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Numerical Study of Resonant Wave Energy Converters in Selected Sites

STRATEGIC USE AND OPTIMAL TUNING OF THE MECHANISM FOR PHASE CONTROL

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J O N A S   J O S E F S S O N

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Abstract

There is an increasing demand for electricity throughout the world. In addition, it is desired to produce a significant amount of this electricity in environmentally friendly ways. One way to do this is to harvest energy from ocean waves. Many different solutions have been proposed worldwide but to this point no solution which is both effective and able to withstand the environment in the oceans has been developed.

The company CorPower Ocean is currently developing a Wave Energy Converter inspired by the pumping principles by the human heart. A newly invented mechanism for phase control called WaveSpring is believed to solve many of the problems that have stopped previous solutions from success.

The purpose of this thesis is to analyze the usage and tuning of the WaveSpring for different wave conditions. Basically the thesis can be divided into two parts. One part is rather analytical and aims to find a good strategy for how the WaveSpring can be used to brake the motion of the WEC when needed. The other part aims to find an optimal strategy for tuning the WaveSpring while considering energy costs for doing so. To do so an optimization problem is formulated and solved.

Sammanfattning

Behovet av elektricitet ökar i hela världen och en stor del av den ökade produktionen bör komma från miljövänliga källor. En förhållandevis outforskad energikälla som är miljövänlig och har stor potential är havsvågor. Många förslag på hur man kan utvinna energi ur havsågor har presenterats men hittills har ingen lyckats kombinera effektivitet och tålighet. Detta på grund av att förhållandena på havet är mycket tuffa och ställer stora krav på hållbarhet, något som ofta leder till dyra lösningar.

Corpower Ocean håller nu på att utveckla en prototyp för att utvinna energi ur havsågor på ett kostnadseffektivt sätt. Denna prototyp är inspirerad av det mänskliga hjärtat. En ny mekanism, kallad WaveSpring, med syfte att kontrollera prototypens fas har uppfunnits och man tror att denna mekanism kommer att lösa många problem som stoppat tidigare försök.

Syftet med detta examensarbete är att undersöka hur denna mekanism kan användas och hur den ska ställas in för att få optimal produktion av elektricitet under olika vågförhållanden. Detta examensarbete kan delas in i två delar. En del går ut på att ta fram en strategi för hur WaveSpring kan användas för bromsa rörelsen. Den andra delen går ut på att ta fram en plan för hur WaveSpring ska ställas in för olika vågförhållanden. För att bestämma en sådan plan har ett optimeringsproblem formulerats och lösts.

Acknowledgements

Personally I think that this thesis work have been very interesting and I am grateful for the opportunity to contribute in this exciting field of engineering. Just knowing that what is done by the company may have a large impact on the future has been a great motivation.

First of all I would like to thank my supervisor Gunnar Steinn Ásgeirsson at CorPower Ocean for his support, enthusiasm and feedback during this thesis. I would also like to thank everyone else working at CorPower Ocean for their support and fruitful discussions.

Finally, I would like to thank my supervisor Xiaoming Hu at KTH for his interest, feedback and support when writing this thesis..

Contents

Nomenclature

η	Wave elevation
h	Water depth
λ	Wave length
H	Wave height
ω	Angular wave frequency
t	Time
k	Wave number
A	Wave Amplitude
θ	Phase
H_S	Significant wave height
F_{pe}	External pressure force
F_{exc}	Excitation force
F_r	Radiation force
F_{hyst}	Hydrostatic force
F_d	Drag force
F_{PTO}	Force from PTO
F	Force from WaveSpring
g	Acceleration of gravity
ρ	Water density
V_{sub}	Submerged volume
A_s	Projected area of body
cd	Drag coefficient
v	Velocity
T_p	Peak period
$\omega_n at$	Natural frequency in radians
S	Stiffness
m	Wave Energy Converter mass
D_{rack}	Displacement of the rack
V_{rack}	Velocity of the rack
A_{rack}	Acceleration of the rack
T_{nat}	Natural period
F_{gas}	Force from pretension spring

p_{fa}	Pressure full area side
p_{rs}	Pressure rod side
A_{fa}	Area full area side
A_{rs}	Area rod side
D_{rack}^{max}	Maximum allowed D_{rack}
P_{elec}	Produced electricity
$\hat{p}_s$	Optimal pressure for sea state s
p_s	Current pressure for sea state s
ΔT	Expected time in a sea state
f	Compressor cost function
d_{max}	Maximum displacement in optimization problem
v_{max}	Maximum velocity in optimization problem
a_{max}	Maximum acceleration in optimization problem
m_{max}	Maximum mechanical forces in optimization problem

Chapter 1

Introduction

1.1 Thesis Background

This thesis is done in collaboration with the company CorPower Ocean (CPO) and the university Kungliga Tekniska Högskolan (KTH). CPO was founded in 2009 with objective to develop a Wave Energy Converter (WEC) for commercial use. This is not an easy task and many different solutions have been suggested worldwide. The common objective for these suggestions is to find a solution that can harvest the energy efficiently and at the same time withstand the harsh environment offered by the oceans.

1.2 Energy and Wave Energy Conversion

As the population of the world increase there is an increase in demand of electricity [1]. There is also an increase in demand per capita which could e.g., be due to increasing standards of living. The production of electricity must therefore increase rapidly to satisfy the demand. In addition it is desired to reduce the emission of carbon dioxide into the atmosphere [2]. Therefore it is of interest to produce an increasing amount of this electricity in environmentally friendly ways. This is referred to as sustainable energy which means energy converted from non-exhaustible resources such as sun, wind and waves.

Ocean waves is a very dense energy resource which is yet to be exploited. The average power flow just below the ocean surface is around 2-3 kW/m^2 [3]. If compared to the more exploited sun and wind energy whose intensity is about 0.1-0.3kW/m^2 and 0.5kW/m^2 respectively it becomes obvious that this is a very interesting resource [3]. In addition, the oceans cover a large area of the earth resulting in enormous amounts of wave energy. However, not all of the oceans are suitable for harvesting energy. Reasons for this are e.g., depth and the requirement of suitable wave profiles in terms of amplitude and frequency. The extraction of this wave energy can be done in different ways, e.g., through oscillating flaps fixed at the bottom or, as in this project, so called point absorbers.

A point absorber is basically a buoy floating on the surface while being moored to the seabed by a cable. Incoming waves makes the buoy oscillate. It is this movement which is then converted to electricity through a generator located inside the buoy. The electricity is then transferred to the grid through wires on the seabed. Obviously, the performance of the device is highly dependent on the waves (amplitude, wave length, etc.) but also on certain parameters of the mechanics inside the device. Such parameters can e.g. be related to the phase control used to obtain resonance in the oscillations. This is for most of the time a desired phenomenon since a larger movement results in a larger production of electricity. The mechanism for controlling the phase of the device is called WaveSpring and is what this thesis is all about.

1.3 Thesis Overview

This thesis work can be divided into two different parts. One part concerns the usage of the WaveSpring mechanism and has a more analytical approach. How the WaveSpring is used has a large impact on the motion and might decide if the device moves in a way that fulfills all mechanical constraints.

The other part concerns the tuning of the WaveSpring in order to obtain an optimal phase for the present sea condition. When finding the optimal tuning of the WaveSpring the costs for performing the tuning should also be included. In order to find the optimal tuning an optimization problem was formulated and solved.

An important tool for the thesis is the Simulink model that has been developed by CPO. The Simulink model will be developed further in this thesis and simulations are used as input for both the analytical part and for the optimization. There is a close connection between these parts since the tuning also affects how effective the WaveSpring can be used for other purposes. Therefore the analytical part was carried out first and the results of this part was later used as input for the second part as constraints in the optimization model.

1.4 Objectives of the Thesis

The WEC is under development and numerous parameters concerning both mechanical parts and usage of the mechanisms have not been determined. Since these parameters will have a very large impact on the results of the tuning it is not of interest for this thesis to find a tuning that will be used for the final device. The main goal is rather to develop a method which can be used in order to find the optimal tuning when all parameters have been determined. The objectives of this thesis have been summarized below.

- Find a way to control the WaveSpring in order to fulfill all constraints on motion.
- Find the optimal way to tune the WaveSpring given a discrete breakdown of sea conditions.
- Perform sensitivity analysis of parameters that could have an effect on the tuning.
- Draw conclusions.
- Final suggestion for usage and tuning.
- Implement new braking strategy in CPO's Simulink model.
- Implement the new tuning scheme in the Simulink model.

Chapter 2

Theoretical Background

In this chapter some of the theory behind harvesting wave energy is presented as well as the more specific details required to fully understand this report. The first sections treats the underlying factors and concepts used in wave technology and the following sections will go into more details.

2.1 Coordinate System

There are 6 degrees of freedoms in which a buoy can move. For the simulation model that is currently in use at the company the two most important degrees of freedom are taken into account. These are

- Heave - up and down
- Surge - back and forth in the same direction as the wave propagation.

A better illustration of the different degrees of freedom is presented Figure 2.1. Here all 6 degrees of freedoms have been included to give the full picture.

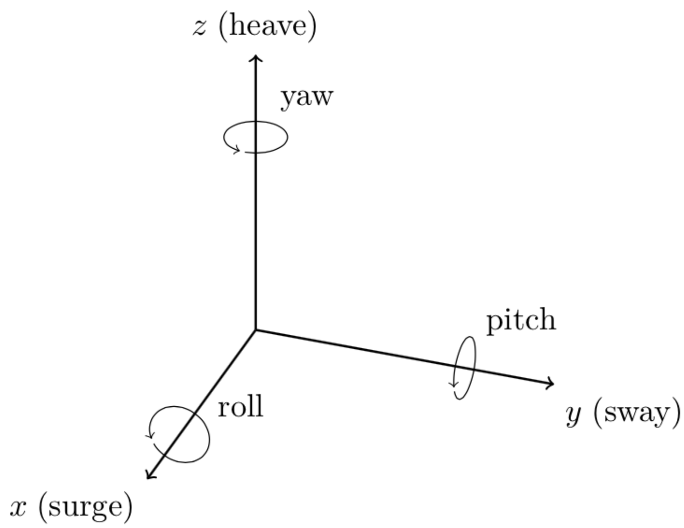

Figure 2.1: The coordinate system used for analyzing the motion of a point absorber. The degrees of freedom taken into account in the model used by CPO is the heave and surge motions.

2.2 Waves

The distribution of sunlight over the earth is uneven, resulting in differences in air pressure [4]. This creates winds that blow from zones with higher pressure to zones with lower pressure. When blowing over the oceans the wind is converted to waves through the friction between the wind and the surface of the ocean. Waves in a zone where the wind is blowing is referred to as wind waves. As the waves leave the regions of wind they are referred to as swells which continue travelling across the oceans with a very small energy loss [5] [6]. There are of course other sources for waves. One example are tidal waves, which are created as an effect of gravitational forces. Other examples include waves generated by boats or other floating structures as well as waves generated by other factors such as earthquakes. However, the WEC developed by CPO is intended to mainly harvest energy from waves generated by winds.

Mathematically, in the two dimensional case the wave elevation, $\eta(x, t)$, can be described by the direction of wave propagation, x, and the time t. Waves are often divided into regular waves and irregular waves, two concepts which will be described in the following sections. In Figure 2.2 a regular wave together with an illustration of different parameters defining the wave is presented. These parameters are the wave length λ, wave height H and water depth h.

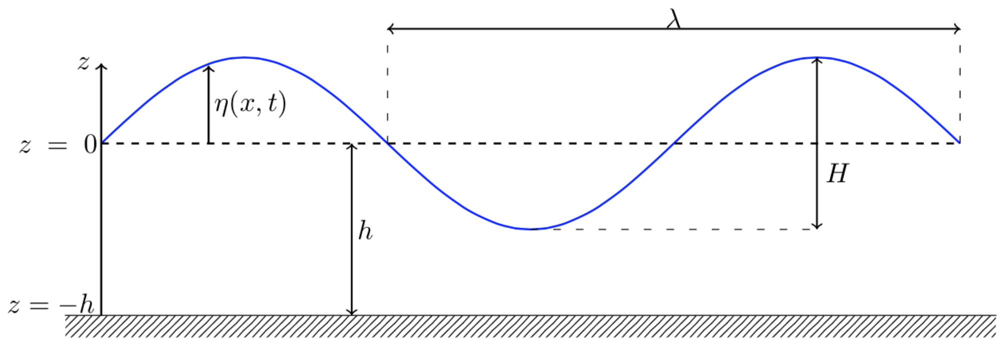

Figure 2.2: A regular wave and some parameters that defines the wave.

2.2.1 Regular and Irregular Waves

The term regular waves refer to waves which in other contexts often are referred to as harmonic. That is, waves with an elevation that varies sinusoidally with time

$$\eta(x,t) = A\cos(\omega t - kx).$$

Here, A is the wave amplitude and $k = 2\pi/\lambda$ is the wave number. Note that the wave amplitude is half of the wave height, i.e., $A = H/2$. The swells described above are rather close to regular waves. However, this model is not sufficient for a mathematical model of ocean waves as they are typically not harmonic but rather of a more stochastic nature.

In order to better model the real ocean waves they are considered as a sum of regular waves [3],

$$\eta(x,t) = \sum_i a_i\cos(\omega_n t - k_n x + \theta_n).$$

This sum of regular waves is considered as a more accurate approximation of ocean waves. Here, instead of using the normal wave height H the so called significant wave height, H_s, is used. The significant wave height have two different definitions which are almost the same. The traditional definition is that the significant wave height is the average wave height among the one third highest waves. Another, more modern, definition says that the significant wave height is defined as four times the standard deviation of the surface elevation [7] [8] [9]. The concept of significant wave height will be explained a bit more thoroughly later in the report. In Figure 2.3 two regular waves can be seen together with the resulting irregular wave that occurs when these two regular waves are super positioned.

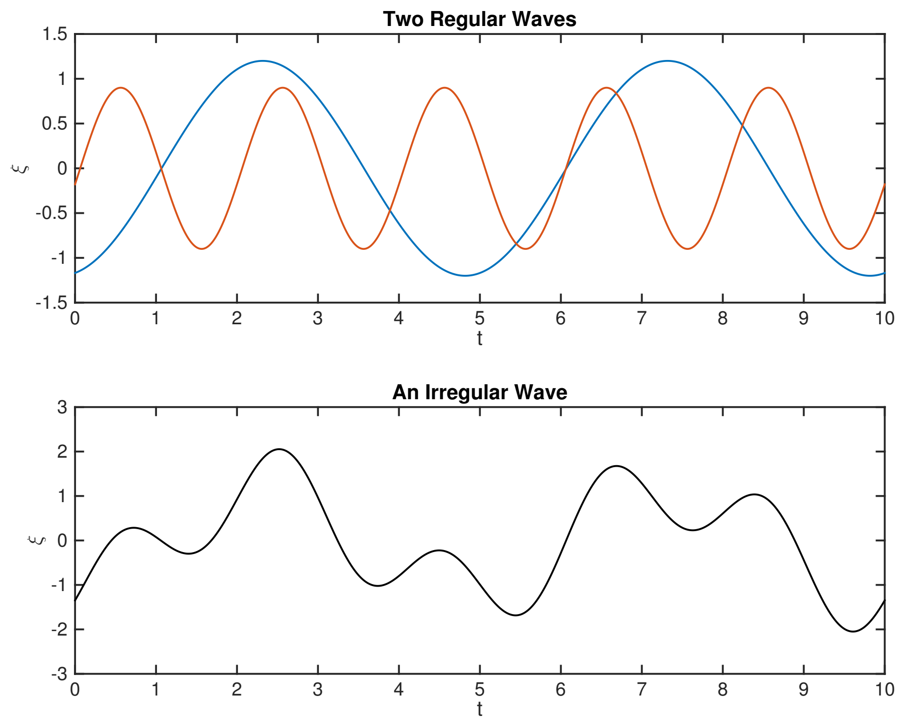

Figure 2.3: *Upper Figure*: Two regular waves with different amplitudes and periods. *Lower Figure*: The superposition of the two regular waves creates this irregular wave.

2.3 Wave Spectrum and JONSWAP spectrum

As mentioned, the waves at a location is a superposition of waves with various origins. This means that the waves will have different properties and together they make up a spectrum of the different components. A spectrum is typically a combination of waves with different periods, amplitudes and direction of propagation [10]. The spectrum can be thought of as a probability distribution of the wave periods. Using such spectrums it is possible to get a better understanding of the sea conditions at a certain location at a certain point in time.

The wave spectrum for the sea is very complicated. Therefore, idealizations are often used to approximate the spectrum. One such idealization is the so called JONSWAP spectrum. It originates from another spectrum called the Pierson-Moskowitz spectrum which assumes a fully developed sea [11]. This means that all waves have been generated by winds blowing over large areas for a long period of time. However, this assumption proved to be inadequate during Joint North Sea Wave Observation Project (JONSWAP) resulting in modifications and the JONSWAP spectrum [12]. In this thesis the JONSWAP spectrum is used to generate the waves when simulating how the buoy performs for irregular waves in different sea states. Mathematically, the JONSWAP spectrum is described as

$$S(\omega) = \frac{\alpha g^2}{\omega^5} exp\Big[-\frac{5}{4}\Big(\frac{\omega_p}{\omega}\Big)^4\Big]\gamma^r, \text{ where } r = exp\Big[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\Big].$$

The constants α, ω_p, γ and σ were determined using data collected during the JONSWAP experiment.

2.4 Wave Period and Peak Period

The time it takes for a wave to make one cycle is called a wave period. More specifically, for a regular wave, the wave period can be seen as the time it takes from one peak to the next peak. When it comes to irregular waves the wave period is less easy to define and to visualize. One such measure is the peak period, T_p, which is defined as the wave period that contains the most energy in the wave spectrum of the current sea conditions [6]. The peak period will play an important role when it comes to classifying the sea conditions in discrete way.

2.5 Wave Height and Significant Wave Height

As was seen in Figure 2.2 the wave height for a regular wave is defined as the vertical distance between a peak and a crest. For irregular waves it is not so straight forward. As mentioned in the previous section, the waves present at a location will contain numerous frequency components and amplitudes. Each of the wave heights will be associated with a probability and therefore a distribution of the heights can be obtained. This distribution has been proven to be approximately a Rayleigh distribution [13]. The significant wave height, H_S, is here defined in the traditional way. Hence, as the average wave height of the one-third highest waves as seen Figure 2.4.

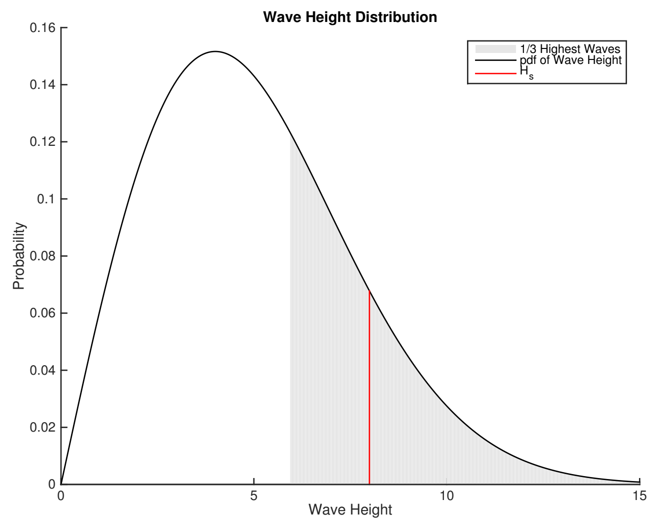

Figure 2.4: The significant wave height, H_S, together with the pdf and 1/3 highest waves for a simulated Rayleigh distribution of wave heights.

2.6 Hydrodynamic Forces

The external forces acting on a WEC are the excitation force F_{exc}, radiation force F_r, hydrostatic forces F_{hyst} and the drag force F_d. These forces are the external pressure forces,

$$F_{pe} = F_{exc} + F_r + F_{hyst} + F_d.$$

Together with inner forces from the power take-off unit, F_{PTO}, and WaveSpring F_{WS} they give rise to the motion of the buoy. The forces F_{PTO} and F_{WS} will be explained later. This section gives a very brief introduction to these external pressure forces.

2.6.1 Excitation Force

The excitation force is produced by the incident wave. It is the force that drives the motion of the buoy.

2.6.2 Radiation Force

Assume that a body is floating in calm water, i.e., where there are no waves present. If this body is forced to move up and down it will generate waves. The force that generates waves is the so called radiation force.

2.6.3 Hydrostatic Forces

The hydrostatic force is the force that makes a body in a fluid to float. It is the force that was once discovered by Archimedes and can be computed according to

$$F_{hyst} = \rho g V_{sub}$$

where ρ is the density of the fluid, g is the gravitational constant and V_{sub} is the submerged volume,.

2.6.4 Drag Forces

When a body moves in a fluid there is force acting in the opposite direction of body movement. This is the so called drag force and can be computed according to

$$F_d = \frac{1}{2} c_d \rho g A_s v^2$$

where c_d is the drag coefficient, ρ is the density if the fluid, A_s is the projected area of the body and v is the velocity of the fluid.

2.7 Energy in the Waves

As mentioned in the background section, the energy in waves is very dense. It can be decomposed into two parts, potential energy and kinetic energy. The potential energy is due to the elevation of water from the throughs to the crests and the potential energy is due to the particles which move as the wave progresses. Most of the energy is located at the surface of the sea and reduces exponentially with depth z according to

$$I(z) = I(0)e^{4\pi z/\lambda}.$$

However, the behavior of the sea differs a lot between sites across the Earth. This is why an optimization of the parameters of the buoy for each location is thought to have a very positive affect on the resulting electricity production.

2.8 Extracting the Energy

One of the most important properties of a good wave absorber is that it should be a good wave maker. In other words, a device which is good at making waves is also good at absorbing waves [9]. This is due to the fact that absorbing energy from the wave is equivalent to sending out a wave that cancels or partly cancels the incoming wave, i.e., destructive interference. That is, the energy, and thus size, of the incoming waves must be reduced by the absorbing device. Parameters that affect the absorbing properties of a device are for example the size and oscillating frequency. A small oscillating device can produce waves of the same size as a larger device with slower oscillations. Several different types of devices for extracting the energy have been proposed. The one developed by CPO is a so called point absorber. The concept of point absorbers is presented in one of the following sections.

2.9 Sea States

A sea state is a way of classifying the wave conditions using the significant wave height, H_S and the peak period, T_p. Mathematically, let $H_S = \{0, 0.25, 0.5, \ldots, 10.25\}$ be the set of significant wave heights and $T_P = 4, 5, \ldots, 18$ be the set of peak periods. Then, $S = H_S \times T_P$ is the set of all sea states used for classification.

The sea state will differ between times and data of how many hours spent in each sea state is available for a number of locations. For each location this data can be summarized nicely in a table as seen in Figure 2.5. This specific table shows the fraction of time in parts per thousand spent in a sea state during the time of observation. Typically the time of observation is about one year. Of course, the table will have different appearance depending on location and can thus be used to classify the behavior of the sea at each location. Based on its behavior it is possible determine whether a location is suitable for deployment of the device.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	0,1	6,4	9,1	7,7	7,4	7,1	4,3	1,4	0,1	0,0	0,0	0,0	0,0	0,0	0,0
1	0,0	6,5	25,0	30,6	38,6	42,2	27,9	15,5	7,1	0,4	0,1	0,1	0,0	0,0	0,0
1,5	0,0	0,1	4,2	18,6	31,8	32,1	24,6	16,6	14,2	9,8	2,0	0,4	0,3	0,0	0,0
2	0,0	0,0	0,0	7,9	25,7	26,9	20,1	16,8	7,4	7,1	2,6	1,3	0,7	0,0	0,0
2,5	0,0	0,0	0,0	0,0	9,2	39,5	37,0	31,1	12,1	8,2	4,9	1,3	0,3	0,0	0,0
3	0,0	0,0	0,0	0,0	1,7	32,1	38,9	28,6	12,9	10,0	6,6	3,6	1,2	0,0	0,0
3,5	0,0	0,0	0,0	0,0	0,0	7,9	33,5	22,0	13,6	7,5	5,9	4,2	2,6	0,9	0,1
4	0,0	0,0	0,0	0,0	0,0	0,4	12,6	14,6	7,5	5,6	3,2	3,2	2,6	1,7	0,4
4,5	0,0	0,0	0,0	0,0	0,0	0,0	3,0	4,9	6,9	2,5	1,4	0,7	0,6	0,0	0,1
5	0,0	0,0	0,0	0,0	0,0	0,0	0,3	2,3	3,9	1,7	0,9	0,1	0,0	0,0	0,0
5,5	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,9	2,7	1,4	0,7	0,0	0,0	0,0	0,0
6	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,4	1,6	1,0	0,0	0,1	0,0	0,0	0,0
6,5	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,3	0,9	0,1	0,0	0,0	0,0	0,0
7	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
7,5	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,3	0,0	0,0	0,0	0,0	0,0

Figure 2.5: Example of how a table of sea states can look. Here the value in each element is the fraction (in parts per thousand) of time spent in that sea state.

Let N be the total number of hours observing the sea and $s(i, j)$ be the number of hours spent in sea state (i, j). Here, i and j corresponds to different values of H_S and T_p respectively. Then, as $N \to \infty$, $s(i, j)/N \to Pr(S = s(i, j))$ for all $s(i, j) \in S$. Hence, for a large number of hours the observed distribution of sea states converges to its actual distribution. As the number of hours observed is roughly a year, i.e., $N = 24 \cdot 365 = 8760$, a fairly good approximation of the distribution can be assumed.

2.10 Point Absorber

A point absorber is by definition a floating body which is small in comparison to the wave length. Typically it consists of a floating buoy which is moored to the seabed by a wire. The idea behind a point absorber is that the heaving and surging motions due to incoming waves will make it oscillate and that this oscillating motion can be converted into electricity through a generator. Another feature is that a larger buoy will be able to absorb more energy [9]. This is of course due to the larger displacement and thus larger forces generated by the waves resulting in more energy within the system. However, it must not be too large as it thus violates the definition which means that the above reasoning is invalid.

2.11 Desired Movement

There are certain factors that affect the oscillating motion of the buoy which can be controlled using phase control. Here is a very short explanation of two physical phenomena which are crucial to this part of the analysis.

2.11.1 Resonance

Oscillating systems have so called natural frequencies. At a natural frequency and its neighboring frequencies the system has a tendency to oscillate with a greater amplitude. This phenomenon is called resonance. The natural frequency, in radians, of a one degree of freedom spring-mass system can be computed according to [14]

$$\omega_{nat} = \sqrt{S/m}.$$

The amount of energy converted to electricity through the PTO is proportional to the amplitude of the oscillations. The amplitude of the oscillations is measured by D_{rack} (displacement of the rack) which will be explained later in the report. For now it can be though of as the same distance as the buoy moves in its heave motion, D_{buoy}. With this in mind it is of interest to match the natural frequency of the system, i.e., the buoy, with the frequency of the incoming waves. By doing so it is possible to make the velocity of the buoy vary in phase with the wave. If done properly this can result in a greatly improved performance in terms of electricity production [15]. Often, the natural period is used instead of the natural frequency. This simply means the period of the oscillation at natural frequency, $T_{nat} = 2\pi/\omega_{nat}$ where ω_{nat} is the natural frequency in radians. It is important to note that for any WEC greater amplitude of the oscillations is not always a good feature. This is because the WEC have a limited size and therefore a limited maximum motion, i.e., a limited maximum displacement of the rack, D_{rack}.

2.11.2 Bandwidth

As mentioned above, there is an interest in matching the natural frequency of the buoy with that of the incoming waves. However, ocean waves are irregular and thus of different wavelength. Therefore it is desired to have a frequency interval in which the oscillations of the system are amplified. This interval is called bandwidth. Assuming no limitations on motion, it is desired to have this interval as wide as possible so that the system oscillates with high amplitude for as many wave lengths as possible [16]. The bandwidth can in a sense be controlled with the same device that controls the phase.

2.12 Phase Control

As mentioned, there is a possibility of using phase control in order to obtain a suitable natural frequency for the system as well as obtaining a fairly wide interval for which the natural frequency has an notable impact. Here is a short description of two such methods. The first method, latching, is well known and have been tested in the company's device [17] [18]. The second method have some advantages such as less forces on the machinery and is therefore the method that will be in focus for both the company and this project. Figure 2.6 shows the a comparison of the time series for D_{rack} when using the different methods for phase control.

2.12.1 Latching

When moving with the waves the buoy will have zero velocity when it is at the top or at the bottom of its motion. The idea behind latching is that by freezing the motion at these two positions and then releasing it at the correct time one can increase the amplitude of the motion, just as with resonance [19] [20]. Results have shown that using this method will heavily increase the amount of produced electricity [21]. However, it will also result in large accelerations and thus large forces. Another drawback is the not so smooth electricity output to the grid. Also, in order to release the buoy at the correct time an algorithm for predicting the next wave in necessary. The accuracy of such a prediction method will of course affect the result. In the left figure of Figure 2.6 the time series of D_{rack} when using the latching method is presented.

2.12.2 WaveSpring

In order to avoid the large forces that arise when using latching another method for phase control has been developed. This method relies on a mechanism called WaveSpring. The idea behind the WaveSpring is to generate a force which cancels the hydrostatic force. That is, only a small force will be needed in order to get a large movement of the buoy. This is equivalent to changing the stiffness, S, of the system and thus, assuming one degree of freedom, the natural period, T_{nat} which is computed according to

$$T_{nat} = \sqrt{S/m}.$$

In figure 2.6 the time series when using latching and WaveSpring phase control methods are visualized. Here the difference in behavior is easy to observe.

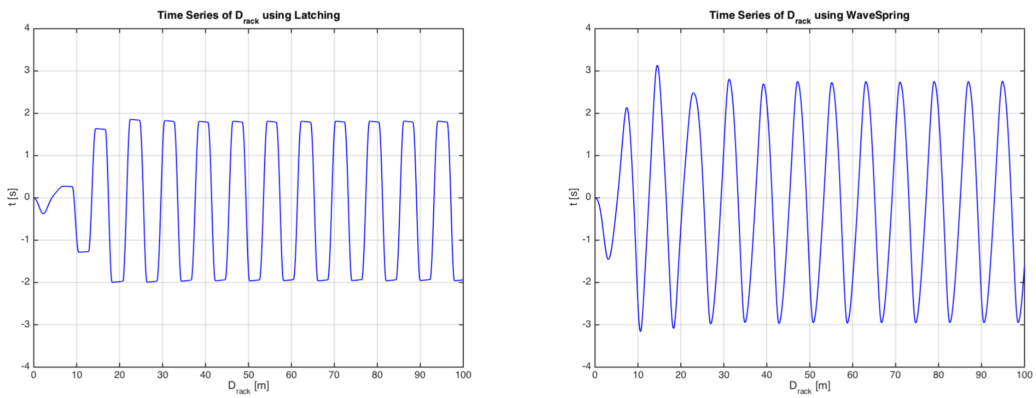

Figure 2.6: *Left figure:* Time series of D_{rack} when using the latching method. *Right figure:* Time series of D_{rack} when using the WaveSpring method.

Since the purpose of both latching and WaveSpring methods is to modify the natural period of the WEC it is interesting to visualize the result of this change. This can be done in Figure 2.7 in which the time interval 50 to 80 seconds of Figure 2.6 is shown together with corresponding forces and velocities. As can be seen both the latching method and the WaveSpring methods make the different lines move in phase. Note also the difference in velocity, V_{rack}, which has a much smoother appearance when using the WaveSpring.

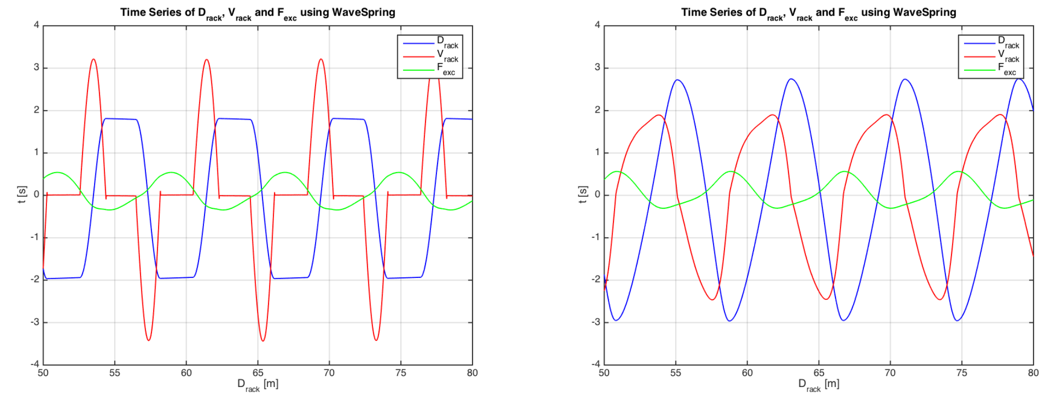

Figure 2.7: *Left figure:* Time series of D_{rack}, V_{rack} and scaled excitation force F_{exc} when using the latching method. *Right figure:* Time series of D_{rack}, V_{rack} and scaled excitation force F_{exc} when using the WaveSpring method.

In Figure 2.8 the different forces affecting the buoy are visualized against the D_{rack}-position. The two main forces are the hydrostatic force, F_{hyst}, and the force from the WaveSpring, F_{WS}. The force F_{gas} which is also included in the figure comes from another mechanism (the pretension spring) which is not part of the project. Hence, no detailed explanation of this force is included here but a short introduction to its purpose is presented later in the report. In the left figure of Figure 2.8 a low stiffness is used for the WaveSpring and in the right figure a significantly larger stiffness is used.

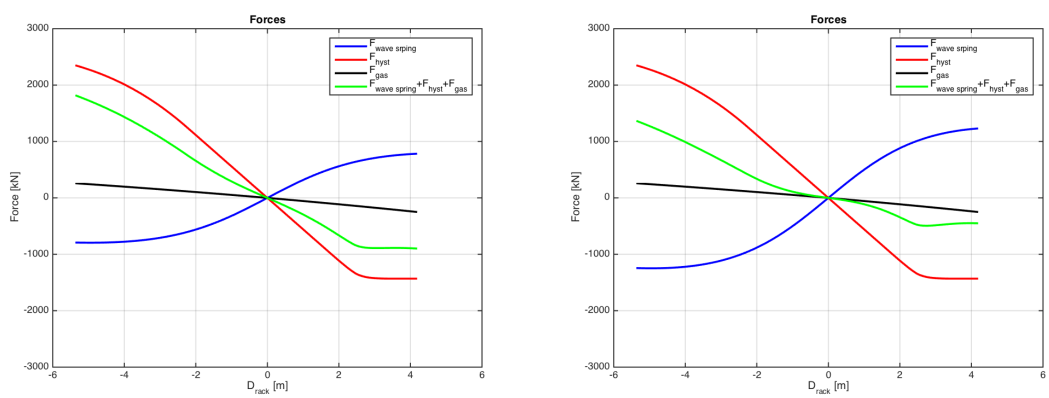

Figure 2.8: *Left figure:* Forces for a rather low WaveSpring stiffness. *Right figure:* Forces for a significantly higher WaveSpring stiffness.

From Figure 2.8 it is obvious that the higher pressure results in a larger interval where the net force, green line, is close to zero. This means that the buoy will move in a desired motion for a greater variation of waves, thus the bandwidth is said to be larger for a higher pressure. In Figure 2.9 the shift in natural period can be seen by comparing the value on the x-axis which corresponds to the peak. In this figure, the change in bandwidth can be seen as the width of the peak. The conclusion from this figure is that for sea states with longer peak period it is desired to have a higher stiffness of the WaveSpring. In addition, having a higher stiffness leads to wider peak and therefore a larger bandwidth.

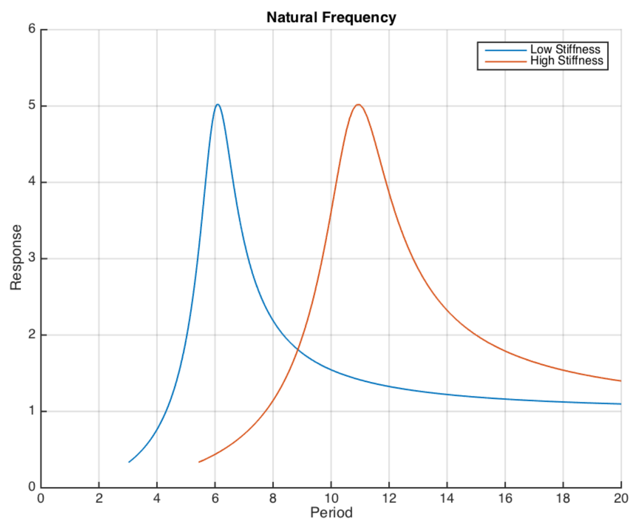

Figure 2.9: Spectrum for the two different selections of stiffness. An increased stiffness results in a longer natural period and a wider bandwidth for the WEC.

In Figure 2.9 the stiffness of the system was increased in order to increase the natural period. The conclusions are in line with the theoretical behavior according to

$$T_{nat} = \sqrt{S/m}.$$

This equation also shows that it is possible to vary the natural period by varying the mass of the system. Varying the mass of the WEC is of course not easy. Therefore having the possibility to change the stiffness is considered extremely good.

A mechanical description of the WaveSpring mechanism and how it is used will be presented later in the report.

2.13 Discrete Markov Process

A Markov process is a stochastic process which is considered to be memoryless [22] [23]. Being memoryless means that whenever the process is in one of its states the transition probabilities to other states are independent on previous states. Mathematically, this can be expressed as

$$Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \ldots, X_0 = x_0) = Pr(X_n = x_n | X_{n-1} = x_{n-1}).$$

A discrete Markov process is a process that every time it is observed is in one of the states. Hence, the jumps occur in a discrete way. Assume that X is a Markov process having state-space S and transition matrix $\mathbf{P}$. If $\boldsymbol{\pi}(x)$ is a probability distribution, i.e, $\sum_j \pi_j = 1$, such that

$$\pi_j = \sum_{i \in S} \pi_i p_{ij} \qquad \forall j \in S$$

then $\boldsymbol{\pi}(x)$ is said to be a stationary distribution [24] [25] [23]. Here p_{ij} is the element on row i and column j in the transition matrix $\mathbf{P}$.

Chapter 3

Wave Energy Converter Developed by CorPower Ocean

The purpose of this chapter is to give a better explanation of the idea behind the WEC developed by CPO and of the components that play an important role for understanding how the WEC works. Note that since the WaveSpring plays such a central part in this thesis it will not be covered in this chapter. Instead, the next chapter is dedicated to give a thorough explanation of the WaveSpring mechanism and how it is used.

3.1 Idea

The WEC currently being developed by CorPower Ocean is a point absorber. Hence, the main idea is that as the waves reach the buoy they will give rise to a motion. This motion is mostly a heaving and surging motion which is why these are the two degrees of freedom included in the Simulink model which is used to simulate the behavior of the WEC. As the buoy moves, certain inner mechanisms and inventions aims to harvest the energy in the most efficient way.

3.2 Pretension Spring and Mooring System

The pretension spring is located at the bottom of the buoy. It is a pneumatic spring with the purpose of pulling the light buoy down to its midpoint position. It is attached to the buoy in one end and the mooring system in the other end. The mooring system can simplified be considered as a wire that connects the buoy with the seabed. How this is done is not important for this thesis but it is a very important part of the WEC. It needs to be able to withstand very large forces that may occur for rough waves and when the buoy is fully submerged. In addition, in some cases the wire might become slack. This is a very risky feature since it might give rise to an extreme snap force as the wire becomes tense again. In the left picture in Figure 3.1 the mooring line can be seen and in the middle picture the pretension spring (pretension module) can be seen.

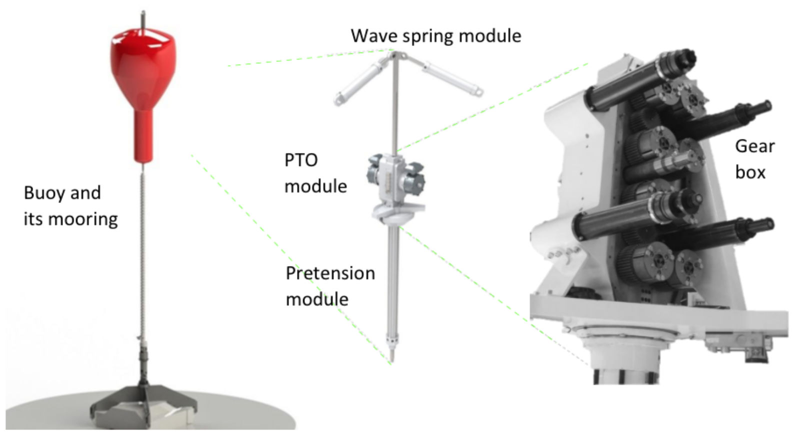

Figure 3.1: The WEC seen from the outside together with inner pictures showing the parts discussed in this chapter.

3.3 PTO

The Power Take-Off unit (PTO) is the system of mechanisms that together works to extract energy from the motion of the buoy. One of the parts of the WEC that will be used for this thesis is the rack which is the part of the WEC that moves through the PTO. In the middle picture of Figure 3.1 the rack is the part where the WaveSpring is attached at the top. As the buoy moves the rack will only move up and down in a linear motion for a reference system fixed at the PTO. Hence, the motion of the rack is a result of the buoy motion in all degrees of freedom. The displacement of the rack, D_{rack} is the term that is used for measuring the motion of the buoy. As the rack moves up and down through the PTO it starts a motion in the gearbox. The gearbox have a unique design that allows all eight cogwheels, seen in the right picture of Figure 3.1, to take an equal load. Through the gearbox the motion of the buoy gives rise to a spinning motion of two flywheels. There are two freewheels in the connection between the gearbox and the flywheels. This means that one flywheel will be accelerated as the rack moves up and the other one as the rack moves down. During the part of the motion that a flywheel is accelerating it is said to be engaged.

Each of the flywheels are connected to a generator which extracts energy from the spinning motion. This is typically done as the flywheel is disengaged (not engaged). Hence, it is possible to extract energy of the flywheel at a pace that allows for a rather smooth power output. This is of course a very nice feature when the electricity is sent to the grid. However, it is also possible to withdraw energy of a flywheel while it is engaged. Therefore, it is possible to control the motion slightly by having a sophisticated strategy for withdrawing energy from an engaged flywheel.

Chapter 4

The WaveSpring Mechanism

This chapter will first explain how the WaveSpring works. That is, the components and what they do. When this has been sorted out there will be an explanation of how the WaveSpring is intended to be used and how it is used. Finally, some issues with the current strategies together with possible remedies will be discussed.

4.1 How the WaveSpring Works

The WaveSpring is a system of three springs connected to the buoy in one end and the rack in the other end. Since there is a relative movement between the buoy and the rack the force of the WaveSpring will have two components. The first component is horizontal and the second is vertical. In the zero-position, where $D_{rack} = 0$, the WaveSpring is perpendicular to the rack making the vertical component zero while the other component has a value equal to the WaveSpring force, F_{WS}. If the the position of the buoy, D_{rack}, goes to infinity the horizontal component goes to 0 and the vertical component goes to the value of the WaveSpring force. However, in the real device the length of the WaveSpring at the original position is almost the same as the maximum allowed $|D_{rack}|$ meaning that the angle between the components is at most around 45 degrees. In Figure 4.1 the buoy and WaveSpring is seen in two different positions.

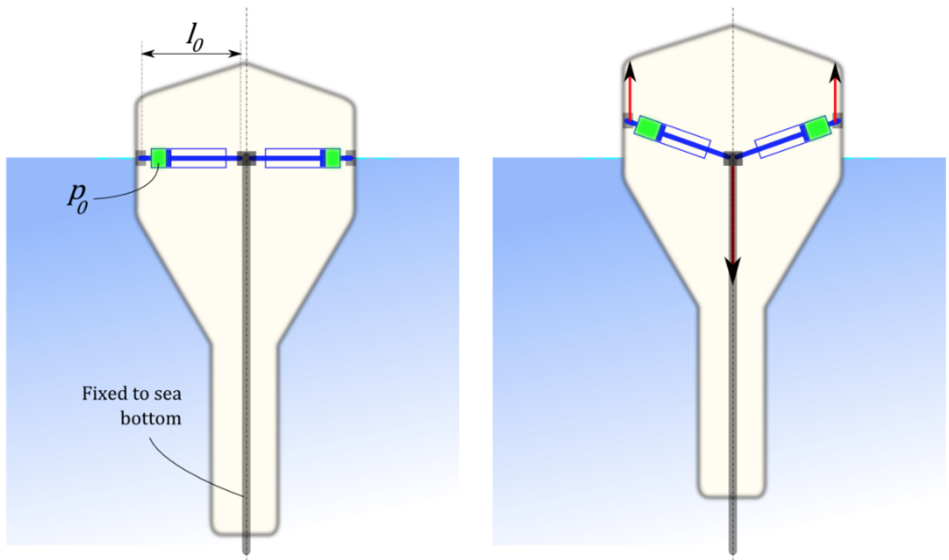

Figure 4.1: The buoy and WaveSpring elongation and angle seen for two different positions. l_0 is the horizontal distance between the two end points for one of the springs in the WaveSping mechanism. It is also the shortest length the springs will have during the motion. p_0 is the initial pressure of the air within the WaveSpring system.

The springs are a so called pneumatic springs which means that the force from a spring arises due to pressures within the spring. Since

$$Force = pressure \cdot area$$

the force from the WaveSpring is entirely dependent on the difference in area and pressure between the two sides of the spring. The net force can thus be computed as

$$\begin{aligned} F_{WS} &= F_{fa} - F_{rs} \\ &= p_{fa}A_{fa} - p_{rs}A_{rs} \end{aligned}$$

where subscriptions rs and fa mean rod side and full area side respectively. The rod side is the side of the wave spring where the rod is. In Figure 4.1 the rod can be seen as the blue line within the spring. The full area side is the other side of the spring. The system is closed and there is a connection between the two sides of the WaveSpring meaning that the pressure is equal on both sides. Therefore, the only parameter which determines the force is how large this pressure is chosen. Within the connection there are chambers whose purpose is to increase the total volume of air in the system. Due to these chambers, the pressure can be considered as constant and, in most cases, equal on both the rod side (side where the rod is) which is connected to the rack and the full area side (side without rod) which is connected to the buoy itself. The result of this is that the force F_{WS} is constant and does not dependent on the extension as it would for a mechanical spring. Since the part of the WaveSpring force that amplifies the motion corresponds to the vertical component its size depends only on the angle between rack and WaveSpring. In Figure 4.2 the routing of the WaveSpring is presented.

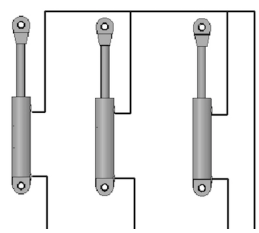

Figure 4.2: Part of the routing of the WaveSpring mechanism.

The WaveSpring mechanism is connected to a compressor that allows the stiffness, which is entirely dependent on the pressure, to be changed. How this fact can be utilized has its background in the theory of resonance and will be discussed in the next section.

4.2 How the WaveSpring is used

As mentioned, the WaveSpring is used as a phase controller in order to modify the natural period of the system. How to modify the period depends on the current sea conditions, that is, which sea state that the sea is currently in. This section aims to explain what is meant by having a tuned or detuned WaveSpring. It also aims to explain how the braking is performed.

4.2.1 Tuning - For low to medium energy sea states

For low energy sea states, meaning those sea states where the buoy can move in resonance without violating the constraints, it is desired to tune the WaveSpring so that resonance occurs. This is done by selecting the pressure to be optimal. Currently, the company is using a table indicating which pressure that should be used for a specific sea state. The resulting electricity output when using this table is not optimal but serves as a good comparison for this thesis.

4.2.2 Detuning - For high energy sea states

For some of the more energy dense sea states, meaning higher peak period and/or higher waves, resonance is not a desired feature. This is simply because being in resonance for these waves would make the device oscillate faster and with higher amplitude than it can handle. This can be solved by detuning, a feature which will now be explained.
By changing the configuration of the WaveSpring it is possible to reverse the force. Hence, instead of amplifying the motion and obtaining resonance the WaveSpring now works to restrict the oscillations and therefore maintaining a safe motion.

4.2.3 Braking

The WaveSpring can also be used to brake the motion. This can be done by utilizing a similar approach as for detuning. The idea is that the force of the WaveSpring can be partially and temporarily reversed whenever some constraint(s) are violated. Note that the way this is done still differs from detuning and hence the braking process can be executed both when the WaveSpring is tuned and when it is detuned. The result of the braking process is a large braking force which will slow down and eventually stop the motion. As this happens, there will be a very large force that aims to push back the buoy. This is due to the mechanical parts of the device. Therefore, it is necessary to quickly reduce the braking force of the system in order to avoid this spring back effect.

By braking the system using the WaveSpring it is possible to have the device running in a tuned mode for higher energy sea states. However, there are limitations on how much braking that can be achieved by this method. The main reason for this is the angle in which the force is working. As mentioned, the maximum angle between the WaveSpring and the rack is about 45 degrees meaning that about half of the WaveSpring force is working in the direction of the rack.

Initialize Braking Process

The current braking strategy utilizes a combination of two parameters, here called A and B, to determine whether or not to initialize the braking process. In order for the device to start braking a few criteria concerning A and B must be fulfilled.

Finalize Braking Process

To avoid the spring back effect it is necessary to have a sophisticated method for finalizing the braking process. If this is not done properly there is a risk that the device can be heavily damaged. In order to construct such a method the parameters A and B are used.

4.3 Issues with Current Strategy

The purpose of this section is to introduce some of the issues that this project aims to solve. First of all, there is no concrete strategy for when to start detuning or how much to detune. In other words, it is not known for which sea states that the system should be tuned and for which it needs to be detuned. Hence, the current strategy for deciding which pressures to use for each sea state is insufficient and there are room for improvements. In addition, it does not take into account for the cost of changing pressure between the sea states. Therefore, it is desired to find a new strategy which takes into account also for this cost, which can easily be expressed in terms of energy or electricity.

There are also room for improvement for the braking strategy. It is believed that the strategy currently in use can be developed and improved significantly by a thorough analysis of the process of initializing the braking. Finally, the strategy for finalizing the braking process has some flaws that are not desired. The major issue is a feature that is referred to as creeping, a common phenomenon for wave conditions with long peak periods. The source of the creeping is the way of finalizing the braking process.

Due to the current strategy, the braking process only have two different states, either it is active or not. In other words, either the WaveSpring is braking the motion or it is not. For waves with long peak period this can cause oscillations between the two states that results is a movement which "creeps" to a higher/lower position. It is believed that this creeping issue can, at least, be reduced by finalizing the braking differently. Another issue that is heavily affected by the creeping is the lack of end stop in the Simulink model. This means that the motion can continue creeping outside what will be allowed in the real device. In other words, there is no feature that simulates the end of the rack at $D_{rack} = 3.5 = D_{rack}^{max}$. It is desired to implement such a feature in order to make the simulations more realistic and it is believed that this will also reduce the length of the creeping. The creeping issue can be seen Figure 4.3. Note in this figure the values on the y-axis showing that the buoy creeps from a position $D_{rack} < 3.5$ m to a position where $D_{rack} > 4$ m.

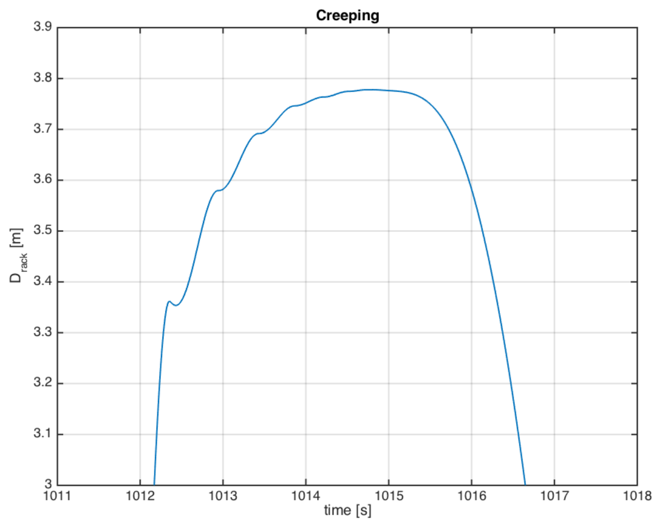

Figure 4.3: The buoy stops for the first time at $D_{rack} \approx 3.37$ and then moves in steps up to $D_{rack} \approx 3.78$. This is the issue which is referred to as creeping.

Chapter 5

Braking Strategy

An improved strategy for braking the motion of the buoy is desired. This strategy must fulfill some mechanical constraints such as maximum movement and maximum forces on components. Typically, it is desired to initialize the braking process as late as possible but still maintain a feasible motion. In other words, brake as late as possible while still keeping the motion within its boundaries in order to loose as little energy as possible. It is also of interest to model the forces when the buoy reaches the end of the rack, i.e., the end stop at $D_{rack} = 3.5$ m. With these things in mind, a strategy for braking should include answers to to the questions

- When should the braking process be initialized?
- When should the braking process be finalized?
- How to model the end stop?

In order to answer these questions it is necessary to clarify some definitions. Braking the motion in time means that $V_{rack} = 0$ at some point during the braking phase and at that point it is required that $D_{rack} \leq 3.5$ m. In this project a peak is defined as the first time $V_{rack} = 0$ in an interval between two following points where $D_{rack} = 0$. This definition of a peaks is illustrated in Figure 5.1. Hence, it is desired to brake so that all peaks are at a position $D_{rack} \leq 3.5$m.

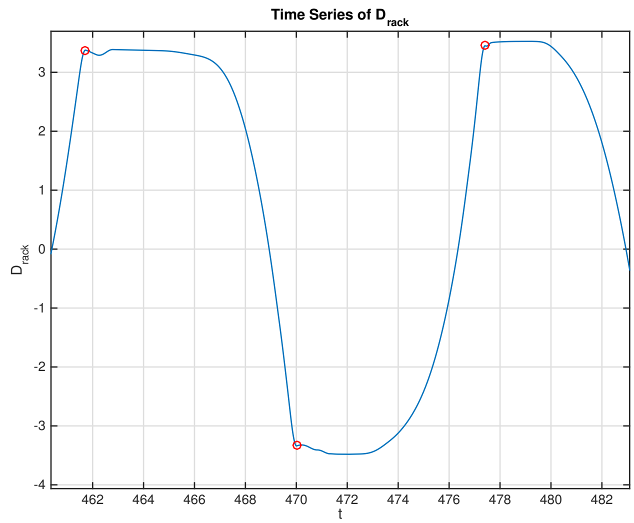

Figure 5.1: In this time series of D_{rack} the definition of a peak, red circles, can be visualized.

Furthermore, the developed strategies will be based on logical functions whose purpose is to make decisions on when to brake and when to stop braking.

5.1 Determine a Strategy for Braking

This section aims to in an analytical way come up with a new, improved, strategy for braking. Firstly, the method for initializing the braking is analyzed. Then the topics of finalizing the braking and the modeling of the end stop are addressed.

5.1.1 When to Initialize the Braking

It is believed that using the previously introduced parameters A and B to determine when to initialize the braking process serves as a good foundation for further investigation. Hence, these are the main parameters being considered during this part of the analysis which aims to determine an expression for when to brake based entirely on these two parameters. The first goal is to find a method that allows the buoy to brake before violating the constraint on D_{rack}. When such a method has been found it is necessary to modify this method so that also the constraint on V_{rack}

remains fulfilled throughout the motion.

Simulations are used to understand how the system behaves when braking. When running the initial simulations the braking strategy in use is very simple. Based on an analysis of the result and identifying possible improvements the strategy is then modified. Using the modified strategy new simulations are performed and analyzed in the same way. By repeating this process of simulating, analyzing and identifying, a new strategy for initializing the braking has been developed.

5.1.2 When to Finalize the Braking

As mentioned, how the braking process is finalized will affect the shape of the peak. The first attempts to find a good strategy for knowing when to finalize the braking uses the parameters A and B. Then the method uses a similar approach as when finding the strategy for initializing the braking process. However, this time it is more focused on the appearance of the peak. In addition to parameters A and B, a need for parameters C and D have been identified and therefore these have been introduced.

5.1.3 End Stop

The end stop is when the rack reaches its highest and lowest position. When this happens in reality there will be some damping or spring force which makes it more smooth. Therefore, in this thesis it will be modelled as a very stiff spring and damper. This is done so that it is possible to monitor the forces that occur to make sure that they do not violate the constraints.

Chapter 6

Tuning Strategy

This chapter will present the approach for determining a strategy for tuning the WaveSpring mechanism according to different sea states.

6.1 Determine a Strategy for Tuning the WaveSpring

The first task is to come up with a suitable mathematical formulation of the problem. At this moment the company is using a scheme that shows how to tune the WaveSpring for each sea state. It is believed that this scheme can be improved by further investigation of which pressure that actually is optimal for each sea state. It is also desired to take into account for the cost, in terms of energy, that it takes to tune the WaveSpring. This cost has its origin in the compressor. One way to approach this problem is to assume that the probability of jumping between the sea states is constant. In other word, whenever a sea state is present, the probabilities of jumping to the nearby sea states have certain values. Hence, the process is memory-less. By making this assumption it is possible to model the process as a Markov process. There will of course be constraints which needs to be fulfilled. These constraints are in many cases the same as the constraints that is present for the analysis of braking strategy. This is due to the fact that the pressure of the WaveSpring will have an impact on the effect of braking. Typically, a higher pressure means a more stiff spring constant which results in a more rapid braking.

The method will then be applied for a location called Yue for which the company have made similar analysis with the previous strategy. Hence, for this location it is possible to compare the results from this analysis with previous results.

6.1.1 Markov

When modeling the problem as a Markov decision process the outcome of the analysis will be a scheme of how to adjust the device only taking into account for which sea state that is present. This means that the number of variables that needs to be determined equals the number of available sea states at a specific location. As the company is requesting such a scheme this method is believed to be suitable. When modeling according to this idea the probabilities of jumping between the discrete sea states, the time expected to stay in a sea state, as well as the cost for adjusting the pressure needs to be taken into account. The resulting model will thus be solved as the optimization problem

$$\begin{aligned} \text{minimize} \quad & Z(\mathbf{p}) = \sum_{s \in S} \left[\Delta T (P_{elec}(\hat{p}_s) - P_{elec}(p_s)) + \sum_{y \in Y(s)} Pr(Y(s) = y) f(p_s, p_y) \right] \\ \text{subject to} \quad & g_i \leq g(\mathbf{p})_{i,max} \quad \forall i \in \mathcal{I} \\ & p^s = p^y \text{ if } s = y \quad \forall s, y \in S \end{aligned}$$

where S is the set of all sea states that can occur at the location and $Y(s)$ is the set of states reachable in one jump from state s. $P_{elec}(p_s)$ and $P_{elec}(\hat{p}_s)$ are the obtained hourly electricity output per hour in sea state s using pressure p_s and the maximum electricity output per hour using optimal pressure $\hat{p}_s$ respectively. and The inequality constraints will be discussed further in the following sections.

6.1.2 Constraints

As mentioned earlier, varying the pressure of the WaveSpring will heavily affect the motion of the buoy. Due to mechanical properties there are limitations on the motion. For example, the maximum amplitude of the motion is limited to d_{max} since exceeding this limit would damage the device. There are also limitations on the velocity, v_{max}, on the amplitude, a_{max}, and mechanical limitations m_{max}. The mechanical constrains are mainly forces on different components and they may not exceed certain limits. These limitations must be taken into account for and the resulting model would look like

$$
\begin{aligned}
\text{minimize} \quad & Z(\mathbf{p}) \\
\text{subject to} \quad & d_{max}^{sim}(\mathbf{p}) \leq d_{max} \\
& v_{max}^{sim}(\mathbf{p}) \leq v_{max} \\
& a_{max}^{sim}(\mathbf{p}) \leq a_{max} \\
& m_{max}^{sim}(\mathbf{p}) \leq m_{max}
\end{aligned}
$$

The notation *sim* means that this is a simulation. Hence, the maximum rack amplitude obtained during the simulation should not exceed the maximum allowed rack motion. The same arguments apply to the velocity and the acceleration. This means that the objective function is constrained by a set of nonlinear functions. However, since these are simulations dependent on $\mathbf{p}$ it is possible to simplify the constraints and make them linear. The idea is that for sea states with low wave height and low wave amplitude these limitations will not have any effect. This is simply due to the fact that there is not enough energy in these waves to make the buoy move close to its limits. Hence, only for higher waves will the constraints be active and they will serve as an upper boundary for the pressure. In fact, having a too low pressure might also lead to violations since a for low pressure the effect of braking of is less powerful. Hence, there will also be lower limits for how the pressure may be chosen.

With this in mind it is possible to reformulate the problem as

$$
\begin{aligned}
\text{minimize} \quad & Z(\mathbf{p}) \\
\text{subject to} \quad & \mathbf{p} \in \mathbf{p}^{sim}(d^{max}, v^{max}, a^{max}, m^{max}).
\end{aligned}
$$

Notice that the limitation from the pressure is now dependent on the amplitude of the rack motion as well as its velocity and acceleration. As seen in the model, these limitations are determined through simulations.

6.1.3 The Model

The previous sections leads to a model based on the Markov assumption

$$\begin{aligned}
\text{minimize} \quad & Z(\mathbf{p}) = \sum_{s \in S} \left[\Delta T(P_{elec}(\hat{p}^s) - P_{elec}(p^s)) + \sum_{y \in Y(s)} Pr(Y(s) = y) f(p^s, p^y) \right] \\
\text{subject to} \quad & \mathbf{p} \in \mathbf{p}^{sim}(d^{max}, v^{max}, a^{max}, m^{max}). \\
& p^s = p^y \text{ if } s = y \quad \forall s, y \in S
\end{aligned}$$

By using this model it is possible to find the optimal settings based on some standard assumptions on the input. Such standard assumptions are for example that the sea state changes every half an hour. Since there are no real probabilities available at this time it is also of interest to investigate how sensitive the model is to variations in this parameter. The objective is to, based on current knowledge and estimations of the inputs, find the optimal strategy and investigate how sensitive the results are on the input.

6.2 Solution

At this stage, some conclusions can be made about the model formulation. First of all it is on a convex space meaning that $p \in C$ where C is convex. This is due to the fact that all constraints are linear. In fact, since all variables have upper and lower limits the space can be thought of as some kind of multi-dimensional cubic space. Note that it is not known whether the objective function is convex.

6.2.1 Determining the Functions

Obviously, the equations that describes the electricity production as a function of pressure needs to be determined. The purpose of this section is to describe how this has been done. First, there is an explanation of how the curves for energy output as a function of WaveSpring pressure are determined. When this have been established there is also an explanation of how the cost for changing pressure have been determined.

The P_{elec} Functions

It is important to always keep in mind that the following study is not meant to be the final one in the sense that the values that are presented is actually going to be in use. This is due to the fact that many parts of the buoy is in development and any variations will lead to the values presented here being out of date. Instead, the goal is rather to prove that the optimization model presented in this report is a good way to approach the problem of choosing pressures. The idea is to first run simulations of a single sea state using different WaveSpring pressures. By doing so it is possible to find the electricity production as a function of pressure. Since running simulations is a rather time consuming procedure it is of interest to limit the amount of simulations. With this in mind it has been decided that simulating over five pressures is enough to capture the most essential behaviour. An example of such a curve can be seen in Figure 6.1.

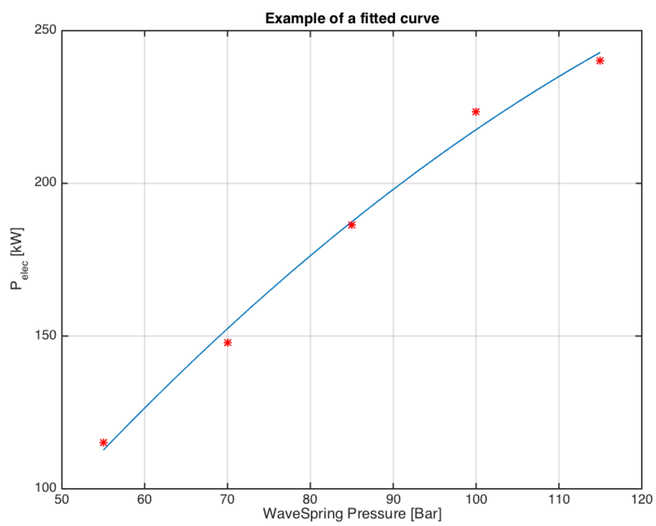

Figure 6.1: An example of the obtained curve fit for one of the simulated curves. Red stars are the simulated values, blue line is the curve fit.

As can be seen from this figure the fit is not perfect. However, as the waves are random the curve still seems to be capturing the essential parts and the accuracy is considered to be sufficient. Typically, this analysis should be performed for all sea states in order to obtain the functions. As mentioned, simulating is time consuming and therefore is is of interest to investigate if the curves can be found by plane fitting. This means that by simulating some well chosen sea states and fitting curves it could be possible to fit a plane describing how these curves vary throughout the table of sea states.

In other words, when the polynomial coefficients have been found for certain sea states it is believed that the coefficients for the not yet simulated coefficients can be found with sufficient accuracy by fitting a plane to the coefficients. The $P_{elec}(p)$-curves are believed to behave as second degree polynomials

$$P_{elec}(p) = c_2 p^2 + c_1 p + c_0$$

where c_0, c_1 and c_2 are the coefficients and therefore three different planes are required. Each plane corresponds to one of the coefficients and can be described on the form

$$c_l(H_s, T_p, FW) = \sum_{j=1}^{m_l} C^l H_s^j + \sum_{i=1}^{n_l} D^l T_p^i + \sum_{k=2}^{3} E^l \cdot FW_k \qquad l = 0, 1, 2.$$

FW_k is an indicator variable taking value 1 if flywheel k is in use. Note that there is no indicator for flywheel 1 and that H_s^j and T_p^i means H_s to the power of j and T_p to the power of i respectively. The plane coefficients C, D and E are determined by the method of least squares. It turns out that the using this method works very well for suitable values for m_l and n_l. Therefore, the results may very well be used in order to save time in this thesis to prove that the method is suitable. Note that from these curves it is possible to find the optimal pressure, $\hat{p_s}$, for each sea state. These optimal pressures will be used as a starting guess for the optimization. For some cases the pressure is below the minimum feasibility limit and for some cases it is above the maximum limit. When this occurs the value in the starting guess is changed from the optimal value to the closest feasible value.

Cost Function

The purpose of the cost function is to describe the cost, in joules, of changing pressure between the sea states. This means that it is very dependent on which compressor that is being used. In addition, there is only a cost when the pressure is to be increased and not when decreased. This means that there is an indicator variable, I, that takes value 1 if the new pressure, p_y, is larger than the current pressure, p_s. In other words,

$$f(p_s, p_y) = f_{compressor}(p_s, p_y) \cdot I\{p_s < p_y\}.$$

When it comes to $f_{compressor}$ the efficiency is not known but an estimation can be made using some thermodynamics. Since the volume is constant the process is isochoric and the gas is air which can be assumed to be an ideal gas. It is believed that an estimation can be done by a linear cost as the one seen in Figure 6.2.

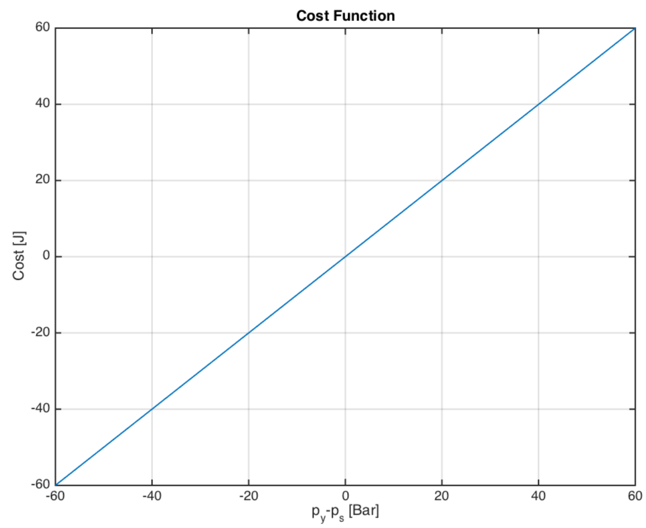

Figure 6.2: The linear estimation of the compressor cost.

As mentioned, this function should be 0 for negative values on the horizontal axis. Therefore the indicator variable is introduced in order to get the function which can be seen in Figure 6.3.

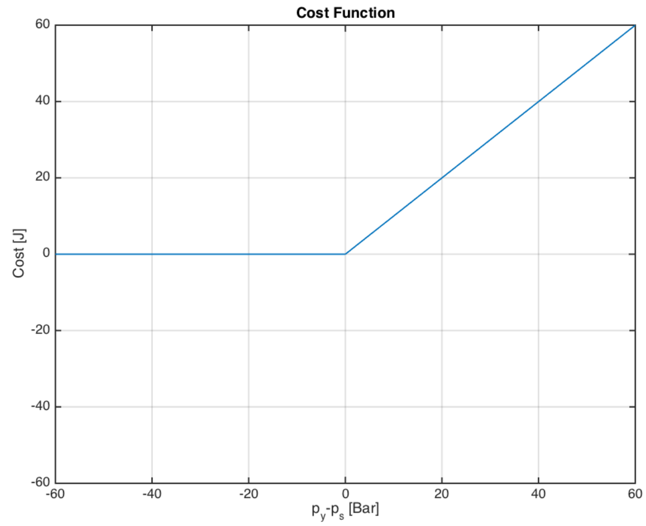

Figure 6.3: The estimation of the compressor cost when the indicator variable has been added.

However, this function is obviously not an ideal one for an optimization problem due to the derivative not being defined for all values on the horizontal axis. Based on this an estimation of the sharp angle is desired. Such an estimation could stretch between 0 and 1 on the horizontal axis and must fulfill certain criterion regarding both function value and derivative in these two points. Typically such a function f is a polynomial of suitable degree and must fulfill

- $f(0) = 0$
- $f(1) = k$
- $f'(0) = 0$
- $f'(1) = k$.

Assuming that $k = 1$ a third degree polynomial with coefficients A, B, C and D results in the function

$$f = -x^3 + 2x^2$$

which fulfill these criterion. This curve is scaled using the so called dead volume, i.e., the volume of air in the WaveSpring system that makes the spring behave more linearly. This curve can be visualized in Figure 6.4.

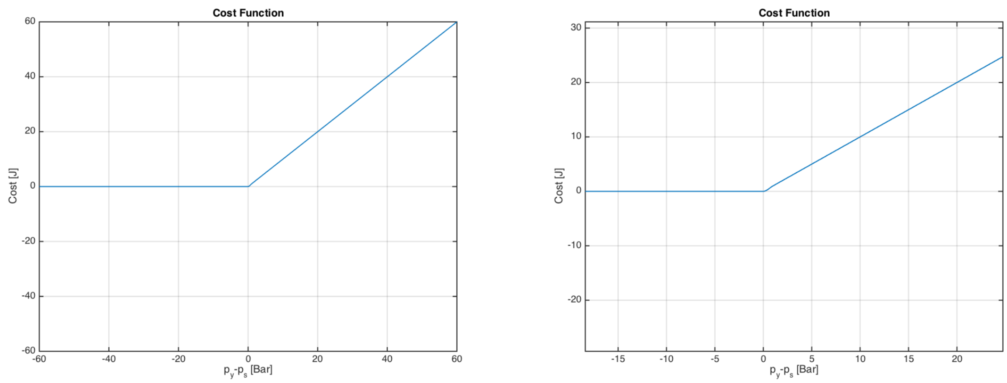

Figure 6.4: *Left:* The estimation of the compressor cost when the polynomial is used to smoothen the corner. *Right:* Zoom in of the corner.

6.2.2 Determining the Constants

Similar to the previous section, this section aims to give an explanation of how the probabilities and the ΔT is determined.

The Probabilities

Determining the probabilities is not an easy task. The process is assumed to be Markovian and there exist a stationary distribution. It is assumed that the measured number of hours in each sea state represents an approximation of the stationary distribution. From the theory it is known that for m states with jump matrix $\mathbf{P} \in \mathbb{R}^{m \times m}$ the stationary distribution can be determined as any row in the matrix [25] [23]

$$\mathbf{P}_\infty = \mathbf{P}^N, \qquad N \to \infty.$$

For the benchmarking location in this thesis the number of states, m, is 106. From each state the system can jump to between 2 and 8 other states. Assume that on average the number of reachable states from a state is 5. Then the total number of probabilities that must be determined are $5 \cdot 106 = 530$. Obviously this is a rather messy problem so before trying to solve it a stability analysis for how the probability profile affects the pressure selection is performed.

The ΔT

The time spent in a sea state, ΔT, is assumed to be 30 minutes since this is standard due to wave properties. However, it is of interest to investigate how sensitive the optimization is to variations in this variable. Typically, if the expected time before moving to another state is very small the cost of changing pressure have a very big impact on the tuning. This is therefore expected to give more or less the same pressure for all sea states. The same applies for the opposite, namely if ΔT is very large. That would lead to a solution where the optimal pressure is alway chosen.

Chapter 7

Results

In this chapter results will be presented first for the braking and then for the wave pressure determination strategy.

7.1 Braking Analysis

Based on simulations a strategy for when to initialize and finalize the braking process has been determined. These strategies are based on a few input variables which, for the real device, easily can be monitored.

7.1.1 Braking without Violating D_{rack}^{max}

Using the developed strategy the system is able to brake without violating the maximum value on D_{rack} for almost all sea states present at the investigated location.
In Figure 7.1 the time series obtained for both the old strategy and the new strategy can be seen. All violations are marked by red circles and each such violation might have the potential of destroying the device. Typically, the new strategy gives a much more controlled behavior which of course is very good. Another feature that should be noticed from this figure is the lack of creeping present for the new strategy. In fact, since creeping is no longer present the buoy does not gather potential energy in a way that is not possible in reality. Therefore the new results can be considered as more realistic.

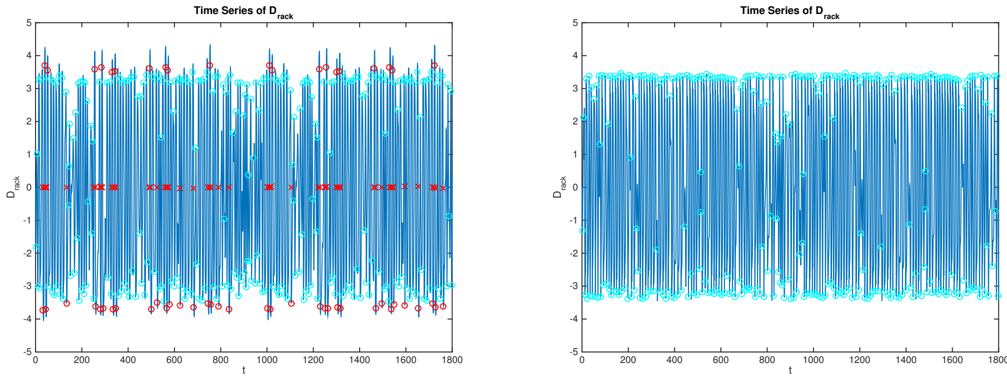

Figure 7.1: *Left:* The times series of the motion when using the old braking strategy. The sea state is a very rough one with$H_s = 6$ and $T_p = 12$. *Right:* The corresponding time series when using the new braking strategy.

7.1.2 Limiting Velocity

This part of the analysis is very difficult. The main reason for this is the rather vague constraints. For example, it is allowed to violate the constraints slightly as long as it does not happen too frequently. This is due to the mechanical properties of the system. In addition, an analysis of the damping that is applied throughout the motion is currently under investigation at the company. As for now, the investigation suggests an increase of damping which will result in lower velocities. What can be said about the performance based on the analysis in this thesis work is that the violations occur at most about 12 times each hour for the very rough sea states. The simulations show that each violation is $\leq 1.09V_{rack}^{max}$ meaning a violation of at most 9%. Since the suggested increase of damping would most likely lower the velocity more than 9% it is believed that the proposed strategy for braking in this thesis is in fact very good.

7.1.3 Finalizing the Braking and Mechanical Forces

The strategy for finalizing the braking process that has been determined in this thesis appears to prevent end stop hits very well. If the pressure of the WaveSpring is chosen properly it appears to be possible to entirely get rid of end stop hits for all sea states. However, the chosen pressure must also result in a sufficient electricity production. It turns out that as long as the pressure is chosen sufficiently large it is possible to limit the end stop forces to about 10% of the maximum allowed force. Such a pressure is expected to be optimal for almost all sea states except the ones with very low energy. However, for these low energy waves the probability of oscillating enough to reach the end stop is for sure zero. Hence, by putting constraints on how the pressure may be chosen for each sea states it is possible to ensure a very solid braking procedure. All other mechanical forces are also observed to be within their limits.

7.1.4 Tuning and Detuning

As mentioned, in the roughest of sea states it is necessary to detune the system. However, it turns out that instead of detuning it is possible to run the device as usual if the strategy for braking is modified slightly. The effect of this is an energy conversion which is about three times as high as when the system is detuned. All constraints are still fulfilled and this solution is of course much more suitable than detuning. In other words, with the new braking strategy there is no need to detune the system. Hence, when running simulations to determine optimal pressure for each sea state it is no longer of interest to investigate detuning. Instead all sea states up to a certain amplitude will be run with a specific strategy. For the rougher sea states safety is prioritized and a more conservative strategy can be applied. Note that the sea states where the most conservative strategy is used occurs only by about 2% of the total time at the investigated location. Note also that a more conservative strategy simply means more restricted and can be achieved by changing parameters of the decision making functions.

7.1.5 Sensitivity to Conservativeness of the Braking Strategy

Simulations with more or less conservative strategies have been performed in order to investigate if it is possible to use a more conservative strategy to keep some margins to the end stop without loosing too much power. Simulations have shown that by using a more conservative strategy it is possible to reduce the motion so that the buoy stops with a very good margin from D_{rack}^{max} without reducing the produced power significantly.

7.2 WaveSpring Pressure Determination

First of all the resulting optimal pressures disregarding mechanical constraints are analyzed. In addition, these pressures have been determined with zero cost for tuning the WaveSpring. In other words, these are the pressures that will give the best output disregarding mechanical constraints and tuning costs. Secondly the resulting optimal pressures when including the constraints and the tuning costs are analyzed. The resulting pressures are confidential and may not be presented here. However, results can briefly be explained as lower pressures in the upper left corner and then higher in the lower right corner. To illustrate the difference in pressures between the optimizations the ratio (constrained divided by unconstrained) has been computed. These ratios are presented in Figure 7.2. The result of the constrained optimization is a smaller spread in pressures between the different sea states. This reduced spread is achieved by increasing the pressure for the sea states with a lower energy, i.e., the upper left corner of the figure.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	1,3	1,3	1,3	1,0	1,1	1,1	1,1	1,0	1,0						
1		1,2	1,2	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0			
1,5			1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0		
2				1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0		
2,5					1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0		
3					1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0		
3,5						1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0
4						1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0
4,5							1,0	1,0	1,0	1,0	1,0	1,0	1,0		1,0
5								1,0	1,0	1,0	1,0	1,0			
5,5								1,0	1,0	1,0	1,0				
6									1,0	1,0		1,0			
6,5										1,0	1,0				
7															
7,5										1,1					

Figure 7.2: The ratio of the pressures. It can be seen that the optimization algorithm reduces the difference in pressure between the sea states by increasing the pressures where a very low pressure is optimal. The sea state with $H_s = 7.5$ and $T_p = 12$ is increased to to a constraint on minimum pressure.

When evaluating the new method where the pressures have been determined optimally according to certain constraints the results in terms of produced electricity are compared to earlier results obtained by the company. These earlier results can be seen in Figure 7.3 and will be referred to as the benchmarking results. As can be seen, the device converts more energy as the wave height increase. When it comes to period there seems to be a maximum at around $T_p = 12s$. Hence, this might be the natural period for the WEC using a WaveSpring pressure of 115 bar.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	0	0	1	2	3	4	5	6	7						
1		1	5	9	14	20	25	29	32	34	34	35			
1,5			12	23	36	48	57	64	64	60	68	67	66		
2				44	66	79	92	89	96	99	102	94	89		
2,5					93	103	128	121	132	139	134	120	118		
3					126	144	167	165	159	157	164	144	140		
3,5						174	191	201	181	202	200	173	168	152	138
4						214	213	216	220	221	229	207	193	176	163
4,5							247	215	221	241	235	217	214		179
5								220	225	245	257	239			
5,5								250	242	284	243				
6									223	285		236			
6,5										268	259				
7															
7,5										254					

Figure 7.3: The benchmarking results in terms of [kW] which have been obtained by the company using the previous braking strategy and the previous pressure selection scheme. Note that these results were obtained for motions that violates the constraints.

In Figure 7.4 the results obtained using the new braking strategy together with the new pressure scheme can be seen. The behavior seems to be rather similar to the previously obtained results. However, it is important to remember that the new strategy makes sure that the constraints are not violated, as was earlier seen in Figure 7.1.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	0	0	1	2	3	4	5	6	6						
1		1	5	9	15	17	23	28	34	34	38	38			
1,5			13	23	36	43	55	65	64	76	74	66	67		
2				44	67	77	90	104	107	106	99	99	95		
2,5					101	103	123	141	131	155	138	133	126		
3					117	132	145	159	161	176	174	153	159		
3,5						174	182	178	181	203	193	171	172	165	155
4						183	207	209	197	223	220	211	189	193	165
4,5							214	224	232	246	239	223	210		178
5								252	236	262	236	249			
5,5								240	241	260	253				
6									237	283		248			
6,5										290	267				
7															
7,5										311					

Figure 7.4: The results in terms of [kW] when using the new strategies for braking and tuning. Note that the results presented in this table are constrained by different mechanical properties. These constraints are fulfilled while obtaining the results presented in this table.

It is not very easy to draw any conclusions from these figures so instead the ratio of the two tables are presented in Figure 7.5. The numbers here are computed as *new/benchmarking* which means that the number 1 indicates no difference while a number > 1 means that the production is now better. Obviously a number < 1 means that the production is now less than before. As can be seen from this figure most of the numbers < 1 occurs for sea states that have a combination of H_S and T_p that puts them close to a line from upper left corner to the lower right corner. This is because sea states along this line have a dangerous combination of H_S and T_p. Consider for example the sea state with $H_S = 4$ and $T_p = 4$. This means that the peak period is rather short while the significant wave height is rather high. Hence, for the old strategy which allowed violations of the constraints it is possible to absorb and extract large amounts of energy for this sea state.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	0,9	1,1	1,0	1,1	1,0	1,3	1,0	1,0	0,9						
1		1,1	1,0	1,0	1,1	0,8	1,0	1,0	1,0	1,0	1,1	1,1			
1,5			1,1	1,0	1,0	0,9	1,0	1,0	1,0	1,3	1,1	1,0	1,0		
2				1,0	1,0	1,0	1,0	1,2	1,1	1,1	1,0	1,1	1,1		
2,5					1,1	1,0	1,0	1,2	1,0	1,1	1,0	1,1	1,1		
3					0,9	0,9	0,9	1,0	1,0	1,1	1,1	1,1	1,1		
3,5						1,0	0,9	0,9	1,0	1,0	1,0	1,0	1,0	1,1	1,1
4						0,9	1,0	1,0	0,9	1,0	1,0	1,0	1,0	1,1	1,0
4,5							0,9	1,0	1,0	1,0	1,0	1,0	1,0		1,0
5								1,1	1,0	1,1	0,9	1,0			
5,5								1,0	1,0	0,9	1,0				
6									1,1	1,0		1,1			
6,5										1,1	1,0				
7															
7,5										1,2					

Figure 7.5: The ratio of the new results divided by the old results. Hence, a number 1 means that the performance are the same for both methods. A number > 1 means that the new strategy performs better while a number < 1 means that the old strategy performs better. Note that randomness is influencing the numbers presented here.

A better comparison is to compare the yearly electricity production for the two strategies. This means that the numbers presented in the table are multiplied by the number of hours that the sea is observed to stay in that specific sea state and then summarizing all numbers. When doing so it turns out that the electricity production using the new strategy is just above 99% of the production from the benchmarking strategy. However, as mentioned, the benchmarking strategy heavily violates the constraints. Therefore the new strategy is considered significantly better.

7.2.1 Sensitivity

This section presents some sensitivity analysis regarding variables such as probabilities, time in a sea state and compressor efficiency. First of all the optimization is performed for two entirely different probability distributions. They can be seen in Figure 7.6. The distribution in the left figure is more or less uniform over the sea states that are present while the right distribution is more random.

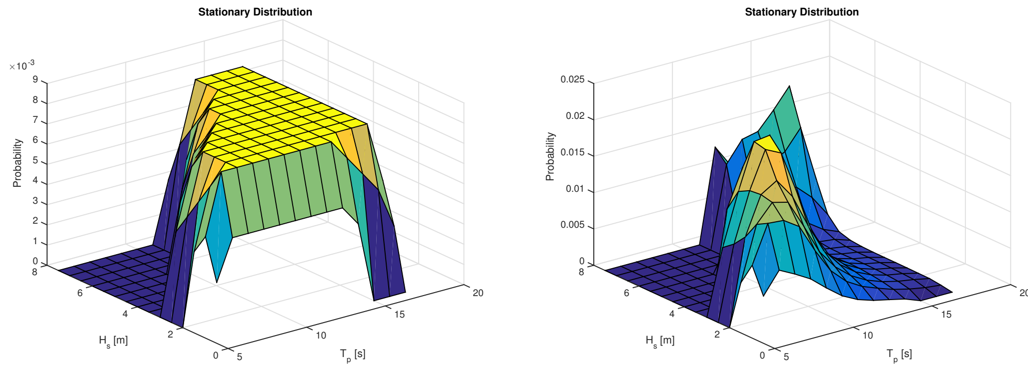

Figure 7.6: Two entirely different distributions for investigating the sensitivity to probabilities of the pressure selection optimization. *Left:* A more or less uniform distribution over the sea states that are present. *Right:* A more random distribution.

The result of the optimization analysis for these two probability distribution is investigated. This investigation is performed by dividing the resulting pressures obtained for the right distribution with the corresponding pressures obtained for the left distribution. The ratio of these pressures are presented in Figure 7.7. From these results it is possible to conclude that the distribution itself does not have a big impact on the solution. The reason for this could be that the obtained curves are not accurate enough. But as mentioned, they are considered good enough to prove that the method works. However, as seen earlier, the resulting production of electricity turns out to be really good which indicates that there should be a good accuracy. Perhaps the probabilities does not have a large impact on the outcome since the selected pressures are rather close to the actual optimal ones.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	1,00	1,00	1,00	1,00	1,01	1,01	1,00	1,00	1,00						
1		1,00	1,00	1,00	1,01	1,00	1,00	1,00	1,00	1,00	1,00	1,00			
1,5			1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2				0,99	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2,5					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3,5						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4,5							1,00	1,00	1,00	1,00	1,00	1,00	1,00		1,00
5								1,00	1,00	1,00	1,00	1,00			
5,5								1,00	1,00	1,00	1,00				
6									1,00	1,00		1,00			
6,5										1,00	1,00				
7															
7,5										1,00					

Figure 7.7: The obtained ratios when dividing the optimal pressures found for the right probability distribution with the optimal pressures found for the left probability distribution.

The next variable that can be varied is the air volume in the system. Varying this variable will have the same effect as varying the compressor efficiency. Therefore it is considered enough to vary this one as the conclusions will be the same if the compressor efficiency is varied. First, the volume is decreased from 1.5 m^3 to 1.0 m^3. To illustrate the difference in result the obtained pressure are now divided by the pressures obtained using a volume of 1.5 m^3. The ratios can be seen in Figure 7.8.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	0,98	0,98	0,98	0,98	0,99	0,98	0,99	1,00	1,00						
1		0,98	0,98	0,98	1,00	0,99	1,00	1,00	1,00	1,00	1,00	1,00			
1,5			1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2				1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2,5					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3,5						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4,5							1,00	1,00	1,00	1,00	1,00	1,00	1,00		1,00
5								1,00	1,00	1,00	1,00	1,00			
5,5								1,00	1,00	1,00	1,00				
6									1,00	1,00		1,00			
6,5										1,00	1,00				
7															
7,5										1,00					

Figure 7.8: The ratios obtained when dividing the optimal pressures found when using a smaller volume by the pressures found when using the original volume.

As can be seen, the difference is in the upper left corner of the table. Once again, the results appear to be very insensitive to variations in the input parameters. Though, the variations that appear are expected since lowering the volume is equivalent to lowering the cost for varying pressure. Therefore the solution should be closer to what is actually optimal if the cost for tuning is disregarded. The next step is to increase the volume from 1.5 m^3 to 2.0 m^3. The resulting ratio of the pressures is presented in Figure 7.9.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	1,02	1,02	1,02	1,02	1,01	1,02	1,00	1,00	1,00						
1		1,02	1,02	1,02	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00			
1,5			1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2				1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2,5					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3,5						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4,5							1,00	1,00	1,00	1,00	1,00	1,00	1,00		1,00
5								1,00	1,00	1,00	1,00	1,00			
5,5								1,00	1,00	1,00	1,00				
6									1,00	1,00		1,00			
6,5										1,00	1,00				
7															
7,5										1,00					

Figure 7.9: The ratios obtained when dividing the optimal pressures found when using a larger volume by the pressures found when using the original volume.

Once again the difference is very small and the motivation to this is the opposite of that when decreasing the volume. Namely that the cost is for tuning the WaveSpring is larger which means that the optimal solution is to increase the lower pressures slightly in order to make the amount of tuning lower. Next, the effect of varying the expected time to stay in a sea state is varied. First, it is decreased from 1800s to 900s and the ratio computed in the same manner as earlier is presented in Figure 7.10.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	1,04	1,04	1,04	1,04	1,01	1,07	1,00	1,00	1,00						
1		1,04	1,04	1,04	1,01	0,99	1,00	1,00	1,00	1,00	1,00	1,00			
1,5			1,02	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2				1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2,5					0,99	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3,5						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4,5							1,00	1,00	1,00	1,00	1,00	1,00	1,00		1,00
5								1,00	1,00	1,00	1,00	1,00			
5,5								1,00	1,00	1,00	1,00				
6									1,01	1,00		1,00			
6,5										1,00	1,00				
7															
7,5										1,00					

Figure 7.10: The ratios obtained when dividing the optimal pressures found when using a smaller ΔT by the pressures found when using the original ΔT.

From this table it can be seen that when the expected time to stay in a sea state is lower the pressures in the upper left corner becomes higher. The reason for this is that the gain of going to the optimal pressure becomes smaller in comparison to the loss of tuning the WaveSpring between the sea states. Finally, increasing the expected time to stay in a sea state from $1800s$ to $2700s$ results in the ratios presented in Figure 7.11.

Hs\Tp	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0,5	0,98	0,98	0,98	0,98	0,99	0,98	0,99	1,00	1,00						
1		0,98	0,98	0,98	1,00	0,99	1,00	1,00	1,00	1,00	1,00	1,00			
1,5			1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2				1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
2,5					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3					1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00		
3,5						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4						1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
4,5							1,00	1,00	1,00	1,00	1,00	1,00	1,00		1,00
5								1,00	1,00	1,00	1,00	1,00			
5,5								1,00	1,00	1,00	1,00				
6									1,00	1,00		1,00			
6,5										1,00	1,00				
7															
7,5										1,00					

Figure 7.11: The ratios obtained when dividing the optimal pressures found when using a larger ΔT by the pressures found when using the original ΔT.

The results of this increase is that the pressures in the upper left corner are now slightly lower. This is simply due tot the fact that now the gain for going closer to the optimal pressure is more worth than reducing the tuning. This result is of course also expected since it is just the opposite as when the expected time was decreased.

7.3 Summary of Results

When using the new strategy for braking it is possible to run the device without violating any of the constraints. In addition, this can be done more or less without reducing the amount of produced electricity thanks to carefully selected pressures for each sea state. There is also a large safety margin when braking meaning that it is extremely unlikely that the device will suffer any mechanical damages even for very rough sea conditions. The main reason for this is the possibility of using a more conservative strategy without a major loss in amount of produced electricity. When determining the pressures using the Markov assumption and solving it as an optimization problem the obtained results are not sensitive to differences in input parameters.

Chapter 8

Conclusions/Discussion

As long as the WaveSpring pressure is kept within certain limits and the new braking strategy is in use it is possible to run the device without violating the constraints. This is achieved without a significant decrease in electricity output, a result that is better than expected. It is also possible to run the device for more energy dense sea states where the peak period is longer and the wave height is higher. This means that instead of detuning the device for these rough sea states it is now possible to use a more conservative strategy and still maintain a movement which fulfills all constraints. In terms of electricity production this is a huge advantage. A detuned device can, in the roughest of sea states that have been in use in this project, produce about 100 kWh. As seen in Figure 7.4, when using a smaller ellipse the production can reach above 300 kWh for these sea states. In other words, for the company's device which have already proven to have enormous potential, it is now possible to increase the production a lot if deploying the device at locations where these rougher sea states are more frequent. In addition to the improvements that have been discussed above, the device now behaves in a more predictable way. The main reason for this is that the creeping have now been reduced by finalizing the braking in a more sophisticated way and by simulating an end stop. Note that the force that occurs when reaching the end stop only appears once for the results presented above and that the force in this case is less than 10 % of the maximum allowed force. In other words, the new strategy for finalizing the braking itself almost entirely eliminates the end stop hits that previously occurred after braking.

When running simulations over different pressures the boundaries within which the pressure must be selected for each sea state have been found. With pressures selected in these intervals it can be guaranteed that the device fulfills the constraints. Typically, a larger pressure means larger forces and more rapid braking which is why the high energy sea states are forced to use higher pressure than the lower sea states. Though, the optimal pressure when using no constraints or tuning costs is in almost all cases above the lowest allowed pressure. In addition, the assumption of having probabilities varying according to a Markov process is considered as a good way to deal with this situation. At least until more knowledge about the behavior of the waves have been obtained.

The accuracy of the curve fitting is of course not exact. Only five different pressures were investigated and used when creating the curves. Using a larger number of simulations including more pressures is expected to increase the accuracy of the curve fitting. In addition, there was a plane fitting in order to avoid simulating all sea states. This feature might have introduced some less well-fitted curves but in order to prove the usefulness of the approach in this thesis it is considered enough. The main reason for this time saving procedure is the sensitivity to changes in other parameters which are currently under investigation. Hence, the analysis performed in this project will have to be repeated when all final parameters and settings have been selected. At that point it might be useful to increase the number of simulations for each sea state as well as simulating each sea state and thus avoid inaccurate curves. Since all limitations on the pressure have been found in this thesis it may be useful to only simulate pressures that are known to be feasible. Of course, which pressures that are feasible might change as the parameters change but it is believed that the found upper and lower limits of the pressure will serve as a good foundation for knowing which intervals to simulate.

As have been proven in this thesis the solution of the pressure selection optimization is not very sensitive to variations in input parameters. However, obtaining more data of how the weather behaves might still be useful. It may for example be so that the ΔT should be different for the sea states. Having data of the jumping probabilities might reveal some features which are not easy to predict. A rather strange phenomena can be seen immediately by looking at the sea state that are present in the investigation. Namely that the sea state $H_s = 7.5$ and $T_p = 12$ occur while none of its neighboring sea states have occurred according to the data. This is considered unlikely and perhaps an indication that better methods for determining the present sea states is desired. Another reason could be the sampling of the data. Perhaps the weather conditions have changed faster than what has expected and the ΔT in the neighboring sea states could have been very small. Anyhow, more accurate data could be useful for obtaining further information of the behavior of the sea and might lead to a better solution.

When performing tests on the real-life device it is strongly suggested to begin using conservative strategies that are chosen to have large margins to the end stop and perhaps a slightly lower electricity production. By doing so it is possible to first evaluate the Simulink model and make sure that things happens in a similar way in reality.

Chapter 9

Future Work

The developed strategy for braking and for optimization have proven to be very useful but of course there is room for improvement. One such improvement could be to introduce more variables to make the braking strategy more complete. By carefully selecting variables to analyze and perhaps introduce to the decision making functions it is believed that the developed strategy can be improved.

A possible improvement for the optimization model is to include leakage of air. It is believed that there will be some leakage of air meaning that there will be another small cost that can be included in the objective function. In addition, the amount of leakage is believed to be dependent on the selected pressure. A high pressure is believed to result in a greater leakage than a small pressure. This means that there is an additional cost to having a large pressure. Hence, introducing this cost may change the solution of the optimization slightly. However, for now the cost of the leakage is believed to be smaller than the accuracy of the pressure curves. Therefore it may be an unnecessary cost at this point.

In the early stages of this project methods which could handle prediction of the following sea states were discussed. It was decided that such methods are not desired at this point but may be useful in the future when the technology is improved. In order to develop and evaluate such methods it is desired to have better data of how the variations between sea states occur. This is another reason why collecting more data might be useful.

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