from the separated case when the market is kinked (i.e non competitive covered) because of the potential asymmetry in prices of component B:

$$BR_{B_1} = d = \begin{cases} \dfrac{1}{8}(4b+4g+t), \text{ if: } \bar{x}_2+\bar{x}_1 > \dfrac{1}{2} & (3.33) \\ \sqrt{-t(b+k-v)}+b-\dfrac{t}{4}, \text{ if: } \bar{x}_2+\bar{x}_1 = \dfrac{1}{2} & (3.34) \\ \dfrac{1}{3}(g-2k+2v), \text{ if: } \bar{x}_2+\bar{x}_1 < \dfrac{1}{2}. & (3.35) \end{cases}$$

The uncovered best response function of the merged entity UB_2 only differs in the level of marginal cost. The downstream division of UB_2 does not pay royalty rate for the use of its own intellectual property. It also internalizes the royalties earned on the sales of component A. As a consequence, its total marginal cost is equal to $c - (k - c) = 2c - k$. Assuming $g = c$ and $k > c$, the inside innovator would charge a lower price than its independent rival (i.e from equations 3.35 and 3.38, we get: $d = \frac{1}{3}(c - 2k + 2v) > \frac{1}{3}(c - 2k - (k - c) + 2v) = b$). Strictly positive royalty rates, are less damaging than in the separated model because the merged entity takes into account the royalty revenue made on market A which results in a less severe horizontal double marginalization problem on system 2. Royalty revenues collected on the sales of the alternative component B do not play a role as it is independent of its price b as long as the market is uncovered.

$$BR_{UB_2} = b = \begin{cases} \frac{1}{8}(4d + 4g + t) & (3.36) \\ \sqrt{-t(d + k - v)} + d - \frac{t}{4} & (3.37) \\ \frac{1}{3}(2c - 3k + 2v). & (3.38) \end{cases}$$

On the other hand, it is worth noting that when the market is competitive, best response functions of both players are symmetric (see equations 3.36 and 3.33). This is due to the fact that the integrated firm internalizes the royalty revenue made on the sales of the rival provider of component B. That is why the difference in marginal cost does not play a role here. Since the market is covered, component A royalty revenue is maximal and independent of the market shares in component B market. An additional sale of component B results in the gain of $b - c$ and the loss of $g - c$. Once we take into account this opportunity cost, we observe that the two firms have the same total marginal cost of production of the component B (and the same profit margin). Marginal costs and players are thus symmetric when the market is covered.

We now characterize Nash equilibria of the pricing stage in the double duopoly structure. There is a unique uncovered equilibrium that is asymmetric (i.e $b \leq d$) and a unique standard competitive covered equilibrium. There is also a multiplicity of kinked equilibria. We characterize the symmetric kinked equilibrium where prices of both goods are equal (i.e $b = d$) and the asymmetric kinked equilibrium which is such that the independent producer charges the uncovered price d while the merged entity chooses the corresponding kinked price b. In equilibrium, the uncovered price of the independent producer is equal to the level of its kinked price. To simplify the exposition, we present the equilibrium conditions in the case where consumers are sufficiently sensitive to the distance in the product space to sustain all types of equilibria (i.e $t > \frac{1}{3}(-8c + 4g + 4k)$):

- the uncovered equilibrium:

$$a = k \tag{3.39a}$$

$$b = \frac{1}{3}(2c - 3k + 2v) \tag{3.39b}$$

$$d = \frac{1}{3}(g - 2k + 2v) \tag{3.39c}$$

$$\text{when: } v < \frac{24t(2c + g + k) + 16(-2c + g + k)^2 + 9t^2}{48t}. \tag{3.39d}$$

- the asymmetric kinked equilibrium:

$$a = k \tag{3.40a}$$

$$b = \frac{1}{12}\left(4\sqrt{3}\sqrt{-t(g + k - v)} + 4g - 8k - 3t + 8v\right) \tag{3.40b}$$

$$d = \frac{1}{3}(g - 2k + 2v) \tag{3.40c}$$

$$\text{when: } \frac{24t(2c + g + k) + 16(-2c + g + k)^2 + 9t^2}{48t} \leq v \leq g + k - \frac{3}{8}\left(2\sqrt{10}\sqrt{t^2} - 7t\right). \tag{3.40d}$$

- the symmetric kinked equilibrium:

$$a = k \tag{3.41a}$$

$$d = \frac{1}{16}(-16k - t + 16v) \tag{3.41b}$$

$$b = \frac{1}{16}(-16k - t + 16v) \tag{3.41c}$$

$$\text{when: } \frac{1}{16}(16g + 16k + 3t) \leq v \leq \frac{1}{16}(16g + 16k + 5t). \tag{3.41d}$$

- the competitive covered equilibrium:

$$a = k \tag{3.42a}$$
$$b = g + \frac{t}{4} \tag{3.42b}$$
$$d = g + \frac{t}{4} \tag{3.42c}$$
$$\text{when: } v > g + k + \frac{5t}{16}. \tag{3.42d}$$

Integration into the differentiated market mainly affects the uncovered equilibrium which is made asymmetric by positive royalty rates. This asymmetry in prices of component B can possibly carry over to the kinked equilibria. The multiplicity of equilibria indeed appears in the double duopoly structure because of the asymmetry in the locations of marginal consumers due to asymmetry in price setting behaviors. From equations 3.40d and 3.41d, we find that the symmetric kinked equilibrium is sustained by higher system valuations than the asymmetric kinked equilibrium. Assuming strictly positive royalty rates are charged, the decrease in the profit margin of the second component B producer following this vertical merger makes the double duopoly structure covered (i.e asymmetric kinked equilibrium) for lower system values than in the separated case. As for the uncovered equilibrium, the price of the system 2 is efficient with respect to the profit of the industry (i.e $p_2 = \frac{2(v+c)}{3}$[11], from equations 3.39a and 3.39b) for all levels of royalties. Positive royalty rates however make the price of the system 1 higher because of vertical double marginalization (i.e $p_1 = \frac{2v+g+k}{3}$, from equations 3.39a and 3.39c).[12] We will now look for price equilibria of the asymmetric structure when the monopoly innovator is active in the differentiated component market.

3.4.1.2 Asymmetric structure

We turn to the analysis of the downstream equilibria in the asymmetric structure (i.e $N_A = 1$ and $N_B = 2$). There are some similarities with the double duopoly structure in

[11] The price of this system corresponds to the fully integrated monopoly price.

[12] The equilibrium price of system 1 is higher than in the fixed fee model where uncovered prices are efficient.

the equilibria that we characterize. The price of component A is not equal to marginal cost anymore and is determined by an independent monopoly producer. The behavior of component B producers remain the same as in the double duopoly except that k is replaced by a in the expression of their respective best response function. We thus present the profit and best response functions of the monopoly producer of the component A:

$$\Pi_A = \begin{cases} (a-k) \times \frac{1}{2}, \text{ if: } \bar{x}_1 + \bar{x}_2 > \frac{1}{2} & (3.43) \\ (a-k)\sqrt{\frac{-a-b+v}{t}} + (a-k)\sqrt{\frac{-a-d+v}{t}}, \text{ otherwise.} & (3.44) \end{cases}$$

$$BR_A = a = \begin{cases} \frac{-16b^2 + 32bd - 8bt - 16d^2 - 8dt - t^2 + 16tv}{16t}, \text{ if: } \bar{x}_1 > \frac{1}{2} - \bar{x}_2 & (3.45) \\ \frac{1}{3}(-2b - 2d - k + 4v) & (3.46) \\ -\frac{2}{3}\sqrt{b^2 - bd + bk - bv + d^2 + dk - dv + k^2 - 2kv + v^2}, \text{ otherwise.} \end{cases}$$

The best response functions of component B producers in the asymmetric structure remain almost identical in the separated and differentiated integration regimes. Both the integrated and independent producers have the same competitive and kinked best response functions.[13] The best response of the integrated B component producer only differs from the function of the independent producer when the market is uncovered. Its marginal cost is equal to $c - (k - c)$ (or $2c - k$ equivalently), whereas the independent producer faces a marginal cost equal to g. The competitive best response function is identical for both players because royalty revenue is maximal and constant when the market is covered. The difference in marginal cost is compensated by the opportunity cost faced by the integrated firm of stealing business to the rival B producer. The kinked best response function of component B producers is also symmetric. Given the prices charged by the two other players (i.e a and b_j), there is a unique symmetric price ensuring that the market is kinked (i.e just covered).

Using these best response functions, we characterize the following downstream equilibria of the asymmetric structure:

[13] The independent producer also has the same best response function as in the separated case.

- the uncovered equilibrium:

$$a = \frac{1}{5}(2\sqrt{4c^2 + 2c(7(g+k) - 9v) - 9v(g+k) + (g+k)^2 + 9v^2} + 4c + 2g + 7k - 4v) \tag{3.47a}$$

$$b = \frac{1}{15}(-4\sqrt{4c^2 + 2c(7(g+k) - 9v) - 9v(g+k) + (g+k)^2 + 9v^2} + 2c - 4g - 19k + 18v) \tag{3.47b}$$

$$d = -\frac{4}{15}\sqrt{4c^2 + 14cg + 14ck - 18cv + g^2 + 2gk - 9gv + k^2 - 9kv + 9v^2} + \frac{1}{15}(-8c + g - 14k + 18v), \tag{3.47c}$$

if and only if:

$$v \leq \frac{72t(2c + g + k) + 16(-2c + g + k)^2 + 45t^2}{144t}. \tag{3.48}$$

- the symmetric kinked equilibrium:

$$a = -g - \frac{3t}{16} + v \tag{3.49a}$$

$$b = g + \frac{t}{8} \tag{3.49b}$$

$$d = g + \frac{t}{8}, \tag{3.49c}$$

if and only if:

$$v \geq \frac{1}{16}(16g + 16k + 5t). \tag{3.50}$$

- the competitive covered equilibrium:

$$a = -g - \frac{5t}{16} + v \tag{3.51a}$$

$$b = g + \frac{t}{4} \tag{3.51b}$$

$$d = g + \frac{t}{4}, \tag{3.51c}$$

if and only if:

$$v \geq \frac{1}{16}(16g + 16k + 7t). \tag{3.52}$$

As one can see in equation 3.48, the boundary on v for the uncovered equilibrium is increasing with the level of royalty rates. This implies that positive royalty rates make the uncovered equilibrium more likely (i.e supported for a larger set of parameters).[14] The price of component B is asymmetric when the market is at the uncovered equilibrium. This is due to the fact that the integrated firm internalizes the royalties perceived on the alternative component B as well as on the component A. Moreover, the independent firm faces a higher marginal cost with positive royalty rates. The price charged by the integrated producer is consequently lower than the price of the independent B component producer.

On the other hand, prices are symmetric at the competitive and symmetric kinked equilibrium. This is due to the fact that once the market is covered, royalty revenue is maximal and constant (equal to $\frac{k-c}{2}$ and $\frac{g-c}{2}$). We deal with the multiplicity of kinked equilibria by focusing on a symmetric kinked equilibrium because it is efficient with respect to transportation costs (i.e consumers purchase their closest variety in equilibrium) and results in higher social surplus and profits for the innovator.

3.4.2 Optimal licensing policy

Given that the innovator is able to commit on the number of licenses and on royalty levels, we know that the upstream innovator is able to capture the entire downstream profit and to maximize it through the use of public two-part tariff licensing contracts.

3.4.2.1 Royalty rates

Assuming the asymmetric structure is chosen by the innovator, we look for its profit-maximizing level of royalties (g^*, k^*). The expression of the profit of the inside innovator

[14] The threshold on the sustaining values of v converges to the fixed fee boundary level as royalties tend to zero.

UB_2 (i.e the profit of the industry) depends on whether or not the downstream market is covered or not in equilibrium:

$$\Pi_{UB_2} = \Pi_{B_1} + \Pi^D_{UB_2} + \Pi_A \tag{3.53}$$

- in the competitive equilibrium:

$$\Pi_{UB_2} = \frac{a-c}{2} + (b-c)\left(\frac{2(d-b)}{2t} + \frac{1}{4}\right) + (d-c)\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) \Leftrightarrow \tag{3.54a}$$

$$\Pi_{UB_2} = \frac{1}{4}\left(2a - \frac{4(b-d)^2}{t} + b - 4c + d\right) \Leftrightarrow \tag{3.54b}$$

$$\Pi_{UB_2} = -2c - \frac{t}{16} + v \tag{3.54c}$$

$$\text{and: } 2c \leq g^* + k^* \leq v - \frac{7t}{16}. \tag{3.54d}$$

- in the symmetric kinked equilibrium:

$$\Pi_{UB_2} = (d-g)\left(\frac{1}{2} - \sqrt{\frac{-a-b+v}{t}}\right) + (b-g)\sqrt{\frac{-a-b+v}{t}}$$

$$+ \frac{a-k}{2} + \frac{g-c}{2} + \frac{k-c}{2} \Leftrightarrow \tag{3.55a}$$

$$\Pi_{UB_2} = \frac{1}{32}\left(-32c - 2t\sqrt{5 - \frac{4t}{\sqrt{t^2}}} + 2\sqrt{t^2}\sqrt{5 - \frac{4t}{\sqrt{t^2}}} - t + 16v\right) \Leftrightarrow \tag{3.55b}$$

$$\Pi_{UB_2} = -2c - \frac{t}{16} + v \tag{3.55c}$$

$$\text{and: } 2c \leq g^* + k^* \leq v - \frac{5t}{16}. \tag{3.55d}$$

- in the uncovered equilibrium:

$$\Pi_{UB_2} = (a-c)\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right) + (b-c)\sqrt{\frac{-a-b+v}{t}}$$
$$+ (d-c)\sqrt{\frac{-a-d+v}{t}} \tag{3.56a}$$
$$\text{and: } g^* = k^* = c \Rightarrow \tag{3.56b}$$
$$\Pi_{UB_2} = -\frac{8(2c-v)\sqrt{\frac{v-2c}{t}}}{5\sqrt{5}}, \text{ if and only if: } v \leq 2c + \frac{5t}{16}. \tag{3.56c}$$

In the appendix C.2.2.1, we show that positive royalties worsen double marginalization problems and depreciate uncovered equilibrium system sales compared to a pure fixed fee contract. Given the expression of the uncovered equilibrium prices presented in equations 3.47a, 3.47b and 3.47c, we find that the profit of the patentee decreases with per unit royalty rates (i.e negative derivative of the uncovered equilibrium profit with respect to royalties). Under two-part tariff contracts, integration into the differentiated market mitigates the double marginalization effects in the uncovered asymmetric structure but it does not enable the full compensation of the negative impact of per unit royalty rates and fails to reach the maximization of the profit of the industry.

On the other hand, we observe in equations 3.54c and 3.55c that the profit of the innovator is independent of the level of royalties when the market is in a covered equilibrium. The patentee is able to capture the full social surplus as soon as the system valuation is sufficiently high. Pure fixed fee contract eliminates asymmetric inefficient kinked equilibria and makes the kinked asymmetric structure efficient. The asymmetry in the behavior of producers of component B only stems from strictly positive royalties. As a result, pure fixed fee licensing is the most profitable strategy in the asymmetric structure for all values of the parameters.

Now assuming that the double duopoly structure is chosen, we look for the optimal royalty levels for each equilibrium we characterized. The profit of the merged entity takes the same expression as in the asymmetric structure except for the price of component A that is equal to marginal cost k. We now present the profit function of the inside innovator and the optimal royalty rates in each equilibrium:

- in the uncovered equilibrium:

$$\Pi_{UB_2} = (k-c)\left(\sqrt{\frac{-k-b+v}{t}} + \sqrt{\frac{-k-d+v}{t}}\right) + (b-c)\sqrt{\frac{-k-b+v}{t}} + (d-c)\sqrt{\frac{-k-d+v}{t}} \tag{3.57a}$$

$$\bar{x}_1 = \sqrt{\frac{\frac{1}{3}(-g+2k-2v)-k+v}{t}}, \text{ depends negatively on royalties,} \tag{3.57b}$$

$$\text{thus: } g^* = k^* = c \tag{3.57c}$$

$$\text{and: } \Pi_{UB_2} = \frac{4(v-2c)^{3/2}}{3\sqrt{3}\sqrt{t}}. \tag{3.57d}$$

- in the asymmetric kinked equilibrium:

$$\Pi_{UB_2} = (d-g)\left(\frac{1}{2} - \sqrt{\frac{-k-b+v}{t}}\right) + (b-g)\sqrt{\frac{-k-b+v}{t}} + \frac{g-c}{2} + \frac{k-c}{2} \Leftrightarrow \tag{3.58a}$$

$$\Pi_{UB_2} = \frac{1}{2}\left(2(b-g)\sqrt{-\frac{b+k-v}{t}} - 2c + 2(d-g)\sqrt{-\frac{d+k-v}{t}} + g + k\right) \Leftrightarrow \tag{3.58b}$$

$$\Pi_{UB_2} = \frac{1}{8}\left(-8c + 2\sqrt{3}\sqrt{-t(g+k-v)} + 4(g+k) - t\right) \tag{3.58c}$$

$$\text{thus: } g^* = \frac{1}{16}(-16k - 3t + 16v) \tag{3.58d}$$

$$\text{and: } \Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}. \tag{3.58e}$$

- in the symmetric kinked equilibrium:

$$\Pi_{UB_2} = (d-g)\left(\frac{1}{2} - \sqrt{\frac{-k-b+v}{t}}\right) + (b-g)\sqrt{\frac{-k-b+v}{t}} + \frac{g-c}{2} + \frac{k-c}{2} \tag{3.59a}$$

$$\bar{x}_1 = \sqrt{\frac{\frac{1}{16}(16k+t-16v)-k+v}{t}}, \text{ then we have:} \tag{3.59b}$$

$$\Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}, \text{ and the innovator is indifferent to royalty levels.} \tag{3.59c}$$

- in the competitive kinked equilibrium:

$$\Pi_{UB_2} = \frac{k-c}{2} + (b-c)\left(\frac{2(d-b)}{2t} + \frac{1}{4}\right) + (d-c)\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) \tag{3.60a}$$

$$g^* = \frac{1}{16}(-16k - 5t + 16v) \tag{3.60b}$$

$$\Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}. \tag{3.60c}$$

Assuming that the market is at the asymmetric kinked equilibrium, we see in equations 3.58d and 3.58e that the inside innovator would find it optimal to stir the equilibrium prices and market shares to symmetry in charging a particular level of royalty rates. This is due to the fact that asymmetric market shares are costly for the upstream monopolist as they generate higher transportation costs and lower surplus (leading to lower profit opportunities).

In the uncovered equilibrium of the double duopoly structure, royalties have negative effects on the total surplus, the profit of the industry and in turn the profit of the inside innovator. Royalties earned on the differentiated market increase marginal costs of production of the independent producer and tend to increase its price (i.e vertical double marginalization). On the other hand, an increase in the marginal cost of production of the complementary good (i.e component A) has a tendency to reduce the price charged by both component B producers. Looking at system prices, $k + b_i$ from equations 3.39a, 3.39b and 3.39c, we see that system 2 is efficiently priced anyway (i.e at the integrated monopoly system price equal to: $\frac{2(v+c)}{3}$). The price of system 1 (i.e $\frac{2v+g+k}{3}$)

is however increasing with both royalties earned on market A and B and is equal to the integrated monopoly price when royalties are equal to zero. This is due to the fact that the independent producer only internalizes a share of the impact of the rise in the price of the complementary good. An increase in its marginal cost is also partially transmitted onto consumers. These two effects lead to an inefficiently high price of system 1. It is desirable for the inside innovator to set royalties to zero in order for both local downstream monopolists to maximize the profit of the industry (i.e elimination of the vertical double marginalization) which is in turn captured by the upstream monopolist.

When the market is in the symmetric kinked equilibrium (i.e non competitive), the inside innovator is indifferent to the levels of royalties. The symmetric best response functions of component B producers do not depend on marginal cost of production. Both producers would fully compensate the increase in the price of the complementary good A resulting from an increase in marginal cost as they charge the residual net surplus in order to make the market just covered. The industry profit is maximized because the symmetric kinked equilibrium prices result in full market coverage and surplus extraction no matter the level of royalties.

Thanks to two-part tariffs, the competitive equilibrium is also efficient in the double duopoly structure. The levels of royalties can be used in order to control system prices through the intensity of competition so that the surplus left to consumers simply allows full participation in the market. The inside innovator can equally charge royalties on one or the other component market. We now compare these two structures of the industry across consistent system valuations and determine the optimal licensing policy of an inside innovator active in the differentiated market.

3.4.2.2 Number of licenses

When the market is uncovered, the asymmetric structure is suboptimal because of the horizontal double marginalization at work between the local monopolists on component B market and the homogeneous monopoly in market A. Even if integration into the differentiated market helps to mitigate this negative effect (compared to the separated regime with strictly positive royalty rates), the inside innovator finds it optimal to completely eliminate horizontal double marginalization by implementing the double duopoly structure.[15]

Because of the asymmetry in component B prices, the multiple kinked equilibria are likely to be suboptimal as well (since transportation costs are minimized when the kinked equilibrium price is symmetric). Differentiated integration with strictly positive royalty rates can lead to asymmetric behaviors and market shares. It results in inefficient consumption decisions and higher average transportation costs. The resulting loss in social surplus is ultimately borne by the upstream innovator. We can see in equation 3.51b that a competitive symmetric equilibrium exists in the asymmetric structure under royalty contracts and differentiated integration. A symmetric non competitive kinked equilibrium is also characterized in equation 3.49b. There is however no way to guarantee coordination on efficient symmetric kinked equilibria. That is why, the upstream monopolist would always prefer to use a pure fixed fee contract in the asymmetric structure. This will ensure that each equilibrium of the asymmetric structure will generate a symmetric efficient allocation of sales in market B. In sum, when the valuation for the system is high (i.e $v > 2c + \frac{5t}{16}$), the inside innovator is indifferent between the asymmetric structure with a fixed fee contract and the double duopoly. Otherwise, the double duopoly is strictly preferred.

Introducing the optimal levels of royalties into equilibrium profits in each structure of the industry, we obtain the following expressions. In the asymmetric structure, equilibrium profits of the upstream innovator are:

- in the competitive equilibrium:

$$\Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}, \tag{3.61}$$

[15]Even though differentiated integration reduces double marginalization effects with respect to the separated royalty model, the fixed fee model still perform better as royalty rates remain to have a negative effect when the market is uncovered.

- in the symmetric kinked equilibrium:

$$\Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}, \tag{3.62}$$

- in the uncovered equilibrium:

$$\Pi_{UB_2} = -\frac{8(2c - v)\sqrt{\frac{v-2c}{t}}}{5\sqrt{5}}, \tag{3.63a}$$

$$\text{if and only if: } 2c < v \leq 2c + \frac{5t}{16}. \tag{3.63b}$$

Equilibrium profits of the innovator in the double duopoly structure are:

- in the uncovered equilibrium, when $v \leq 2c + \frac{3t}{16}$, and $g = k = c$:

$$\Pi_{UB_2} = \frac{4(v - 2c)^{3/2}}{3\sqrt{3}\sqrt{t}}, \tag{3.64}$$

- in the asymmetric kinked equilibrium, when $v \geq \frac{1}{16}(32c + 3t)$, and $g = \frac{1}{16}(-16k - 3t + 16v)$:

$$\Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}, \tag{3.65}$$

- in the symmetric kinked equilibrium, when $\frac{1}{16}(32c + 3t) \leq v \leq \frac{1}{16}(32c + 5t)$, the profit of the industry is also maximal:

$$\Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}, \tag{3.66}$$

- in the competitive equilibrium, when $k \leq \frac{1}{16}(-16g - 5t + 16v)$, $g \leq \frac{1}{16}(-16k - 5t + 16v)$ and:

$$v \geq \frac{1}{16}(32c + 5t), \tag{3.67a}$$

$$\text{we have : } \Pi_{UB_2} = -c - \frac{t}{32} + \frac{v}{2}. \tag{3.67b}$$

When the sensitivity of consumers to distance is high (equivalently when v is low as in the equation 3.63b), the market is uncovered in the asymmetric structure. We showed in the appendix C.2.2.1 that royalties are inefficient and have a negative impact on the profit of the innovator. Thus royalties are optimally set to zero. Fixed fee contracts are chosen and the double duopoly structure remains strictly more profitable in uncovered equilibrium. The uncovered asymmetric structure features inefficient double marginalization on complementary goods.

To the contrary, the double duopoly structure enables the elimination of the horizontal double marginalization through the implementation of perfect competition in the component A market. Otherwise (i.e when v is sufficiently high and equation 3.63b is not satisfied), the outside innovator is indifferent between the kinked asymmetric and the competitive double duopoly structure which allow an efficient system price to be charged in equilibrium. If the asymmetric structure is chosen, fixed fee contracts are used in order to avoid the occurrence of an inefficient asymmetric price equilibrium. On the other hand, positive royalty rates are necessary for the inside innovator in the competitive equilibrium of the double duopoly structure (i.e high level of v as in equation 3.66) in order to reduce the intensity of competition.

Proposition 3.4.1. *A monopoly innovator active in the production of the differentiated component chooses to deliver as many licenses as possible in both component markets, implementing the double duopoly structure with a pure fixed fee licensing contract when the technology is used in a niche market (i.e $v < 2c + \frac{5t}{16}$).*

When the technology is used in a mass market in which the valuation for the final system is high (i.e $v > 2c + \frac{5t}{16}$), the inside innovator is indifferent between:

- *the double duopoly structure (i.e $N_A = N_B = 2$) with a two-part tariff licensing contract with royalty rates such that $g + k = v - \frac{5t}{16}$,*
- *a pure fixed fee contract (i.e $g = k = c$) including an exclusive license on the homogeneous component market (i.e $N_A = 1$ and $N_B = 2$).*

Proof. See in the appendix. □

The introduction of two-part tariff licensing contracts makes the implementation of the double duopoly structure a weakly dominating strategy for the outside or inside innovator.[16] The use of per unit royalty rates is strictly profitable in the double duopoly structure when the valuation for the system is high. This remains true when the upstream monopolist is integrated with a differentiated component producer. Differentiated integration seems to make the upstream monopolist more sensitive to royalty rates in kinked equilibria than under separation and integration into the homogeneous market. Efficient kinked prices can indeed be reached in some particular cases through a specific use of royalties (e.g stirring to symmetry effect in the double duopoly structure, pure fixed fee licensing in the asymmetric structure).

3.5 Double vertical integration

We will now consider the structure of the industry where the monopoly holder of a technology is active in each of the complementary good markets in which its technology is used. This structure is the result of two mergers between the upstream innovator and one of the producers of each downstream component. The second integration can also be considered as a conglomerate merger between two complementary good producers with different vertical ownership structures (i.e one vertically integrated and one separated producer).

In the following section, we characterize the optimal two-part tariff licensing policy and show that it is strictly profitable for the inside innovator active in both downstream complementary markets to implement the double duopoly structure for all values of the parameters.[17] This is in contradiction with the case where the technology is used in a single homogeneous downstream market in which the upstream monopolist is indifferent between downstream monopoly and duopoly when two-part tariff licensing contracts are available. In the double integration regime, we show that the asymmetric structure is unable to maximize the profit of the industry. Symmetric kinked equilibria do not

[16] See the summarizing diagram C.1 page 230.

[17] See the summarizing diagram C.1 page 230.

exist in the post conglomerate merger asymmetric structure. The resulting asymmetry in market shares makes it inefficient with respect to transportation costs[18], social surplus and the profit of the industry.

3.5.1 Downstream equilibria

3.5.1.1 Double duopoly structure

The conglomerate merger does not change the set of price equilibria of the double duopoly structure compared to the case where the inside innovator is active in the differentiated market.[19] Perfect Bertrand competition leads to a price of component A equal to the marginal cost of the independent producer (i.e $a = k$). The competition constraint is so strong that the conglomerate merger does not allow an effective control of the price of component A. We also know that the direct control of the price a can be redundant under two-part tariff licensing. In the separated double duopoly structure, the inside innovator (i.e U_{B_2}) is able to control the price a through the level of royalties charged on the homogeneous component market. The conglomerate merger does not change the set of equilibria of the double duopoly structure.

3.5.1.2 Asymmetric structure

In this section, we show that there is no symmetric kinked equilibrium allocation of sales of component B in the asymmetric structure. This results in higher total transportation costs, lower social surplus and profits for the innovator. The asymmetry in component B prices is due to the subsidization behavior of the integrated firm which allows it to capture a higher share of the social surplus through an increase in the price of component

[18] This is also true in the pure fixed fee licensing model studied in chapter two.
[19] This is true under both fixed fee and two-part tariff licensing.

A without reducing the demand.[20] The market is made uncovered if:

$$d > \sqrt{3}\sqrt{tv - 2ct} + \frac{1}{4}(4g - 3t). \tag{3.68}$$

Then the best response function of the integrated firm takes the following form[21]:

$$BR_{UAB_2} = \begin{cases} a = \frac{1}{3}(2c - 2d - g + 2v) & (3.69) \\ b = \frac{2d}{3} + \frac{g}{3}. & (3.70) \end{cases}$$

Otherwise, the market is kinked and its best response function is equal to:

$$BR_{UAB_2} = \begin{cases} a = -\frac{16d^2 + 8d(15t - 4g) + (4g + 3t)^2 - 144tv}{144t} & (3.71) \\ b = \frac{1}{3}(2d + g). & (3.72) \end{cases}$$

Combining this expression with the standard best response function of the independent producer of the component B, we characterize the following price equilibria when the monopoly innovator directly produces both components:

- the uncovered equilibrium:

$$a = \frac{6c}{5} - g + \frac{2v}{5} \tag{3.73a}$$
$$b = -\frac{8c}{15} + g + \frac{4v}{15} \tag{3.73b}$$
$$d = -\frac{4c}{5} + g + \frac{2v}{5} \tag{3.73c}$$
$$p_1 = \frac{2}{5}(c + 2v) \tag{3.73d}$$
$$p_2 = \frac{2(c + v)}{3}, \tag{3.73e}$$

[20]This result is first derived in chapter two. In this section, we show that this analysis carries over to the two-part tariff licensing contract framework.

[21]The best response function of the integrated firm is identical to the one presented in chapter two except for the marginal cost term of the independent firm (i.e g instead of c) that now includes the royalty rate charged by the integrated firm.

if and only if:

$$2c < v < 2c - \frac{15}{8}\left(\sqrt{15} - 4\right)t, \tag{3.74}$$

- the strategic kinked equilibrium:

$$a = -g + \frac{9}{8}\sqrt{15}\sqrt{t^2} - \frac{9t}{2} + v \tag{3.75a}$$

$$b = g - \frac{1}{2}\sqrt{15}\sqrt{t^2} + 2t \tag{3.75b}$$

$$d = g - \frac{3}{4}\sqrt{15}\sqrt{t^2} + 3t \tag{3.75c}$$

$$p_1 = \frac{3}{8}\sqrt{15}\sqrt{t^2} - \frac{3t}{2} + v \tag{3.75d}$$

$$p_2 = \frac{5}{8}\sqrt{15}\sqrt{t^2} - \frac{5t}{2} + v, \tag{3.75e}$$

if and only if:

$$v > 2c - \frac{15}{8}\left(\sqrt{15} - 4\right)t. \tag{3.76}$$

From the observation of the best response function of the merged entity (see equation 3.72), we know that its subsidization behavior prevents the existence of any symmetric equilibria. Price externalities between complementary goods are internalized within the integrated firm which makes it willing to charge a lower price for its component B resulting in asymmetric market shares. We present here a particular asymmetric kinked equilibrium (i.e the strategic kinked equilibrium) where the uncovered and kinked best response sub-functions of the independent firm are equal. It implies that in equilibrium, the uncovered best response sub-function of the independent firm makes the market just covered (i.e kinked).

When the market is uncovered, the levels of sales of the two systems are independent. There are two local monopolists in the differentiated downstream market. The profit-maximizing prices of the component A and B_1 (i.e a and d) are the same as in the separated case whereas b is lowered by the conglomerate merger. The inside innovator chooses to subsidize the component B produced in-house in order to implement the efficient integrated monopoly price. We now turn to the analysis of the optimal two-part tariff licensing policy of the innovator active in both component markets.

3.5.2 Optimal licensing policy

3.5.2.1 Royalty levels

In the double duopoly structure, the optimal levels of royalties remain the same as in the previous section (i.e integration into the differentiated market) because the conglomerate merger has no impact in this structure of the industry. Royalties are strictly profitable in the competitive equilibrium. Otherwise, the inside innovator does not charge positive royalty rates. The asymmetric structure is in the uncovered equilibrium when the valuation for the technology is sufficiently low. We can see in equations 3.75b and 3.75c that equilibrium prices are such that the royalty charged on the independent firm is fully transmitted to consumers through an increase in the price of its variety of component B. This is fully compensated by the price of component A making system prices independent of royalty rates. This is the case because the integrated firm internalizes the royalty revenue it earns on the sales of the independent producer. Total profit of the industry is independent of the level of royalty which makes the conglomerate firms indifferent to its level.

On the other hand, when the valuation for the technology is high so that the market is fully covered, the allocation of sales remain asymmetric and inefficient for all levels of the royalty rate which has no effect on system prices nor on profits.[22] The inside innovator being indifferent to the level of the royalty rate in the asymmetric structure, we consider that it chooses to use a pure fixed fee contract.

It is worth noting that the conglomerate merger raises the efficiency of the uncovered asymmetric structure making the price of the integrated system efficient (see equation 3.73e). Taking the licensing policy as exogenous, the conglomerate merger is profitable for the upstream firm when the valuation for the system is low (i.e in a niche market).[23]

[22]This is in contradiction with the differentiated integration case where the level of royalty rates matters to the inside innovator in kinked equilibria.
[23]We derive this result in chapter two. Chapter three shows that it is robust to the availability of royalty rates.

3.5.2.2 Number of licenses

We compare the profit of the innovator across structures in order to determine the most profitable number of licenses. We consider the framework that is the most favorable to the conglomerate firm. We assume that the multiplicity of equilibria does not prevent the innovator to fully extract the profit of the independent firm through the use of fixed fee. This implies a coordination and commitment mechanism enabling the realization of a specific equilibrium which ensures that licensees are ready to pay up front despite the multiplicity of equilibria.

Under these circumstances, we compare the profit of the inside innovator across structures. Knowing that it is always profitable to allow the production of two varieties of the differentiated component, we only compare the asymmetric and double duopoly structures for all levels of valuation for the final system good. In the asymmetric structure, the profit of the inside innovator is:

- in the uncovered equilibrium (i.e when $v \leq 2c - \frac{15}{8}\left(\sqrt{15}-4\right)t$):

$$\Pi_{UAB_2} = \frac{1}{225}(-2)\left(25\sqrt{3}+18\sqrt{5}\right)(2c-v)\sqrt{\frac{v-2c}{t}}, \tag{3.77}$$

- in the strategic kinked equilibrium (i.e only if $v \geq 2c - \frac{15}{8}\left(\sqrt{15}-4\right)t$)):

$$\begin{aligned}\Pi_{UAB_2} =& \frac{\sqrt{4-\frac{\sqrt{15}t}{\sqrt{t^2}}}}{16\sqrt{2}} \times (-8\left(\sqrt{3}+\sqrt{5}\right)(2c-v) \\ &+ 25\sqrt{3}\sqrt{t^2} + 9\sqrt{5}\sqrt{t^2} - 4\left(3\sqrt{3}+5\sqrt{5}\right)t).\end{aligned} \tag{3.78}$$

In the double duopoly structure, the profit of the patentee is:

- in the uncovered equilibrium (i.e when $v \leq 2c + \frac{3t}{16}$ and with $g = k = c$):

$$\Pi_{UAB_2} = \frac{4(v-2c)^{3/2}}{3\sqrt{3}\sqrt{t}}, \tag{3.79}$$

- in the asymmetric kinked equilibrium (i.e only if $v \geq \frac{1}{16}(32c + 3t)$ and $g = \frac{1}{16}(-16k - 3t + 16v)$):

$$\Pi_{UAB_2} = -c - \frac{t}{32} + \frac{v}{2}, \tag{3.80}$$

- in the symmetric kinked equilibrium (i.e if $\frac{1}{16}(32c + 3t) \leq v \leq \frac{1}{16}(32c + 5t)$), the profit of the industry is maximal for all values of royalties k and g:

$$\Pi_{UAB_2} = -c - \frac{t}{32} + \frac{v}{2}, \tag{3.81}$$

- in the competitive equilibrium equilibrium (i.e if $v \geq \frac{1}{16}(32c + 5t)$, with $k \leq \frac{1}{16}(-16g - 5t + 16v)$ and $g \leq \frac{1}{16}(-16k - 5t + 16v)$):

$$\Pi_{UAB_2} = -c - \frac{t}{32} + \frac{v}{2}. \tag{3.82}$$

When both structures are uncovered, the double duopoly is preferred (i.e equations 3.77 and 3.79). When both structures are kinked, we find in comparing equations 3.78 and 3.80 that the double duopoly is also preferred. Moreover, knowing that the double duopoly is covered for a wider range of parameters than the asymmetric structure, it implies that the double duopoly structure is always preferred by the conglomerate firm (i.e the covered double duopoly also dominates the uncovered asymmetric structure).

Thanks to two-part tariff licensing contracts, the double duopoly structure is never dominated even in a mass market (i.e high v/t). The inside innovator active in both complementary good markets finds it more profitable to deliver as many licenses as possible in both component markets when the valuation for the system is low (i.e mass niche market). The double duopoly structure avoids the double marginalization of the uncovered asymmetric structure. It is also preferred when the valuation for the system is high (i.e in all covered equilibria) because its allocation of sales is symmetric. Royalty rates indeed allow the double duopoly structure to remain efficient and profitable in the competitive equilibrium.

Proposition 3.5.1. *The double duopoly structure is always chosen by the innovator directly producing both downstream complementary goods. In this particular vertical structure, there is no more range of parameters such that the indifference between monopoly and duopoly on the homogeneous market which holds in the case of an isolated downstream good, remains in the case of downstream complementary goods.*

Proof. This proposition directly results from the comparison of equilibrium profits of the inside innovator (i.e equal to the industry profit) in the asymmetric and double duopoly structures. More details are available in the appendix. □

The optimal licensing policy of the inside innovator active in each of the complementary good is affected by the conglomerate merger. It makes the implementation of the double duopoly structure a strictly profitable strategy on the whole range of parameters. It is the only ownership structure where the indifference between the asymmetric and the double duopoly does not arise under two-part tariff licensing (see the summarizing diagram C.1 page 230). The optimal licensing policy of such an inside innovator strongly differs when its technology is used in downstream complementary markets. The indifference across competitive structures that appears with two-part tariff contracts in the case of an isolated homogeneous market is completely broken when downstream complementary goods are considered and the upstream monopolist is active in both markets. Comparing the effect of royalties on the optimal licensing policy across vertical structures, we see that two-part tariff licensing consistently makes the double duopoly structure efficient in mass markets (i.e competitive equilibrium). [24]

3.6 Conclusion

Under fixed fee licensing, we consistently find across integration regimes that in a mass market (i.e low values of the sensitivity of consumers to the distance between available

[24] In the post conglomerate merger industry, the asymmetric structure is preferred for very high system valuations under fixed fee licensing but not when two-part tariffs are available (see the summarizing diagram C.1 page 230).

and ideal varieties), the use of an exclusive license is desirable for the upstream innovator whose technology is used in perfectly complementary markets. The asymmetric structure (i.e monopoly in the homogeneous and duopoly in the differentiated market) is indeed preferred in order to avoid the dissipation of profits due to intense downstream competition. High downstream market power is not damaging in this case because system valuation is high enough to fully cover the market. As a result the fixed fee licensing policy of the upstream innovator in perfectly complementary markets may differ from the one in a single homogeneous downstream market where an exclusive license would be optimal.

When we extend our analysis to the case of two-part tariff contracts, we find that the efficiency of the double duopoly structure (i.e no exclusive license) in a mass downstream market is restored. As a consequence, the double duopoly structure is never strictly dominated (see the summarizing diagram C.1 page 230). Under two-part tariff licensing, we do not find a clear rationale for the use of an exclusive license since the double duopoly structure is doing as well as the asymmetric structure when the latter is covered in equilibrium whereas it outperforms the asymmetric structure when it is uncovered.

Now consider the benchmark where a monopoly innovator licenses its technology to a single homogeneous downstream market. While its optimal licensing policy would be to deliver an exclusive license under pure fixed fee contracting, two-part-tariff licensing restores the efficiency of downstream competition. The upstream monopolist is then able to determine the marginal cost of production of its licensees and thus the downstream prices. Competition with positive royalties is performing as well as pure fixed fee exclusive licensing. The innovator is then indifferent between monopolized and competitive downstream market structures.

Comparing our results to this single homogeneous benchmark, we find that under two-part tariff, the licensing policy on an homogeneous market may also differ when we consider the differentiated complementary market using the same technology. When the upstream firm produces both components in-house (i.e in the double vertical integration case), this indifference is indeed completely broken in favor of downstream competition (i.e the double duopoly structure is more profitable for all range of parameters). In other vertical structures of the industry, the indifference remains when the valuation for the system is high (i.e in mass markets). Otherwise, the innovator finds it strictly profitable to implement downstream competition.

In chapter one, we show that with pure fixed fee contracts, the number of licenses delivered in equilibrium for the production of the homogeneous component strictly decreases as the varieties of the complementary good become closer substitutes. This is consistent with Doganoglu and Inceoglu (2014) and Hernández-Murillo and Llobet (2006) but in contradiction with Arora and Fosfuri (2003). Under two-part tariffs however, the intensity of competition can also be regulated through the use of per unit royalty rates. As a consequence, the number of licenses does not necessarily decrease with the valuation for the system (or the degree of substitutability). With the exception of the double vertical integration case, we find that the innovator is made indifferent between decreasing the number of licenses using pure fixed fee contracts or maintaining the number of active firms and use per unit royalty rates.

Arora and Fosfuri (2003) find in a model with multiple innovators that the optimal number of licenses is lower when an innovator is active in the product market. Doganoglu and Inceoglu (2014) find that it is optimal for an innovator of a drastic innovation using per unit royalty two-part tariffs to remain outside the final market when there is downstream product differentiation. This article generalizes the result of Sandonis and Fauli-Oller (2006) showing that the outside innovator is able to achieve the multiproduct monopoly outcome. In the case of pure fixed fee licensing, we find that the innovator is indifferent between being outside and inside one of the component market. Producing both components is however profitable in a niche market when the structure of the industry and the number of licenses are given and correspond to the asymmetric structure. This remains true under two-part tariff licensing contracts.

In contrast with Doganoglu and Inceoglu (2014) and Sandonis and Fauli-Oller (2006), we show that (double) vertical integration is desirable for a patentee who licenses its technology to complementary markets when only one firm is ready to produce the homogeneous component good. Moreover, if we exogenously determine the structure of the industry to be the asymmetric structure (i.e $N_A = 1$, $N_B = 2$) and assume that the licensing contract is required to include a strictly positive royalty rate, we find that single vertical integration on one of the downstream complementary markets is profitable for the monopoly innovator in niche markets (i.e low levels of system valuation). Integration with the monopoly producer of the homogeneous good is more profitable than integration into the differentiated markets as it allows to fully compensate the negative effects of positive royalty rates in the uncovered equilibrium. This is consistent with the results of Lemarié (2005) showing in a differentiated Bertrand model (i.e linear demand system) that the innovator of a technology increasing the valuation of consumers, finds it optimal to merge when the licensing contract takes the form of a per unit royalty contract.

In this chapter, we find that per unit royalties are profitable in order to dampen downstream competition. Colombo and Filippini (2015) show that the strategic effect of royalties reducing the intensity of downstream competition and increasing aggregate industry profit explains the profitability of per unit royalty rates. They argue that per unit royalty rates will be more frequently used in markets where profits of licensees are rather low which seems to be consistent with our findings. In the uncovered equilibrium however, we find that per unit royalties are always privately and socially undesirable. This prediction seems to be in line with the empirical work of Vishwasrao (2007) which concludes that royalties are more commonly observed when sales are high whereas fluctuating sales and higher profitability are correlated with the use of a fixed fee.

San Martín and Saracho (2016) study in their article the respective properties of various types of royalties in the context of patent licensing. They consider an inside innovator licensing its technology to a downstream industry where differentiated producers compete in quantities. Moreover, they endogenize the elasticity of substitution which allow them to characterize the optimal licensing policy for different types of goods. The profitability of the royalty base depends on the degree of product differentiation and the type of goods produced by the industry. Focusing on the case of per unit royalty contract, this article shows that in the case of complements, the inside innovator chooses to charge a pure fixed fee contract (i.e with zero per unit royalty rate). To the contrary, when goods are substitutes, a positive per unit royalty is charged. This is due to the collusive or double marginalization effects of per unit royalties (depending on the sign of the elasticity of substitution). We find the same patterns for the optimal level of per unit royalty rate in this chapter.

Stamatopoulos and Tauman (2008) analyze the licensing of a quality enhancing technology by an outside monopoly innovator to differentiated duopolists. They use a logit model of product differentiation. The innovator is free to determine the level of per unit royalty rate and to decide how many licenses (i.e one or two) it wishes to auction, as well as the minimum level of bids for each license. Assuming that all consumers purchase one of the two varieties (either low or high quality), the optimal licensing policy is to issue two licenses and to use a pure royalty contract (i.e zero fixed fee). This contract allows the patentee to capture the full extent of the increase in social surplus generated by its technology. When the consumers are allowed not to consume the good (i.e relaxing the market covered assumption), it is still optimal for the innovator not to issue an exclusive license if the degree of product differentiation is sufficiently high. The optimal licensing agreement remains a full royalty contract if the valuation for the outside option is sufficiently low.

Poddar and Sinha (2004) studies the licensing of a cost-reducing innovation to spatially differentiated independent downstream producers. Under the market covered condition, they show that the outside innovator profitably diffuse the technology to all downstream producers by the means of a full royalty contract which is consistent with the findings of Stamatopoulos and Tauman (2008). In our licensing model, we obtain the same result in the covered case. All licenses are issued and royalties are used. We find a positive fixed fee because the outside option of the licensees is equal to zero in our model of drastic innovation. In the uncovered case, in which an outside option for consumers is introduced (i.e outside good), the authors find that the licensing policy remains identical when the valuation for the outside option is sufficiently low. In contrast, we find that per unit royalties are both privately and socially undesirable in our uncovered equilibrium.

We believe that the difference in our results concerning the optimal licensing policy in the uncovered case can be explained by the way the outside option is taken into account. In the logit model used by Stamatopoulos and Tauman (2008) the outside option is an additional fictitious variety that enters in a symmetric competition. The uncovered equilibrium in their model maintain competition within varieties. The demand addressed to each variety only depends on relative prices. As a result, a zero royalty generates excessive competition and low profits.

In our model using the circular model of product differentiation (Salop, 1979), the value of the outside option of consumers is equal to zero for all consumers no matter their location. The consumers are not heterogeneous in their perception of their outside option. In the uncovered equilibrium of the circular city model, the producers of the variants act as local monopolists and independently set their price facing a symmetric elastic demand (i.e outside option). There is no competition between varieties in the uncovered equilibrium of this model. As a consequence, the per unit royalty rate unambiguously deters the industry profit in amplifying the double marginalization effects and increasing the relative attractiveness of the outside option against both variants.

In the logit model however, the per unit royalty rate has two opposite effects on the industry profit. On the one hand, it dampens downstream competition between the two high quality variants which allows the firms and the innovator to extract more surplus from consumers. On the other hand, it raises the equilibrium price of high quality products which increases the demand for the outside option variant. The authors argue that for a sufficiently high degree of product differentiation, the innovator chooses to issue two licenses through a pure royalty contract if the valuation for the outside good is sufficiently low. A positive fixed fee will be included otherwise. Our results on the desirability of per unit royalty rates (i.e only used in the competitive double duopoly structure) sharply differ. In chapter four, we will introduce ad valorem royalties, study their effects in comparison with per unit royalties and characterize the optimal licensing policy in complementary markets.

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Chapter 4

Three-part tariff licensing and demand uncertainty in complementary markets

4.1 Introduction

In the first chapter, we characterized the optimal licensing policy of a monopoly innovator whose technology is used in a perfectly complementary market (i.e system market). Considering the number of licenses to be issued, a monopoly innovator finds it optimal to capture downstream profits through an asymmetric licensing policy (implying an exclusive license on the homogeneous downstream market) when the technology is used in a mass component market (i.e when the sensitivity of consumers to component differentiation is low). On the other hand, when the technology is used in a niche component market (i.e when consumers have a high sensitivity to the distance with their ideal component variety), the outside monopoly innovator chooses a symmetric licensing policy (with two producers on each market). We also previously explored the implications of single vertical integration of the upstream monopolist into one of the downstream component markets. We found that in this framework (i.e leaving aside the time dimension), single vertical integration had no effects on downstream competition nor on the optimal licensing policy of the innovator. Moreover, we studied in Chapter three the role played by royalties and integration in the distribution of technology licenses in perfectly complementary markets. We focused on the case in which royalties take the form of a transfer (from the licensee to the owner of the technology) of a given amount of money for each unit of the final good sold (i.e per unit royalties). The use of royalties were found to restore the profitability of downstream competition when the sensitivity of consumers to the distance from their ideal variety is low (i.e mass market) which makes it a weakly dominating strategy for both outside and inside monopoly innovator.

In the following chapter, we aim at introducing ad valorem royalties which are based on downstream sales revenue. Following Bousquet et al. (1998), we study the role played by uncertainty on the profit-maximizing licensing policy with a monopoly patentee and downstream complementary goods. Bousquet et al. (1998) show that in the French telecommunication industry ad valorem royalties are in practice used in the majority of licensing agreements (i.e 63% of fixed fee contracts are combined with ad valorem royalties and 96% of royalties used in licensing contracts are ad valorem royalties). This chapter contributes to show that demand uncertainty can explain the use of royalties in licensing agreements. The profit-maximizing licensing contract will generally combine a fixed fee with an ad valorem royalty. In a model with one potential licensee, Bousquet et al. (1998) show that a license contract with zero royalties is sub optimal in the presence of uncertainty. In the case of demand uncertainty at most two instruments are used in equilibrium and ad valorem royalties are always profitably charged (i.e dominate per unit royalties). The authors explain that the ad valorem royalty rate generates price inefficiency through its effect on downstream marginal cost. It also has an income effect on the downstream licensee which is balanced with a decrease in the fixed fee in order to guarantee its participation. Because of risk aversion, the decrease in the fixed fee which is independent of the state of nature will be smaller than the income captured through the state dependent royalty payment. A per unit royalty can be replaced by an ad valorem royalty which implies the same level of downstream marginal cost (i.e and the same output distortion) while enabling a better allocation of risk.

Llobet and Padilla (2016) suggest that ad valorem royalties tend to generate higher incentives for innovation and lower downstream prices in a model with successive monopolies. Under homogeneous Cournot competition, ad valorem royalties would make a better commitment mechanism, for the licensing of a non-drastic cost-reducing technology allowing downstream prices to rise more easily than would per unit royalties (San Martín and Saracho, 2010). The opposite is found to be true under Bertrand competition by Colombo and Filippini (2015). In this article, the authors show that a per unit royalty contract is more profitable for an inside innovator of a non-drastic cost-reducing technology. The per unit royalty performs better because it represents a more effective commitment to preserve downstream profits. The strategic effect of royalties that reduces the intensity of downstream competition and increases aggregate industry profit is stronger with per unit than ad valorem royalty rates which explains the profitability of the former. In a differentiated Bertrand model, competition tends to be more intense than in a Cournot model lowering equilibrium price and sales revenue while increasing equilibrium quantities. Per unit royalty is better at making the inside innovator internalize the effect of its pricing decision on its royalty revenue because the profit of the licensee (i.e ad valorem profit component) is less significant under Bertrand competition while the increase in price and sales profits generated by per unit royalty is made more beneficial.

Niu (2013) investigates the properties of another type of licensing (i.e profit-sharing licensing) which implies a transfer in equity from the licensee to the patentee as well as a fixed fee. This type of patent licensing is similar to partial integration. The main effect of the profit sharing component in this differentiated Cournot model is a collusive effect making the inside innovator less eager to increase its output. This is due to the transfer in equity that makes the profit of the patentee depend on the profit of its rival. On the other hand, the per unit royalty contract would also achieve the dampening of downstream competition through the increase in the marginal cost of the licensee which would make it behave less aggressively. The market share of the inside innovator would be higher under per unit royalty licensing. The author argues that assuming the technology allows to reach zero marginal cost, the profit sharing licensing can be compared to pure ad valorem royalty licensing. In particular, the article of San Martín and Saracho (2010) showing the superiority of pure ad valorem royalty contracts over per unit royalty contracts for non-drastic innovations is consistent with the results of Niu (2013).

San Martín and Saracho (2016) introduce in a differentiated Cournot model, the possibility for a downstream competitor to potentially develop an alternative technology. The inside innovator finds it desirable to use ad valorem royalties when the degree of substitutability is low or when the cost of developing a new technology is high. The ad valorem (per unit) royalty indeed makes the patentee (licensee) less aggressive. The lower the degree of substitutability is, the less aggressive the ad valorem royalties will induce the patentee to behave. In another article, San Martín and Saracho (2015) also study the respective properties of ad valorem and per unit royalties in the context of patent licensing. They consider an inside innovator licensing its technology to a downstream industry where differentiated producers compete in quantities. Moreover, they endogenize the elasticity of substitution which allows them to characterize the profit-maximizing licensing policy for different types of goods. The profitability of the royalty base depends on the degree of product differentiation and the type of good produced by the industry. In the case of a non-drastic innovation the inside innovator benefits from ad valorem royalties when the technology is used to produce complements and substitutes when the degree of substitution is sufficiently low.

Our model studies the impact of demand uncertainty on the profit-maximizing three-part tariff licensing contract in perfectly complementary downstream markets. We focus on a specific structure of the industry in which two producers are active in each of the downstream complementary good markets, namely the double duopoly structure which is shown to be the weakly dominating structure under two-part tariff contracts (see in Chapter three). Given this structure of the industry, we characterize the three-part tariff licensing contract that maximizes the profit of the upstream monopolist in two distinct types of downstream differentiated markets (and equilibrium): the niche markets (uncovered equilibrium) in which the valuation for the system good is sufficiently low and the mass markets (competitive equilibrium) where the valuation for the system is sufficiently high. We show that the innovator prefers to use an ad valorem royalty in combination with a fixed fee. Per unit royalty rates are not used in the uncovered nor in the competitive equilibria no matter whether the innovator is active in one of the downstream markets or not. In the following section, we present the general framework of our model. Then we characterize the licensing strategy of an outside innovator in section three as well as an integrated innovator in section four. We will see how our results in complementary markets differ from the ones presented above.

4.2 Framework

Following our previous work, we build a model with a monopoly patent holder whose technology is required for the production of two perfectly complementary goods. One of the downstream component is homogeneous as it is considered to be the basis of the final good (e.g the CPU, hidden in the device). For a given equal quality of this component, consumers have no preferences on the identity of its producer. On the other hand, the various varieties of the second component are spatially differentiated (i.e circular city model) because consumers directly interact with the final good through this component (i.e an interface, software or operating system). The indifferent consumer $\hat{x}$ such that it is indifferent between the closest two varieties is located at a distance $\hat{x}$ from B_0:

$$\hat{x} = \frac{1}{2N} + \frac{N}{2t} \times (p_{B_1} - p_{B_0}). \tag{4.1}$$

The marginal consumer of B_0, $\bar{x}$ such that it earns zero net surplus from consumption of a variety is located at a distance $\bar{x}$ from B_0:

$$\bar{x} = \sqrt{\frac{v - p_A - p_{B_0}}{t}}. \tag{4.2}$$

In this chapter, we consider that three-part tariff licensing contracts are available to the monopoly innovator. Fixed fees are paid by licensees upfront and royalties are earned on the sales of the end products using the technology of the innovator. Per unit royalties are taking the form of a price charged by the upstream monopolist on each unit of the end product sold by its licensees. These royalties increase the marginal cost of production (i.e from level c to g) of downstream producers. Ad valorem royalties correspond to a share of the sales revenue made by each licensee using the technology. When licensing contracts are signed, the volume of sales is unknown. The probability distribution however is common knowledge. We do not allow for state contingent contracts to be signed. We do not consider ex post renegotiation. We restrict the analysis to the double duopoly structure of the industry (i.e $N_A = 2$ and $N_B = 2$) which we found to be weakly preferred under two-part tariff licensing with a deterministic demand function.

We focus on demand uncertainty. The volume of sales is unknown until the downstream pricing stage. We choose to represent this uncertainty by considering that the mass of consumers uniformly distributed along the unit circle is a random variable. It appears to be the simplest way to introduce some risk sharing problem in our setting. The other parameters are common knowledge which allows the upstream monopolist to anticipate the downstream market structure and equilibria depending on its licensing policy. For given values of v and t, it is able to determine how many licenses to issue. It is worth noting that market shares only depend on prices. Best response functions of licensees seem to be independent of the realization of the random parameter of mass of consumers. It only influences the extent of profits. Following Bousquet et al. (1998), we assume that licensees are risk averse. The uncertainty on the levels of profits makes it more difficult for the upstream innovator to capture downstream profits through the use of fixed fees. A trade-off between surplus maximization and appropriation is likely to appear. Optimal risk sharing will require lower fixed fees than in a deterministic demand setting. Risk aversion reduces the attractiveness of a licensing agreement including a fixed fee. In this chapter, we also introduce an additional pricing instrument which is an ad valorem royalty rate. We restrict our analysis to the case where royalty rates are symmetric across downstream markets.

In the deterministic demand model developed in chapter three, we show that per-unit royalties are unprofitable for the upstream monopolist except in situations where downstream competition is fierce. In this case, the distortion of downstream marginal costs allows the maximization of the profit of the industry which is recouped through the use of fixed fees. In the present chapter, the licensing policy consists in the choice of the respective levels of fixed fee, per unit and ad valorem royalty rates. It will determine downstream marginal cost and price equilibria which in turn determines downstream and upstream profits. We will explore the licensing policy of such a monopoly innovator and study the profitability of vertical merger in two distinct market structures which are the niche and the mass component markets.

The timing of the game is as follows:

1. the terms of the two licenses offered on each component market are determined by the innovator (i.e three-part tariff contracts are set),
2. acceptance decisions of potential licensees,
3. uncertainty is resolved: downstream pricing stage and realization of utilities and profits.

We consider a simple binary random variable for the mass of consumers uniformly distributed along the circle. It is equal to $\frac{3}{2}$ in the favorable state or $\frac{1}{2}$ in the low demand outcome. The two states of nature have the same probability of occurrence. We use the power function in order to represent the risk aversion of the licensees which we consider to be of intermediate degree (i.e equal to $\frac{1}{2}$). We use the following notations:

1. a, the price of component A,
2. b, the price charged by producer B_1,
3. d, the price charged by producer B_2,
4. c, the marginal cost of producing a compatible component faced by a licensee,
5. g, the sum of per unit royalty rate and marginal production cost c,
6. s, the ad valorem royalty rate,
7. f, the fixed fee charged to differentiated producers for the technology.

We first characterize downstream equilibria (i.e third stage) then the participation decision of potential downstream firms and finally the profit-maximizing contract for the uncovered and competitive equilibria (i.e first stage) in the case of an outside innovator.

4.3 Outside innovator

4.3.1 Downstream equilibria

We consider the structure of the industry with a duopoly on each downstream market (i.e $N_A = 2$ and $N_B = 2$). We derive the best response function of the licensees for each market structure and corresponding demand function of the differentiated market. In the double duopoly structure, the price a will be fully determined by perfect Bertrand competition. We replace this variable by the level of price satisfying the zero licensee profit (and utility) condition. When downstream firms set their level of price, all uncertainty is resolved. The only difference with the deterministic framework is the fact that the mass of consumers will vary (i.e be different from one) according to the state of nature (i.e high or low mass of consumers). Moreover, we must take into account the availability of a symmetric ad valorem royalty rate (s). Perfect competition makes the expected utility of homogeneous producers equal to zero implying that the patentee is unable to charge a positive fixed fee for the licensing of its technology on this particular market. On the other hand, the expected utility function of a differentiated downstream producer can be strictly positive and depends on the location of the marginal consumer when the market is uncovered and on the location of the indifferent consumer otherwise (see equations 4.1 and 4.2) takes the following form:

$$EU_{B_1} = \begin{cases} \frac{1}{2}\sqrt{(d(1-s)-g)\sqrt{\frac{-a-d+v}{t}}\times\frac{3}{2}-f} & (4.3) \\ +\frac{1}{2}\sqrt{(d(1-s)-g)\sqrt{\frac{-a-d+v}{t}}\times\frac{1}{2}-f} & \\ \frac{1}{2}\sqrt{(d(1-s)-g)\left(\frac{b-d}{t}+\frac{1}{4}\right)\times\frac{3}{2}-f} & (4.4) \\ +\frac{1}{2}\sqrt{((d(1-s)-g)\left(\frac{b-d}{t}+\frac{1}{4}\right)\times\frac{1}{2}-f}. & \end{cases}$$

Symmetric downstream differentiated producers set their price in order to maximize their profit conditioned on the mass of consumers and the level of fixed fee. The latter constitutes a sunk cost that we ignore at the downstream competition stage:

$$\Pi_{B_1} = \begin{cases} (d(1-s)-g)\sqrt{\frac{-a-d+v}{t}} & (4.5) \\ (d(1-s)-g)\left(\frac{(b-d)}{t}+\frac{1}{4}\right). & (4.6) \end{cases}$$

The mass of consumers does not matter for the determination of profit-maximizing levels of price and the characterization of downstream price equilibria (i.e it only enters in the profit function as a positive multiplicative factor of the market share). For a given set of tariff instruments (i.e f, g, s) the first order conditions for each symmetric profit function lead to the following best response functions:

- uncovered best response functions

$$b = -\frac{2a}{3} + \frac{g}{3-3s} + \frac{2v}{3} \tag{4.7a}$$

$$d = -\frac{2a}{3} + \frac{g}{3-3s} + \frac{2v}{3} \tag{4.7b}$$

- competitive best response functions

$$b = \frac{-4ds + 4d + 4g - st + t}{8 - 8s} \tag{4.8a}$$

$$d = \frac{-4bs + 4b + 4g - st + t}{8 - 8s}. \tag{4.8b}$$

For a specific price level, the differentiated market is just covered (i.e the marginal and indifferent consumers are identical). In this situation each symmetric differentiated producer charges the price making the consumer located in the center of the representative segment indifferent between consuming one component, the other or non of them. This constitutes a Nash equilibrium for intermediate values of system valuation (i.e v) and distance sensitivity (i.e t). The level of the following kinked best response function depends on the price of the homogeneous component which is determined by the Bertrand marginal cost pricing condition:

- kinked best response functions

$$b = \sqrt{-at - dt + tv} + \frac{1}{4}(4d - t) \tag{4.9a}$$

$$d = \sqrt{-at - bt + tv} + \frac{1}{4}(4b - t) \tag{4.9b}$$

Combining the best response function of each player and demand consistency conditions, we derive the following Nash equilibria.

Lemma 4.1. *Compared to the perfect information model, we observe in the following equations that ad valorem royalty increases the influence that per unit royalty and production cost terms have on prices (i.e g becomes $\frac{g}{1-s}$):*

- *competitive price equilibrium*

$$a = \frac{g}{1-s} \tag{4.10a}$$
$$b = \frac{g}{1-s} + \frac{t}{4} \tag{4.10b}$$
$$d = \frac{g}{1-s} + \frac{t}{4} \tag{4.10c}$$
$$v \geq \frac{2g}{1-s} + \frac{5t}{16} \tag{4.10d}$$

- *uncovered price equilibrium*

$$a = \frac{g}{1-s} \tag{4.11a}$$
$$d = \frac{2v}{3} - \frac{g}{3(1-s)} \tag{4.11b}$$
$$b = \frac{2v}{3} - \frac{g}{3(1-s)} \tag{4.11c}$$
$$v \leq \frac{2g}{1-s} + \frac{3t}{16} \tag{4.11d}$$

- *kinked price equilibrium*

$$a = \frac{g}{1-s} \tag{4.12a}$$
$$d = v - \frac{g}{1-s} - \frac{t}{16} \tag{4.12b}$$
$$b = v - \frac{g}{1-s} - \frac{t}{16} \tag{4.12c}$$
$$\frac{2g}{1-s} + \frac{3t}{16} < v < \frac{2g}{1-s} + \frac{5t}{16} \tag{4.12d}$$

We can see in equations 4.7a and 4.7b that in the uncovered equilibrium case, an increase in the ad valorem royalty has a direct positive effect on price (i.e the pass-through of g is positive and lower than one). On the other hand, there is an indirect negative effect through the price of the complementary good (i.e component prices are strategic substitutes). The result of these two effects appears in the expression of the uncovered equilibrium price of component B in equations 4.11b and 4.11c showing that it decreases with s (i.e the complementarity effect dominates). Nevertheless, the increase in the price of component A makes the uncovered equilibrium system price an increasing function of both royalty rates.

In the competitive equilibrium, g is fully passed onto consumers and complementarity does not matter for the price equilibrium level (see equations 4.10b and 4.10c). As a result, the competitive equilibrium price of component B is an increasing function of both royalty rates. The associated system price unambiguously behaves in the same way. To the contrary, in the kinked equilibrium, differentiated producers fully compensate the increase in g (i.e zero pass-through of g in equations 4.9a and 4.9b) and in the price of the complementary good, making the price of the system independent of royalty rates. For sustaining values of the kinked equilibrium, it is indeed profitable for differentiated producers to capture the residual surplus of the marginal consumer.

As one can see in equation 4.11d, the ad valorem royalty also increases the range of sustaining values of system valuation (i.e parameter v) for the uncovered equilibrium. Everything else being equal, the uncovered equilibrium is made more likely by high ad valorem royalties.

4.3.2 Expected utility and acceptance decisions

We simply compute the utility derived by licensees from the equilibrium profits presented in the previous section. The utility of a licensee is equal to the square root of net profits (i.e their degree of risk aversion is equal to $\frac{1}{2}$). The level of utility reached in each state of nature is weighted by its probability of occurrence. Downstream firms accept a contract if and only if it leads to non negative expected utility.

When there is no uncertainty nor risk aversion, the licensees decide on whether or not to accept the licensing contract in comparing their anticipated gross profit with the fixed fee charged by the innovator. The participation constraint then imposes that the fixed fee must be lower or equal to the gross profit of the licensee made in the subsequent subgame. In the presence of uncertainty and risk aversion, the net profit (i.e including fixed fees) and the utility of a licensee must be evaluated for each outcome of the game in order to determine the participation constraint based on its expected utility. The latter must be positive in order to guarantee its participation.

Perfect Bertrand competition makes it impossible to charge a strictly positive fixed fee on the homogeneous component market A. We focus on the fixed fee charged on the differentiated component market (i.e f) and express the corresponding participation constraint for both uncovered and competitive pricing equilibria:

$$z\sqrt{(d(1-s)-g)\,h\sqrt{\frac{-a-d+v}{t}}}-f+(1-z)\sqrt{(d(1-s)-g)\,l\sqrt{\frac{-a-d+v}{t}}}-f\geq 0 \tag{4.13a}$$

$$z\sqrt{(d(1-s)-g)\,h\left(\frac{b-d}{t}+\frac{1}{4}\right)}-f+(1-z)\sqrt{((d(1-s)-g)\,l\left(\frac{b-d}{t}+\frac{1}{4}\right)}-f\geq 0. \tag{4.13b}$$

Observing the valuation of consumers for the final good (i.e v), for variety (i.e t) and the levels of royalties, downstream producers are able to foresee the price equilibrium of the subsequent stage and derive their expected utility. They accept the licensing contract if and only if it is non negative.

4.3.3 Three-part tariff licensing contract

In this section, we use the above price equilibrium values and look for the profit-maximizing licensing contract of the outside monopoly innovator under the participation constraint of the licensees and the demand consistency condition (i.e covered or uncovered market condition). This section studies the first stage of the game given the uncovered and competitive Nash equilibria and participation constraints stemming from previous stages.

We assume that the patentee is risk neutral as in the article of Bousquet et al. (1998). This assumption is sensible as long as we consider that the licensor is less sensitive to risk taking than its licensees. We believe it is reasonable to think that the upstream monopoly innovator has more ability (e.g human, financial or technological capital) to manage and support the risk associated with uncertain demand for its technology. We start by deriving the three-part tariff licensing contract maximizing the profit of the upstream monopolist when the downstream system market is in the competitive equilibrium.

4.3.3.1 Competitive equilibrium contract

With an outside innovator and symmetric downstream competitive price equilibrium, the total volume of sales and market shares are (for a given range of price instruments values) constant and independent of royalty rates. Total surplus remains unaffected as long as the indifferent consumer wishes to participate in the market. This is due to the fact that the equilibrium is symmetric and supported by high system valuations making all consumers willing to purchase.

The profit function of the patentee is a convex function of royalty rates. Increased royalty rates yield to higher component and system prices, constant volume of sales and enhanced upstream profits. The upstream monopolist finds it profitable to charge the highest royalty rates and fixed fee levels consistent with participation constraints. The equilibrium licensing contract is entirely determined by binding participation constraints of licensees and consumers. Given the competitive price equilibrium, we can express both constraints in the following way:

$$v \geq \frac{2g}{1-s} + \frac{5t}{16} \qquad \Leftrightarrow g \leq \frac{1}{32}(5st - 16sv - 5t + 16v) \tag{4.14a}$$

$$EU_{B_i} \geq 0 \qquad \Leftrightarrow f \leq \frac{1}{32}(t - st). \tag{4.14b}$$

We observe in these equations that there is some substitution between tariff instruments. If the ad valorem royalty rate s increases, constraints on per unit royalty rate and fixed fee respectively tighten (and reciprocally). Both constraints are profitably binding so that we can substitute the variables f and g by their respective expressions given by each binding constraint. We then derive the level of s leading to the maximum level of profit and substitute it back into the expressions of the remaining instruments. We obtain the following equilibrium contract:

Proposition 4.3.1. *In the presence of demand uncertainty, the outside innovator chooses to transfer its technology to risk averse licensees through an ad valorem two-part tariff licensing contract (i.e with a zero per unit royalty rate) when the valuation for the system is sufficiently large to sustain the competitive equilibrium. In this case, the profit-maximizing contract of the upstream monopolist in the double duopoly structure takes the following form:*

$$s = \frac{32c + 5t - 16v}{5t - 16v} \tag{4.15a}$$

$$g = c \tag{4.15b}$$

$$f = \frac{ct}{16v - 5t}. \tag{4.15c}$$

Proof. Details of the proof are presented in the appendix. □

This proposition results from the fact that the profit of the innovator is strictly increasing with the levels of both royalty rates and the fixed fee. The upstream monopolist does not wish to leave unnecessary surplus to consumers and then makes the market just covered (i.e binding indifferent consumer participation constraint). Taking into account the substitution effects between the three pricing instruments (i.e the binding constraints), we find that the profit function of the innovator depends positively on the ad valorem royalty rate. The patentee charges the highest feasible level of ad valorem consistent with participation and non negativity of per unit royalty rate constraints. In order to charge the highest level of ad valorem royalty, the innovator profitably sets the per unit royalty rate to zero. The fixed fee is set to the highest level acceptable for the downstream differentiated producers.

In order to understand why the ad valorem royalty rate is favored by the patentee, we must analyze the respective effects of royalty rates on total marginal cost, component price and profits of the differentiated producers. We can rewrite the expression of the profit of a differentiated producer:

$$\Pi^{B_i} = (d(1-s)-g)\left(\frac{(b-d)}{t}+\frac{1}{4}\right) \Leftrightarrow (d-sd-g)\times\left(\frac{b-d}{t}+\frac{1}{4}\right). \tag{4.16}$$

We observe that with a positive ad valorem royalty rate, the full marginal cost of producing component B is an increasing function of its own price (i.e equal to $sd+g$). As we mentioned earlier, an increase in the per unit royalty rate would be fully transmitted to consumers. It means that downstream producers will not bear the cost of an increase in per unit royalty rate and will increase component price in order to just maintain their profit and utility. In fact, this implies that the pass-through on g is greater than one (i.e the derivative of the equilibrium price of component B with respect to g is equal to $\frac{1}{1-s}$). The increase in price more than compensates the direct increase in marginal cost. It takes into account the indirect effect of the per unit royalty rate through the level of the component price which partially determines the ad valorem royalty payment. In sum, the per unit royalty has no effect on the net profit nor on the utility of the licensees.

To the contrary, an increase in ad valorem royalty rate will generate a reduction in the profit of the differentiated producers because it makes it impossible for them to pass onto consumers the full effective increase in marginal cost associated with a marginal rise in s. The ad valorem royalty payment indeed introduces a positive relationship between price and full marginal cost (i.e through the sd term). An increase in ad valorem royalty would generate a loop of price and marginal cost increases ultimately resulting in a reduction of their profit margin and utility. This reduction of the vertical double marginalization is desirable for the innovator because it loosens the participation constraint of the indifferent consumer.

In order to satisfy the market covered and licensee participation conditions, an increase in ad valorem royalty must be followed by a decrease in per unit royalty and fixed fee but the generated royalty revenue more than outweigh this loss. This strategic move leads to an ad valorem two-part tariff competitive equilibrium licensing contract (i.e no per unit royalty). The fact that differentiated producers cannot fully pass onto consumers their increase in marginal cost (due to the rise in ad valorem royalty rate) and reduce their profit margin leaves more space for the innovator to increase ad valorem royalty before hitting the market covered constraint (i.e indifferent consumer participation). To sum up, ad valorem royalty allows the patentee to extract surplus directly (avoiding the risk aversion of its licensees) more efficiently than per unit royalty because it generates a lower price increase. In the competitive equilibrium, price efficiency is not an issue because the level of sales is constant, only the ability of the innovator to extract surplus matters for the shape of the profit-maximizing contract.

Risk aversion reduces the ability of the patentee to capture surplus through the fixed fee. Royalty rates are able to attenuate this loss in circumventing the risk aversion of its licensees by decreasing the amount of surplus passing through their utility function. In this model, the upstream monopolist can indeed easily implement the industry profit-maximizing price but remains unable to fully recoup downstream profits. The vertical double marginalization does not generate price inefficiency when the market is fully covered but the ability of the monopolist to capture surplus is limited by the risk aversion of its licensees. It is consequently desirable for the innovator to reduce the profit made by its licensees (i.e the share of surplus affected by the risk aversion constraint). Bousquet et al. (1998) explain in their article that the ad valorem royalty features a per unit royalty component denoted r such that: $c + r = \frac{c}{1-s}$ (with c denoting the marginal cost of licensees). If a strictly positive per unit royalty rate is charged, we obtain: $g + r = \frac{g}{1-s}$, that we find in the competitive equilibrium price expressions in equations 4.9a and 4.9b. The ad valorem royalty rate influences downstream prices through its per unit component r. There exists an equivalence in the marginal cost distortion generated by a given ad valorem and a per unit royalty rate. Llobet and Padilla (2016) argue that the ad valorem royalty rate also shares some features with the fixed fee. If the marginal cost of production is equal to zero, the ad valorem royalty would not distort equilibrium price just as a fixed fee.

The outside innovator has no trouble capturing downstream profit under risk neutrality of licensees. In contrast with our unique competitive equilibrium contract, there would be a multiplicity of three-part tariff equilibrium agreements constituted of all contracts stipulating levels of licensing instruments such that the participation constraint of the indifferent consumer and the differentiated component B producers are binding. These would implement efficient pricing and ensure that no unnecessary surplus is left to consumers nor licensees.

4.3.3.2 Uncovered equilibrium contract

Using the uncovered price equilibrium values that we previously obtained, we look for the profit-maximizing contract (i.e f, s, g) of the outside monopoly innovator under the participation constraint of licensees and demand consistency condition. The profit of the innovator is strictly increasing with the level of fixed fee which induces the patentee to charge the highest acceptable fixed fee. We replace f by its expression given by the binding participation constraint of licensees and look for the profit-maximizing levels of royalty rates under the demand consistency condition. We find that the market remains uncovered in equilibrium (i.e unbinding demand consistency constraint) and that ad valorem royalties are strictly more profitable than per unit royalty rates.

Proposition 4.3.2. *In the presence of demand uncertainty, the outside innovator chooses to transfer its technology to risk averse licensees through an ad valorem two-part tariff licensing contract (i.e with a zero per unit royalty rate) when the valuation for the system is sufficiently low to sustain the uncovered equilibrium.*

Proof. See in the appendix for the proof and the explicit expression of the profit-maximizing contract of the upstream monopolist in the double duopoly structure. □

In the uncovered equilibrium, an increase in per unit royalty would not be fully transmitted to consumers and the increase in the price of the complementary good makes downstream producers reduce their price. The price of component A just increases enough to maintain non negative profit on the homogeneous market. As we previously mentioned, an increase in the full marginal cost of production of differentiated producers induces a small increase in component B price. But this is more than outweighed by the complementarity effect that compensates the rise in the price of component A and leads to a decrease in the price of component B. As me mentioned earlier, the ad valorem royalty rate introduces a positive relationship between price and full marginal cost. If a positive ad valorem royalty is charged, a price cut yields to a decrease in full marginal cost which benefits the licensee. Because ad valorem royalty payment is based on the sales revenue, the effort made by licensees to mitigate the negative effect of increased royalties (i.e decrease in sales due to the rise in the price of the complementary good) are accounted for and rewarded. As a consequence, a contract with a high level of ad valorem royalty and a low level of per unit royalty is more favorable for the licensees because it results in a higher reduction in full marginal cost of production of the component B. This results in the loosening of the constraint on the level of fixed fee which in turn benefits the patentee. The difference between ad valorem and per unit lies in their respective impacts on the participation constraint of the licensee.

Concerning the precise level of ad valorem royalty, the innovator faces a trade-off between surplus maximization and extraction as royalty rates harm social surplus generated by the technology. Ad valorem royalty rates allows to extract surplus directly (i.e circumventing risk aversion of its licensees) without requiring as much of a decrease in fixed fee as the per unit royalty does. The profit-maximizing level of ad valorem royalty rate is characterized by the right balance between the increase in ad valorem and decrease in fixed fee revenues generated by the increase in ad valorem royalty rate.

Under risk neutrality, royalty rates are set to zero and the entire net social surplus is captured by the outside innovator by the mean of the fixed fee charged to the differentiated producers of component B. The profit of the outside innovator is greater under risk neutrality. On the other hand, the use of a fixed fee does not enable the full extraction of the profit of licensees under risk aversion because it does not depend on the realization of the demand random variable. The ability of the innovator to extract surplus is reduced. Royalty rates decrease industry profit in the uncovered equilibrium but are more effective tools to recoup the surplus generated by the technology. There is a trade-off between the maximization of the profit of the industry and the ability to capture it.

4.3.3.3 Conclusion

The ad valorem royalty rate is profitable for the outside innovator in both competitive and uncovered equilibria of the double duopoly structure. It allows the increase in the ability of the innovator to extract downstream profits in the presence of uncertainty and risk aversion (i.e substituted for fixed fee). It is superior to the per unit royalty because it is less harming for consumers in the competitive equilibrium (i.e lower pass-through) and for the licensees in the uncovered equilibrium (i.e lower full marginal cost) enabling more surplus to be extracted while satisfying both participation constraints. We will now see if this remains true when the patentee is active in the downstream market and evaluate the profitability of vertical mergers under demand uncertainty in complementary markets.

4.4 Inside innovator

We consider now a different structure of the industry where the monopoly innovator produces one of the downstream component in-house. The vertical merger between the patentee and one of the producers of the homogeneous component (i.e A) would have no effect on downstream equilibrium prices and profits nor on the shape of the profit-maximizing licensing contract. Perfect Bertrand competition prevents any profit opportunity outside of royalty based revenues (i.e marginal cost difference for an inside innovator). We will focus in this section on the analysis of a vertical merger between the monopoly innovator and one of the differentiated downstream producers (i.e component B makers). As in the previous section, we first characterize the downstream equilibria for the competitive and uncovered market structures and then derive the corresponding profit-maximizing licensing contracts.

4.4.1 Downstream equilibria

4.4.1.1 Competitive equilibrium

Under two-part tariffs with per unit royalty rates, there is no difference between the price charged by an independent and inside innovator producer of the component B. This is due to the fact that when the market is covered, the innovator would earn the equivalent value of the per unit royalty rate in cost saving having directly sold the component or in royalty payment otherwise. With the introduction of the ad valorem royalty rate, there appears to be a positive relationship between the price of the competitor and royalty revenue of the innovator. For a given level of the B_1 component, the incentive for the inside innovator to undercut or match the price of its rival is lowered by the opportunity cost of earning high royalty payment.

In order to characterize the competitive downstream price equilibrium, we determine the best response function of the inside innovator in maximizing its second period profit function (conditioned on the state of nature, i.e the mass of consumers) with respect to price b:

$$\Pi_2^{UB_2} = \frac{as}{2} + \frac{g-c}{2} + (g-c)\left(\frac{b-d}{t} + \frac{1}{4}\right) + ds\left(\frac{b-d}{t} + \frac{1}{4}\right) + (b-c)\left(\frac{d-b}{t} + \frac{1}{4}\right). \quad (4.17)$$

We focus on the terms depending on component price b and re-express them in order to highlight the difference between the integrated and a separated B component producer:

$$(g-c)\left(\frac{b-d}{t} + \frac{1}{4}\right) - c\left(\frac{d-b}{t} + \frac{1}{4}\right) + ds\left(\frac{b-d}{t} + \frac{1}{4}\right) + b\left(\frac{d-b}{t} + \frac{1}{4}\right). \quad (4.18)$$

We change the expression of the royalty revenues by introducing the market share of the inside innovator ($\hat{x} = \frac{b-d}{t} + \frac{1}{4} = \frac{1}{2} - (\frac{d-b}{t} + \frac{1}{4})$). We observe in the following expression that royalty revenues can be considered as an opportunity cost of not earning profit directly through system sales:

$$-(g-c)\left(\frac{d-b}{t}+\frac{1}{4}\right)-c\left(\frac{d-b}{t}+\frac{1}{4}\right)-ds\left(\frac{d-b}{t}+\frac{1}{4}\right)+b\left(\frac{d-b}{t}+\frac{1}{4}\right)+\frac{g-c}{2}+\frac{ds}{2}. \tag{4.19}$$

This can be compared to the gross profit function of an independent producer expressed in equation 4.6. The expanded profit function of the independent B_2 producer is:

$$-(g-c)\left(\frac{d-b}{t}+\frac{1}{4}\right)-c\left(\frac{d-b}{t}+\frac{1}{4}\right)-bs\left(\frac{d-b}{t}+\frac{1}{4}\right)+b\left(\frac{d-b}{t}+\frac{1}{4}\right). \tag{4.20}$$

Lemma 4.2. *The terms of the gross profit functions of the integrated and separated B_2 producers that depend on b are exactly identical except that for the integrated firm, the ad valorem royalty cost term is based on the given rival price d (instead of own price b). This results in a higher component price charged by the integrated firm. We can observe this gap in the respective best response functions:*

$$b=-\frac{g}{s-1}-\frac{(s+3)t}{4(s-3)} \tag{4.21a}$$

$$d=-\frac{g}{s-1}-\frac{3t}{4(s-3)} \tag{4.21b}$$

$$b-d=\frac{st}{12-4s}. \tag{4.21c}$$

The fact that the ad valorem revenue is based on the rival price makes its derivative with respect to b constant and positive. For the inside innovator, increasing b generates more sales revenue up to a point where it starts to decrease. On the other hand, it generates a higher ad valorem revenue based on the given price d as this leaves more demand to the rival. The profit-maximizing level of b is reached when both effects cancel out.

The inside and outside producer face the same sales revenue function. An increase in b would also decrease their total cost as it lowers their demand. But contrary to the inside innovator, the independent producer faces an additional negative effect when increasing its component price. The full marginal cost of production would follow the price charged by the independent producer due to the ad valorem royalty charged by the outside innovator (i.e based on sales revenue). As a consequence, it is not desirable for the independent producer to increase its price as much as the inside innovator because the gain in total cost reduction following an increase in price is reduced by the increased marginal ad valorem royalty payment. The point where the loss in sales revenue and the gain in cost savings is reached will be lower for the outside innovator. This is why in equilibrium, the price charged by the inside innovator is higher. The best response function of the independent producer remains identical to the one in the case of separated ownership. We obtain the following competitive price equilibrium:

$$a = \frac{g}{1-s} \tag{4.22a}$$

$$b = \frac{g}{1-s} + \frac{3t}{4(3-s)} + \frac{st}{4(3-s)} \tag{4.22b}$$

$$d = \frac{g}{1-s} + \frac{3t}{4(3-s)} \tag{4.22c}$$

4.4.1.2 Uncovered equilibrium

Following the merger between the upstream monopolist and one of the differentiated component producers (i.e B_2), the downstream uncovered price equilibrium is modified. The inside innovator changes its behavior (i.e its best response function). It appears that contrary to the competitive equilibrium we previously analyzed, the inside innovator charges a lower equilibrium price than its rival (and than in the separated case). Looking at the following expressions for the best response function of the independent and integrated producers, we observe that the price charged by the latter will be lower as soon as g is greater than c (i.e under non negative per unit royalty rate):

$$a = \frac{g}{1-s} \tag{4.23a}$$

$$d = \frac{1}{3}\left(2v - \frac{g}{1-s}\right) \tag{4.23b}$$

$$b = \frac{1}{3}\left(2v - \frac{g}{1-s}\right) - \frac{1}{3}\left(\frac{2g}{1-s} - 2c\right). \tag{4.23c}$$

In order to characterize the uncovered downstream price equilibrium, we determined the best response function of the inside innovator in maximizing its second period profit function (conditioned on the state of nature, the mass of consumers) with respect to price b:

$$\begin{aligned}\Pi_2^{UB_2} =&(g-c)\left(\sqrt{\frac{-a-b+v}{t}}+\sqrt{\frac{-a-d+v}{t}}\right)+(b-c)\sqrt{\frac{-a-b+v}{t}} \\ &+ds\sqrt{\frac{-a-d+v}{t}}+as\left(\sqrt{\frac{-a-b+v}{t}}+\sqrt{\frac{-a-d+v}{t}}\right)+(g-c)\sqrt{\frac{-a-d+v}{t}}.\end{aligned} \tag{4.24}$$

We focus on the terms depending on component price b and re-express them in order to highlight the difference between an integrated and independent component B_2 producer:

$$(g-c)\sqrt{\frac{-a-b+v}{t}}+(b-c)\sqrt{\frac{-a-b+v}{t}}+as\sqrt{\frac{-a-b+v}{t}}, \tag{4.25a}$$

$$(b-g)\sqrt{\frac{-a-b+v}{t}}-bs\sqrt{\frac{-a-b+v}{t}}. \tag{4.25b}$$

Lemma 4.3. *The difference between the two producers is that the integrated firm internalizes royalty revenues which eliminates double marginalization. It allows the implementation of an efficient price for the integrated system (i.e $a+b$) and the increase in the total revenue generated by the technology which is directly captured through the (increased) sales revenue. Vertical double marginalization makes the independent system more expensive than the integrated system. In equilibrium, system prices take the following form:*

$$a+d=\frac{2}{3}\left(v+\frac{g}{1-s}\right) \tag{4.26a}$$

$$a+b=\frac{2(c+v)}{3}. \tag{4.26b}$$

4.4.2 Acceptance decisions

The participation conditions remain the same as in the case of independent downstream producers. There will be only one license issued on the differentiated component market as the downstream division of the inside innovator has a direct access to the technology. Given the participation constraints of licensees and consumers, we look for the three-part tariff licensing contract which maximizes the profit of the inside innovator first when the market is in the competitive equilibrium and second when it is in the uncovered equilibrium.

4.4.3 Three-part tariff licensing contract

4.4.3.1 Competitive equilibrium

Turning now to the determination of the profit-maximizing licensing contract of an inside innovator. In the separated case, we found that it is desirable for the upstream monopolist to use ad valorem royalty in order to increase its surplus extraction ability. The downstream equilibrium is symmetric and the efficiency of the industry structure is guaranteed as long as both (indifferent consumer and licensees) participation constraints are binding. In the integrated case, the internalization of ad valorem royalty leads to asymmetric downstream prices and market shares. This implies inefficient transportation costs and a loss in social surplus. It is also worth noting that the vertical merger naturally solves part of the extraction problem (i.e on sales of component B_2).

The licensing strategy of the inside innovator remains essentially the same. It charges the highest ad valorem royalty rate and fixed fee consistent with market covered and licensee participation constraints. The precise expressions are presented in the appendix D.2.1.1. The ad valorem royalty is superior to the per unit royalty because it prevents the licensee from fully transmitting cost increases to consumers. As a consequence, the consumer participation constraint will be binding less rapidly using the ad valorem royalty rate, allowing the patentee to directly extract more profit. It ensures a decrease in the amount of profit affected by risk aversion and an increase in royalty revenue.

The inefficiency in transportation costs makes the market covered constraint to tighten. It is binding for a lower level of ad valorem royalty rate. As a consequence, the fixed fee charged in equilibrium (i.e binding participation condition of independent producer of component B) is higher and ad valorem royalty is lower in the integrated structure. Relatively more surplus is left to the independent B component producer.

It is worth noting that the inside innovator does not aim at reducing the asymmetry in prices but at extracting surplus more efficiently (i.e increasing s). The market shares and component B prices are symmetric with zero ad valorem royalty rate. But because of the risk aversion of licensees, fixed fee would be inefficient to capture their profits. The strategy consisting in making downstream prices and market shares symmetric is dominated. In the competitive equilibrium, the ability to extract surplus is more important for the innovator than surplus maximization.

Proposition 4.4.1. *Comparing the competitive equilibrium profit of an inside and outside innovator, we find that it would be undesirable for the innovator to merge with one of the differentiated producers of the component B. This is the case for all values of parameters consistent with the competitive equilibrium.*

Proof. See in the appendix. □

We found that the vertical merger is undesirable for the innovator in the competitive equilibrium. The price of system 1 is lower under integration while the opposite is true for system 2 making sales asymmetric. The revenue made on system 1 increases because of a higher market share. But the decrease in sales and profits on system 2 is too sharp to make integration desirable. There is a decrease in surplus due to inefficient transportation costs. Moreover, the asymmetry in shares induces a lower level of s (due to a tighter indifferent consumer participation constraint) and higher level of f which implies that more surplus is left to the independent licensee under integration.

Integration is undesirable under risk aversion of licensees when ad valorem royalty rate are available. It is indeed an excellent tool of surplus appropriation which limits the risk aversion problem. On the other hand, the ad valorem royalty creates inefficiency in the distribution of sales to consumers which deters the surplus and the industry profit. This in turn generates a tighter consumer participation constraint which reduces the feasible level of ad valorem royalty rate. As a result, the ability of extracting surplus from the independent system sales is decreased.

4.4.3.2 Uncovered equilibrium

Looking for the uncovered equilibrium licensing contract, we find the same kind of strategy as in the separated case. The level of the per unit royalty is set to its lowest level (i.e $g = c$) because the derivative of profit with respect to the per unit royalty rate is negative for consistent profitable values of royalties. The profit of the patentee increases in the level of fixed fee so that the variable f can be replaced by the expression given by the binding participation constraint of the independent differentiated producer. The desirable level of ad valorem royalty is derived from the corresponding first order condition for constrained profit maximization (i.e uncovered market structure). The binding participation constraint of the independent producer of component B determines the associated level of the fixed fee.

The ad valorem remains superior to the per unit royalty because it is based on sales revenue and not just on the volume of sales. It is preferable for the licensee which in turn allows a greater fixed fee level to be charged by the patentee. Ad valorem royalty indeed takes into account the cut in component price profitably made by the independent producer in order to mitigate the decrease in sales caused by the increase in royalty and subsequent rise in price of component A.

Proposition 4.4.2. *The ad valorem two-part tariff remains the profit-maximizing contract across integration regimes for both uncovered and competitive equilibria (i.e mass and niche downstream component markets).*

Proof. See in the appendix. □

Proposition 4.4.3. *Comparing the uncovered equilibrium profit of an inside and outside innovator, we find that it would be desirable for the innovator to merge with one of the differentiated producers of the component B. This is the case for all values of parameters consistent with the uncovered equilibrium.*

Proof. See in the appendix D.2. □

4.5 Conclusion

In this chapter, we explored the effects of demand uncertainty on the licensing of a patented technology used in downstream perfectly complementary markets by a monopoly innovator. We focused on a specific form of uncertainty affecting the size of the demand for the system good based on the patented technology. We introduced the demand uncertainty dimension in our model with one spatially differentiated component market through the mass of consumers distributed along the circular city. A particular feature of this modelization is the fact that demand size (i.e the mass of consumers) does not affect downstream price equilibria. We assumed risk aversion of licensees which is modeled with a simple square root utility function. We characterized the profit-maximizing three-part licensing contract with potentially an ad valorem (i.e based on sales revenue) and a per unit royalty rate as well a fixed fee for both an outside and inside innovator whose technology is used in a mass or a niche differentiated system market.

We showed that the introduction of demand uncertainty changes the shape of licensing contracts making the use of the ad valorem royalty rate strictly profitable in the case of risk averse downstream component producers. The innovator charges the highest feasible level of ad valorem royalty rate in order to compensate for its decreased ability to capture downstream profit through fixed fees due to the risk aversion of licensees (i.e their participation constraint). An ad valorem two-part tariff is profitable for outside and inside monopoly innovator because it allows a more efficient extraction of surplus (i.e increasing ad valorem royalty and decreasing fixed fee). This is true both when the downstream differentiated component market is a mass and a niche component market (i.e both when the valuation for the system is such that the innovator is willing to fully cover the market and the contrary).

In the case of the competitive covered equilibrium, the precise levels of tariff instruments are determined by the binding participation conditions of consumers and licensees. The ad valorem royalty rate is superior to the per unit royalty because it is less harming for consumers (i.e lower downstream pass-through) which allows more surplus to be extracted while satisfying both participation constraints. In the uncovered equilibrium, both types of royalties decrease sales and social surplus. The ad valorem royalty is preferred by both licensees and patentee because they are based on sales revenue and take into account the equilibrium price resulting from complementarity (i.e strategic substitutability) between downstream goods. Licensees favor ad valorem royalties which results in the loosening of their participation constraint and more surplus extraction.

The vertical merger into the differentiated market is desirable for horizontal double marginalization reasons in the uncovered market equilibrium. It is undesirable in the competitive equilibrium due to the asymmetry in market shares induced by ad valorem royalty and the pricing strategy of the inside innovator. It leads to inefficiency in the allocation of sales across differentiated producers making this structure of the industry a second best in terms of social surplus and extraction capacity of the innovator. When there is only vertical double marginalization and surplus extraction issues as in the double duopoly competitive equilibrium, there is no room for vertical integration under risk aversion of licensees and ad valorem royalties. The fact that in the competitive equilibrium, the royalty revenue depends on the price charged by the inside innovator creates an incentive for price increase and inefficient asymmetric market shares. This is not the case in the uncovered equilibrium where market shares are independent of the price of the rival. This incentive for asymmetric market shares will in turn hurt the inside innovator. It faces a commitment problem on the price charged by its downstream division that does not take into account the effect of the tightening of the participation constraint of the consumer resulting from its pricing strategy (i.e asymmetric market shares).

One of the findings of Bousquet et al. (1998) stating that demand uncertainty can explain the use of royalties in licensing agreements and that the profit-maximizing licensing contract generally combine a fixed fee and an ad valorem royalty remain valid in our framework of downstream complementary markets.

San Martín and Saracho (2015) shows that in the case of a differentiated Cournot duopoly, a drastic innovation would be profitably licensed through a pure ad valorem royalty contract. In the case of a non-drastic innovation, an ad valorem two-part tariff contract would be used when goods are complements and for some degree of product differentiation when goods are substitutes. In their model with zero marginal cost of production, the ad valorem royalty only influences the behavior of the internal patentee making it less (more) aggressive in the case of substitutes (complements) as it internalizes the effect of its price on the volume of sales of its rival and in turn on the ad valorem royalty licensing revenue.

We observe the same mechanism in our model when comparing the impact of the ad valorem royalty on the price charged by the inside innovator in the uncovered and competitive equilibria. In the uncovered equilibrium, the complementarity dimension dominates and the equilibrium component price is a decreasing function of the ad valorem royalty. To the contrary, the price of the inside innovator would increase with the ad valorem royalty in the competitive equilibrium where the complementarity vanishes behind intense competition between component producers.

In our model the ad valorem royalty rate is profitable for an inside innovator for both uncovered and competitive equilibria. One of the differences between the two models is the fact that in our model, the ad valorem royalty rate is found to be always used in combination with a fixed fee. This may be caused by the market covered constraint in the case of the competitive equilibrium which prevents further increase (and decrease) in royalties (fixed fee). In the uncovered equilibrium however it results from a trade-off with the negative impact of ad valorem royalty rate on the volume of sales due to increased marginal cost of component production and the strategic substitutability between component prices (i.e complementarity between components).

Contrary to San Martín and Saracho (2015) our results rely on the demand uncertainty and risk aversion hypotheses. Under perfect information and in competitive equilibrium, the inside innovator would favor a two-part tariff based on per unit royalty. Ad valorem royalty would imply asymmetric downstream prices and market shares which generate inefficient transportation costs and lower profits and welfare. This misallocation of product variety due to the spatial differentiation framework does not appear in models with a system of demand functions.

As regards vertical integration, we found in the three-part licensing model under demand uncertainty, that it can be profitable for the innovator to be active in the final good market by producing one of the differentiated component. In equilibrium, the innovator uses ad valorem royalty in combination with a fixed fee. This seems in contradiction with the findings of Doganoglu and Inceoglu (2014) and Sandonis and Fauli-Oller (2006) in the context of per unit royalty two-part tariff licensing where the innovator prefers to remain outside the final market.

A natural extension of this model would be to consider another structure of the industry (e.g with a monopoly on the homogeneous component market). In this case we could analyze the scope for conglomerate merger and characterize the profit-maximizing licensing contract charged by an innovator producing one variant of each downstream components. It would be interesting to see if our results carry over to other modelizations of demand uncertainty. We could for example use a random system valuation (i.e v). The analysis of the impact of cost uncertainty on patent licensing in complementary markets might also be a promising extension of our framework.

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General Conclusion

Patent licensing and the market for ideas are nowadays a significant economic activity and a source of revenues for institutions active in R&D. It is particularly the case in the IT industry where patent agreements have a key strategic dimension. The recent patent war between large IT firms such as Samsung and Apple is a good illustration of this. The U.S. Supreme Court has indeed ruled in the end of 2016 that Samsung had to pay damages based on the value of the patent infringing components of the smartphones but not on their total value. Antitrust authorities have also been paying more attention to acquisitions of large patent portfolios and creations of patent pools, imposing remedies and Fair Reasonable And Non Discriminatory (FRAND) licensing for essential patents. The European competition authority raised concerns on the availability of technological information at a reasonable price for the production of complementary goods in the context of conglomerate mergers in the IT industry such as Intel/McAfee, Google/Motorola or Microsoft/Nokia as well as the Cisco/Tandberg mergers. The complementarity between goods and services is also a very important and strategic feature of the IT industry. Consumers navigate in a stream of complementary services forming an environment which requires interoperability. It appears that patent licensing and complementarity between services are two important characteristics of the IT industry. This dissertation aims at linking these two aspects and connect two separate vast literatures.

We explicitly model the patent licensing problem of a monopoly innovator whose technology is used in complementary markets. This enables the characterization of the profit-maximizing licensing policy of the patentee for various structures of the industry.

In particular, we study the impact on licensing strategies of the degree of product differentiation, vertical integration and the availability of per unit and ad valorem royalty rates in complementary markets. Vertical integration plays an important role in the IT industry and in this dissertation. Firms like Intel, that are active in the R&D conceiving and manufacturing their products, are vertically integrated. Others like AMD, focus on their research activities, sell their designs and intellectual properties and remain outside the product market.

In the first chapter, we look for the profit-maximizing licensing strategy of an outside innovator (i.e research laboratory) using pure fixed fee contracts. In a model where only one of the component market is spatially differentiated, we find that under the assumption of limited number of potential downstream firms (i.e $(N_A, N_B) \leq 2$), the number of licenses on the homogeneous market increases with the degree of product differentiation. The double duopoly structure ($N_A = N_B = 2$) is optimal for the innovator when the sensitivity of consumers to product differentiation is high. This policy enables the elimination of the horizontal double marginalization effect. In mass component market however, the asymmetric structure ($N_A = 1, N_B = 2$) is chosen by the patentee in order to avoid the dissipation of the profit of the industry resulting from intense downstream competition in the double duopoly structure. Assuming that the number of potential downstream firms is unlimited, the optimal licensing strategy is to deliver an exclusive license for the monopoly production of the homogeneous good and to implement perfect competition on the differentiated market by issuing as many licenses as possible. This allows the maximization of social surplus through the minimization of transportation costs and its full extraction by the mean of a fixed fee for the exclusive license. In contradiction with the previous model, the number of licenses on the differentiated market decreases (increases) with the degree of product differentiation (the valuation for the system) when there is an infinite number of potentially active firms. Extending this model to the case where both components are spatially differentiated (i.e double circular city model), we find that the same rationale remains valid. In the case of unlimited number of potential firms, the asymmetric structure remains profitable. Otherwise, the double duopoly structure dominates in niche component markets.

In the second chapter, we contribute to the literature studying the effect of vertical integration (e.g Sandonis and Fauli-Oller, 2006) on the incentives for the innovator to deliver licenses to other market participants. We study various integration regimes of our fixed fee licensing model assuming a limited number of potential downstream firms. We find that single vertical integration does not have any effect on downstream equilibria except delaying the earning of the profit of the downstream division to the end of the product development stage. We find that an imperfectly impatient patentee will transfer the rent away from the market where it is active. However, when the innovator is active in both component markets (i.e in the situation following two vertical mergers), downstream equilibria and licensing strategies are altered even in the case of a perfectly patient patentee. The integrated firm charges a lower price for the differentiated component, making market shares asymmetric and the licensing of an exclusive license less profitable than in the separated regime. As a result, the double duopoly (asymmetric) structure dominates for a larger (smaller) range of parameters corresponding to niche (mass) component markets.[1] Moreover, when only one firm is willing to produce the homogeneous component, making the double duopoly structure unavailable, directly producing both components becomes profitable for the innovator.

The empirical literature (e.g Taylor et al., 1973, Rostoker, 1984, Macho-Stadler et al., 1996, Degnan and Horton, 1997, Bousquet et al., 1998) shows the prevalence of the use of royalties in licensing agreements. This is explained in theoretical work by changes in the structure of the industry, the type of goods, the degree of product differentiation or the availability of information (e.g Kamien and Tauman, 2002, Muto, 1993, Gallini and Wright, 1990). In the third chapter, we introduce per unit royalty rates and look for the two-part tariff profit maximizing licensing policy. We find that per unit royalty rates can appear in licensing contracts when the valuation for the system is high (i.e mass markets). Positive royalty rates are desirable in the competitive equilibrium of the double duopoly structure and make it as profitable as the asymmetric structure in mass markets. Vertical integration matters for downstream competition and equilibria but the two-part tariff profit-maximizing licensing policy remains fairly robust to changes in the structure of the ownership. Assuming there is only one potential producer of the

[1] See the summarizing diagram C.1 page 230.

homogeneous component, we show that the double vertical integration is profitable for the innovator in niche component markets. The effects of the double vertical integration under pure fixed fee licensing (e.g making the double duopoly strictly dominating for a wider range of parameters) carries over to the case of two-part tariff contracts. In the latter instance, the double duopoly structure is indeed chosen by the patentee for all levels of the valuation for the final system good.[2]

Recent articles have been exploring the optimal base for royalty rates in patent licensing (e.g Llobet and Padilla, 2016). Royalty rates can be based on the number of sales made by the licensee (i.e per unit royalties) or on the generated sales revenue in the case of ad valorem royalties. Bousquet et al. (1998) shows that demand uncertainty can explain the use of ad valorem royalty rates. In the fourth chapter, we analyze the impact of demand uncertainty on the profit-maximizing licensing strategy in complementary markets, allowing for the use of a fixed fee, per unit and ad valorem royalty rates. We show in two distinct equilibria of the double duopoly structure that a two-part tariff contract including an ad valorem royalty rate is profitable for an outside innovator licensing its technology to risk averse licensees. As regards vertical integration, we find in this three-part tariff licensing model under demand uncertainty, that it is profitable for the innovator to be active in the production of the differentiated niche component market (i.e for sufficiently low levels of system valuation). In equilibrium, the inside innovator also uses ad valorem royalty in combination with a fixed fee.

Overall, we characterize the profitable strategies of a monopoly innovator with respect to the number of licenses, the pricing instruments as well as vertical and conglomerate mergers. We show that in general, the number of licenses delivered in equilibrium differs from the standard model with a single downstream market. In particular, we consistently find, for various forms of licensing contracts that more licenses are issued in niche markets when the number of potential licensees is capped. The opposite is true when there is an infinite number of potentially active downstream firms. Vertical integration and conglomerate mergers are found to be unprofitable except when there is only one firm likely to acquire the technology or when there is uncertainty on demand.

[2] See the summarizing diagram C.1 page 230.

On the other hand, per unit royalty rates are only used in the most competitive structure of the industry for high valuations of the final good. Finally, sales revenue (i.e ad valorem) is found to be a more profitable royalty base than the number of sales (i.e per unit royalties) when demand is uncertain and licensees are risk averse. Our results suggest that complementarity influences the way in which technologies are transfered and that vertical mergers do not generate foreclosing behaviors in this framework.

There are of course several limitations to the work presented in this dissertation. The use of spatial product differentiation allows us to make the number of licensee and varieties endogenous in a downstream system market where the net valuation for the final good depends on the price charged by complementary good producers. Nevertheless, the fact that the size of the market is constant is a limitation to our model. Demand is inelastic in covered equilibria where the price of the component is fully determined by competition within the component market. The price of the complementary good has no effect in the interval sustaining this equilibrium. Our model is characterized by some kind of separation between the effects of complementarity (and demand elasticity) and competition within component markets. It would be desirable to be able to solve a model where all these effects are simultaneously at work.

On the other hand, we have assumed a simple vertical relation framework in which the innovator does not suffer from any commitment problem. We assume that contracts are public and the innovator is able to commit to the number of licenses offered to each downstream market. The patentee is then made capable of capturing downstream profits. In this context, there is very little room for vertical integration to be profitable. Introducing commitment problems into our framework could be an interesting avenue of research. Moreover, we focused on the analysis of drastic innovation with no alternative provider of technology, leaving no bargaining power to potential downstream firms. The literature on patent licensing have shown the importance of the size of innovation for the design of the licensing strategy. For non-drastic cost-reducing innovations, the outside option of potential licensees depends on the type of contracts used by the patentee making the participation constraint and the effects of licensing instruments more complex. The analysis of the interaction between these effects and those of complementarity could be interesting.

We would also wish to develop our analysis of the potential anti-competitive effects of conglomerate mergers in vertically related markets. In our framework, the licensees could face a cost of making their product compatible that would be endogenous and strategically determined by the patentee. This would allow the analysis of the scope for profitable interoperability degrading strategies (e.g Gans, 2011). This assessment is in the heart of the concerns raised by the competition authority in some recent merger cases involving complementary goods and services in the IT industry. Finally, the analysis of the profitability of tying strategies could also represent a promising extension of our model especially in a framework where the monopoly innovator faces commitment problems. These issues are left for future research.

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Appendix A

Optimal fixed fee licensing in complementary markets

A.1 Single differentiated market model

A.1.1 Homogeneous monopoly downstream equilibria

A.1.1.1 Kinked differentiated market

We give more details on the third kind of equilibrium price candidate in which the market B would be covered but competition between neighbors would actually not take place. Given the price a, we verify that this kinked price of market B is the best response of B component producers and an price equilibrium candidate. Given that the rival charges $b = v - a - \frac{t}{4N^2}$, does B_1 find it optimal to charge the same price? We look for profitable deviations. If B_1 sets a higher price, then the market becomes uncovered and the profit-maximizing price previously determined is: $d = \frac{2v+c-2a}{3}$. We obtain the following optimal deviation profit:

$$\Pi_{B_1}^D = \frac{2t\left(\frac{v-c-a}{t}\right)^{\frac{3}{2}}}{3\sqrt{3}}. \tag{A.1}$$

We compare it to the equilibrium profit

$$\Pi_{B_2}^K = \frac{v-c-a-\frac{t}{4N^2}}{2N}. \tag{A.2}$$

We find that the kinked equilibrium profit is dominated when: $a \leq v-c$. However, the deviation profit is consistent with the uncovered market demand if the location of the marginal consumer $\bar{x}$ is such that $\bar{x} < \frac{1}{2N}$. Deviating demand and profit functions are only valid when:

$$v-c-\frac{3t}{4N^2} < a \leq v-c. \tag{A.3}$$

If B_1 deviates from the kinked equilibrium candidate with a lower price, the market becomes covered and demand addressed to B_1 is given by the location of the indifferent consumer. Assuming that the price of the rival firm is given and equal to the kinked price equilibrium candidate, the profit function of B_1 is:

$$\Pi_{B_1}^D = (d-c)\times\left(\frac{1}{2N}+\frac{N\left(v-a-\frac{t}{4N^2}-d\right)}{2t}\right). \tag{A.4}$$

Comparing it to the kinked equilibrium profit, we find that such a deviation is only profitable if:

$$a < v-c-\frac{5t}{4N^2}. \tag{A.5}$$

When a is high, the kinked equilibrium price charged by the neighbor B_2 is lower, which reduces the competitive deviation profit. We obtain the same condition if we maximize the deviation profit and compare it to the equilibrium candidate and finally crossing profitable conditions with competitive demand consistency condition. Combining the two conditions for unprofitable deviations, we characterize the following kinked equilibrium of the market B:

$$b = d = v-a-\frac{t}{4N^2} \tag{A.6}$$

$$\text{if: } v - c - \frac{5t}{4N^2} < a < v - c - \frac{3t}{4N^2}. \tag{A.7}$$

A.1.1.2 Equilibrium characterization

We add some details to the characterization exposed in chapter one. We first characterize the uncovered market equilibrium. Best response functions:

$$b = \frac{(2v + c - 2a)}{3} \text{ if: } a > v - c - \frac{3t}{4N^2} \tag{A.8a}$$
$$a = \frac{2v + c - 2b}{3} \text{ if: } b > v - c - \frac{3t}{4N^2}. \tag{A.8b}$$

The uncovered Nash equilibrium price is:

$$a = b = \frac{2v + c}{5} \tag{A.9}$$

$$\text{if: } b > v - c - \frac{3t}{4N^2}, \text{ and } a > v - c - \frac{3t}{4N^2}, \tag{A.10}$$

$$\text{that is if, } v < 2c + \frac{5t}{4N^2}. \tag{A.11}$$

This is the condition for the market to be indeed uncovered in equilibrium (i.e location of the marginal consumer $\bar{x} < \frac{1}{2N}$). Profits made in the entire market (i.e around the circle) in the uncovered equilibrium Π_i^U are respectively:

$$\Pi_A^U = \frac{4N}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}} \tag{A.12}$$

$$\Pi_B^U = \frac{4(v - 2c)^{\frac{3}{2}}}{5\sqrt{5t}} \tag{A.13}$$

$$\Pi_U^U = \frac{8N}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}}. \tag{A.14}$$

We turn to the characterization of the competitive covered equilibrium. We previously determined an equilibrium candidate of the market B such that it is competitive and component B producers play their best response: $b = c + \frac{t}{N^2}$, if $a < v - c - \frac{5t}{4N^2}$. We know that if $b < v - c - \frac{3t}{4N^2}$, A's best response function is given by: $a = v - b - \frac{t}{4N^2}$. Replacing b by its competitive equilibrium value, A charges $a = v - c - \frac{5t}{4N^2}$ if $b < v - c - \frac{3t}{4N^2}$. As a consequence, we obtain the following competitive covered equilibrium characterization:

$$\left(a = v - c - \frac{5t}{4N^2}, b = c + \frac{t}{N^2}\right) \tag{A.15}$$

$$\text{if: } v > 2c + \frac{7t}{4N^2}. \tag{A.16}$$

Competitive covered market equilibrium total profits Π_i^C of A, B producers and U are respectively:

$$\Pi_A^C = v - 2c - \frac{3t}{4N^2} \tag{A.17}$$

$$\Pi_B^C = \frac{t}{2N^3} \tag{A.18}$$

$$\Pi_U^C = v - 2c - \frac{t}{4N^2}. \tag{A.19}$$

We finally characterize the kinked equilibrium. We previously found the kinked equilibrium of the market B characterized by: $b = v - a - \frac{t}{4N^2}$, if $v - c - \frac{5t}{4N^2} < a < v - c - \frac{3t}{4N^2}$. On the other hand, we know that if $b \leq v - c - \frac{3t}{4N^2}$, A's best response function is given by: $a = v - b - \frac{t}{4N^2}$. We look for a value of (a, b) such that $a = BR_A(b)$ and $b = BR_B(a)$. In the kinked situation, this system is collinear and there is multiplicity of equilibria. Thus, we have the following set of kinked equilibria satisfying all above conditions:

$$\left(a = v - b - \frac{t}{4N^2}, b = v - a - \frac{t}{4N^2}\right) \tag{A.20}$$

$$\text{if: } v - c - \frac{5t}{4N^2} < a < v - c - \frac{3t}{4N^2} \tag{A.21}$$

$$\text{and: } b < v - c - \frac{3t}{4N^2}. \tag{A.22}$$

Knowing that $a + b = v - \frac{t}{4N^2}$ needs to be smaller than $2 \times \left(v - c - \frac{3t}{4N^2}\right)$, there exists a kinked equilibrium if and only if:

$$v > 2c + \frac{5t}{4N^2}. \tag{A.23}$$

Kinked market equilibria total profit functions Π_i^K of A, B producers and U are respectively:

$$\Pi_A^K = v - b - \frac{t}{4N^2} - c \tag{A.24}$$

$$\Pi_B^K = v - a - \frac{t}{4N^2} - c \tag{A.25}$$

$$\Pi_U^K = v - 2c - \frac{t}{4N^2}. \tag{A.26}$$

A.1.2 Optimal licensing with limited number of potential downstream firms

A.1.2.1 Double monopoly structure

Given the value of a, the price of the component B is characterized in the following way. When a is such that the market is uncovered, the differentiated monopolist optimally charges:

$$b = \frac{2v + c - 2a}{3} \tag{A.27}$$

$$\text{if: } a > v - c - \frac{3t}{4}. \tag{A.28}$$

Otherwise, the market becomes covered and the differentiated monopolist charges the highest price compatible with full participation. Given the price charged by the producer of the complementary good a, b is set so that the market is just covered and no unnecessary surplus is left to consumers:

$$b = v - a - \frac{t}{4} \tag{A.29}$$

$$\text{if: } a < v - c - \frac{3t}{4}. \tag{A.30}$$

Since the competitive equilibrium does not exist in the differentiated monopolist case, there is no profitable deviation towards the competitive demand implying no lower bound on the price a compatible with the market to be just covered (i.e kinked component B market). We derive the following equilibria of the game in the double monopoly case ($N_A = N_B = 1$). The uncovered Nash equilibrium price is:

$$a = b = \frac{2v + c}{5} \tag{A.31}$$

$$\text{if: } v < 2c + \frac{5t}{4}. \tag{A.32}$$

This is the condition for the market to be indeed uncovered in equilibrium (i.e location of the marginal consumer $\bar{x} < \frac{1}{2}$). Otherwise, the market is covered and given the price of the complementary good, each monopolist charges the highest consistent price. Monopolists face a coordination problem and we have a multiplicity of covered equilibria. These covered equilibria are necessarily kinked (i.e just covered). Thus, we have the following set of kinked equilibria:

$$\left(a = v - b - \frac{t}{4}, b = v - a - \frac{t}{4}\right) \tag{A.33}$$

$$\text{if: } a < v - c - \frac{3t}{4} \tag{A.34}$$

$$\text{and: } b < v - c - \frac{3t}{4}. \tag{A.35}$$

Knowing that the system price $a + b = v - \frac{t}{4}$ needs to be smaller than $2 \times \left(v - c - \frac{3t}{4}\right)$, there is a kinked equilibrium if $v > 2c + \frac{5t}{4}$. Kinked equilibrium prices, a and b are necessarily included in the following interval:

$$(a, b) \in \left[c + \frac{t}{2}, v - c - \frac{3t}{4}\right]. \tag{A.36}$$

A.1.2.2 Optimal licensing

We derived the conditions under which each structure is uncovered in equilibrium and ranked the structures in decreasing likelihood of being uncovered. The condition for the double monopoly structure to be uncovered is wider (the upper bound on v is higher and the set of v values for which the market is uncovered is larger) whereas the condition on v in the double duopoly structure is the most restrictive of the available structures. We now determine the optimal licensing policy which maximizes the upstream monopolist's profit for each value of system valuation v. When all structures are covered, the structure that maximizes the upstream monopolist profit is the homogeneous monopoly with the differentiated duopoly. This corresponds to the constrained asymmetric structure. The upstream monopolist indeed chooses to sell only one exclusive license. We simply compare equilibrium profits when all structures are covered (i.e when the double monopoly structure is covered). If, $v > 2c + \frac{5t}{4}$ profit of U is:

1. under the double monopoly structure:

$$\Pi_U^K = v - 2c - \frac{t}{4} \tag{A.37}$$

2. under the differentiated monopoly structure:

$$\Pi_U^K = v - 2c - \frac{t}{4} \tag{A.38}$$

3. under the asymmetric structure:

$$\Pi_U^C = \Pi_U^K = v - 2c - \frac{t}{16} \tag{A.39}$$

4. under the double duopoly structure:

$$\Pi_U^C = \frac{t}{4}. \tag{A.40}$$

The asymmetric structure is clearly preferred to both structures involving an exclusive differentiated producer. This is due to the fact that the transportation cost (social surplus) decreases (increases) as the number of active firms increases. On the other hand, the asymmetric structure is preferred to the double duopoly structure as soon as the system valuation is sufficiently high:

$$\Pi_U^3 = v - 2c - \frac{t}{16} > \Pi_U^4 = \frac{t}{4} \tag{A.41}$$

$$\text{if: } v > 2c + \frac{5t}{16}. \tag{A.42}$$

Since the condition for the double monopoly to be covered is more restrictive, the upstream monopolist chooses the asymmetric structure. If, $2c + \frac{3t}{4} < v < 2c + \frac{5t}{4}$, the upstream profit chooses the asymmetric structure. All structures are covered except the double monopoly. We showed earlier that asymmetric structure was preferred to all other covered structures. Moreover, we show that in this case the uncovered double monopoly structure is dominated.

$$\Pi_U^1 = \frac{8}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}} < \Pi_U^3 = v - 2c - \frac{t}{16} \tag{A.43}$$

$$\text{if: } 2c + 1.82 \times t > v > 2c + \frac{5t}{16}. \tag{A.44}$$

This condition is necessarily satisfied when $2c + \frac{3t}{4} < v < 2c + \frac{5t}{4}$. Now if $2c + \frac{5t}{16} < v < 2c + \frac{3t}{4}$, the differentiated monopoly structure is also uncovered. Comparing the covered asymmetric profit with the uncovered differentiated monopoly profit, we find that the asymmetric structure is more favorable for consistent values of v:

$$\Pi_U^2 = \frac{4}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}} < \Pi_U^3 = v - 2c - \frac{t}{16} \tag{A.45}$$

$$\text{if: } 2c + \frac{5t}{16} < v < 2c + \frac{3t}{4}. \tag{A.46}$$

Moreover, from the observation of U's profit function, we know that when both structures are uncovered, the differentiated monopoly leads to a higher profit than the double monopoly structure (i.e $\frac{8}{5\sqrt{5t}} < \frac{4}{3\sqrt{3t}}$). As a consequence, the asymmetric is necessarily preferred to the double monopoly structure. Since the asymmetric structure is also the most favorable covered structure, we characterized the optimal licensing policy for the considered values of v. If, $2c + \frac{3t}{16} < v < 2c + \frac{5t}{16}$, then the asymmetric becomes uncovered and the double duopoly structure moves from the competitive equilibrium to the unique kinked equilibrium. Looking at the profit function of U, we observe that the uncovered asymmetric structure is more favorable than uncovered structures 1 and 2 (i.e $\frac{16}{5\sqrt{5t}} > \frac{4}{3\sqrt{3t}} > \frac{8}{5\sqrt{5t}}$). On the other hand, the asymmetric structure is dominated by the double duopoly structure in the interval of system valuation considered:

$$\Pi_U^3 = \frac{16}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}} < \Pi_U^4 = v - 2c - \frac{t}{16}, \tag{A.47}$$

as long as, $2c + \frac{5}{512} \times (9 + \sqrt{17})t < v < 2c + \frac{5t}{16}$ which is satisfied because $\frac{5}{512} \times (9 + \sqrt{17}) \approx 0.128 < \frac{3}{16}$. As a consequence, the condition on v for the double duopoly to yield to more profit than the asymmetric structure is necessarily satisfied if $2c + \frac{3t}{16} < v < 2c + \frac{5t}{16}$. When all structures available are uncovered, U chooses to set the double duopoly structure which leads to the highest uncovered profit. This is due to the fact that double marginalization is avoided allowing participation and thus social surplus to be higher than under the other uncovered structures. If, $v < 2c + \frac{3t}{16}$,

$$\Pi_U^3 = \frac{16}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}} < \Pi_U^4 = \frac{8}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}}, \tag{A.48}$$

thus the double duopoly structure is optimal for the upstream monopolist. In conclusion, the asymmetric structure is the most favorable structure of the industry for the upstream monopolist U as soon as the valuation for the system is sufficiently high (i.e when $v > 2c + \frac{5t}{16}$). Otherwise, the double duopoly structure is preferred. This is the case, because the asymmetric structure always dominates both the double monopoly and the differentiated monopoly. As a consequence, the optimal licensing policy in the limited case is determined by the comparison between the asymmetric and the double duopoly structures. We found that when consumers are highly sensible to the distance with their ideal variety (i.e $t > (v - 2c) \times \frac{16}{5}$), which characterizes a component niche market, the optimal licensing policy is symmetric. Despite the symmetry of the optimal licensing policy, profits are only generated in one of the component markets. All the rent is indeed transferred to one downstream market and extracted by two local monopolists.

A.2 Double circular city model

A.2.1 Double circular city model under market covered assumption

Within the symmetric licensing policies, the profit of the innovator decreases with the number of licenses: $\Pi_U = \frac{t_B}{N_B^2} + \frac{t_A}{N_A^2}$. The double monopoly consequently maximizes profit within this family of structures. In the case of the balanced covered equilibrium, the profit upstream monopolist takes the following value:

$$v - \frac{t_A + t_B}{4} - 2c. \tag{A.49}$$

Proposition 1.6.1 then results from direct comparison of the profit of the upstream innovator between the asymmetric and the double monopoly structure:

$$v - 2c - \frac{t_A}{4} > v - 2c - \frac{t_A + t_B}{4}. \tag{A.50}$$

We observe that the profit of the asymmetric structure does not depend on the number of producers on the competitive market.

Appendix B

Vertical integration and optimal fixed fee licensing in complementary markets

B.1 Vertical integration

B.1.1 Homogeneous vertical integration

B.1.1.1 Polar values of the degree of patience

Proof of proposition 2.2.2:

We derive the optimal policy for each range of v assuming the favorable (the most favorable to B and thus to the extremely impatient UA) equilibrium is selected when there is multiplicity of equilibria. When all structures are covered (i.e $2c + \frac{5t}{4} \leq v$), the differentiated monopoly structure (i.e $N_A = 2$, $N_B = 1$ denoted by the subscript D) dominates both the asymmetric (i.e subscript A) and the double duopoly (i.e subscript

$2D$) as well as the double monopoly (i.e subscript $2M$) structures:

$$\Pi_{D}^{UA} = v - 2c - \frac{t}{4} \geq \Pi_{A}^{UA} = \Pi_{2D}^{UA} = \frac{t}{4}, \text{ if and only if:} \tag{B.1}$$

$$2c + \frac{2t}{4} \leq v, \text{ which is satisfied in this covered case.} \tag{B.2}$$

$$\text{And, } \Pi_{D}^{UA} = v - 2c - \frac{t}{4} \geq \Pi_{2M}^{UA} = v - 2c - \frac{3t}{4}, \text{ for all levels of } v. \tag{B.3}$$

When only the double monopoly structure is uncovered, the latter is doing worse than in the covered case which does not challenge the dominance of the differentiated monopoly structure. When only the asymmetric and double duopoly structures are covered (i.e $2c+\frac{3t}{4} \geq v \geq 2c+\frac{5t}{16}$), the differentiated monopoly structure is preferred if v is sufficiently large:

$$\Pi_{D}^{UA} = \frac{4}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}} \geq \Pi_{A}^{UA} = \Pi_{2D}^{UA} = \frac{t}{4}, \tag{B.4a}$$

$$\text{if and only if: } 2c + \frac{3 \times \sqrt[3]{2}t}{8} \leq v \leq 2c + \frac{3t}{4}. \tag{B.4b}$$

Otherwise the differentiated monopoly structure is dominated by both the double duopoly and the asymmetric structure. Assuming the favorable to B kinked equilibrium is selected, the asymmetric structure leads to the same innovator's profit as the double duopoly structure when it is at the competitive covered equilibrium. We assume that this indifference is broken in favor of the double duopoly structure as it avoids multiplicity of equilibria and possible coordination problems. The double duopoly structure is thus preferred when the valuation is sufficiently low (i.e $2c + \frac{3\times\sqrt[3]{2}t}{8} \geq v$). When only the double duopoly structure is covered. We only need to compare the (unique) kinked covered double duopoly structure with the uncovered asymmetric one. We find that the double duopoly remains the most profitable structure:

$$\Pi_{2D}^{UA} = v - 2c - \frac{t}{16} \geq \Pi_{A}^{UA} = \frac{8}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}}, \tag{B.5a}$$

$$\text{if and only if: } v \leq 2c + \frac{5t}{8 \times \sqrt[3]{2}} = 2c + 0,496 \times t, \tag{B.5b}$$

which is satisfied on this specific interval because v is sufficiently low (i.e $v < 2c + \frac{5t}{16} = 2c + 0,3125 \times t$). When all structures are uncovered, the double duopoly structure dominates the asymmetric structure:

$$\Pi_{2D}^{UA} = \frac{8}{3\sqrt{3t}} \times (v - 2c)^{\frac{3}{2}} \geq \Pi_{A}^{U} = \frac{8}{5\sqrt{5t}} \times (v - 2c)^{\frac{3}{2}}. \qquad \text{(B.6)}$$

As a result, the double duopoly structure is preferred when the valuation for the system is sufficiently low (i.e $2c + \frac{3\times\sqrt[3]{2}t}{8} \geq v$).

B.1.1.2 Intermediate values of the degree of patience

Proof of proposition 2.2.3:

The unfavorable to B kinked differentiated monopoly profit of UA is equal to:

$$\Pi_{D}^{UA} = \Pi_{D}^{B} = v - 2c - \frac{t}{4}. \qquad \text{(B.7)}$$

Whereas the unfavorable to B kinked equilibrium of the asymmetric structure profit of UA is:

$$\Pi_{A}^{UA} = \frac{t}{8} + \delta\left(-2c - \frac{3t}{16} + v\right). \qquad \text{(B.8)}$$

Direct comparison between these two expressions leads to the expression of the threshold on δ presented in the proposition. The uncovered differentiated monopoly profit of UA is equal:

$$\Pi_{D}^{UA} = \frac{4(v - 2c)^{3/2}}{3\sqrt{3t}}. \qquad \text{(B.9)}$$

Unfavorable to B kinked equilibrium of the asymmetric structure profit of UA:

$$\Pi_{A}^{UA} = \frac{t}{8} + \delta\left(-2c - \frac{3t}{16} + v\right). \qquad \text{(B.10)}$$

Direct comparison between these two expressions leads to the expression of the threshold on δ presented in the proposition. The competitive double duopoly profit of UA is equal to:

$$\Pi_{2D}^{UA} = \Pi_{A}^{B} = \frac{t}{4}. \tag{B.11}$$

Whereas the unfavorable to B kinked equilibrium of the asymmetric structure profit of UA is:

$$\Pi_{A}^{UA} = \frac{t}{8} + \delta\left(-2c - \frac{3t}{16} + v\right). \tag{B.12}$$

Direct comparison between these two expressions leads to the expression of the threshold on δ presented in the proposition.

B.1.2 Vertical integration into the differentiated component market

B.1.2.1 Polar values of the degree of patience

Proof of lemma 2.2:

The proof is straight forward from observation of the profit functions under each structure. For exposition purposes, we only give elements of the proof when the equilibrium that is the most unfavorable to B producers is selected. This equilibrium is sustained by all values of the parameter v such that a kinked equilibrium exists. It is also worth noting that this equilibrium minimizes the discounted profit and is the most favorable to the inside innovator. When all structures are covered:

$$\Pi_{A}^{UB} = \frac{t\delta}{16} + \frac{t}{16} + \left(v - 2c - \frac{3t}{16}\right) = v - 2c - \frac{t(2-1\delta)}{16} \geq \Pi_{D}^{UB} = \delta\left(v - 2c - \frac{t}{4}\right), \tag{B.13}$$

$$\text{and } \Pi_{A}^{UB} = v - 2c - \frac{t(2-1\delta)}{16} \geq \Pi_{2M}^{UB} = v - 2c - \frac{t(3-2\delta)}{4}. \tag{B.14}$$

When all structures are uncovered (i.e no kinked equilibria):

$$\Pi_{A}^{UB} = \frac{12}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} + \frac{4\delta}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} \geq \Pi_{D}^{UB} = \frac{4\delta}{3\sqrt{3t}}(v-2c)^{\frac{3}{2}}, \text{ and:} \tag{B.15}$$

$$\Pi_{A}^{UB} = \frac{12}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} + \frac{4\delta}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} \geq \Pi_{2M}^{UB} = \frac{4}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} + \frac{4\delta}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}}. \tag{B.16}$$

When only some structures are uncovered, as the market covered asymmetric structure constraint is the less restrictive, the asymmetric structure would remain covered whereas the alternatives would be uncovered and do even worse than when they ensured full coverage of the market.

Proof of proposition 2.2.4:

The asymmetric structure is optimal for intermediate values of v (i.e $2c + \frac{3t}{16} \leq v \leq 2c + \frac{5t}{16}$) when the inside innovator is sufficiently impatient:

$$\begin{aligned} \delta \leq \bar{\delta} = & -1 + \frac{2}{7}\left(1 - \frac{5t}{128c+5t-64v}\right) \\ & +1280\sqrt{5}\sqrt{+\frac{t(16v-t-32c)^2(v-2c)^3}{(128c+5t-64v)^2\left(1024c^2+960ct+25t^2-32(32c+15t)v+256v^2\right)^2}} \\ & +\frac{128(32v-5t-64c)(v-2c)}{7\left(1024c^2+960ct+25t^2-32(32c+15t)v+256v^2\right)} \end{aligned} \tag{B.17}$$

When both structures are covered (i.e $v \geq 2c + \frac{5t}{16}$),

$$\Pi_{2D}^{UB} = \frac{t\,(1+\delta)}{8} \leq \Pi_{A}^{UB} = v - 2c - \frac{t(2-1\delta)}{16}, \tag{B.18}$$

if and only if: $v \geq 2c + \frac{(4-\delta)t}{16}$, which is satisfied for all values of δ. When $\delta = 0$, which is the most restritive case, v must be such that: $v \geq 2c + \frac{t}{4}$, which is necessarily satisfied when both structures are covered. When both structures are uncovered:

$$\Pi_{2D}^{UB} = \frac{4}{3\sqrt{3t}}(v-2c)^{\frac{3}{2}} + \frac{4\delta}{3\sqrt{3t}}(v-2c)^{\frac{3}{2}} \leq \Pi_{A}^{UB} = \frac{12}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} + \frac{4\delta}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}}, \tag{B.19}$$

$$\text{if and only if: } 0 \leq \delta \leq 0,737. \tag{B.20}$$

When v takes intermediate values, the asymmetric structure is uncovered whereas the double duopoly one is at its unique kinked covered equilibrium. Comparing the profit of the inside innovator in both cases and solving for δ leads to proposition 3.

$$\Pi_{2D}^{UB} = (v - 2c - \frac{t}{16}) \times \frac{1+\delta}{2} \leq \Pi_{A}^{UB} = \frac{12}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}} + \frac{4\delta}{5\sqrt{5t}}(v-2c)^{\frac{3}{2}}, \tag{B.21}$$

$$\text{if and only if: } 0 \leq \delta \leq \bar{\delta}. \tag{B.22}$$

B.1.3 In-house development

B.1.3.1 Proof of proposition 1

This proposition directly results from comparison of consistent profit functions across structures and along the interval on system valuation v. We first consider the case where all structures are covered. We show that the asymmetric structure dominates the differentiated monopoly and double monopoly structures and then that the very asymmetric structure (denoted VAS) dominates the asymmetric and double duopoly structures. Second, we repeat this procedure in the case where all structures are uncovered. Last, we compare the very asymmetric and the differentiated oligopoly structures.

When structures are all covered (i.e $v > 2c + \frac{5t}{36}$): The asymmetric structure dominates the double monopoly structure:

$$\Pi_M^U = \left(-2c - \frac{3t}{4} + v\right) + \frac{\delta t}{2} < \Pi_A^U = \left(-2c - \frac{3t}{16} + v\right) + \frac{\delta t}{16} + \frac{t}{16}. \tag{B.23}$$

The aymmetric structure dominates the differentiated monopoly structure:

$$\Pi_D^U = \delta\left(-2c - \frac{t}{4} + v\right) < \Pi_A^U = \left(-2c - \frac{3t}{16} + v\right) + \frac{\delta t}{16} + \frac{t}{16}. \tag{B.24}$$

The VAS dominates the asymmetric structure:

$$\Pi_A^U = \left(-2c - \frac{3t}{16} + v\right) + \frac{\delta t}{16} + \frac{t}{16} < \Pi_{VAS}^U = -2c + \frac{\delta t}{54} - \frac{3t}{36} + \frac{2t}{54} + v. \tag{B.25}$$

VAS dominates the double duopoly structure (dominated by the asymmetric structure when both structures are covered):

$$\Pi_{2D}^{UA} = \frac{\delta t}{8} + \frac{t}{8} < \Pi_{VAS}^{UA} = -2c + \frac{\delta t}{54} - \frac{3t}{36} + \frac{2t}{54} + v. \tag{B.26}$$

We now study the case in which the VAS is uncovered (i.e $v < 2c + \frac{5t}{36}$): The uncovered asymmetric structure dominates the double monopoly and differentiated monopoly:

$$\Pi_M^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{4(v-2c)^{3/2}}{5\sqrt{5t}} < \Pi_A^{UA} = \frac{12(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{4\delta}{5\sqrt{5t}}(v-2c)^{3/2}, \quad (B.27)$$

$$\frac{(4\delta)(v-2c)^{3/2}}{3\sqrt{3t}} < \frac{(4\delta)(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{12(v-2c)^{3/2}}{5\sqrt{5t}}. \quad (B.28)$$

We show that the VAS dominates both the asymmetric and the double duopoly structures. The VAS dominates the asymmetric structure:

$$\Pi_A^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{12(v-2c)^{3/2}}{5\sqrt{5t}} < \Pi_{VAS}^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{20(v-2c)^{3/2}}{5\sqrt{5t}}, \quad (B.29)$$

The VAS dominates the double duopoly structure:

$$\Pi_{2D}^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{3\sqrt{3t}} + \frac{4(v-2c)^{3/2}}{3\sqrt{3t}} < \Pi_{VAS}^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{20(v-2c)^{3/2}}{5\sqrt{5t}}. \quad (B.30)$$

We now compare the very asymmetric and the differentiated oligopoly structures (denoted *DO*) first considering the case where both are covered. The VAS dominates the differentiated oligopoly:

$$\Pi_{DO}^{UA} = \frac{1}{27}(\delta+2)t < \Pi_{VAS}^{UA} = -2c + \frac{\delta t}{54} - \frac{3t}{36} + \frac{2t}{54} + v \quad (B.31)$$

$$v > 2c + \frac{5t}{36}. \quad (B.32)$$

The uncovered VAS dominates the uncovered differentiated oligopoly structure:

$$\Pi_{VAS}^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{5\sqrt{5t}} + \frac{(v-2c)^{3/2}}{5\sqrt{5t}} > \Pi_{DO}^{UA} = \frac{(4\delta)(v-2c)^{3/2}}{3\sqrt{3t}} + \frac{8(v-2c)^{3/2}}{3\sqrt{3t}}, \quad (B.33)$$

If and only if:

$$2c + \frac{3t}{36} > v > 2c \quad (B.34)$$

$$0 < \delta < \frac{45\sqrt{5} - 50\sqrt{3}}{25\sqrt{3} - 9\sqrt{5}} \tag{B.35}$$

$$\Leftrightarrow 0 < \delta < 0.604941. \tag{B.36}$$

The uncovered VAS dominates the covered differentiated oligopoly structure:

$$\begin{aligned} \delta \leq \bar{\delta}_3 = \quad & -\frac{432(2c-v)(144c+5t-72v)}{7\left(5184c^2-216v(24c+5t)+2160ct+25t^2+1296v^2\right)} + 6480\sqrt{5} \\ & \times \sqrt{-\frac{t(2c-v)^3(72c+t-36v)^2}{(288c+5t-144v)^2\left(5184c^2-216v(24c+5t)+2160ct+25t^2+1296v^2\right)^2}} \\ & + \frac{3}{7}\left(1 - \frac{5t}{288c+5t-144v}\right) - 2. \end{aligned} \tag{B.37}$$

Appendix C

Two-part tariff licensing and vertical integration in complementary markets

C.1 Separated model

C.1.1 Downstream equilibria

C.1.1.1 Double duopoly

Profit functions:

- Uncovered

$$\Pi_{B_1} = (d - g)\sqrt{-\frac{a + d - v}{t}} \tag{C.1}$$

$$\Pi_{B_2} = (b - g)\sqrt{-\frac{a + b - v}{t}} \tag{C.2}$$

- Kinked

$$\Pi_{B_1} = (d - g)\sqrt{-\frac{a + d - v}{t}} \tag{C.3}$$

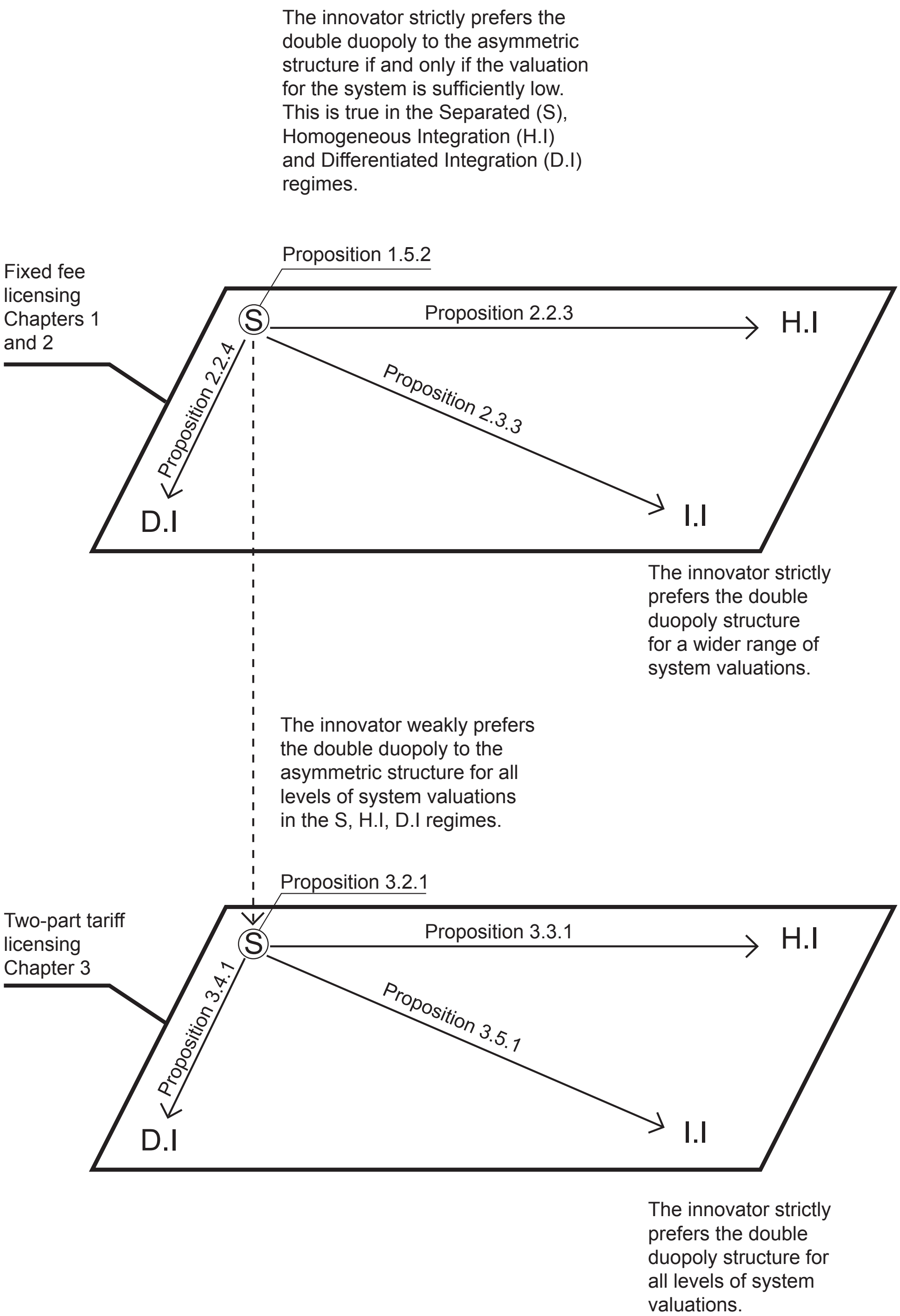

Figure C.1: Licensing policy across types of contracts and integration regimes

$$\Pi_{B_2} = (b-g)\sqrt{-\frac{a+b-v}{t}} \tag{C.4}$$

$$\text{iff:} \quad \sqrt{-\frac{a+d-v}{t}} + \sqrt{-\frac{a+b-v}{t}} = \frac{1}{2} \tag{C.5}$$

- Competitive

$$\Pi_{B_1} = (d-g)\left(\frac{b-d}{t} + \frac{1}{4}\right) \tag{C.6}$$

$$\Pi_{B_2} = (b-g)\left(\frac{d-b}{t} + \frac{1}{4}\right) \tag{C.7}$$

Equilibrium Profits:

B producers:

- Uncovered

$$\Pi_{B_i} = \frac{2t\left(-\frac{g+k-v}{t}\right)^{3/2}}{3\sqrt{3}} \tag{C.8}$$

- Kinked

$$\Pi_{B_i} = \frac{1}{4}\left(-g-k-\frac{t}{16}+v\right) \tag{C.9}$$

- Competitive

$$\Pi_{B_i} = \frac{t}{16} \tag{C.10}$$

Upstream monopolist U:

- Uncovered

$$\Pi_U = \frac{2(-6c+g+k+2v)\sqrt{-\frac{g+k-v}{t}}}{3\sqrt{3}} \tag{C.11}$$

- Kinked

$$\Pi_U = -c - \frac{t}{32} + \frac{v}{2} \tag{C.12}$$

- Competitive

$$\Pi_U = \frac{t}{8} + \frac{k-c}{2} + \frac{g-c}{2} \tag{C.13}$$

C.1.1.2 Asymmetric structure

Equilibria and profits

- Uncovered

$$d = \frac{3g}{5} - \frac{2k}{5} + \frac{2v}{5} \tag{C.14}$$

$$b = \frac{3g}{5} - \frac{2k}{5} + \frac{2v}{5} \tag{C.15}$$

$$a = -\frac{2g}{5} + \frac{3k}{5} + \frac{2v}{5} \tag{C.16}$$

$$v < \frac{1}{16}(16g + 16k + 5t) \tag{C.17}$$

$$\Pi_A = \frac{4t\left(-\frac{g+k-v}{t}\right)^{3/2}}{5\sqrt{5}} \tag{C.18}$$

$$\Pi_{B_1} = \Pi_{B_2} = \frac{2t\left(-\frac{g+k-v}{t}\right)^{3/2}}{5\sqrt{5}} \tag{C.19}$$

$$\Pi_U = \frac{2(-10c + g + k + 4v)\sqrt{-\frac{g+k-v}{t}}}{5\sqrt{5}} \tag{C.20}$$

- Kinked (e.g most favorable to A)

$$a = \frac{1}{16}(-16g - 3t + 16v) \tag{C.21}$$

$$b = d = g + \frac{t}{8} \tag{C.22}$$

$$\Pi_A = \frac{1}{32}(-16g - 16k - 3t + 16v) \tag{C.23}$$

$$\Pi_{B_1} = \Pi_{B_2} = \frac{t}{32} \tag{C.24}$$

$$\Pi_U = \frac{1}{2}(-2c - \frac{t}{16} + v) \tag{C.25}$$

- Competitive

$$a = -g - \frac{5t}{16} + v \tag{C.26}$$

$$d = b = g + \frac{t}{4} \tag{C.27}$$

$$\Pi_A = \frac{1}{2}\left(-g - k - \frac{5t}{16} + v\right) \tag{C.28}$$

$$\Pi_{B_1} = \Pi_{B_2} = \frac{t}{16} \tag{C.29}$$

$$\Pi_U = \frac{1}{2}(-2c - \frac{t}{16} + v) \tag{C.30}$$

C.1.2 Optimal licensing policy

C.1.2.1 Royalty rates

The observation of the profit function of the innovator and the sign of its derivative with respect to per unit royalty rates. Asymmetric uncovered profit function depending negatively on g and k:

$$\Pi_U = \frac{2(-10c + g + k + 4v)\sqrt{-\frac{g+k-v}{t}}}{5\sqrt{5}}. \tag{C.31}$$

Double duopoly uncovered profit function depending negatively on g and k:

$$\Pi_U = \frac{2(-6c + g + k + 2v)\sqrt{-\frac{g+k-v}{t}}}{3\sqrt{3}}. \tag{C.32}$$

In the case of kinked equilibria, the profit of the innovator does not depend on royalty rates in the asymmetric as well as double duopoly structure:

$$\Pi_U = \frac{1}{2}(-2c - \frac{t}{16} + v). \tag{C.33}$$

In the competitive equilibrium of the double duopoly structure, the profit of the patentee is strictly increasing with royalty rates. The innovator thus profitably charge the highest consistent rate of royalty which is in the symmetric case given by:

$$g = k = \frac{v}{2} - \frac{5t}{32}. \tag{C.34}$$

C.1.2.2 Number of licenses

Proof of proposition 3.2.1:

The proof of proposition 1 comes directly from the comparison between the profit function of the innovator in the asymmetric and double duopoly structure. The covered double duopoly profit is equal to:

$$\frac{1}{2}(-2c - \frac{t}{16} + v) \tag{C.35a}$$

$$\text{with } g = k = \frac{v}{2} - \frac{5t}{32}, \tag{C.35b}$$

which is exactly equal to the profit in the covered asymmetric structure. Double duopoly uncovered profit is greater than asymmetric uncovered profit for consistent values of parameters:

$$\frac{2(-6c + g + k + 2v)\sqrt{-\frac{g+k-v}{t}}}{3\sqrt{3}} > \frac{2(-10c + g + k + 4v)\sqrt{-\frac{g+k-v}{t}}}{5\sqrt{5}}. \tag{C.36}$$

As a consequence, the double duopoly strictly dominates the asymmetric structure when the market is uncovered and both structures are equivalent otherwise.

C.2 Differentiated vertical integration

C.2.1 Downstream equilibria

C.2.1.1 Asymmetric structure

Profit functions and resulting best response functions take the following forms:

$$\Pi_{B_1} = \begin{cases} (d-g)\left(\frac{(b-d)}{t}+\frac{1}{4}\right) & \text{(C.37)} \\ (d-g)\sqrt{\frac{-a-d+v}{t}},\ \text{if } \bar{x}_1 = \frac{1}{2}-\bar{x}_2 & \text{(C.38)} \\ (d-g)\sqrt{\frac{-a-d+v}{t}} & \text{(C.39)} \end{cases}$$

If $\bar{x}_1 \geq \frac{1}{2} - \bar{x}_2$:

$$\Pi_{B_2} = (g-c)\left(\frac{(b-d)}{t}+\frac{1}{4}\right) + (b-c)\left(\frac{(d-b)}{t}+\frac{1}{4}\right) + \frac{k-c}{2} \tag{C.40}$$

If $\bar{x}_1 = \frac{1}{2} - \bar{x}_2$:

$$\Pi_{B_2} = (b-c)\sqrt{\frac{-a-b+v}{t}} + (g-c)\sqrt{\frac{-a-d+v}{t}} + \frac{k-c}{2}. \tag{C.41}$$

$$\Leftrightarrow \Pi_{B_2} = (b-g)\sqrt{\frac{-a-b+v}{t}} + \frac{g-c}{2} + \frac{k-c}{2}. \tag{C.42}$$

If $\bar{x}_1 \leq \frac{1}{2} - \bar{x}_2$:

$$\begin{aligned}\Pi_{B_2} = &+ (k-c)\sqrt{\frac{-a-b+v}{t}} + (b-c)\sqrt{\frac{-a-b+v}{t}} + (g-c)\sqrt{\frac{-a-d+v}{t}} \\ &+ (k-c)\sqrt{\frac{-a-d+v}{t}}.\end{aligned} \tag{C.43}$$

$$\Pi_A = \begin{cases} (a-k)\times\frac{1}{2}, \text{ if } \bar{x}_1 > \frac{1}{2} - \bar{x}_2 & \text{(C.44)} \\ (a-k)\sqrt{\frac{-a-b+v}{t}} + (a-k)\sqrt{\frac{-a-d+v}{t}}. & \text{(C.45)} \end{cases}$$

C.2.2 Optimal licensing policy

C.2.2.1 Optimal Royalty levels

The uncovered equilibrium profit function of the innovator in the asymmetric structure and under symmetric royalty rates $g = k$:

$$\begin{aligned} \Pi_U = {} & (2(\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}+3v) \\ & \times(\sqrt{-2\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}-14c-4k+9v} \\ & + \sqrt{-2\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}-4c-14k+9v}) \\ & + k(2\sqrt{-2\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}-14c-4k+9v} \\ & + 7\sqrt{-2\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}-4c-14k+9v}) \\ & - c(8\sqrt{-2\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}-14c-4k+9v} \\ & + 13\sqrt{-2\sqrt{4(c^2+7ck+k^2)-18v(c+k)+9v^2}-4c-14k+9v})) \times \frac{1}{15\sqrt{15}\sqrt{t}}. \end{aligned} \tag{C.46}$$

The derivative with respect to the symmetric royalty rate assuming $g = k = c$ takes the following form:

$$\frac{-26\sqrt{(v-2k)^2}-48k+24v}{5\sqrt{5}\sqrt{t\left(-2\sqrt{(v-2k)^2}-6k+3v\right)}}. \tag{C.47}$$

Which is negative. It is also the case when $g = \frac{6c}{5}$ or $g = 2c$. We conclude that per unit royalties have a negative effect on the uncovered profit. As a result the optimal royalty rate in the uncovered asymmetric structure is equal to zero. Moreover, we know that the total demand depends negatively on royalty rates. The price of system 1 depends positively on royalties whereas the opposite is true for the price of system 2. The price of system 1 is higher with positive royalties than with a pure fixed fee contract. Both system prices are higher than the efficient system price because of double marginalization. The condition on v such that the uncovered equilibrium exists in the asymmetric structure is:

$$v \leq \frac{72t(2c + g + k) + 16(-2c + g + k)^2 + 45t^2}{144t}. \tag{C.48}$$

The value of this boundary is higher than the one in the pure fixed fee contract model (i.e $v \leq 2c + \frac{5t}{16}$). As royalty rates tend to zero this boundary converges to the one in the pure fixed fee contract model. Royalties make the uncovered equilibrium more likely (more generally sustained). This is bad news for the upstream monopolist aiming to maximize downstream profits and to capture it. There would be excessive prices for a greater range of parameters with positive royalty rates.

We know that the boundary on v ensuring the non existence of profitable deviations from the uncovered equilibrium of the asymmetric structure (i.e $\bar{v}_U$) is such that:

$$\bar{v}_U \geq 2c + \frac{5t}{16}. \tag{C.49}$$

On the other hand, we know that no matter the levels of royalty rates, the double duopoly structure is at a kinked covered equilibrium as soon as the valuation for the system is such that:

$$v > 2c + \frac{3t}{16}, \tag{C.50}$$

which is a lower boundary than $\bar{v}_U$. It means that the inside innovator can choose between a kinked covered equilibrium of the double duopoly structure and the uncovered equilibrium of the asymmetric structure. It is at least weakly optimal for the inside innovator to choose the double duopoly structure.

Given that the double duopoly structure is chosen, it is profitable for the conglomerate firm to use a pure fixed fee contract when the valuation for the system takes intermediate values (i.e $2c + \frac{5t}{16} \geq v \geq 2c + \frac{3t}{16}$) in order to avoid the multiplicity of equilibria and the existence of possibly inefficient asymmetric equilibrium allocation of market shares in the differentiated component market. When the valuation is low, the inside innovator also charges a pure fixed fee contract in order not to worsen the vertical double marginalization problem, to maximize downstream profit and to capture it through the use of the fixed fee. Finally, when the valuation for the system is high, the conglomerate firm will use a two-part tariff contract in order to avoid the dissipation of downstream profits due to intense competition. There is a multiplicity of kinked equilibria in the asymmetric structure.

C.2.2.2 Optimal number of licenses

Profit of the upstream innovator in the double duopoly structure is equal to the profit of the industry (i.e sum of B_1 and UB_2's profits):

$$\begin{aligned}
\Pi_U =&(g-c)\left(\frac{b-d}{t}+\frac{1}{4}\right)+(b-c)\left(\frac{d-b}{t}+\frac{1}{4}\right)+\frac{k-c}{2}+(d-g)\left(\frac{b-d}{t}+\frac{1}{4}\right)\\
&+(b-c)\sqrt{-\frac{b+k-v}{t}}+(k-c)\sqrt{-\frac{b+k-v}{t}}+(g-c)(\sqrt{-\frac{d+k-v}{t}})\\
&+(k-c)\sqrt{-\frac{d+k-v}{t}}+(d-g)\sqrt{-\frac{d+k-v}{t}} \qquad \text{(C.51a)}\\
=&(b-c)\sqrt{-\frac{b+k-v}{t}}+(k-c)\sqrt{-\frac{b+k-v}{t}}+(g-c)\sqrt{-\frac{d+k-v}{t}}\\
&+(k-c)\sqrt{-\frac{d+k-v}{t}}+(d-g)\sqrt{-\frac{d+k-v}{t}} \qquad \text{(C.51b)}
\end{aligned}$$

We can rewrite the above expressions in the covered cases, using the fact that: $\sqrt{-\frac{d+k-v}{t}} = \frac{1}{2}-\sqrt{-\frac{b+k-v}{t}}$:

$$\begin{aligned}
&\frac{g-c}{2}+(b-g)\left(\frac{d-b}{t}+\frac{1}{4}\right)+\frac{k-c}{2}+(d-g)\left(\frac{b-d}{t}+\frac{1}{4}\right)\\
&(b-g)\sqrt{-\frac{b+k-v}{t}}+\frac{g-c}{2}+\frac{k-c}{2}+(d-g)\left(\frac{1}{2}-\sqrt{-\frac{b+k-v}{t}}\right) \qquad \text{(C.52)}
\end{aligned}$$

C.3 Double vertical integration

C.3.1 Downstream equilibria

C.3.1.1 Asymmetric structure

Profit functions:

- Uncovered

$$\Pi_{UAB_2} = (a-c)\left(\sqrt{-\frac{a+b-v}{t}} + \sqrt{-\frac{a+d-v}{t}}\right) \tag{C.53a}$$

$$+ (b-c)\sqrt{-\frac{a+b-v}{t}} + (g-c)\sqrt{-\frac{a+d-v}{t}} \tag{C.53b}$$

$$\Pi_{B_1} = (d-g)\sqrt{-\frac{a+d-v}{t}} \tag{C.53}$$

- Kinked

$$\Pi_{UAB_2} = (g-c)\left(\frac{1}{2} - \sqrt{\frac{-a-b+v}{t}}\right) + (b-c)\sqrt{\frac{-a-b+v}{t}} + \frac{a-c}{2} \tag{C.54}$$

$$\Pi_{UAB_2} = (b-g)\sqrt{\frac{-a-b+v}{t}} + \frac{a-c}{2} + \frac{g-c}{2} \tag{C.55}$$

$$\Pi_{B_1} = (d-g)\sqrt{-\frac{a+d-v}{t}} \tag{C.56}$$

- Competitive

$$\Pi_{UAB_2} = \frac{a-c}{2} + (b-g)\left(\frac{d-b}{t} + \frac{1}{4}\right) + \frac{g-c}{2} \tag{C.57}$$

$$\Pi_{B_1} = (d-g)\left(\frac{b-d}{t} + \frac{1}{4}\right) \tag{C.58}$$

Best response function of the integrated firm:

- Uncovered

$$a = \frac{1}{3}(2c - 2d - g + 2v) \tag{C.59}$$

$$b = \frac{2d}{3} + \frac{g}{3} \tag{C.60}$$

$$d > \sqrt{3}\sqrt{tv - 2ct} + \frac{1}{4}(4g - 3t) \tag{C.61}$$

- Kinked

$$a = -\frac{16d^2 + 8d(15t - 4g) + (4g + 3t)^2 - 144tv}{144t} \tag{C.62}$$

$$b = \frac{2d + g}{3} \tag{C.63}$$

$$d < \sqrt{3}\sqrt{tv - 2ct} + \frac{1}{4}(4g - 3t) \tag{C.64}$$

- Competitive

$$a = -\frac{16d^2 + 8d(15t - 4g) + (4g + 3t)^2 - 144tv}{144t} \tag{C.65}$$

$$b = \frac{2d + g}{3} \tag{C.66}$$

$$d < \sqrt{3}\sqrt{tv - 2ct} + \frac{1}{4}(4g - 3t) \tag{C.67}$$

Standard best response function of the independent firm:

- Uncovered

$$d = \frac{-2a + g + 2v}{3} \tag{C.68}$$

$$\frac{-16b^2 + 32bd - 16d^2 - 8bt - 8dt - t^2 + 16tv}{16t} \leq a \tag{C.69}$$

- Kinked

$$d = \sqrt{-at - bt + tv} + \frac{1}{4}(4b - t) \tag{C.70}$$

$$\frac{-4b - t + 4v}{4} < a < \frac{-16b^2 + 32bd - 16d^2 - 8bt - 8dt - t^2 + 16tv}{16t} \tag{C.71}$$

- Competitive

$$d = \frac{1}{8}(4b + 4g + t) \tag{C.72}$$

$$a \leq \frac{1}{4}(-4b - t + 4v) \tag{C.73}$$

Downstream equilibria:

- Uncovered

$$a = \frac{6c}{5} - g + \frac{2v}{5} \tag{C.74}$$

$$b = -\frac{8c}{15} + g + \frac{4v}{15} \tag{C.75}$$

$$d = -\frac{4c}{5} + g + \frac{2v}{5} \tag{C.76}$$

$$p_1 = \frac{2}{5}(c + 2v) \tag{C.77}$$

$$p_2 = \frac{2(c + v)}{3} \tag{C.78}$$

$$2c < v < 2c - \frac{15}{8}\left(\sqrt{15} - 4\right)t \tag{C.79}$$

- Asymmetric Uncovered Kinked

$$a = -g + \frac{9}{8}\sqrt{15}\sqrt{t^2} - \frac{9t}{2} + v \tag{C.80}$$

$$b = g - \frac{1}{2}\sqrt{15}\sqrt{t^2} + 2t \tag{C.81}$$

$$d = g - \frac{3}{4}\sqrt{15}\sqrt{t^2} + 3t \tag{C.82}$$

$$p_1 = \frac{3}{8}\sqrt{15}\sqrt{t^2} - \frac{3t}{2} + v \tag{C.83}$$

$$p_2 = \frac{5}{8}\sqrt{15}\sqrt{t^2} - \frac{5t}{2} + v \tag{C.84}$$

$$v > 2c - \frac{15}{8}\left(\sqrt{15} - 4\right)t \tag{C.85}$$

- Asymmetric Competitive Kinked

$$a = -g - \frac{57t}{256} + v \tag{C.86}$$

$$b = g + \frac{t}{8} \tag{C.87}$$

$$d = \frac{1}{16}(16g + 3t) \tag{C.88}$$

$$p_1 = a + d = v - \frac{9t}{256} \tag{C.89}$$

$$p_2 = a + b = v - \frac{25t}{256} \tag{C.90}$$

$$v > 2c + \frac{75t}{256} \tag{C.91}$$

C.3.2 Optimal licensing policy

C.3.2.1 Number of licenses

The proposition 3.5.1 directly results from the comparison of the profit of the inside innovator (i.e equal to the industry profit which is captured through the fixed fee) across structures for each equilibrium.

Comparisons between uncovered asymmetric and double duopoly:

$$\frac{4(v-2c)^{3/2}}{3\sqrt{3}\sqrt{t}} \geq \tag{C.92}$$

$$\frac{1}{225}(-2)\left(25\sqrt{3} + 18\sqrt{5}\right)(2c - v)\sqrt{\frac{v-2c}{t}}. \tag{C.93}$$

Comparisons between asymmetric kinked and double duopoly kinked:

$$-c - \frac{t}{32} + \frac{v}{2} \geq \tag{C.94}$$

$$\frac{\sqrt{4-\frac{\sqrt{15}t}{\sqrt{t^2}}}\left(-8\left(\sqrt{3}+\sqrt{5}\right)(2c-v)+25\sqrt{3}\sqrt{t^2}+9\sqrt{5}\sqrt{t^2}-4\left(3\sqrt{3}+5\sqrt{5}\right)t\right)}{16\sqrt{2}}. \tag{C.95}$$

Comparisons between asymmetric kinked and double duopoly competitive:

$$\Pi^{U}_{UB_2} = -c - \frac{t}{32} + \frac{v}{2} \geq \tag{C.96}$$

$$\frac{\sqrt{4-\frac{\sqrt{15}t}{\sqrt{t^2}}}\left(-8\left(\sqrt{3}+\sqrt{5}\right)(2c-v)+25\sqrt{3}\sqrt{t^2}+9\sqrt{5}\sqrt{t^2}-4\left(3\sqrt{3}+5\sqrt{5}\right)t\right)}{16\sqrt{2}}. \tag{C.97}$$

Appendix D

Three-part licensing contracts and demand uncertainty in complementary markets

D.1 Separated model

D.1.1 Competitive equilibrium contract

The total profit of the outside innovator in the competitive equilibrium takes the following form:

$$\Pi_U = \frac{3}{8}as + \frac{1}{8}as + \frac{3}{4}bs\left(\frac{2(d-b)}{2t} + \frac{1}{4}\right) + \frac{3}{4}ds\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) + \frac{1}{4}bs\left(\frac{2(d-b)}{2t} + \frac{1}{4}\right)$$

$$+ \frac{1}{4}ds\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) + \frac{3}{8}(g-c) + \frac{1}{8}(g-c) + \frac{3}{8}(g-c) + \frac{1}{8}(g-c) + 2f. \qquad \text{(D.1)}$$

The profit of the innovator is convex and depends positively on both royalty rates and fixed fee. The patentee charges the highest feasible level for each pricing instrument. We substitute the per unit royalty rate g and fixed fee f by their respective expressions given by the participation constraint of licensees and the competitive equilibrium condition:

$$g = \frac{1}{32}(5st - 16sv - 5t + 16v) \tag{D.2a}$$

$$f = \frac{1}{32}(t - st). \tag{D.2b}$$

We obtain the following expression which strictly depends positively on the ad valorem royalty rate s:

$$\Pi_U = \frac{1}{32}(-32c + (2s - 3)t + 16v). \tag{D.3}$$

Thus the patentee finds it profitable to charge the highest ad valorem royalty rate consistent with both constraints and consistent values of parameters (i.e non negativity of the per unit royalty rate). This is given by the following conditions:

$$0 < c < \frac{1}{32}(16v - 5t) \tag{D.4a}$$

$$v > 0 \tag{D.4b}$$

$$0 < t < \frac{16v}{5} \tag{D.4c}$$

$$g = \frac{1}{32}(5st - 16sv - 5t + 16v) \tag{D.4d}$$

$$f = \frac{1}{32}(t - st) \tag{D.4e}$$

$$s = \frac{32c + 5t - 16v}{5t - 16v}. \tag{D.4f}$$

We finally obtain the values of the competitive equilibrium contract presented in section 4.3.3.1 in replacing the optimal ad valorem royalty rate s in the expressions of f and g.

D.1.2 Uncovered equilibrium contract

The total profit of the outside innovator in the uncovered equilibrium takes the following form:

$$\Pi_U = \frac{3}{4}(g-c)\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right) + \frac{1}{4}(g-c)(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}})$$
$$+ \frac{3}{4}(g-c)\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right) + \frac{1}{4}(g-c)\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right)$$
$$+ \frac{3}{4}as\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right) + \frac{1}{4}as\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right)$$
$$+ \frac{3}{4}bs\sqrt{\frac{-a-b+v}{t}} + \frac{1}{4}bs\sqrt{\frac{-a-b+v}{t}} + \frac{3}{4}ds\sqrt{\frac{-a-d+v}{t}} + \frac{1}{4}ds\sqrt{\frac{-a-d+v}{t}} + 2f. \tag{D.5}$$

The fixed fee is set to the highest level consistent with the participation of component B producers:

$$\bar{f} = \frac{1}{9}\sqrt{\frac{24g^3 + 36g^2sv - 36g^2v + 18gs^2v^2 - 36gsv^2 + 18gv^2 + 3s^3v^3 - 9s^2v^3 + 9sv^3 - 3v^3}{(s-1)t}}. \tag{D.6}$$

The per unit royalty is set to zero (i.e $g = c$) because of the non negativity constraint on per unit royalty rate. It is indeed shown that it is impossible to have a positive per unit royalty consistent with the first order conditions of the constrained optimization problem. Then the outside innovator charges the following ad valorem royalty level given by the corresponding first order condition:

$$s = \frac{1}{3}\left(\frac{\sqrt[3]{c^2v^3(17c-81v) + 9\sqrt{c^4v^6\left(20c^2-34cv+81v^2\right)}}}{v^2}\right) \tag{D.7a}$$
$$+ \frac{1}{3}\left(-\frac{11c^2}{\sqrt[3]{c^2v^3(17c-81v) + 9\sqrt{c^4v^6\left(20c^2-34cv+81v^2\right)}}} - \frac{c}{v} + 3\right). \tag{D.7b}$$

The optimal contract for the monopoly innovator makes the market uncovered in equilibrium. The demand consistency constraint (i.e non covered market condition) is unbinding. It is profitable for the innovator not to serve all consumers. Explicit expressions of the uncovered equilibrium contract are the following:

$$g = c \tag{D.8a}$$

$$s = \frac{1}{3}\Big(\frac{\sqrt[3]{c^2v^3(17c - 81v) + 9\sqrt{c^4v^6\,(20c^2 - 34cv + 81v^2)}}}{v^2} - \frac{11c^2}{\sqrt[3]{c^2v^3(17c - 81v) + 9\sqrt{c^4v^6\,(20c^2 - 34cv + 81v^2)}}} - \frac{c}{v} + 3\Big) \tag{D.8b}$$

$$f = \frac{\sqrt{\dfrac{\left(v\left(-\frac{11c^2}{3\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}+\frac{\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}{3v^2}-\frac{cv-3v^2}{3v^2}\right)+2c-v\right)^3}{t\left(-\frac{11c^2}{3\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}+\frac{\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}{3v^2}-\frac{cv-3v^2}{3v^2}-1\right)}}}{3\sqrt{3}}. \tag{D.8c}$$

D.2 Integrated model

D.2.1 Competitive equilibrium

D.2.1.1 Equilibrium licensing contract

The total profit of the inside innovator in the competitive equilibrium takes the following form:

$$\begin{aligned}\Pi_U = &\frac{3}{8}as + \frac{1}{8}as + \frac{3}{4}(g-c)\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) + \frac{1}{4}(g-c)\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) \\ &+ \frac{3}{4}(b-c)\left(\frac{2(d-b)}{2t} + \frac{1}{4}\right) + \frac{1}{4}\left((b-c)\left(\frac{2(d-b)}{2t} + \frac{1}{4}\right)\right) \\ &+ \frac{3}{4}ds\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) + \frac{1}{4}ds\left(\frac{2(b-d)}{2t} + \frac{1}{4}\right) + \frac{3}{8}(g-c) + \frac{1}{8}(g-c) + f. \qquad \text{(D.9)}\end{aligned}$$

The same rationale as in the separated model applies here. The profit of the inside innovator is a positive function of royalty rates and fixed fee. We substitute the binding constraints expression and look for the highest consistent level of ad valorem royalty:

$$f = \frac{9t - 9st}{32s^2 - 192s + 288} \qquad \text{(D.10a)}$$

$$g = \frac{-16s^3v - 12s^2t + 112s^2v + 57st - 240sv - 45t + 144v}{32s^2 - 192s + 288}. \qquad \text{(D.10b)}$$

The equilibrium competitive contract charged by the inside innovator takes the following form:

$$f = \frac{9t - 9st}{32s^2 - 192s + 288} \qquad \text{(D.11a)}$$

$$g = \frac{-16s^3v - 12s^2t + 112s^2v + 57st - 240sv - 45t + 144v}{32s^2 - 192s + 288}. \qquad \text{(D.11b)}$$

The equilibrium competitive contract charged by the inside innovator takes the following form:

$$g = c \tag{D.12a}$$

$$\begin{aligned} s = {} & ((-1024c^3 - 1152c^2t - 3072c^2v + 9\sqrt{3}(-16384c^3tv^2 - 20032c^2t^2v^2 \\ & - 49152c^2tv^3 - 8112ct^3v^2 - 22784ct^2v^3 - 49152ctv^4 - 1089t^4v^2 - 132t^3v^3 \\ & + 12800t^2v^4 - 16384tv^5)^{\frac{1}{2}} - 432ct^2 - 1224ctv - 3072cv^2 - 54t^3 - 27t^2v \\ & + 1872tv^2 - 1024v^3)^{\frac{1}{3}}) \times (\frac{1}{12\sqrt[3]{2}v}) - (-1024c^2 - 768ct - 2048cv - 144t^2 \\ & - 48tv - 1024v^2) \times ((96\, 2^{\frac{2}{3}} v(-1024c^3 - 1152c^2t - 3072c^2v \\ & + 9\sqrt{3}(-16384c^3tv^2 - 20032c^2t^2v^2 - 49152c^2tv^3 \\ & - 8112ct^3v^2 - 22784ct^2v^3 - 49152ctv^4 - 1089t^4v^2 - 132t^3v^3 \\ & + 12800t^2v^4 - 16384tv^5)^{\frac{1}{2}} - 432ct^2 - 1224ctv - 3072cv^2 - 54t^3 - 27t^2v \\ & + 1872tv^2 - 1024v^3)^{13})^{-1}) - \frac{8c + 3t - 28v}{12v} \end{aligned} \tag{D.12b}$$

$$f = -(9t(((-1024c^3 - 1152tc^2 - 3072vc^2 - 432t^2c - 3072v^2c - 1224tvc - 54t^3$$
$$- 1024v^3 + 1872tv^2 - 27t^2v + 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 - 49152ctv^4$$
$$- 132t^3v^3 - 22784ct^2v^3 - 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2$$
$$- 16384c^3tv^2)^{1/2})^{1/3}) \times \frac{1}{12\sqrt[3]{2}v} - \frac{8c + 3t - 28v}{12v} - (-1024c^2 - 768tc - 2048vc$$
$$- 144t^2 - 1024v^2 - 48tv) \times (96\ 2^{2/3}v(-1024c^3 - 1152tc^2 - 3072vc^2$$
$$- 432t^2c - 3072v^2c - 1224tvc - 54t^3 - 1024v^3 + 1872tv^2 - 27t^2v + 9\sqrt{3}$$
$$(-16384tv^5 + 12800t^2v^4 - 49152ctv^4 - 132t^3v^3 - 22784ct^2v^3 - 49152c^2t$$
$$v^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2 - 16384c^3tv^2)^{1/2})^{1/3})^{-1} - 1))$$
$$\times (32(((-1024c^3 - 1152tc^2 - 3072vc^2 - 432t^2c - 3072v^2c - 1224tvc - 54t^3$$
$$- 1024v^3 + 1872tv^2 - 27t^2v + 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 - 49152ctv^4$$
$$- 132t^3v^3 - 22784ct^2v^3 - 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2$$
$$- 16384c^3tv^2)^{1/2})^{1/3}) \times \frac{1}{12\sqrt[3]{2}v} - \frac{8c + 3t - 28v}{12v} - (-1024c^2 - 768tc - 2048vc$$
$$- 144t^2 - 1024v^2 - 48tv) \times (96\ 2^{2/3}v(-1024c^3 - 1152tc^2 - 3072vc^2$$
$$- 432t^2c - 3072v^2c - 1224tvc - 54t^3 - 1024v^3 + 1872tv^2 - 27t^2v$$
$$+ 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 - 49152ctv^4 - 132t^3v^3 - 22784ct^2v^3$$
$$- 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2 - 16384c^3tv^2)^{1/2})^{1/3})^{-1}$$
$$- 3)^2)^(- 1) \qquad \text{(D.12c)}$$

D.2.1.2 Merger undesirability

The competitive equilibrium profit of the outside innovator takes the following form:

$$\Pi_U = \left(\frac{1}{64}\left(\frac{v(16v - 5t)}{c} + 64c\left(\frac{2t}{5t - 16v} - 1\right) - 2t + 32v\right)\right. \qquad \text{(D.13)}$$

We compare it to the competitive equilibrium profit of the inside innovator:

$$
\begin{aligned}
\Pi_U = {} & \frac{1}{32} t(-4 - 21 \times (((-1024c^3 - 1152tc^2 - 3072vc^2 - 432t^2c - 3072v^2c - 1224tvc \\
& - 54t^3 - 1024v^3 + 1872tv^2 - 27t^2v + 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 - 49152ctv^4 \\
& - 132t^3v^3 - 22784ct^2v^3 - 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2 \\
& - 16384c^3tv^2)^{1/2})^{1/3}) \times \frac{1}{12\sqrt[3]{2}v} - \frac{8c + 3t - 28v}{12v} \\
& - (-1024c^2 - 768tc - 2048vc - 144t^2 - 1024v^2 - 48tv) \times (96\, 2^{2/3} v \\
& (-1024c^3 - 1152tc^2 - 3072vc^2 - 432t^2c - 3072v^2c - 1224tvc - 54t^3 - 1024v^3 \\
& + 1872tv^2 - 27t^2v + 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 - 49152ctv^4 - 132t^3v^3 \\
& - 22784ct^2v^3 - 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2 \\
& - 16384c^3tv^2)^{1/2})^{1/3})^{-1} - 3)^{-1}) \\
& + c(-((-1024c^3 - 1152tc^2 - 3072vc^2 - 432t^2c - 3072v^2c - 1224tvc - 54t^3 \\
& - 1024v^3 + 1872tv^2 - 27t^2v + 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 - 49152ctv^4 \\
& - 132t^3v^3 - 22784ct^2v^3 - 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 - 20032c^2t^2v^2 \\
& - 16384c^3tv^2)^{1/2})^{1/3}) \times \frac{1}{12\sqrt[3]{2}v} \\
& + \frac{8c + 3t - 28v}{12v} + (-1024c^2 - 768tc - 2048vc - 144t^2 - 1024v^2 - 48tv) \\
& \times (96\, 2^{2/3} v(-1024c^3 - 1152tc^2 - 3072vc^2 - 432t^2c - 3072v^2c - 1224tvc \\
& - 54t^3 - 1024v^3 + 1872tv^2 - 27t^2v + 9\sqrt{3}(-16384tv^5 + 12800t^2v^4 \\
& - 49152ctv^4 - 132t^3v^3 - 22784ct^2v^3 - 49152c^2tv^3 - 1089t^4v^2 - 8112ct^3v^2 \\
& - 20032c^2t^2v^2 - 16384c^3tv^2)^{1/2})^{1/3})^{-1} + 1)^{-1} - 1).
\end{aligned} \tag{D.14}
$$

We find that the merger with one of the differentiated producer would be undesirable.

D.2.2 Uncovered equilibrium

D.2.2.1 Uncovered equilibrium licensing contract

The total profit of the inside innovator in the uncovered equilibrium takes the following form:

$$\Pi_U = \frac{3}{4}(g-c)\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right) + \frac{1}{4}(g-c)\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right)$$
$$+ \frac{3}{4}(b-c)\sqrt{\frac{-a-b+v}{t}} + \frac{1}{4}(b-c)\sqrt{\frac{-a-b+v}{t}} + \frac{3}{4}as\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right)$$
$$+ \frac{1}{4}as\left(\sqrt{\frac{-a-b+v}{t}} + \sqrt{\frac{-a-d+v}{t}}\right) + \frac{3}{4}(g-c)\sqrt{\frac{-a-d+v}{t}} + \frac{1}{4}(g-c)\sqrt{\frac{-a-d+v}{t}}$$
$$+ \frac{3}{4}ds\sqrt{\frac{-a-d+v}{t}} + \frac{1}{4}ds\sqrt{\frac{-a-d+v}{t}} + f. \tag{D.15}$$

Substituting f by the binding expression of the participation condition of licensees:

$$f = \frac{\sqrt{\frac{8g^3+12g^2sv-12g^2v+6gs^2v^2-12gsv^2+6gv^2+s^3v^3-3s^2v^3+3sv^3-v^3}{(s-1)t}}}{3\sqrt{3}}. \tag{D.16}$$

The derivative of the resulting expression with respect to g is negative for consistent values of the parameters which leads to a zero per unit royalty rate to be profitably charged. The first order condition with respect to the ad valorem royalty rate s and licensee participation constraint give us the uncovered equilibrium contract charged by the inside innovator:

$$s = -\frac{11c^2}{3\sqrt[3]{17c^3v^3 - 81c^2v^4 + 9\sqrt{20c^6v^6 - 34c^5v^7 + 81c^4v^8}}} + \frac{\sqrt[3]{17c^3v^3 - 81c^2v^4 + 9\sqrt{20c^6v^6 - 34c^5v^7 + 81c^4v^8}}}{3v^2} - \frac{cv - 3v^2}{3v^2}$$

$$g = c$$

$$f = \frac{\sqrt{\frac{\left(v\left(-\frac{11c^2}{3\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}+\frac{\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}{3v^2}-\frac{cv-3v^2}{3v^2}\right)+2c-v\right)^3}{t\left(-\frac{11c^2}{3\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}+\frac{\sqrt[3]{17c^3v^3-81c^2v^4+9\sqrt{20c^6v^6-34c^5v^7+81c^4v^8}}}{3v^2}-\frac{cv-3v^2}{3v^2}-1\right)}}}{3\sqrt{3}} \tag{D.17}$$

D.2.2.2 Merger desirability

We cannot present the expressions of the uncovered equilibrium profit of the outside nor inside innovator that are too large and complex. The proposition on the desirability of the vertical merger on the differentiated component market directly results from the comparison of these expressions of the profit of the innovator for sustaining values of the parameters.

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Licences de Brevets et Intégration Verticale dans les Marchés Complémentaires

Le secteur des TIC est caractérisé par des arrangements stratégiques de transferts de technologies tels que les licences et les regroupements de brevets. Par ailleurs, les produits et services ont souvent de fortes relations de complémentarité dans ce secteur. Afin de garantir un niveau satisfaisant d'interopérabilité aux utilisateurs, les producteurs de biens complémentaires doivent échanger des informations techniques. Cette thèse cherche à prendre en compte ces deux dimensions et à produire de nouveaux éclairages sur les cas de politique de concurrence impliquant des marchés complémentaires (e.g Intel/McAfee, Google/Motorola). Nous étendons la littérature sur les licences de brevets en modélisant des marchés avals différenciés et complémentaires. En utilisant les méthodes de l'économie industrielle, nous caractérisons les stratégies de licences profitables pour un innovateur en situation de monopole concernant le nombre de licences, les instruments tarifaires ainsi que l'intégration verticale et conglomérale. Nous montrons que le nombre de licences attribuées diffère généralement de celui observé lorsque la technologie est utilisée dans un marché aval isolé. En particulier, nous obtenons que le nombre de licences distribuées est plus élevé dans les marchés de niche lorsque le nombre de firmes intéressées par la technologie est limité. Dans ce cadre d'analyse, l'intégration verticale n'est pas profitable à l'exception des cas où, une seule firme est susceptible d'acquérir une licence sur le marché homogène, ou lorsque la demande pour le produit final est incertaine. Par ailleurs, les royalties unitaires perçus sur le nombre de produits vendus en aval ne sont utilisés que dans la structure de l'industrie la plus concurrentielle et lorsque la valorisation pour le bien final est élevée. Enfin, nous montrons que lorsque la demande est incertaine et que les acquéreurs de la technologie sont réticents à la prise de risque, l'innovateur préfère utiliser des royalties ad valorem qui portent sur les revenus issus des ventes de produits finals. Nos résultats montrent que les relations de complémentarité entre les marchés finals influent sur la manière dont sont transférées les technologies et que les fusions verticales et conglomérales ne semblent pas générer de comportements de forclusion.

Mots clés : Brevets; Intégration verticale; Intégration conglomérale; Bien système.

Patent Licensing and Vertical Integration in Complementary Markets

IT industries are characterized by strategic patent agreements such as patent licensing or patent pools. Products and services frequently have strong potential complementarity relations in this industry. To guarantee a satisfactory level of interoperability to users, the exchange of technical information is required between complementary producers. This dissertation aims at taking into account these two dimensions of the IT sector in order to provide new insights on competition policy cases involving high technology complementary products (e.g Intel/McAfee, Google/Motorola). We extend the literature on patent licensing by explicitly modeling downstream differentiated complementary goods. Using industrial organization methods, we characterize the profitable strategies of a monopoly innovator with respect to the number of licenses, the pricing instruments as well as vertical and conglomerate mergers. We show that the number of licenses delivered in equilibrium can differ from the standard model with a single downstream market. In particular, we consistently find, for various forms of licensing contracts that more licenses are issued in niche markets when the number of potential licensees is capped. Overall vertical integration and conglomerate mergers are found to be unprofitable except when there is only one firm likely to acquire the technology or when there is demand uncertainty. On the other hand, per unit royalty rates are only used in the most competitive structure of the industry for high valuations of the final good. Finally, sales revenue (i.e ad valorem) is found to be a more profitable royalty base than the number of sales (i.e per unit royalties) when demand is uncertain and licensees are risk averse. Our results show that complementarity influences the way in which technologies are transferred and that vertical mergers do not generate foreclosing behaviors in this framework.

Keywords : Patent Licensing; Vertical Integration; Conglomerate merger; System good.