

# Institutionen för systemteknik

## Department of Electrical Engineering

**Examensarbete**

## **A network based algorithm for aided navigation**

Examensarbete utfört i Reglerteknik  
vid Tekniska högskolan vid Linköpings universitet  
av

**Daniel Magnusson**

LiTH-ISY-EX--11/4534--SE

Linköping 2011



**Linköpings universitet**  
**TEKNISKA HÖGSKOLAN**



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|---|---|---|---|
|    | <b>Avdelning, Institution</b><br>Division, Department<br><br>Division of Automatic Control<br>Department of Electrical Engineering<br>Linköpings universitet<br>SE-581 83 Linköping, Sweden |   | <b>Datum</b><br>Date<br><br>2011-12-04  |
|   | <b>Språk</b><br>Language<br><br><input type="checkbox"/> Svenska/Swedish<br><input checked="" type="checkbox"/> Engelska/English<br><br><input type="checkbox"/> _____                      | <b>Rapporttyp</b><br>Report category<br><br><input type="checkbox"/> Licentiatavhandling<br><input checked="" type="checkbox"/> Examensarbete<br><input type="checkbox"/> C-uppsats<br><input type="checkbox"/> D-uppsats<br><input type="checkbox"/> Övrig rapport<br><input type="checkbox"/> _____ | <b>ISBN</b><br>_____<br><b>ISRN</b><br>LiTH-ISY-EX--11/4534--SE<br><b>Serietitel och serienummer ISSN</b><br>Title of series, numbering _____ |
| <b>URL för elektronisk version</b><br><a href="http://www.control.isy.liu.se">http://www.control.isy.liu.se</a><br><a href="http://www.ep.liu.se">http://www.ep.liu.se</a>  |   |   |   |
| <b>Titel</b> En nätverksbaserad algoritm för navigeringsunderstöd<br><b>Title</b> A network based algorithm for aided navigation<br><br><b>Författare</b> Daniel Magnusson<br>Author  |   |   |   |
| <b>Sammanfattning</b><br>Abstract<br><br><p>This thesis is concerned with development of a navigation algorithm primarily for the aircraft fighter SAAB JAS 39 Gripen, in swarms of other units. The algorithm uses information from conventional navigation systems and additional information from a radio data link as aiding information, relative range measurements. As the GPS can get jammed, this group tracking solution can provide an increased navigation performance in these conditions.</p> <p>For simplicity, simplified characteristics are used in the simulations where simple generated trajectories and measurements are used. This measurement information can then be fused by using filter theory applied from the sensor fusion area with statistical approaches. By using the radio data link and the external information sources, i.e. other aircraft and different types of landmarks with often good performance, navigation is aided when the GPS is not usable, at e.g. hostile GPS conditions.</p> <p>A number of scenarios with operative sense of reality were simulated for verifying and studying these conditions, to give results with conclusions.</p> |   |   |   |
| <b>Nyckelord</b><br>Keywords    Sensor fusion, Navigation, Swarms, Scenarios, Group tracking, Link, LINK-16   |   |   |   |



# Abstract

This thesis is concerned with development of a navigation algorithm primarily for the aircraft fighter SAAB JAS 39 Gripen, in swarms of other units. The algorithm uses information from conventional navigation systems and additional information from a radio data link as aiding information, relative range measurements. As the GPS can get jammed, this group tracking solution can provide an increased navigation performance in these conditions.

For simplicity, simplified characteristics are used in the simulations where simple generated trajectories and measurements are used. This measurement information can then be fused by using filter theory applied from the sensor fusion area with statistical approaches. By using the radio data link and the external information sources, i.e. other aircraft and different types of landmarks with often good performance, navigation is aided when the GPS is not usable, at e.g. hostile GPS conditions.

A number of scenarios with operative sense of reality were simulated for verifying and studying these conditions, to give results with conclusions.

# Sammanfattning

Det här examensarbetet syftar till utveckling av en algoritm för navigering, primärt för stridsflygplanet SAAB JAS 39 Gripen, i svärmar av andra enheter. Algoritmen använder information från konventionella navigeringssystem och ytterligare information från en radiodatalänk som ger understödande information, relativa avståndsmätningar. Då den förlitade GPS:en kan störas ut, kan denna gruppspårande lösning öka navigeringsprestandan i dessa förhållanden.

För enkelhetens skull, används förenklade karaktäristiker i simuleringarna där enkla genererade trajektorier och mätningar används. Denna mätinformation kan sedan ihopviktas genom att använda filterteori från statistisk sensorfusion. Genom att använda radiodatalänkar och den tillförda informationen från externa informationskällor, således andra flygplan och olika typer av landmärken som väldigt ofta har god prestanda, är navigeringen understödd när GPS inte är användbar, t.ex. i GPS-fientliga miljöer.

Ett antal scenarion med operativ verklighetsanknytning simulerades för att verifiera och studera dessa förhållanden, för att ge resultat med slutsatser.





# Acknowledgments

Without all the support from all people concerned with this master thesis it would have been hard to perform this thesis. First of all, I would like to thank SAAB Aeronautics allowing me to do my thesis in an inspiring and exciting area and environment.

Many thanks to my supervisor at Linköping University, Zoran Sjanic, and my supervisor at SAAB Aeronautics, Mikael Axelsson, for a lot of useful inputs in terms of help and ideas when I got stuck. I would also like to thank for the outlooks in the research and development area, and I have learnt a lot during the thesis.

I would also like to thank my examiner Fredrik Gustafsson too, providing this master thesis through Linköping University.

Lots of thanks to my family for their support during my studies and this thesis. There have sometimes been harder moments, but their support has been very helpful.

Finally, thanks to everyone else during my years of studying at Linköping University. I probably would not been able to start the thesis or possibly it would not been performed without their support.

*Daniel Magnusson*  
Linköping, October 2011



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# Chapter 1

## Introduction

This chapter gives an introductory overview of this master thesis. Mainly, it defines the purpose, a problem formulation, problem limitation, and goals.

### 1.1 Background

Today the navigation systems for many civilian and military aircraft rely on solutions of the *Inertial Navigation System (INS)* supported by the *Global Positioning System (GPS)*, which is controlled by the United States (US). To become more independent of the GPS, other methods need to be used in GPS jammed conditions to achieve improved *absolute* and *relative* position estimates between different *nodes*; such as other aircraft and different landmarks.

### 1.2 Aim and Goal

The aim of this Master Thesis is to develop and evaluate a navigational algorithm where the GPS is jammed in e.g. hostile environments where it can be used as a backup system. This algorithm is primarily developed for the aircraft fighter *SAAB JAS 39 Gripen*, which is performed by applying sensor fusion theory. It uses a radio data link which provides information from external sources where the information is used in this algorithm together with information from existing conventional solutions, where the navigational performance can be improved in the GPS jammed conditions.

By simulating and evaluating the algorithm, where scenarios related to realistic and operational scenarios are used, the results are achieved. These results may help to point out if it is worth to put more effort in this area. To do this, feasible physical parameters and other feasible properties in the algorithm are used, where sensor fusion principles are applied by creating a filter solution. By using this filter solution in each node, which is the decentralized case, position updates can be achieved for each node at different times using this algorithm. Evaluations of the algorithm are achieved from these simulations, which are performed to be able

to make any conclusions of the navigational performance, if and when it is useful, and if it is worth to put any further work in this area for improvements.

### 1.3 Problem

The main problem is to achieve useful navigational position updates, primarily for the aircraft fighter *SAAB JAS 39 Gripen*, in different scenarios by using data from other nodes as external information. In these scenarios the *Global positioning system (GPS)* is jammed or not usable of any other reason. Since the *Inertial navigation system (INS)* in the aircraft are drifting away unlimited in position over time due to measurement errors the navigational performance decreases unlimited over time in these conditions, which means that the aircraft will get a more faulty position and increased position error the longer time it flies.

By using external measurement sources this performance can be improved and the INS measurement errors estimated, which in turn improves the dead reckoning of the INS. To be able to increase the performance a network of the available nodes is used by using radio data links, which is a network solution. By using other aircraft INS and GPS blended information as external information of their positions, together with information of transmission times, the problem is to develop an algorithm which considers this information too. When using different nodes in this network such as other aircraft and landmarks, e.g. base stations, the estimated absolute and relative positions for each aircraft can be improved by applying sensor fusion theory, decreasing the absolute and relative position errors in GPS jammed conditions.

### 1.4 Limitation

The Master Thesis focuses mainly on the sensor fusion part, including simulation and evaluation of the position estimations from realistic scenarios. This is performed by using models together with dynamical localization in an algorithm.

Therefore, e.g. simple models in a simple earth frame are used to describe the aircraft, where no consideration of the earth's curvature affecting the dynamics are taken into prior. This need to be considered in practice, to be able to navigate in a correct way.

Also, the aircraft descriptions in 3D are not in prior more than an independent decoupled description in altitude from the other two horizontal dimensions. This simple model can still describe the motion of the aircraft quite well, since it follows the motion characteristics.

The thesis does not compare different solutions. There are many different solutions and several kinds of filter solutions itself, but here is only one filter solution applied which uses one model.

Also, transmitting data between the nodes, consisting of different data, is a problem to handle in the reality. E.g. sending data over a data bus which is quantified creates problems if it is not taken into account in the real system, since this causes time errors such as time delays, which is not taken into account in



this thesis. In practice, for example when a node is about to receive these signals, each node that transmits to the receiving node must send their signals at different times, since they might be using the same frequency. This is not taken into account either.

## 1.5 Target Group

Graduate and undergraduate engineer students, and employees at SAAB Aeronautics are the main target groups.

## 1.6 References

Mainly, theory related to the sensor fusion issue are referred to [9], which gives a covering theoretical background in the sensor fusion related area. Navigational theory are mainly referred to [4], which covers the navigational area.

## 1.7 Previous Work

There are a lot of published work in this area of navigation which can be found, and a lot of research in progress. Generally, many different sources can be found which handles the sensor fusion and the navigation issue of for example the INS, GPS, and different extended solutions related to this thesis. Examples can be given both from the used sources and other sources referred in this thesis. For example in previous master theses this is handled, like in [3, 12]. There are also good books which handles this, like [11].

## 1.8 Abbreviations

A list of common abbreviations in this master thesis are here written.

NAV = Navigation frame  
BODY = Body frame  
INS = Inertial Navigation System  
GNSS = Global Navigation Satellite System  
GPS = Global Positioning System  
TADIL J = Tactical Digital Information Link J  
MIDS = Multifunctional Information Distribution System  
JTIDS = Joint Tactical Information Distribution System  
TDMA = Time Division Multiple Access  
RELNAV = Relative navigation  
TOA = Time Of Arrival  
RMSE = Root Mean Squared Error  
nmi = nautical miles  
M = Mach

# Chapter 2

## Navigation

This chapter describes basic definitions of the navigation notion and navigation systems, and related theory, briefly. Theory related to navigation and navigation systems in this chapter mainly originates from [4], from where more and deeper knowledge can be obtained.

### 2.1 Navigation, Guiding and Guidance

There are several definitions and interpretations of the *navigation* notion. A commonly used explanation is the ability of answering questions about where my own current position is, and to make a diagnosis of further desired direction or way to reach a specified location. This common explanation can be split into three new different notions; *navigation*, *guiding* and *guidance*. Navigation is the most interesting notion in this work, since it is strictly related to the purpose.

The issue of navigation is to measure and estimate the *current* position and other kinematic quantities, in a specified measurement reference. These quantities are called navigation quantities, which need a suitable reference, to increase the precision and relating the kinematics in this reference. Definition 2.1 defines navigation and the navigation problem.

**Definition 2.1** *Navigation is the same as keeping kinematic relations between different specified references in the current time instance. The navigation problem is a function measuring a motion and calculates at least some of the kinematical motion states in the desired reference system.*

*Navigation* can therefore be divided into two types; *terrestrial* and *celestial*. In terrestrial navigation the earth is used as reference model, while celestial navigation uses space (the solar system). Terrestrial navigation is the case in this work, and the only one discussed, where the measured motion in the navigation problem is commonly from a craft or vehicle. In this area, the motion is measured in the aircraft reference and related to the earth frame reference system. In turn, there are different kinds of earth references using different earth models. These

coordinate systems are body fixed coordinate systems which are converted to an earth centered and earth fixed reference coordinate system. There are also local fit reference systems, locally adapted to different locations, which is the case in this work.

Briefly, *guiding* and *guidance* are related to answer how to get to requested targets. Guiding is how the motion is affected by applying forces, and guidance is the decision making of how to guide. Simply expressed, navigation is required for guiding, which in turn is required for guidance.

## 2.2 Reference Systems

In the reference models different coordinate systems has to be specified and defined. These coordinate systems are for the earth frame and the inertial space frame, which got two different coordinate systems fixed in the earth center that has to be defined when using earth models in a more global perspective. Those are commonly the *Earth Centered Inertial frame (ECI)* (also *Inertial frame*, where the inertial space is modeled), and the *Earth Centered Earth Fixed frame (ECEF)* (also *Earth frame*, following the earths rotation). Definition 2.2 and 2.3 gives the definitions of these coordinate systems. The rotation of the earth in the ECEF observed from the ECI is called *earthrate*, where the ECI and the ECEF coincides if the rotation of the earth is disregarded. To relate the ECEF and the ECI, a polar coordinate system can be defined using longitude, latitude and altitude, which is another common coordinate system expressing the ECEF instead of cartesian coordinates.

**Definition 2.2** *The ECI got the origin in the earth center of mass, where the coordinate system is fixed to the space frame, and the earth rotates about one of the base vectors.*

**Definition 2.3** *The ECEF got the origin in the earth center of mass and is fixed to the surface, where instead the coordinate system follows the rotational motion of the earth.*

Another two frames need to be introduced, the *body frame (BODY)* and the *navigation frame (NAV)* (also local geodetic frame), which are needed to be able to relate the aircraft dynamics to the earth frame commonly. Base vectors of BODY are  $x^B, y^B, z^B$ , which also defines the *roll, pitch* and *yaw* rotational axes, which are the *attitude Euler angles*. These frames can be defined in different ways, but here for this thesis, NAV is defined as in Definition 2.4, and BODY as in Definition 2.5.

**Definition 2.4** *The origin of the navigation frame (NAV) coordinate system is fixed in the aircraft. The base vectors are  $x^N, y^N$  and  $z^N$ .  $x^N$  is pointing in the east direction,  $y^N$  in the north direction, and  $z^N$  in the upward direction.*

**Definition 2.5** *The origin of the body frame (BODY) coordinate system is fixed in the aircraft. The base vectors are  $x^B, y^B$  and  $z^B$ .  $x^B$  is pointing in the aircraft*

forward direction,  $y^B$  in the aircraft left direction, and  $z^B$  in the aircraft upward direction.

## 2.3 Earth Models

Basically, an earth model describes the earth mathematically in some way, and is the basis of terrestrial navigation. An earth model can describe *geometry*, *gravitation*, *kinematics* related to inertial space, and *magnetics* of the earth. In this case, only models describing geometry, gravitation and kinematics are described further, since they are more relevant and related to this work, where the geometrical model is the most important in terrestrial navigation. To be able to model gravitation and kinematics in a single way, a model describing geometry needs to be selected first, since they are functions of geometry.

A simple but often enough suitable model of the earth is a *sphere* or an *ellipsoid*. There are a lot of different ellipsoid models, designed to fit local or global geometries well. One of them, a common global model, is the *World Geodetic System 1984 (WGS84)*, which is an ellipsoid using the mean sea level. This is the standard model, used by NATO and civilians. Locally, WGS84 can deviate several hundred meters, why local models can be preferred. An example of a local model, which is adapted to Sweden, is the planar fitted model *Rikets Triangelnät (RT90)*. A difference of several hundred meters in altitude between the WGS84 and the RT90 model is the local case in Sweden.

## 2.4 Navigation Systems

Systems applied for navigation are called *navigation systems*, and gives information about navigation quantities. A navigation system uses sensors to measure kinematic quantities related to navigation and navigation quantities, which can be used for calculations. To define a navigation system more strictly, a definition of a navigation system is given in Definition 2.6, where in this case the navigation systems measures the aircraft motion.

**Definition 2.6** *A navigation system is an application which measures and calculates at least some of the navigation quantities related to a desired reference system.*

Since the earth is rotating about its own axis, the earth fixed coordinate system is not fixed in space. This motion together with the curvature of the earth affects the measurements of motion for an aircraft. Therefore the effects caused by the rotation of the earth has to be in mind when navigating using measurements of the aircraft motion in *navigation systems*. Examples of different navigation systems which can be used in navigation are the *Global positioning system (GPS)* and the *Inertial navigation systems (INS)*, which are used in this work.

## 2.5 Inertial Navigation and the Inertial Navigation System

This section describes the essentials of *inertial navigation* theory and how a *Inertial Navigation System (INS)* works and the theory behind inertial navigation, which can be seen as an introduction to inertial navigation and inertial navigation systems. An additional reference of inertial navigation is [17], while [15] and [16] are additional references for inertial navigation systems.

### Inertial Navigation

Inertial navigation concerns kinetics acting in an inertial frame, which are related to motion. This inertial frame is in this case the space fixed frame, since the motion of the earth itself causes forces acting in the earth frame together with the explicit gravity. These forces acting together, is the sensed gravitation force of the earths gravitational field and the earths motion itself. As a definition of the already explained forces, these two force components are results of the *centrifugal force (CF)*, caused by the earths rotation and the law of *mass attraction (MA)*. By studying these components a model of the earths gravitation field can be derived. There is a simple way to derive this gravitation model, by simply adding the components. This can result in a function of longitude and latitude, a simple and common gravity model with a "fair" approximation from  $-1$  to  $20$  km of altitude, which is feasible enough. As a result of modeling, there is a remaining error in gravity, which is called *gravity deflection*. This is because of in the modeled ideal case the gravity only acts in the altitude direction, which in practice is not the case.

### Inertial Navigation Systems

An *Inertial Navigation System (INS)* is a device which contains navigation sensors of *gyros* and *accelerometers*, sealed by a small box. A common case in three dimensions is that three accelerometers respectively three gyros, are used and mounted orthogonally to each other. These orthogonal mountings are called *triads*, measuring from each dimension. Hence, the accelerometer placement is called *accelerometer triad*, and the gyro placement is called *gyro triad*.

The INS device gives navigation quantities as output signals, using raw measurement data from each gyro and accelerometer, commonly in three dimensions, measuring from the true specific force and angular momentum acting on the inertial navigation system due to angular velocity and acceleration. These outputs from the INS are position, velocity, acceleration, attitude and angular velocity, derived by only using the aircraft as measurement reference, and no other external sources. That is an extra advantageous property in the military, why the INS is very popular and common in military applications.

To achieve correct outputs from the INS, the force due to the rotation and gravitation of the earth must be subtracted from the raw sensor measurements, before calculations to velocity, position and orientation can be performed. This is

why the earth's gravitation field is very significant for an INS, it simply would not work properly without a model describing the field.

Even if the effects from the gravitation field is taken into account, the INS is affected by different error sources, affecting the measurements and calculations. These effects are mainly due to measurement errors and gravitation model errors, where also e.g. errors in the initial position affects the performance. This causes erroneous calculations, resulting in a position error which is drifting over time.

E.g. if a constant bias error in acceleration and angular velocity are measured, the velocity error and direction error will increase linearly, and the position error will increase quadratically over time. That is why the INS is a bad position sensor, and need to be supported by external sensors, e.g. GNSS.

Due to errors in the gravity model, the calculated altitude will also contain model errors, why the altitude need to be supported by external measurements. Five different methods can be applied; measuring the air pressure (barometer) or the altitude (radar, laser, GNSS, and fixed height). It is called *altitude stabilization*.

The positional drift is most often about  $1500\text{-}2000\text{ m/h}$  ( $0.8\text{-}1.1\text{ nmi/h}$ ) while the attitude error often is less than  $0.1^\circ/\text{h}$ , for a contemporary aircraft INS, in well calibrated conditions. By calibration of the INS, e.g. different distances, coordinate systems and biases are calibrated, related to its mounting place in the aircraft.

## 2.6 Global Navigation Satellite Systems

This section describes the global navigation satellite system (GNSS), where an additional reference can be [7]. A GNSS is an umbrella term for satellite navigation systems, e.g. the US global positioning system (GPS), which is concerned in this thesis. These navigation systems uses earth orbiting system satellites, with very accurate orbital positions, to be able to navigate by using a receiver, which can be done by using different methods and settings. For this purpose, first of all at least four system satellites need to be available for the system being possible to use in a proper way, estimating both receiver position and time, which is needed since the receiver otherwise has an unknown time error from registering measurements. Secondly, signals from at least four different system satellites, instantly at the receiver, are required to be able to perform these estimations, resulting in a singular position estimate. If only three satellites are used, two candidates of the receiver position can be achieved by triangulation, where one of them can be disqualified, since it is far from the surface of the earth. In the case of five or more satellites available the least square solution is the estimate. Also the velocity of the receiver can be estimated, but not discussed any further on.

The GNSS uses measurements from radio signals (radio waves). These measurements can e.g. be time difference measurements for ranging, which is the most common and popular method, and used in e.g. the GPS. This means that the ranges from different transmitters to a receiver are measured and then multiplied with the speed of light, and used to perform the estimations.

The GPS service can be divided into two services using different bandwidths; the *Standard positioning service (SPS)* and the *Precise positioning service (PPS)*, which was the original purpose of the GPS. These services differs in precision and sensitivity to disturbances, where the PPS uses more bandwidth, which gives the highest available precision and is harder to disturb. It is used by the US and their allied, while the SPS is available for all civilian users in all other countries.

For estimation of the GPS position, the GPS uses range measurements, where two measurement methods are used. The first one is measuring the transmission time in the GPS receiver of transmitted GPS satellite code the system uses, while the second one is measuring phases of carrier waves instead. Both methods renders in very good estimation accuracy, which mostly are less than a few meters. But several issues affects the precision of the GPS position. Mainly varying precision depends of following circumstances; like different varying satellite properties (e.g. availability, US settings and supporting applications like differential systems), the receiver, the terrain and the atmosphere (ionosphere and troposphere) which affects the performance very differently depending on the location.

## 2.7 INS and GPS Solutions

A common navigation solution in conventional navigation, is a blended solution of the commonly used information from the conventional navigation systems, the INS and the GPS, which also can be supported by other information sources. This solution can be made by using the INS and the GPS measurements and apply sensor fusion theory, to fuse the information and improve the performance. This is what is already done in already existing common solutions and implementations of the blended solutions, in e.g. contemporary aircraft like Gripen, by using filter theory and the *Extended Kalman filter (EKF)*, which is introduced later on. These filters can estimate kinematical quantities like e.g. position, velocity, and errors in the INS sensors, by using the GPS support. [11] can here be used as an additional reference, which describes these solutions.

Since the GPS uses, as typically, lower measurement frequency than the INS, a measurement update using new GPS information are performed not as often as using new INS information. But it still works as a very good position reference through a longer time lap, catching the slower dynamics, while the INS works well through a shorter time lap, catching the faster dynamics, without the GPS support. This means that the position uncertainty increases rapidly from when the latest GPS measurement update was performed, and the uncertainty shrinks after a new GPS measurement update. But if the GPS is not usable through a relatively long time of flight, the performance is decreased, since the INS measurements renders in a quadratical drift in the filter too. Even if this solution decreases the position drift, the performance is still not satisfying in these conditions. Usable backup systems of today are quite poor in this context, where the position by e.g. geographical information from a database and beacons simply can be used to correct the position if possible.

# Chapter 3

## Radio Communication Links

This chapter concerns wireless communication using radio communication links for aided navigation in the general case, where the purpose is to explain and describe the functionality and the error characteristics of a radio communication link, from which new navigational information can be obtained, and the usage of them. The error sources of radio propagation affecting the performance of a radio communication link are described briefly too in this chapter. There are a lot of good references in the radio propagation and radio communication link area, but here the chosen reference in this work is [2]. Also the LINK-16 standard is described as an example of a radio communication link, commonly used in military applications nowadays. References of LINK-16 are [1], [5], and [14]; where lots of more information about LINK-16 can be retrieved.

### 3.1 Introduction

The radio communication link is a quite wide concept, applicable and used in many applications in all kinds of environments. But the purpose of a radio communication link is always the same, which is to connect different units wireless in some way, where electromagnetic waves are transmitted and received wireless, transferring e.g. different information data or measuring arrival times.

Basically, a radio communication link consists of a transmitting and a receiving terminal, together with a propagation channel, where the propagation channel is the physical medium that electromagnetic waves travel in, from the transmitter to the receiver. Several transmitters, receivers and at least one propagation channel creates a wireless communication system, a wireless communication network.

There are different categories of radio communication links, using different communication channels such as terrestrial, atmospheric and ionospheric, covering different physical conditions using different principles and processes. These physical conditions are atmospheric and ionospheric properties in this case, where the radio propagation conditions in different environments basically affects the output of the radio communication links. Mainly the output performance of a radio communication link depends basically on the noise in the receiving antenna,



noise in the transmitting antenna, noise in the electronics communicating with the antennas, and the ambient and background noise. Another important issue is the frequency bandwidth which is important when predicting the performance of the radio communication link. To achieve a good output performance, the transmitting antenna operation, the antenna connecting propagation channel properties, and the receiving antenna, are the main issues that need to be studied to achieve a well performing radio communication link, where the issues are electromagnetic and electronically.

To explain the properties of the propagation channel, affecting the output performance, the fundamentals of radio propagation phenomena need to be considered, which depends on both the radio wave itself and the local environment. Since this thesis does not focus on the noise caused by the terminals itself and signal processing aspects, the terminal noise and their specific sources is not focused and more briefly discussed.

## **3.2 Radio Communication Link Terminal Noise**

The noise in the receiver, are both generated by the receiver itself, internal noise, and by ambient and background sources, external noise, where the noise in the receiving terminal also depends on the transmitting terminal. These disturbances affects the electromagnetic signals in different ways, in turn affecting the output performance, which has to be considered.

In idealistic environments where external noise is not considered, the outcome of using the terminals is that the terminals themselves still can cause noise, where the internal noise basically originates from the electronic devices, the electronic components, such as thermal noise due to the random motion of the electrons. This thermal noise can be summarized by it is related to the background temperature as additive thermal Gaussian noise, which is not explained in more detail in this thesis.

Sources of the external noise are all ambient natural sources, such as atmospheric phenomena in the ionosphere and troposphere, e.g. clouds and cosmic effects; and also sources made by man from e.g. power stations causes noise, affecting the output performance. Its phenomenons contains different processes and physical principles, where the most affecting sources are the effects from both terminals antennas directional characteristics, absorption, scattering, diffraction and reflection. These processes are caused by various obstructions placed in the surroundings or between the receiver and the transmitter, both natural and artificial ones, which creates radio wave propagation properties, described further in the next section.

## **3.3 Radio Wave Propagation Properties**

When an electromagnetic signal is transmitted from a transmitter, the signal can be affected in different ways, e.g. by already given examples. The arrival of this received signal, radio wave, is not arriving as it could be expected. It can arrive

simultaneous through several different paths, which is the multipath case, where each path can differ in distance. This causes a resulting signal at the receiver which is the combination of arriving radio signals, varying depending on the distribution of phases of the total radio signals.

The radio wave propagation properties are laws of nature, where the physical processes that cause these phenomena are the spreading of electromagnetic waves radiated outward in space by the transmitting antenna and affected by the obstructing effects of any natural or man-made objects in the vicinity of the antenna. For these obstructions, the effects causing the errors are statistically distributed in different ways, where the distribution varies depending of location, in which the terrestrial, atmospheric and ionospheric effects differs.

## 3.4 LINK-16

In this thesis a general link is used in the simulations, and no special functionality for a specific link are used. But, as an example and since the most interesting link related for this thesis, is LINK-16, the LINK-16 standard is studied and briefly described in this section. This description focuses on the communication and the relative navigation issues in own subsections, and the architecture and the encryption are not dealt with.

### 3.4.1 General Functionality

LINK-16, also often referred to as TADIL J, is an international military tactical data link used in e.g. many contemporary military aircraft like Gripen, as a military standard nowadays, which is issued by NATO. An example of a military aircraft using the LINK-16 standard is Gripen, which also uses many other links but here LINK-16 are considered as the most current one to study. This standard provides anti-jam communications by several techniques, such as frequency hopping and pseudo noise spreading. Briefly, this standard is an improvement of recent data links functionality, which e.g. are more sensitive to disturbances.

The LINK-16 standard ensures providing of almost real time information to different systems, where these different systems are different defence systems; in the army, the navy, and the air force; which are provided with the same information through a link network. This information is tactical information, e.g. own location and location of unidentified units. There is also no limitation of network participants number set in this case, which can be a limitation in previous links.

For operational use of LINK-16 there are e.g. functionality for surveillance, electronic warfare, air control, navigation, identification, and fighter networking. This LINK-16 functionality can also operate together with other data links which complements and improves the others, which operations are called multi-link operations.

### 3.4.2 Communication

The radio communication terminal components of LINK-16, which distributes the information, is the *Multifunctional Information Distribution system (MIDS)*; or the predecessor, the *Joint Tactical Information Distribution System (JTIDS)*. These terminals use the time division multiple access (TDMA) principle and the TDMA protocol, allocating time slots in the LINK-16 network architecture, in which the requirements for a net control station are eliminated due to the use of TDMA. Its distribution of the information exchange is range dependent, due to propagation issues, and dependent of terminal frequency. If the information can not be exchanged, a terminal can, by a relay design of the terminals, be allowed to send the received information back using another net to be able to circumvent the range dependency.

To be a little bit more precise, the communication uses 1536 time slots in frames of 12 seconds, where the frames are provided by the TDMA scheme. For every transmitted pulse (signal), the terminal transmission frequency is changed and changes pseudorandomly, preventing jamming. Here, the hopping pattern of the frequency defines a net, where all possible nets has different hopping patterns, and uses 51 different frequencies in the *ultra high frequency (UHF)* band. These frequencies changes 77000 times per second, which render an avoidance of narrow jamming, against a jammer (which causes the disturbances) in the narrow band. Another factor is the used waveform, which is designed to improve matching jamming resistance, against a jammer in the matching band. This makes operating in electromagnetic locations possible.

### 3.4.3 Relative Navigation Functionality

Another rendering result of the LINK-16 integration, is that there are accurate LINK-16 TOA measurements. These TOA measurements are here achieved by measurements of the transmissions arrival times. This requires time synchronization of the network for the functionality to work and when units are entering the network, which can be achieved by one unit in the network. This unit acts as one single time synchronization source, for all units, which is called the *Network time reference (NTR)* unit, and is needed to initialize the network functionality, which after synchronization can operate for hours. In this functionality, the network provided position data can be combined with the TOA measurements for identification, where the time synchronization is maintained if the network terminals are provided with this information. This functionality is an automatic functionality and called *relative navigation (RELNAV)*. RELNAV also provides the (geodetic) position data in other units, which results in a position perception of the units in every unit.

The rendering TOA measurements, considered in this thesis, can be used as extra information in e.g. new implementations of navigational solutions, together with corresponding INS/GPS solutions, where the TOA measurements can give a more robust navigation solution in GPS jammed conditions. This solution can use the information from e.g. other aircraft, and land based units which can be added in LINK-16 as tracks or statical points marking (geodetical) positions, since the

land based units can be stationary or translocating. Examples of land based units are military headquarters, bridges, trucks and tanks.

# Chapter 4

## Filter Theory

This chapter gives a theoretical introduction of the applied filter in this master thesis, which is theory coupled to the sensor fusion area. The theory related to the sensor fusion issue is mainly referred to [9], where [8, 13] can be additional references. [13] is more specified to the navigation issue.

### 4.1 Modeling

The following section contains general modeling theory of both dynamics and measurements, which is introduced before introducing any modeling or other theory applying such models, which is the case in the model based filter theory later on.

#### 4.1.1 Modeling of Dynamics

A general non-linear continuous state space model, a dynamical model, is defined as in (4.1), where all variables are continuous.  $x(t)$  are the states,  $u(t)$  the input signals,  $w(t)$  the process noise, and  $\theta$  are the model parameters. This process noise,  $w(t) \sim p_w$ , can affect the states in different ways.

$$\dot{x}(t) = f(t, x(t), u(t), w(t); \theta), \quad (4.1)$$

In discrete time the corresponding general non-linear discrete state space model is following (4.2), which follows the same notation but in each time step.

$$x_{k+1} = f(k, x_k, u_k, w_k; \theta), \quad (4.2)$$

#### 4.1.2 Sensor Modeling

A general non-linear continuous sensor model, modeling the sensor measurements, is modeled as in (4.3), where all variables are in discrete time since measurements are sampled.  $x$  is the continuous state vector,  $u$  the input signals,  $e$  the measurement noise and  $\theta$  the model parameters. This measurement noise  $e \sim p_e$ , affects the measurements in different ways.

$$y(t_k) = h(t_k, x(t_k), u(t_k), e(t_k); \theta), \quad (4.3)$$

The measurement noise is often assumed to be explicit additive and independent, in the special case white noise, where the sensor gives measurement samples as raw data, measured from a measurement reference.

### 4.1.3 Error Modeling

The sensor model and the dynamical model can include error models, i.e. using states representing the errors which are useful to be able to estimate the errors. Therefore, the error models are used to improve the estimations by considering error sources, extending the dynamics of the models without any error modeling. This can for example be external disturbing signals or errors in the sensors.

An error model in the process can be derived from  $\hat{x} = x + \epsilon_w(x, u)$ , where  $x$  is the true states and  $\hat{x}$  the estimated states. An error model in the measurements can be derived from  $\epsilon_e(x, u) = y - h(x, u)$ , where  $y$  is the measurements and  $h(k, u)$  the sensor model without error modeling.

## 4.2 Filter

This section explains briefly how information can be fused by using measurement and dynamical models in a filter solution, applied in this thesis further on. It deals with the theory of the *Kalman filter (KF)*, and mostly the Kalman filter derived *Extended Kalman filter (EKF)* which is the only filter applied in this work. This dynamical fusion process uses the motion model together with the sensor models to predict and estimate the states in these models.

### 4.2.1 The Kalman Filter

The *Kalman Filter (KF)* is the optimal model based filter, if the estimates are unbiased and the process and the measurement noise is explicit additive independent Gaussian noise. In these cases, the Kalman filter minimizes the variances. Therefore it is the best linear unbiased filter, where states in a linear state space model are estimated. This requires that all applied models are linear, since the Kalman filter can not be applied directly when there are non-linearities. Basically the Kalman filter periodically uses one time update using the dynamical models, and one measurement update using the measurement models.

### 4.2.2 The Extended Kalman Filter

But in this case not all models are linear, why the problems of the Kalman filter need to be circumvented. Since the Kalman filter cannot be applied directly, the non-linearities can be linearized by using the Taylor expansion technique before applying the Kalman filter, which gives the *Extended Kalman Filter (EKF)*. It instead estimates the states in a nonlinear state-space model. The EKF algorithm is presented in Appendix A, where the applied measurement jacobian

is defined as  $H_k = \left. \frac{\partial h(x,e)}{\partial x} \right|_{x=\hat{x}_{k|k-1}, e=0}$ , and the measurement noise jacobian as  $G_{e,k} = \left. \frac{\partial h(x,e)}{\partial e} \right|_{x=\hat{x}_{k|k-1}, e=0}$ .

A main difference between the KF and the EKF, is the convergence properties. The KF does always converge independently of initial data, while the EKF might diverge if it is affected to much by the approximations or initial data which is not good enough. This depends on how non-linear the models are.

# Chapter 5

## Modeling

This chapter includes all modeling and describes all models. As a theoretical reference [9] is mainly used here.

### 5.1 Introductory Overview

First of all this chapter concerns modeling of the true aircraft dynamics which generates e.g. true aircraft trajectories, and measurement generation which is used as measurement information, since there are no useful data gathered and to be able to simulate all thinkable scenarios. Secondly, modeling concerning the models applied by the filter solution, that are used in the simulations further on.

As an introductory block scheme overview of the modeling, Figure 5.1, can be inspected, which is divided into two parts. The first one is "Trajectory" where the true aircraft dynamics and trajectories are the generated true kinematical quantities. These true quantities are used as measurement references in the consecutively modeling. The dashed second part is where the true quantities are used to model a modified truth, measurement generation and a model of the already existing blended solution of the INS and GPS information. In "Navigation sensor measurements" the generation of the raw measurements are performed, while the INS and GPS blended solution are modeled in "INS/GPS solution model" to catch the characteristics. These raw measurements are used in the new navigational solution, the developed algorithm, which is denoted as "Algorithm".

### 5.2 True Aircraft Dynamics and Trajectories

This section explains the modeling of the true aircraft dynamics and the generation of the true aircraft trajectories.

Modeling dynamics in 3D is generally quite complicated, which includes the aircraft case. Generally, the aircraft rotational dynamics in 3D can be modeled by using *Euler angles* or *quaternions*, related to the earth. These Euler angles and quaternions forms a transformation matrix each, which relates the rotation



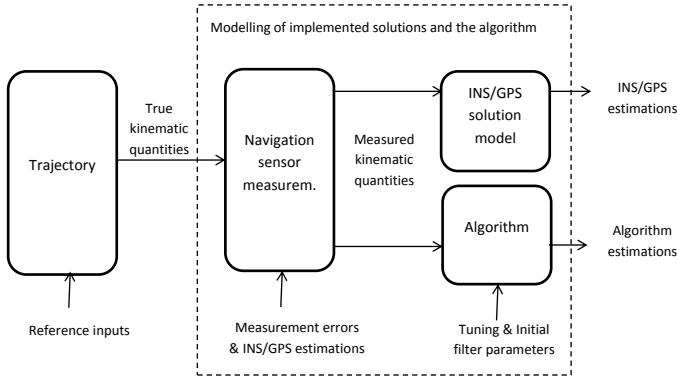


Figure 5.1. An overview of the system

of the aircraft to the earth frame. Briefly, the Euler angle description are easier to understand than the quaternion description, but is not completely satisfying since it can suffer from the disadvantage of not being able to describe all extreme cases of orientations, why quaternions are preferred when describing aircraft orientation.

Basically, the longitudinal aircraft motions are independent of the lateral aircraft motions, but the lateral motions depends of the longitudinal motions. These motion dependencies, and other different aspects related to the motions, can be considered and controlled by using rudders as controllers, which improves the performance of the aircraft dynamics. This improved performance of the dynamics gives the true aircraft motion which is the true trajectory of the aircraft, and the modeled aircraft motion.

### 5.2.1 A Simplified Model in 3D

For simplicity assumptions need to be done, since this work does not focus on aircraft motion modeling, which else would render in far more model complexity. These assumptions can be seen under the assumption paragraph below, which can render in the simplified model which here is described.

The first step is to define the needed coordinate systems the model are using, before the model is presented. As the aircraft here is considered as a dynamical particle,  $z^B$  and  $z^N$ , base vectors from the previously defined BODY and NAV frame, can be defined parallel and both pointing upwards in the local cartesian frame. Then also the horizontal planes of  $\{x^N, y^N\}$  and  $\{x^B, y^B\}$  are parallel, where NAV is fixed in the local cartesian frame

#### Inputs

Here is the second step, where the inputs to the true model below are chosen and defined. These inputs are 1D reference input signals, which are  $a(t)$ ,  $a_z(t)$ , and

$\omega(t)$ .  $a(t)$  is the acceleration in the velocity direction of the aircraft.  $a_z(t)$  is the acceleration in the altitude direction,  $z^B$ .  $\omega(t)$  is the angular velocity about the  $z^B$  axis.

## Continuous Time

As next step the true model can be presented, which uses the different true input references, seen under the input paragraph above. This model decouples the motion in the vertical direction ( $z$ ) from the motions in the horizontal directions ( $x$  and  $y$ ), where the aircraft motion in this local 2D horizontal plane of the local frame is modeled according to a *Coordinated Turn model (CT)*, in this case a polar CT. This continuous model is written in (5.1).

$$\dot{x}^N(t) = v_h(t) \cos(\Psi(t)) \quad (5.1a)$$

$$\dot{y}^N(t) = v_h(t) \sin(\Psi(t)) \quad (5.1b)$$

$$\dot{z}^N(t) = v_z(t) \quad (5.1c)$$

$$\dot{\Psi}(t) = \omega(t) \quad (5.1d)$$

$$\dot{v}(t) = a(t) \quad (5.1e)$$

$$\dot{v}_z(t) = a_z(t) \quad (5.1f)$$

$$\theta(t) = \arcsin\left(\frac{v_z(t)}{v(t)}\right) \quad (5.1g)$$

$$v_h(t) = v(t) \cos(\theta(t)) \quad (5.1h)$$

$v_h(t)$  is the velocity in the  $x^B$  direction, the velocity in the horizontal plane.  $v_z(t)$  is the velocity in the  $z^B$  direction, defined upwards.  $v(t)$  is the velocity in the aircraft motion direction.  $\Psi(t)$  is the angle defined about the  $z^B$  axis, and is defined from the  $x^B$  axis.  $\theta(t)$  is the angle defined about the  $y^B$  axis, defined from the  $x^B$  axis.

## Discrete Time

The continuous CT model above is discretized using the integral definition of discretization. This gives the corresponding discretized CT model and yields as (5.2).

$$x_{k+1}^N = x_k^N + \frac{2v_{h,k}}{\omega_k} \sin\left(\frac{\omega_k T}{2}\right) \cos\left(\frac{\omega_k T}{2} + \Psi_k\right) \quad (5.2a)$$

$$y_{k+1}^N = y_k^N + \frac{2v_{h,k}}{\omega_k} \sin\left(\frac{\omega_k T}{2}\right) \sin\left(\frac{\omega_k T}{2} + \Psi_k\right) \quad (5.2b)$$

$$z_{k+1}^N = z_k^N + T v_{z,k} \quad (5.2c)$$

$$\Psi_{k+1} = \Psi_k + T \omega_k \quad (5.2d)$$

$$v_{k+1} = v_k + T a_k \quad (5.2e)$$

$$v_{z,k+1} = v_{z,k} + T a_{z,k} \quad (5.2f)$$

$$\theta_k = \arcsin\left(\frac{v_{z,k}}{v_k}\right) \quad (5.2g)$$

$$v_{h,k} = v_k \cos(\theta_k) \quad (5.2h)$$

$a_k$ ,  $a_{z,k}$  and  $\omega_k$  are the corresponding discrete input signals. This model is used in the simulations.

### Kinematics

Here useful kinematics are written, the acceleration and velocity expressed in BODY are given by (5.3), where this velocity in the  $y^B$  direction is 0. The position, velocity and acceleration in NAV are written in (5.5). [6] can be used as a reference and introduction to the kinematics.

$$a^B = [a \quad v_h \omega \quad a_z]^T \quad (5.3a)$$

$$v^B = [v_h \quad 0 \quad v_z]^T \quad (5.3b)$$

By using the rotational matrix from BODY to NAV

$$R(\Psi) = \begin{bmatrix} \cos \Psi & -\sin \Psi & 0 \\ \sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5.4)$$

the acceleration and velocity in NAV can be written as (5.5).

$$a^N = R(\Psi) a^B \quad (5.5a)$$

$$v^N = R(\Psi) v^B \quad (5.5b)$$

$$p^N = [x^N \quad y^N \quad z^N]^T \quad (5.5c)$$

### Assumptions

The aircraft motion which gives the true trajectory is in the simulations modeled as a dynamical particle in a local fit cartesian frame, a motion constrained model,

using independent lateral and longitudinal motions. This means that the aircraft can be considered as it does not roll, pitch or slide in the lateral direction. Also, the aircraft altitude dynamics is decoupled, and modeled using the angle  $\theta(t)$  against the horizontal plane.

Different aspects related to the motions can be controlled by using rudders as controllers. E.g. motions due to the motion dependencies can be controlled, which improves the performance of the aircraft dynamics. There are no motions due to the motion dependencies, the motions caused by the dependencies are controlled completely.

## 5.3 Measurement Generation

Here all modeling of the measurement data generation are presented, which are used as measurement information in the filter solution later on.

### 5.3.1 INS Measurements

The measurements of acceleration and angular velocity from the INS has several different errors due to sensor errors in these sensors. These measurements derives from accelerometer and gyro measurements, which got bias and scale factor errors.

A general sensor model for the INS is simply expressed a model of euler angles or quaternions, using INS measurements in the model of the earlier described and general aircraft dynamics.

#### A Simplified Model

The INS model, used to generate INS measurements, can be simplified and modeled as in (5.6).

$$p_{k+1}^N = p_k^N + T_{INS}v_k^N + \frac{T_{INS}^2}{2}a_k^N \quad (5.6a)$$

$$v_{k+1}^B = v_k^B + T_{INS}a_k^B \quad (5.6b)$$

$$\Psi_{k+1} = \Psi_k + T_{INS}\omega_k \quad (5.6c)$$

$$v_k^N = R(\Psi_k)v_k^B \quad (5.6d)$$

$$a_k^N = R(\Psi_k)a_k^B \quad (5.6e)$$

$$\omega_k = \omega_k^T + b_0^{INS,\omega} + e_k^{INS,\omega} \quad (5.6f)$$

$$a_k^B = a_k^T + b_0^{INS,a} + e_k^{INS,a} \quad (5.6g)$$

The input signals are  $\omega_k$  and  $a_k^B$ , which are the measured angular velocity and accelerations in BODY. They are the pure sensor models of the navigation sensors, modeled by using constant biases and Gaussian noise added to the true references.  $\omega_k^T$  is the true angular velocity in the horizontal plane,  $b_0^{INS,\omega}$  the constant bias and  $e_k^{INS,\omega}$  its Gaussian measurement noise.  $a_k^T = a_k$  is the true acceleration in

BODY,  $b_0^{INS,a}$  the constant bias and  $e_k^{INS,a}$  its Gaussian measurement noise. This is a common simple way to model the INS.

All generated measurements from an INS is shown in (5.7), which are used in the simulations with a sample frequency of  $f_{INS} = 60 Hz$ . There are 5 outputs for the simplified INS model; position and velocity in NAV, acceleration in BODY, direction angle in the horizontal plane and angular velocity in the horizontal plane.

$$y_k^p = p_k^N \quad (5.7a)$$

$$y_k^\Psi = \Psi_k \quad (5.7b)$$

$$y_k^v = v_k^N \quad (5.7c)$$

$$y_k^\omega = \omega_k \quad (5.7d)$$

$$y_k^a = a_k^B \quad (5.7e)$$

### INS Error Characteristics

The generated INS measurement errors in  $a_k^B$ ,  $a_0$ , and  $\omega_k$ ,  $\omega_0$ , are modeled with a bias error and independent Gaussian noise as already denoted. This bias characteristics of  $a_0$  is modeled as a uniform distribution,  $\mathcal{U}(-a_{0,max}, a_{0,max})$ , where  $a_{0,max}$  is a parameter given by a *Circular error probability (CEP)* radius, defined in Definition 5.1. These characteristics of  $\omega_0$  is modeled as an uniform distribution, but the default value in the simulations is chosen to be  $0 rad/s$ .

**Definition 5.1** *The Circular error probability of  $P\%$  ( $CEP_P$ ) is the radius in which  $P\%$  of the radial position errors can be found.*

A usual assumption is to assume independent distributed errors in  $x$  and  $y$ . If the distribution is Gaussian which is a common case, the error radius  $\rho$ , is Rayleigh distributed. Here, it is assumed uniform instead.

The probability density function of the used circular uniform distribution in 2 dimensions is

$$f(\rho) = \begin{cases} \frac{1}{\pi R^2} & 2\rho^2 \leq R^2 \\ 0 & 2\rho^2 > R^2 \end{cases} \quad (5.8)$$

where  $\rho$  is the error radius, and  $R$  is the maximum radius.

The probability of a maximum value of  $\rho$  can, by using  $f(\rho)$ , be derived to

$$P(x < \rho) = \begin{cases} \frac{2\rho^2}{R^2} & 2\rho^2 \leq R^2 \\ 1 & 2\rho^2 > R^2 \end{cases}$$

Then the  $CEP_{50\%}$  radius,  $\rho$ , is  $CEP_{50\%} = \frac{1}{\sqrt{2}}R$ , where the transformation to  $x^B$  and  $y^B$  in BODY, is performed just by using a drawn angle from a uniform distribution between 0 and  $2\pi$ .

## Assumptions

Assumptions in the simulations has to be made since this work does not focus on the INS, and a lot of effort would otherwise be needed. The most essential characteristics from the INS are included in the simulations. All INS issues, except a small and constant bias in acceleration and angle velocity, are assumed solved.

There are always measurements available from the INS, available at each time step, where the measurement frequency is assumed constant and exact at  $f_{INS} = 60 \text{ Hz}$ . All INS devices has the same dynamics, where the measurement noise  $e_k \sim \mathcal{N}(0, \sigma)$  is assumed.  $a_0$  and  $\omega_0$ , the measurements biases, in each INS are assumed to be constant. These biases are assumed to be drawn from a uniform distribution,  $\mathcal{U}(-a_{0,max}, a_{0,max})$ , where the values differs for each INS.  $a_{0,max}$  follows the  $CEP_{50}$  value of maximum position error per hour.

### 5.3.2 GPS Measurements

The measurements in position from the GPS, in beneficial conditions, mostly got a relatively very small position error characterized by a bias and some noise. This is the result of the ranging the GPS uses to estimate the position.

Each satellite, which receiver measures the time difference can be modeled by using ranging. The resulting position estimate, is relating the receiver to the earth frame, which can be modeled by using geodetic coordinates.

## A Simplified Model

Since the positions of the satellites are not known here, the model is simplified to just contain the position in 3D; in the  $x$ ,  $y$  and  $z$  direction.

In this work the simplified sensor model (5.9), is used in the simulations, with absolute position instead of ranging.

$$y_k = p_k + b_0^{GPS,p} + e_k^{GPS,p} \quad (5.9a)$$

$$p_k = \begin{cases} x_k \\ y_k \\ z_k \end{cases} \quad (5.9b)$$

$$b_0^{GPS,p} = \begin{cases} x_0^{GPS,p} \\ y_0^{GPS,p} \\ z_0^{GPS,p} \end{cases} \quad (5.9c)$$

$p_k$  is the true position, while  $b_0^{GPS,p}$  is the bias in position and  $e_k^{GPS,p}$  the measurement noise which is assumed to be white noise,

To note here is, if two or several receivers are close to each other, the position bias  $b_0^{GPS,p}$ , should be similar. Different position biases is due to the local atmosphere properties.

### Assumptions

Assumptions in the simulations has to be made since this work does not focus on the GPS, and a lot of effort would else be needed. The most essential characteristics from the GPS are included in the simulations. There are always measurements available from the GPS, when the GPS is available. The measurement sample frequency is  $f_{GPS} = 1Hz$ , assumed to be constant and exact.

The measurement bias in each direction is set to a maximum of 3 meters, since the precision is smaller in reality. In the simulation the bias is drawn from a uniformly distribution,  $\mathcal{U}(-3, 3)$ , in each dimension. This bias is assumed to be constant, and equal if the initial positions are close enough, since e.g. the clock errors are assumed to be constant, there is no time drift in the clocks.

#### 5.3.3 The INS and GPS Solution

The characteristics of the conventional INS and GPS blended solution, includes the characteristics of the INS and GPS measurement information, as described in previous description of the solution.

The earlier description of the INS and GPS solution, points out that the solution generally is a filter solution, an EKF filter, which generates own estimations including e.g. position.

#### A Simplified Model

The existing INS and GPS solution in the simulations is simplified by modeling the characteristics. These characteristics are modeled, in a very simple way, to be able catch the characteristics of the filter behavior. A simple model used in the simulations is written in (5.10), which models the behavior of the position estimations.

$$\hat{p}^{INS/GPS} = \begin{cases} p_t^{GPS}, & t = k^{GPS} \\ p_k^{INS} - b_t^{INS}, & k = k^{INS} \end{cases} \quad (5.10)$$

Here,  $k^{GPS}$  is the time instance of a new GPS measurement, and  $k^{INS}$  is the time instance of a new INS measurement. At each  $k^{GPS}$  the INS bias in position,  $b_t^{INS}$ , is estimated and fixed to  $b_t^{INS} = p_t^{INS} - p_t^{GPS}$ .

### Assumptions

The model used above in (5.10) is approximately correct, catching the main characteristics, why the performance might be decreased compared to the filter solution. The arrival time of the GPS measurements is assumed to arrive at the same time as INS measurements arrives.

#### 5.3.4 Radio Communication Link

The TOA measurements, transformed to range measurements, are characterized by different errors caused by different error sources described previously.

By considering these error sources, the range measurements can be generated, by characterizing all of them. The characteristics can be generated by using descriptions of different stochastic processes, which in its nature can be very complicated in the general case.

### A Simplified Model

The generated range measurements are here modeled as in (5.11), where the TOA sensors of all aircraft forms a TOA network. This catches the basics of the fundamental measurement characteristics in a very simple way.

$$r_k^{i,Link} = r_k^i + b_0^{i,r} + e_k^{i,r} \quad (5.11a)$$

$$r_k^i = \| p^o - p^i \| \quad (5.11b)$$

$$e_k^{i,r} \sim \mathcal{N}(0, \sigma_r) \quad (5.11c)$$

The range measurements are modeled by using a constant range bias and Gaussian noise, added to the true range, for each measurement source.  $r_k^i$  is the true range and  $r_k^{i,Link}$  is the measured range from the own aircraft (o) to aircraft  $i$ .  $b_0^{i,r}$  is the measurement bias and  $e_k^{i,r}$  the Gaussian measurement noise related in the own aircraft to aircraft  $i$ , due to link characteristics.

### Assumptions

Assumptions in the simulations has to be made since this work does not focus on the modeling of the communication noise issues, and a lot of effort would otherwise be needed. The most essential characteristics from the communication issue are included in the simulations.

There are a lot of methods to estimate the errors by modeling, where a remaining error is still present, why an assumption of a pure stochastic distribution can be made. Also, in a complex and mixed environment, the noise is pure stochastic and hard to predict. Therefor a pure probability density function can be assumed to fit.

Gaussian distributions and independent Gaussian noise are assumed to fit for simplicity, and used in the simulations. This stochastical distribution is the same through the simulation, where the TOA measurements from each aircraft are assumed to have a constant range bias independently of location and geometry. Often a slow changing error term is included in the TOA measurements, as in [5]. But in this work the changes is regarded as negligible, since the term is seen as changing slowly enough. The slow changing term is due to several combined influences; such as influence of misreports, propagation delay and clock drift.

The TOA sensor network is assumed synchronized, and the receivers well calibrated, which is a requirement for the information to be usable. No stochastical random measurement losses are assumed to be present in the network; due to signal processing, location, geometry or other aspects rendering random losses. Also, there are always measurements available from the link when the link is usable,



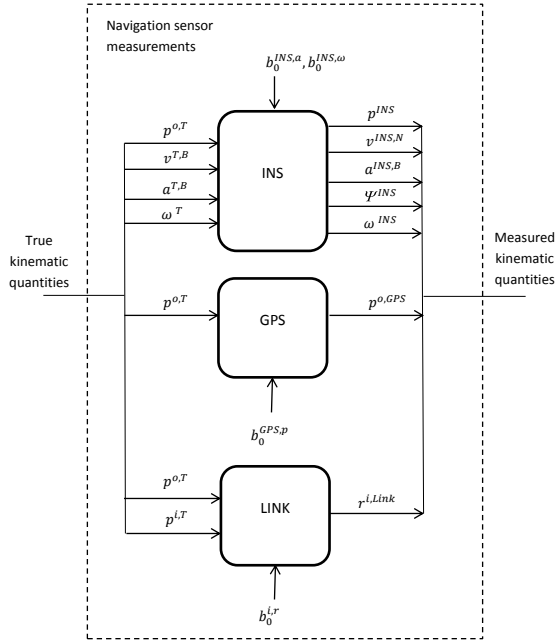


Figure 5.2. An overview of the sensor model block, "Navigation sensor measurements"

where the measurement frequency  $f_{link} = 15 \text{ Hz}$  is assumed constant and exact in the simulations.

## 5.4 Filter

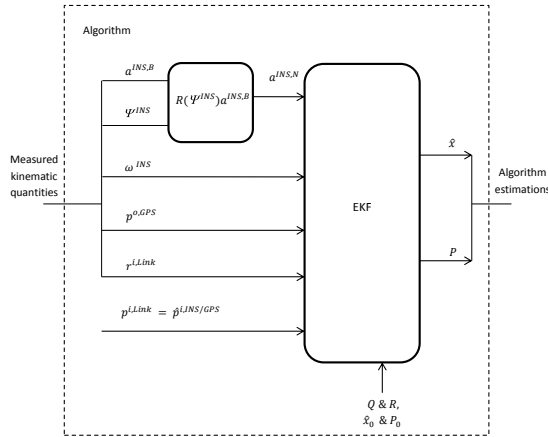
In this section models concerning the new filter solution are derived, which are applied by the EKF algorithm, to be able to perform estimations of the positions and other states. This algorithm uses both data from the INS, the GPS and the link, when available. This algorithm is applied in the decentralized case, which is the case in this thesis.

### 5.4.1 Overview of the Generated Measurements

As a summarizing overview, the previously described generated measurements are shown, which are the used information in the filter. The sensor measurement block is according to Figure 5.1 in this chapter, and Figure 5.2. These sensor model outputs are used as measurement sources in the algorithm measurements.

### 5.4.2 Introductory Overview of the Algorithm

The algorithm block is according to Figure 5.1 previously in this chapter, and Figure 5.3.



**Figure 5.3.** An overview of the algorithm block, "Algorithm"

In this case, no considerations of false improvements need to be taken, since the used information are independent, why the aspect is not discussed. If so, other cautions would have to be taken.

### 5.4.3 Algorithm Filter States

The chosen states in the filter algorithm consists of different kinematical states, describing both translational and rotational dynamics, together with different bias states describing error dynamics, are defined in equation (5.12). These translational kinematic states for each aircraft itself are the translational positions ( $p^o$ ), velocities ( $v$ ) and accelerations ( $a$ ) in NAV, in 3 dimensional cartesian coordinates. Its rotational states are the angle in the horizontal plane ( $\Psi$ ), and its angular velocity ( $\omega$ ). The bias states for the aircraft itself; needed to be able to estimate the biases related to the INS measurement biases; are the acceleration measurement bias ( $b^a$ ) in NAV in 3 dimensional cartesian coordinates, and the measurement bias in angular velocity ( $b^\omega$ ) in the horizontal plane.

Also, if measurements are used from other aircraft, the position for each of them ( $p^i$ ) are included in the state vector together with one range bias state for each of them ( $b^{i,r}$ ). Each bias state is the range bias related to the aircraft measured from, needed to be able to estimate the bias related to the range measurements.

$$x = [x^o \quad x^N]^T \quad (5.12a)$$

$$x^o = [p^o \quad v \quad a \quad \Psi \quad \omega \quad b^a \quad b^\omega] \quad (5.12b)$$

$$x^N = [p^1 \quad \dots \quad p^N \quad b^{1,r} \quad \dots \quad b^{N,r}] \quad (5.12c)$$

$x^o$  are the states for the aircraft itself, and  $x^N$  are the states for all other  $N$  signaling aircraft. The complete state vector in one aircraft filter is  $x$ .

### 5.4.4 Dynamics

Here, both dynamics of the specific aircraft and another aircraft in the specific aircraft are considered and modeled. These dynamical models are applied by the EKF.

#### A Specific Aircraft

The kinematic translational states gives the state space representation for the translational kinematic motion model in 3 dimensions. This model is general and is in the ideal case a pure double integrator. In this case, the translational motions are independent in the motion model. Equation (5.13) defines the used model in continuous time, where  $x^{o,tr} = [p^o \quad v \quad a]^T$ .

$$\dot{x}^{o,tr} = \begin{bmatrix} 0 & I_3 & 0 \\ 0 & 0 & I_3 \\ 0 & 0 & 0 \end{bmatrix} x^{o,tr} + \begin{bmatrix} 0 \\ 0 \\ I_3 \end{bmatrix} w^a \quad (5.13)$$

In discrete time, the used translational kinematic model is defined in equation (5.14).

$$x_{k+1}^{o,tr} = F^{o,tr} x_k^{o,tr} + G_w^{o,tr} w_k^a \quad (5.14a)$$

$$F^{o,tr} = \begin{bmatrix} I_3 & TI_3 & \frac{T^2}{2}I_3 \\ 0 & I & TI_3 \\ 0 & 0 & I_3 \end{bmatrix} \quad (5.14b)$$

$$G_w^{o,tr} = \begin{bmatrix} \frac{T^3}{6}I_3 \\ \frac{T^2}{2}I_3 \\ TI_3 \end{bmatrix} \quad (5.14c)$$

$$w_k^a \sim \mathcal{N}(0, Q^a) \quad (5.14d)$$

Similarly, the used rotational kinematic model in continuous time, where  $x^{o,rot} = [\Psi \quad \omega]^T$ , is defined in equation (5.15).

$$\dot{x}^{o,rot} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x^{o,rot} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w^\omega \quad (5.15)$$

This is in discrete time, defined as in equation (5.16).

$$x_{k+1}^{o,rot} = F^{o,rot} x_k^{o,rot} + G_w^{o,rot} w_k^\omega \quad (5.16a)$$

$$F^{o,rot} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad (5.16b)$$

$$G_w^{o,rot} = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} \quad (5.16c)$$

$$w_k^\omega \sim \mathcal{N}(0, Q^\omega) \quad (5.16d)$$

The used error model for the specific aircraft in continuous time, where  $x^{o,b} = [b^a \ b^\omega]^T$  and  $w^{o,b} = [w^{b,a} \ w^{b,\omega}]^T$ , defined as in equation (5.17).

$$\dot{x}^{o,b} = \begin{bmatrix} I_3 & 0 \\ 0 & 1 \end{bmatrix} w^{o,b} \quad (5.17)$$

This used error model in discrete time for the specific aircraft is (5.18), where  $w_k^{b,a} \sim \mathcal{N}(0, Q^{b,a})$  and  $w_k^{b,\omega} \sim \mathcal{N}(0, Q^{b,\omega})$

$$x_{k+1}^{o,b} = F^{o,b} x_k^{o,b} + G_w^{o,b} w_k^{o,b} \quad (5.18a)$$

$$F^{o,b} = \begin{bmatrix} I_3 & 0 \\ 0 & 1 \end{bmatrix} \quad (5.18b)$$

$$G_w^{o,b} = \begin{bmatrix} T I_3 & 0 \\ 0 & T \end{bmatrix} \quad (5.18c)$$

$$Q^{o,b} = \begin{bmatrix} Q^{b,a} & 0 \\ 0 & Q^{b,\omega} \end{bmatrix} \quad (5.18d)$$

The complete model in discrete time of the specific aircraft is given by equation (5.19).

$$x_{k+1}^o = F^o x_k^o + G_w^o w_k^o \quad (5.19a)$$

$$F^o = \begin{bmatrix} F^{o,tr} & 0 & 0 \\ 0 & F^{o,rot} & 0 \\ 0 & 0 & F^{o,b} \end{bmatrix} \quad (5.19b)$$

$$G_w^o = \begin{bmatrix} G_w^{o,tr} & 0 & 0 \\ 0 & G_w^{o,rot} & 0 \\ 0 & 0 & G_w^{o,b} \end{bmatrix} \quad (5.19c)$$

The complete filter tuning parameter  $Q^o$  of the specific aircraft dynamics is given by (5.20).

$$Q^o = \begin{bmatrix} Q^a & 0 & 0 \\ 0 & Q^\omega & 0 \\ 0 & 0 & Q^{o,b} \end{bmatrix} \quad (5.20a)$$

### Another Associated Aircraft

Another aircraft states, the states associated in the specific aircraft, are in continuous time modeled as in (5.21) for aircraft  $i$ .

$$\dot{p}^i = w^{i,p} \quad (5.21)$$

This model is in discrete time, defined as in (5.22).

$$p_{k+1}^i = F^{i,p} p_k^i + G_w^{i,p} w_k^{i,p} \quad (5.22a)$$

$$F^{i,p} = I_3 \quad (5.22b)$$

$$G_w^{i,p} = I_3 \quad (5.22c)$$

$$w_k^{i,p} \sim \mathcal{N}(0, Q^{i,p}) \quad (5.22d)$$

The used error model, for another aircraft in continuous time, is (5.23).

$$\dot{b}^{i,r} = w^{i,b} \quad (5.23)$$

This used error model in discrete time is defined as in (5.24).

$$b_{k+1}^{i,r} = F^{i,b} b_k^{i,r} + G_w^{i,b} w_k^{i,b} \quad (5.24a)$$

$$F^{i,b} = 1 \quad (5.24b)$$

$$G_w^{i,b} = T \quad (5.24c)$$

$$w_k^{i,b} \sim \mathcal{N}(0, Q^{i,b}) \quad (5.24d)$$

### The Complete Model

The complete model is given by (5.25), in the corresponding order and according to the complete estimated state vector.

$$x_{k+1} = F x_k + G_w w_k \quad (5.25a)$$

$$F = \begin{bmatrix} F^o & 0 \\ 0 & F^N \end{bmatrix} \quad (5.25b)$$

$$F^N = \begin{bmatrix} F^p & 0 \\ 0 & F^b \end{bmatrix} \quad (5.25c)$$

$$F^p = \text{diag}(F^{1,p}, \dots, F^{N,p}) \quad (5.25d)$$

$$F^b = \text{diag}(F^{1,b}, \dots, F^{N,b}) \quad (5.25e)$$

$$G_w = \begin{bmatrix} G_w^o & 0 \\ 0 & G_w^N \end{bmatrix} \quad (5.25f)$$

$$G_w^N = \begin{bmatrix} G_w^p & 0 \\ 0 & G_w^b \end{bmatrix} \quad (5.25g)$$

$$G_w^p = \text{diag}(G_w^{1,p}, \dots, G_w^{N,p}) \quad (5.25h)$$

$$G_w^b = \text{diag}(G_w^{1,b}, \dots, G_w^{N,b}) \quad (5.25i)$$

The filter tuning parameter  $Q$ , is given by (5.26).

$$Q = \begin{bmatrix} Q^o & 0 & 0 \\ 0 & Q^p & 0 \\ 0 & 0 & Q^b \end{bmatrix} \quad (5.26a)$$

$$Q^p = \text{diag}(Q^{1,p}, \dots, Q^{N,p}) \quad (5.26b)$$

$$Q^b = \text{diag}(Q^{1,b}, \dots, Q^{N,b}) \quad (5.26c)$$

### 5.4.5 Measurements

The chosen measurement models, used as models by the filter, are considered and presented here. They are describing the internal INS and GPS measurements, together with the external supporting INS/GPS position and the TOA measurements achieved from the link data from the other aircraft as external sources, by using the filter states.

#### INS

The own internal INS measurements are in the filter modeled, by using the filter states, as in (5.27), where the internal bias states are included, as added biases to the kinematical states of acceleration and angular velocity.

$a_k^m$  is the, in the filter, modeled acceleration measurement of the INS transformed into NAV; and  $\omega_k^m$  is the, in the filter, modeled angular velocity measurement in the horizontal plane. And, the noise  $e_k^a$  and  $e_k^\omega$  are assumed Gaussian, which covariances are tuning parameters in the filter.

$$a_k^m = a_k + b_k^a + e_k^a \quad (5.27a)$$

$$\omega_k^m = \omega_k + b_k^\omega + e_k^\omega \quad (5.27b)$$

$$e_k^a \sim \mathcal{N}(0, R^a) \quad (5.27c)$$

$$e_k^\omega \sim \mathcal{N}(0, R^\omega) \quad (5.27d)$$

Its measurement jacobian derived from these measurement models in  $h_k^{o,INS} = [a_k + b_k^a \quad \omega_k^m + b_k^\omega]^T$ , are then written as in (5.28), which follows the state vector representation, and where  $H^{o,INS} = \left. \frac{\partial h^{o,INS}}{\partial x} \right|_{x=\hat{x}_{k|k-1}, e=0}$ .

$$H^{o,a} = [I_3 \quad 0_{3 \times 2} \quad I_3 \quad 0_{3 \times 1}] \quad (5.28a)$$

$$H^{o,\omega} = [0_{1 \times 4} \quad 1 \quad 0_{1 \times 3} \quad 1] \quad (5.28b)$$

$$H^{o,INS'} = [H^{o,a} \quad H^{o,\omega}]^T \quad (5.28c)$$

$$H^{o,INS} = [0_{4 \times 6} \quad H^{o,INS'}] \quad (5.28d)$$

Following the assignments above, the noise vector is  $e_k^{INS} = [e_k^a \quad e_k^\omega]^T$ , which gives the covariance matrix with the tuning parameters,

$$R^{o,INS} = \begin{bmatrix} R^a & 0_{3 \times 1} \\ 0_{1 \times 3} & R^\omega \end{bmatrix}.$$

The acceleration measurements from the INS in BODY, are transformed by using the INS derived angle  $\Psi_k^{INS}$ , into  $a_k^{INS} = R(\Psi_k^{INS})a_k^{INS,B}$  in NAV, and  $\omega_k^{INS}$ , which are used as filter measurement information inputs. A measurement vector, which is used and includes these measurement inputs derived from the INS, is  $y_k^{INS} = [a_k^{INS} \quad \omega_k^{INS}]^T$ .

Notice that the direction angle,  $\Psi_k^{INS}$ , here is used when transforming the acceleration from BODY to NAV. As an option, and commonly in navigation, the filter estimate  $\hat{\Psi}_k$  can be used instead. But the chosen angle is for this purpose good enough.

## GPS

The own internal GPS measurements are modeled as in (5.29), which jacobian is derived in the same way as for the INS measurements above, where  $h_k^{o,GPS} = p_k^o$ , and  $R^{o,GPS} = R^{o,p}$ .

$$p_k^{o,m} = p_k^o + e_k^{o,p} \quad (5.29a)$$

$$H^{o,p} = I_3 \quad (5.29b)$$

$$H^{o,GPS} = [H^{o,p} \quad 0_{3 \times 12}] \quad (5.29c)$$

$$e_k^{o,p} \sim \mathcal{N}(0, R^{o,p}) \quad (5.29d)$$

The actual used measurements derived from the GPS, used as measurement information input in the filter, is the estimated GPS position  $y_k^{GPS} = p_k^{GPS}$ .

## Range

The TOA measurements for aircraft  $i$  are, after transforming to range, modeled as in (5.30), where  $h_k^{i,r} = \|p_k^o - p_k^i\| + b_k^{i,r}$  and  $R^{i,r}$  is a tuning parameter.

$$r_k^{i,m} = \|p_k^o - p_k^i\| + b_k^{i,r} + e_k^{i,r} \quad (5.30a)$$

$$e_k^{i,r} \sim \mathcal{N}(0, R^{i,r}) \quad (5.30b)$$

$h_k^{i,r}$  is the measurement model used by the filter, which is the modeled range measurement from aircraft  $i$ , shown in (5.31). Its corresponding tuning parameter of the measurement covariance is  $R^{i,r}$ , where all tuning parameters of different  $i$  can be assumed to be equal. This means that the tuning parameter is  $R^{Link,r}$ .

$$r_k^m = [r_k^{1,m} \quad \dots \quad r_k^{N,m}]^T \quad (5.31a)$$

$$e_k^{Link,r} = [e_k^{1,r} \quad \dots \quad e_k^{N,r}]^T \quad (5.31b)$$

$$R^{Link,r} = \text{diag}(R^{1,r}, \dots, R^{N,r}) \quad (5.31c)$$

Derivation of the modeled range measurements jacobian  $H_k^{Link,r}$ , is performed in the same way as previously, which in this case results in (5.32), where  $N$  is the available measurement sources.

$$H_k^{Link,r} = [H_k^o \quad H_k^r \quad H_k^b] \quad (5.32a)$$

$$H_k^o = [H_k^{1,o} \quad \dots \quad H_k^{N,o}]^T \quad (5.32b)$$

$$H_k^r = [H_k^{1,r} \quad \dots \quad H_k^{N,r}]^T \quad (5.32c)$$

$$H_k^b = \text{diag}(H_k^{1,b}, \dots, H_k^{N,b}) \quad (5.32d)$$

$$H_k^{i,o} = \left[ \frac{(p_k^o - p_k^i)^T}{\|p_k^o - p_k^i\|} \quad 0_{1 \times 12} \right] \quad (5.32e)$$

$$H_k^{i,r} = - \left[ 0_{1 \times 3(i-1)} \quad \frac{(p_k^o - p_k^i)^T}{\|p_k^o - p_k^i\|} \quad 0_{1 \times 3(N-i)} \right] \quad (5.32f)$$

$$H_k^{i,b} = 1 \quad (5.32g)$$

$$i = 1, \dots, N \quad (5.32h)$$

The actual used range measurements from the network used as measurement input to the filter is  $y_k^{TOA} = [r_k^{1,TOA}, \dots, r_k^{N,TOA}]^T$ .

### INS/GPS Position

The INS/GPS position measurements for aircraft  $i$ , are modeled as in (5.33), where  $h_k^{i,p} = p_k^i$  and the measurement covariance matrix,  $R_k^{i,p}$ , is a tuning parameter.

$$p_k^{i,m} = p_k^i + e_k^{i,p} \quad (5.33a)$$

$$e_k^{i,p} \sim \mathcal{N}(0, R_k^{i,p}) \quad (5.33b)$$

A vector containing the filter modeled INS/GPS position measurements, is the measurement model used by the filter, which is  $h_k^{Link,p}$ , and is written in (5.34).

The tuning parameter,  $R_k^{i,p}$ , only depends on if the GPS is usable for aircraft  $i$  or not, which here means it can be assigned two different values. These different values can be assumed to be equal for all of the tuning parameters in all aircraft, which gives only one tuning parameter with two possible values. There is a tuning parameter matrix in each aircraft, containing the value of each aircraft  $i$  in the diagonal, which is  $R_k^{Link,p}$ . If the measurement source got usable GPS available,  $R_k^{i,p} = R_k^{i,GPS}$  is used. Else, if GPS is not usable,  $R_k^{i,p} = R_k^{i,-GPS}$  is used, which is chosen to be a greater value, where an explanation can be given in the filter tuning paragraph below.

$$p_k^m = [p_k^{1,m} \quad \dots \quad p_k^{N,m}]^T \quad (5.34a)$$

$$e_k^{Link,p} = [e_k^{1,p} \quad \dots \quad e_k^{N,p}]^T \quad (5.34b)$$

$$R_k^{Link,p} = \text{diag}(R_k^{1,p}, \dots, R_k^{N,p}) \quad (5.34c)$$



The jacobian  $H_k^{Link,p}$  is derived, like previously, which in this case results in (5.35). For now, the internal states,  $x^o$ , are not included in  $H_k^{Link,p'}$  since here they are not used in  $h_k^{Link,p}$ .

$$H_k^{Link,p'} = [H_k^p \quad 0_{3N \times N}] \quad (5.35a)$$

$$H_k^p = \text{diag}(H_k^{1,p}, \dots, H_k^{N,p}) \quad (5.35b)$$

$$H^{i,p} = I \quad (5.35c)$$

$$i = 1, \dots, N \quad (5.35d)$$

The actual measurement vector, with measurements from the INS/GPS models and used by the filter as measurement input is  $y_k^{INS/GPS} = [p_k^{1,INS/GPS}, \dots, p_k^{N,INS/GPS}]^T$ .

### 5.4.6 The Complete Model

To summarize and end the filter measurement modeling, the earlier derived models are nested together as one measurement model. This complete measurement model follows (5.36), if all of the measurements are available.

$$h_k = [h_k^{INS} \quad h_k^{GPS} \quad h_k^{Link,r} \quad h_k^{Link,p}]^T \quad (5.36)$$

The jacobians required by the filter measurement update are  $H_k = \frac{\partial h(x,e)}{\partial x} \Big|_{x=\hat{x}_{k|k-1}, e=0}$  and  $G_{e,k} = \frac{\partial h(x,e)}{\partial e} \Big|_{x=\hat{x}_{k|k-1}, e=0}$ , where  $e$  is assumed to be independent, explicit and additive Gaussian noise, is given by (5.37).

$$H_k = [H^{INS} \quad H^{GPS} \quad H_k^{Link,r} \quad H_k^{Link,p}]^T \quad (5.37a)$$

$$H^{INS} = [H^{o,INS} \quad 0_{3 \times 4N}] \quad (5.37b)$$

$$H^{GPS} = [H^{o,GPS} \quad 0_{3 \times 4N}] \quad (5.37c)$$

$$H_k^{Link,p} = [0_{3N \times 15} \quad H_k^{Link,p'}] \quad (5.37d)$$

$$G_{e,k} = I \quad (5.37e)$$

The earlier described measurement tuning parameters are combined together into a complete tuning parameter matrix  $R = R_k$ , which is given by (5.38).

$$R_k = \begin{bmatrix} R^{o,INS} & 0 & 0 & 0 \\ 0 & R^{o,GPS} & 0 & 0 \\ 0 & 0 & R^{Link,r} & 0 \\ 0 & 0 & 0 & R_k^{Link,p} \end{bmatrix} \quad (5.38)$$

## 5.5 Filter Tuning

The parameters that has to be set are the covariance of the measurement noise,  $e$ , and the process noise in the state propagation,  $w$ , where  $w$  for example can be interpreted as external disturbances affecting the dynamics. These parameters are the state propagation covariance matrix,  $Q$ , and the measurement noise covariance matrix,  $R$ .

As a usual assumption of the filter models is that  $e$  and  $w$  are represented as white noise. Then if, in the linear case,  $e \sim \mathcal{N}(0, R)$  and  $w \sim \mathcal{N}(0, Q)$  is the real case, the Kalman filter is the optimal filter as already denoted. These circumstances are not correct, but the measurement noise is often reasonable to be approximated as white noise, while the process noise commonly cannot be. It is mostly too far from the truth to assume that the process noise is white, and it can often be seen as a design variable. E.g. all used bias models in this thesis uses random walk descriptions, which of course is not the case, why the process noise can be seen as design variable instead.

Basically,  $Q$  states how reliable the applied model is considered to be, and  $R$  states how reliable the measurements are considered to be. As  $Q$  increases the more unreliable the model is stated to be, and more considerations of the measurements are taken. Similarly, by increasing  $R$ , the measurements are stated to be affected more by the measurement noise, where more consideration of the model is taken. This means that a fast and sensitive filter is achieved when a large  $Q$  is chosen, while a slow and unsensitive filter is achieved when a large  $R$  is chosen, and a compromise has to be done.

Since the dynamical models applied in this filter solution does not use any measurements as input signals, the process noise is not described by using any measurement noise description at all. The filter instead uses pure process noise to describe the uncertainties in the dynamics, by the consideration of uncertainties, as in e.g. the acceleration model,  $\dot{a}^N = w^a$ . This process noise covariance,  $Q$ , in this work is assumed to be constant, while some measurement noise covariances in  $R$  can differ as already described in previous paragraphs in this section. For simplicity an assumption of all covariance matrices being seen as diagonal, which means that the sensors respectively the process noises are uncorrelated.

The initial data required to be set for the initialization of the filter, the initial states and its uncertainties has to be chosen. These are in this thesis given by using the corresponding true quantities of the kinematics, setting the bias states to 0, and by using uncertainties corresponding reasonable values from e.g. the use of the INS and the GPS in beneficial conditions. Commonly for the initial uncertainties, they can be set very low in this case since the initial states are true.

## 5.6 Practical Algorithm Issues

There is a lot of practical aspects and issues related to and concerning usage of filters. Time aspects such as irregular arrivals of the measurements, both asynchronous measurements and irregularities in measurement frequency from one specific sensor are some examples, which are not dealt with. But, in practice issues

like these are common, and there are modified filters and algorithms which handles the problems. These modifications are needed to, i.e. be able to handle unsynchronized processes and handle the case of no measurements available.

In this thesis, none of these aspects are involved. The filter handles the measurements as synchronous processes, where at least the measurements from the INS is available every measurement update. In this case no distortion is present and other aspects concerning the sampling are assumed to be ok. All measurement frequencies are synchronized with the filter update frequency.

Also the differences in positions caused by the motion during the data transfers are neglected. It is assumed momentarily, hence a high frequency is used and the speed of light is relatively much higher than the speed of the motion.

# Chapter 6

## The Simulation Environment

The chapter gives an explanation of the simulation environment and describes how the simulation environment is built up and implemented, mostly in terms of implementation structure. A helpful manual as a reference can here be [10], which e.g. deals with the implementation issue of Kalman filter algorithms in MatLab<sup>®</sup>. It deals with both theoretical and practical aspects of Kalman filtering, but mainly the practical aspects.

### 6.1 Introduction

Basically, the simulation environment is modular and consists of three block modules; initialization, running and evaluation. A complete and not detailed structure is shown in Figure 6.1, which shows how the simulation environment is built up. It also shows how one simulation uses the initialization and running blocks, and how several simulations are performed, basically. Simulated position estimations and true trajectories are saved and able to be used in the evaluation block, after all simulations are performed.

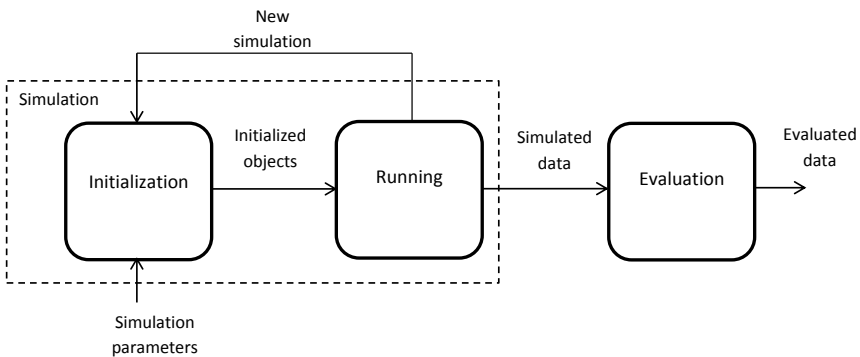


Figure 6.1. Overview of the simulation blocks

Briefly, the initialization block initializes all objects and properties, the running block runs and updates the simulation, and the evaluation block is a separate script performing the evaluation of the simulation data. In the running block, which is an open loop implementation, the simulation is updated by using the inputs given from the initialization.

## 6.2 Description

This section gives a more detailed description of how the simulation environment is implemented and what it consists of. Details of the initialization block is put in focus, since it is the most important block to understand, because it handles all of the simulation parameters which are required to be able to define the simulations.

The simulation environment is object oriented, and uses the object oriented programming in MatLab<sup>®</sup>, in which it is implemented. These objects are implemented by using the MatLab<sup>®</sup> class definition function *classdef*. They consists of several properties and methods (object functions), where the properties and methods are necessary to be able to initialize, and run the simulations. For a deeper explanation of how the object oriented programming works in MatLab<sup>®</sup>, see the MatLab<sup>®</sup> help manual.

As a basic explanation, the object oriented implementation of the simulation environment uses two different objects, and additional functions which are required to perform the simulations. In this case, a general object for each physical node, and one link object for calculating the true relative distances and handling the link data between all nodes are used. These physical nodes are aircraft objects, which uses the link object as in Figure 6.2. More about this implementation and many details can be found in the text files appended to the implementation.

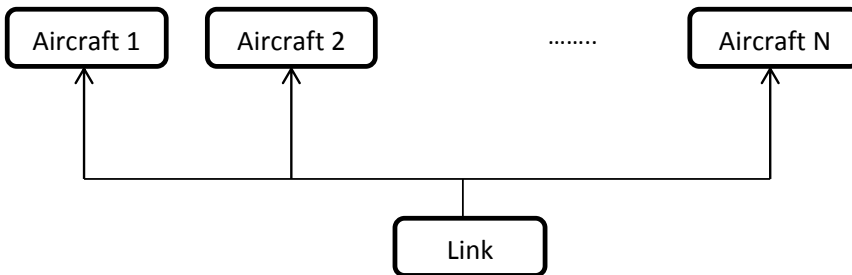
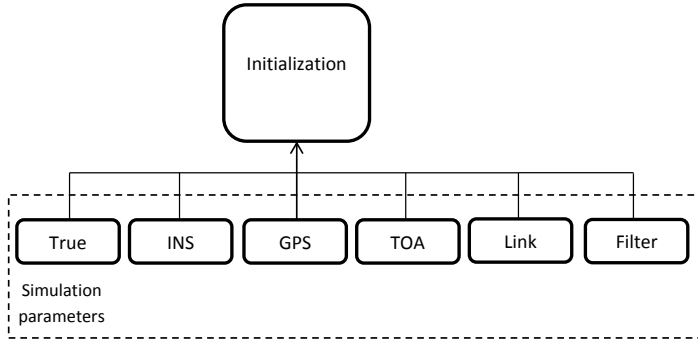


Figure 6.2. Overview of the simulation objects

### 6.2.1 Initialization

To be able to get started with simulations, inputs defined in different input files need to be defined. When the simulation initializes, the objects are created and

initialized by using these configured input parameters from external functions as in Figure 6.3.



**Figure 6.3.** Overview of the initializing simulation block

The input parameters in "Simulation parameters" defines the simulations, the objects and their properties; i.e. how many aircraft objects the simulation shall contain, how the simulation shall behave and for how long the simulation will run. In the initialization block all necessary parameters, needed to be able to run the simulation, are allocated. These parameters comes from the "True" block which gives all true inputs, the "INS", "GPS", and "TOA" block which gives all measurement parameters, the "Link" block gives information about how the communication during the simulation are to be handled given as parameters, and finally the "Filter" block which gives all of the filter parameters. More descriptions of e.g. how to use this can be found in the text files in the implementation.

## True

From here, the parameters needed to generate specific trajectories due to the aircraft motion modeling are specified. This includes initial data of the kinematics and the true inputs like the number of aircraft. The input information are defined in a separate file called from each aircraft object.

## INS

Here, the INS characteristics are specified including how the noises and the biases in acceleration and angular velocity are drawn. An input file is to be used for desired decisions of the parameters, called from each aircraft object.

## GPS

The GPS characteristics are here defined, by choosing how the GPS position biases and noises are drawn. These biases can also be chosen as identical for several

different aircraft, and are to be defined in a separate input file which is called from each aircraft object.

### TOA

The TOA measurements characteristics in the aircraft are chosen from here, by choosing how the TOA biases and noises are drawn, which are defined in a separate input file called from each aircraft object.

### Link

Defining the communication of different aircraft objects are performed from here. By defining the allowance of transmitting and receiving for different aircraft, both initially and during the simulation, a general communication availability is achieved. The input file is the only input file called from the link object, but the link object in turn provides the aircraft object with some of the data. The link object controls everything related to receiving, while the aircraft controls whether it is transmitting or not during the simulation.

### Filter

The filter parameters are defined in one separate file, from which it is possible to call and use already available true parameters as initial data input, if desired. From here, the algorithm in the aircraft is provided with initial data and tuning parameters.

## 6.3 Algorithm Implementation

This section gives a briefly explanation of how the algorithm is implemented. Basically, the EKF algorithm itself is implemented straight forward. But the measurement models and sensor data handling need to be explained briefly, to be able to proceed with extensions of the algorithm with additional measurement models using additional sensor data.

The EKF algorithm gets new measurements from pipes, which are filled when new measurements are put in them. External and internal measurements has their own pipes in each aircraft object, which is filled with new data after every new measurement. By creating new pipes similar to the already implemented ones, the algorithm can be provided with new measurements. These pipes can be arbitrary long if the fill frequency is larger than the get frequency, e.g. it can be used to avoid aliasing if required. Already implemented pipe generally contains pipe data which consists of the measurements and the absolute measurement times. But the link pipes containing the external link pipe data also consist of aircraft ID's and GPS indicator flags telling if the source of the INS/GPS transmitted position got GPS available or not, together with the external measurements.

In the already implemented algorithm, the filter measurement model can vary in size dependently of what measurement update there is to perform. An exten-

sion of the measurement model will work in the same manner, by extending the measurement model, vectors and matrices, in the same way if there are measurements available. Extensions of the already implemented measurement model are therefor easy to perform.



# Chapter 7

## Simulations and Scenarios

To be able to verify the implementations and to perform studies of interesting scenarios, simulations need to be performed. This chapter presents and describes different scenarios as examples, some mostly to verify the implementation and some mostly related to real operative scenarios which are simulated and studied.

### 7.1 Definition of Scenarios

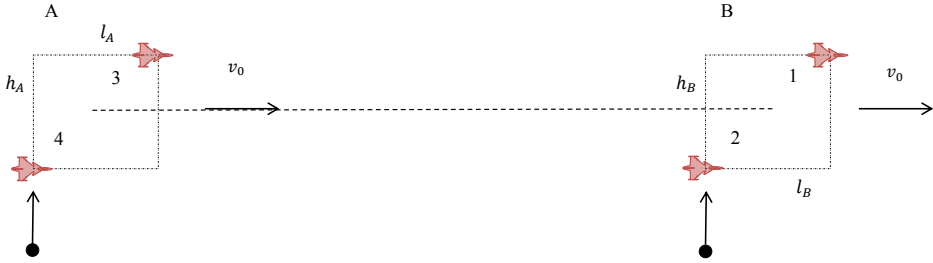
Several different types of scenarios are brought and studied, where the basics of the scenarios related to the real operative scenarios are originally defined by SAAB, from which further definitions are done. These are the most interesting scenarios to study.

Also some, completely own defined, scenarios are brought and studied. These scenarios are developed to be able to study and verify the implementation in more beneficial conditions. But these scenarios can also be seen as related to realistic operative scenarios. These are also interesting to study, since it gives hints of how well the navigational performance in these conditions can be.

#### 7.1.1 General Definitions

In the scenarios related to the realistic operative scenarios one basic formation consisting of a group of 4 aircraft, divided into 2 groups of 2 aircraft each, is defined and used. This formation follows Figure 7.1. In this case,  $h = l = 200\text{ m}$  is set for both  $A$  and  $B$ , and the distance between them is set to  $50\text{ km}$ , where all aircraft are located and flies at the same constant altitude.

Another studied case, is when only one aircraft are used and where landmarks are used to support the aircraft navigational performance. Also an extended combination of the scenarios are defined and studied, which includes the group of 4 aircraft and supporting landmarks. For these cases the altitude is considered as constant and equal for all of the aircraft and landmarks too. This is of course not the case in practice, but can be considered as the case because of e.g. the relatively long distances between the landmarks and the group of aircraft.



**Figure 7.1.** The base formation where 4 aircraft are used

The default speed are constant and chosen to be  $v = 350 \text{ m/s}$  ( $\approx 1 M$ ), and the headings to be constant and equal for all aircraft moving in the horizontal plane. By choosing the speed to approximately  $1 M$ , an optimal speed due to other aspects is not taken into account, like control system aspects. But, since the navigational aspects studied in this thesis for simulation, are not affected by the speed, the choice is still suitable.

Initially, in all studied simulations all aircraft got GPS performance. During the simulations different events happens, related to the GPS and the link communication. Here, the common case is that the aircraft loses their GPS performance during the simulations.

## 7.2 Scenario A

As an introductory scenario a simple scenario is tried out in a beneficial geometry, where landmarks are used as navigational support.

### Description

A describing figure of scenario A is Figure 7.4. The purpose of the scenario is mostly to verify the implementation by a beneficial scenario, where 5 landmarks are used to support 1 aircraft.

### Data

- GPS is lost when the aircraft is at  $30 \text{ km}$
- Simulation is completed when the aircraft has reached  $400 \text{ km}$

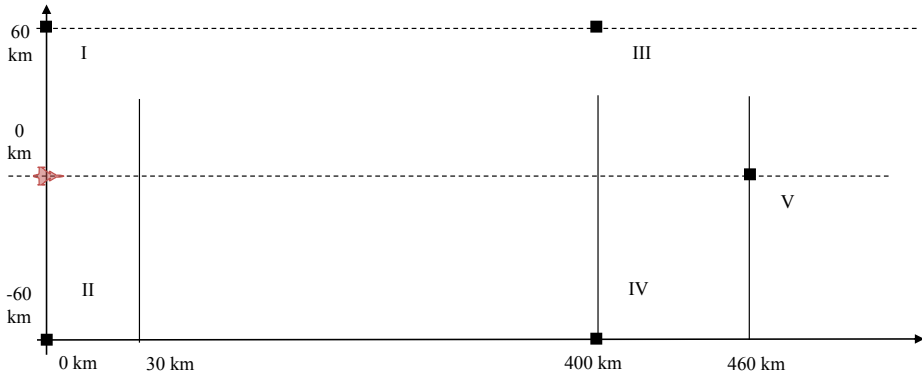


Figure 7.2. Describing figure of scenario A

### 7.3 Scenario B

As an additional and introductory scenario this scenario is brought in another beneficial geometry.

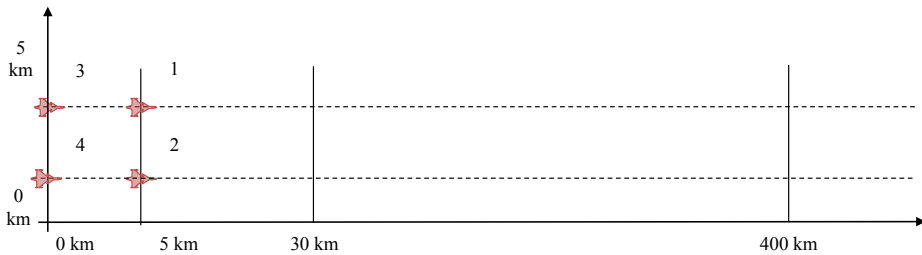


Figure 7.3. Describing figure of scenario B

#### Description

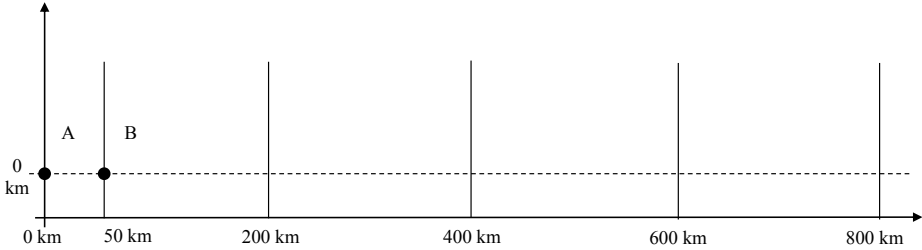
A describing figure of scenario B is Figure 7.3. The purpose of the scenario is mostly to verify the implementation by a beneficial scenario, where 4 single aircraft are used, and each aircraft uses information from the other aircraft as navigational support.

#### Data

- GPS is lost for all aircraft when the last 2 aircraft are at 30 km
- Simulation is completed when the 2 last aircraft has reached 400 km

## 7.4 Scenario 1

In this scenario only the 4 group is used, which uses each other for navigational support.



**Figure 7.4.** Describing figure of scenario 1

### Description

A describing figure of scenario 1 is Figure 7.4.

Using the 4 aircraft as in Figure 7.1, the purpose is to study and evaluate the performance when the GPS becomes not usable in different time instances for aircraft 1 to 3.

### Data

- GPS is lost for aircraft 1, 2 and 3
  - Aircraft 1:** When aircraft 2 is at 200 km
  - Aircraft 2:** When aircraft 2 is at 400 km
  - Aircraft 3:** When aircraft 4 is at 600 km
- Simulation is completed when A reached 800 km

## 7.5 Scenario 2

This scenario uses only one aircraft where landmarks are used as navigational support.

### Description

A describing figure of scenario 2 is Figure 7.5.

The purpose is to study and evaluate how landmark support affects the performance by using this geometry, where the aircraft GPS malfunctions, and later on the aircraft loses radio contact with landmark 1 in the simulation.

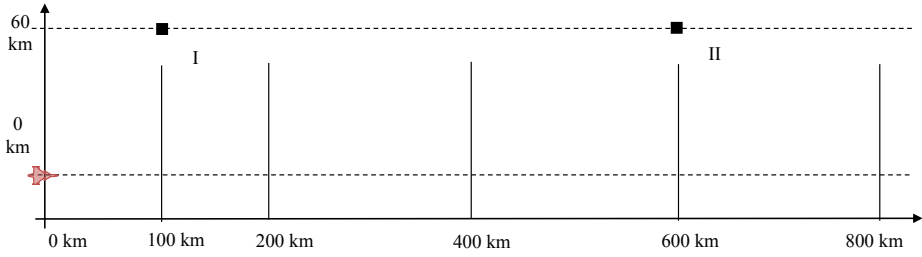


Figure 7.5. Describing figure of scenario 2

Data

- The GPS in the aircraft malfunctions at 200 km
- Radio contact with landmark I is lost at 400 km
- Simulation is completed at 800 km

## 7.6 Scenario 3

This scenario also uses only one aircraft where landmarks are used as navigational support, but in a different geometry and where other events occur.

Description

A describing figure of scenario 3 is Figure 7.6.

The purpose is to study and evaluate effects due to geometry of the landmarks. Initially the aircraft signals with landmark I. During the simulation the aircraft GPS malfunctions and later on starts to signal with landmark II.

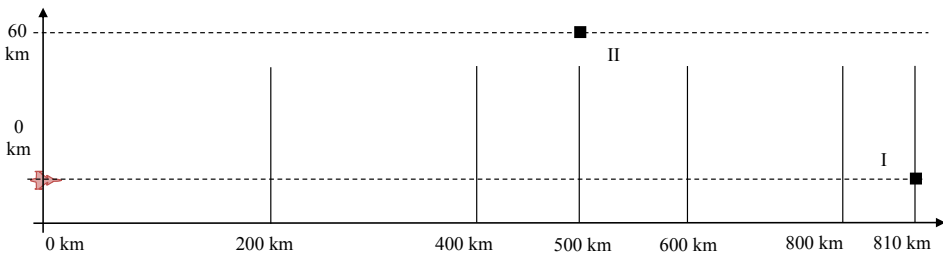


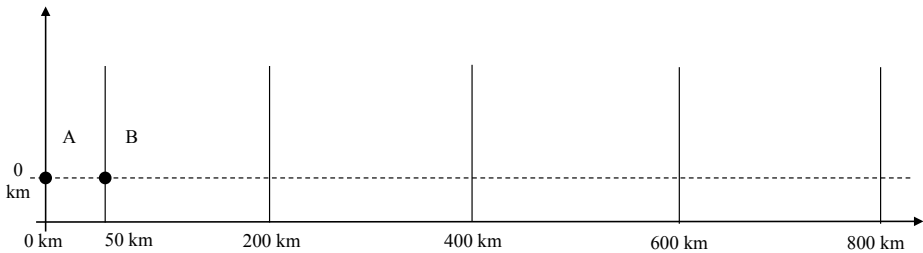
Figure 7.6. Describing figure of scenario 3

**Data**

- The GPS in the aircraft malfunctions at 200 *km*
- Radio signaling with landmark *II* starts at 400 *km*
- Simulation is completed at 800 *km*

**7.7 Scenario 4**

In this scenario only the 4 group is used, which uses each other for navigational support. This is similar to Scenario 1, where the geometry is identical, but where other events occur.



**Figure 7.7.** Describing figure of scenario 4

**Description**

A describing figure of scenario 4 is Figure 7.7.

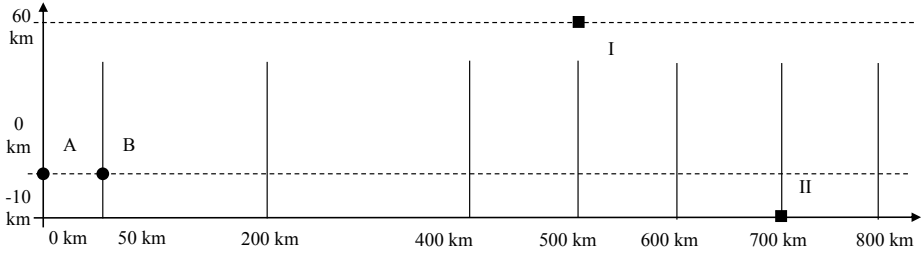
The purpose is to study and evaluate how the performance is affected when the group of 4 aircraft GPS is disturbed, and they all loses their GPS performance instantly.

**Data**

- GPS is disturbed for all aircraft when *A* at 200 *km*
- Simulation is completed when *A* at 800 *km*

**7.8 Scenario 5**

In this scenario the 4 group and 2 supporting landmarks are used, where each aircraft uses both information from the other aircraft and the landmarks for navigational support. This uses the occasions from Scenario 4, but the aircraft are here using information from landmarks too, where the landmarks are arranged in a similar way as in Scenario 3.



**Figure 7.8.** Describing figure of scenario 5

### Description

A describing figure of scenario 5 is Figure 7.8.

The purpose is to study and evaluate the performance after the GPS is disturbed for all aircraft, and starts to communicate with landmarks. During the simulation the GPS is disturbed for all aircraft. The aircraft does not communicate with the landmarks initially.

### Data

- GPS is disturbed for all aircraft when  $A$  at  $200\text{ km}$
- Landmark  $I$  starts to signal with all aircraft when  $A$  at  $400\text{ km}$
- Landmark  $II$  starts to signal with all aircraft when  $A$  at  $600\text{ km}$
- Simulation is completed when  $A$  reached  $800\text{ km}$

# Chapter 8

## Results and Conclusions

In this chapter the achieved results from the simulations of the scenarios are presented. The results are the evaluations of the simulations, and these results are here used to make conclusions of the performance in each of the scenarios, where also general conclusions are done and expectations discussed.

### 8.1 Introduction

The defined scenarios are each simulated by using *Monte Carlo* simulations, using the defined models, to be able to evaluate the performance in terms of mean values during the simulations. Many of the scenarios are simulated  $N = 20$  times while some are only simulated once as a verifying result. These scenarios only simulated once are scenario A and B. But in all cases the performance when using link aiding information, are to be compared with the performance when this information is not used. It is performed by simulating the same number of simulations again, where the link information is not used. This is what is done when comparing the absolute position performance, where each comparison is shown in the same figure, and the unaided results drifts away quadratically and smooth.

The filter data from all simulations of one scenario, are used to evaluate the performance of the scenario. A theoretical best case of absolute position error growth using link and INS data in favorable geometries without the GPS, is a scaled error growth when using only INS data. This theoretical scaling is in the best case proportional to  $1/\sqrt{N}$ , where  $N$  here is the number of external data sources, derived from statistical sensor fusion criteria. But here often the most interesting results to study are the relative position performance.

### 8.2 Evaluation

The evaluation concerns position data which are each aircraft own position data in  $2D$ , and the relative position between them in  $2D$ . These errors in relative



and absolute position during the simulations, are the most interesting quantities, which are the main results.

By applying the *root mean squared error (RMSE)*, a statistical method to estimate the standard deviation, the position data can be used to evaluate how accurate each position estimation are through the simulations. But to note from the simulations generally, there is a drift in altitude because of no data for altitude stabilization are present, opposite from reality. Only the already chosen sensors in this work are used. This, of course, affects the other results.

### 8.3 Scenario A

As there is only one aircraft to study, and the positions of the landmarks absolute positions are assumed to be exact, the absolute position performance both represents the absolute position performance and the relative position performance to the landmarks.

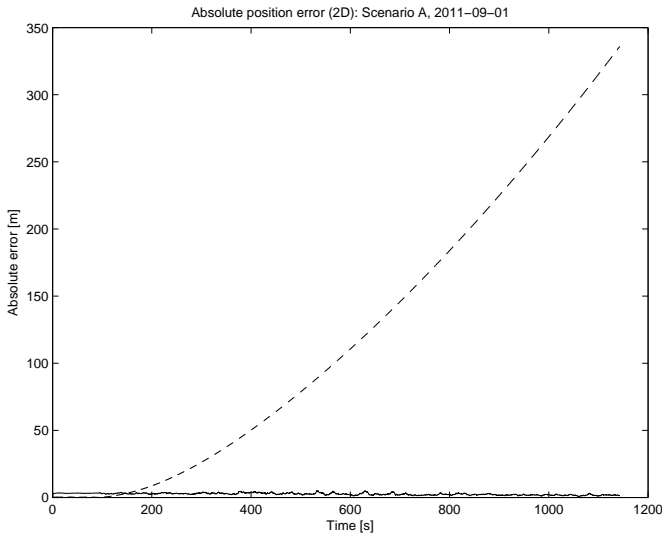


Figure 8.1. Aided and unaided (dashed) absolute position performance

#### Absolute Position

Figure 8.1 shows the result of one performed simulation using the aiding information, comparing the result of when it is not used. As seen in the figure, the performance is improved, where the aided result is bounded after loosing the GPS, while the unaided result grows.

## Conclusion

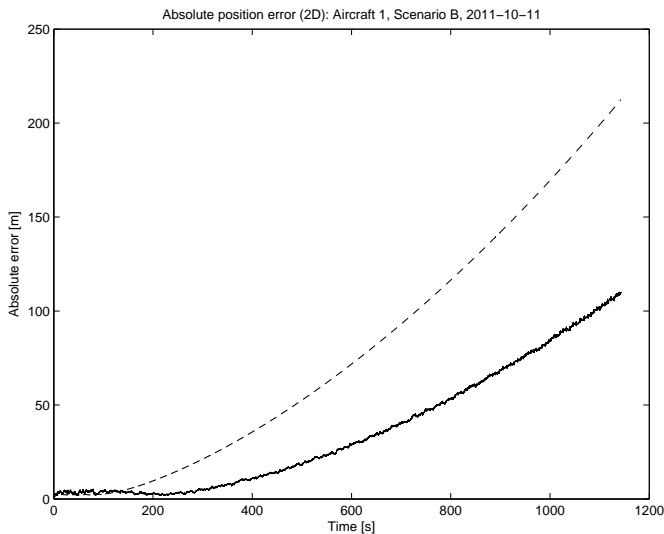
It is shown that the performance can be improved in this case, where the geometry is beneficial. Several simulations has been done, which all points out that the performance is improved, which verifies the expected result and the implemented functionality when landmarks are used as good absolute position references.

## 8.4 Scenario B

Some of the results of this scenario are here presented, which shows some plots from the simulation. These plots shows the absolute position for one aircraft, and some of its relative positions to the others. The more interesting aspect in this scenario is to study the relative positions, which mainly are presented. Else, it would be a lot of plots, which shows similar results.

### Absolute Position

An example of the absolute position performance for aircraft 1 is here presented, and shown in Figure 8.2. This performance is shown since aircraft 1 is chosen to be studied, here by only picking one of them.

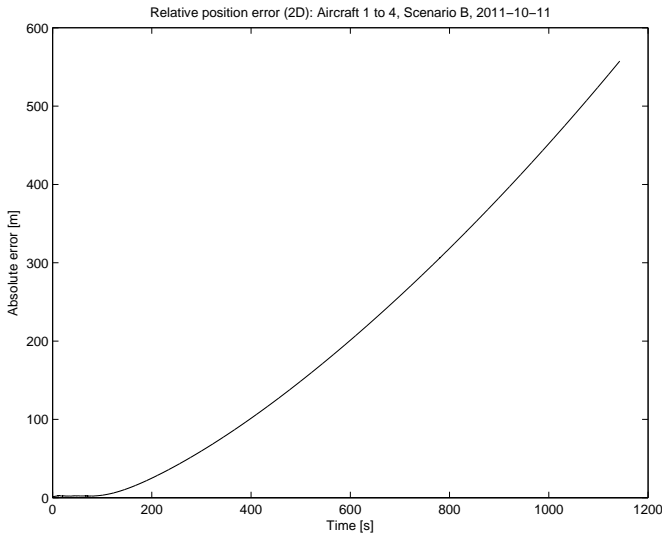


**Figure 8.2.** Aided and unaided (dashed) absolute position performance

### Relative Position

As the relative position performance is the most interesting among the studied aspects, and it is possible to relate different aircraft here, it is studied a little bit more further. But like already denoted, it is concentrated to aircraft 1, since the other aircraft gave similar results.

To give an example of how the relative position performance is when not using any aiding information, Figure 8.3 is shown. This performance can be an example of a result of the absolute positions drifting away in completely different directions. It of course depends on the characteristics of each simulation, why the performance not necessarily is improved in every simulation. But a degraded performance of this size has not been observed at all when using aiding information.



**Figure 8.3.** *An example of unaided relative position performance*

Here the corresponding relative position performances of aircraft 1 when using the aiding information are studied, which are shown in Figure 8.4-8.6. As seen the performance can be improved.

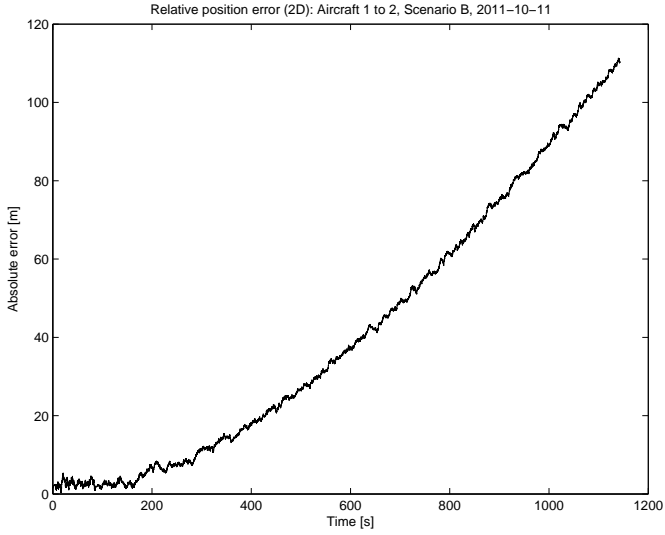


Figure 8.4. Aided relative position performance

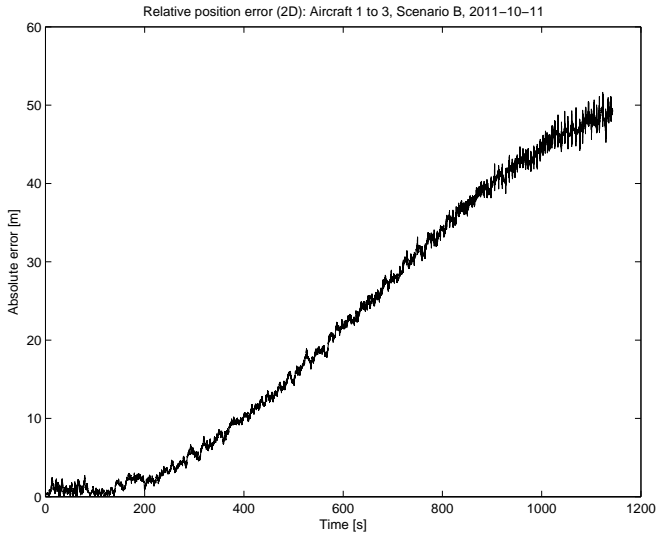


Figure 8.5. Aided relative position performance

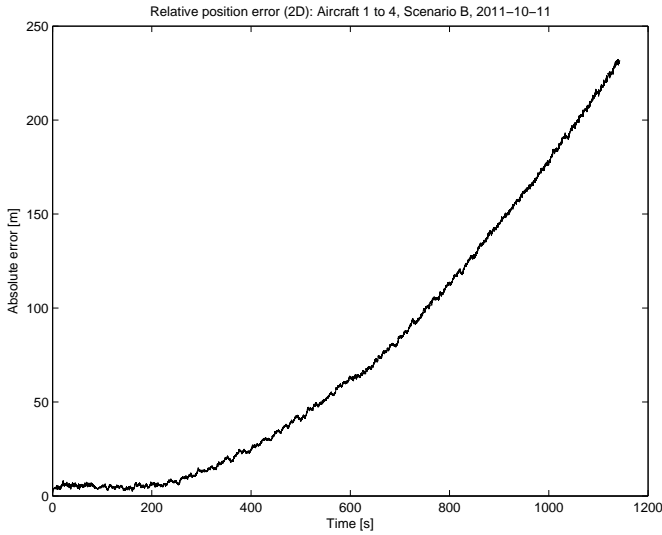


Figure 8.6. Aided relative position performance

### Conclusion

As a conclusion of these results the navigational performance can be improved, where the geometry is beneficial. The absolute position performance got no absolute position reference, but the performance can get improved. For the relative positions the most of the improvements can be achieved, which is the case here, and is an expected result. Several simulations has been done, which all points out that the performance is improved, which verifies the expected result and the implemented functionality when several aircraft are used as relative position references.

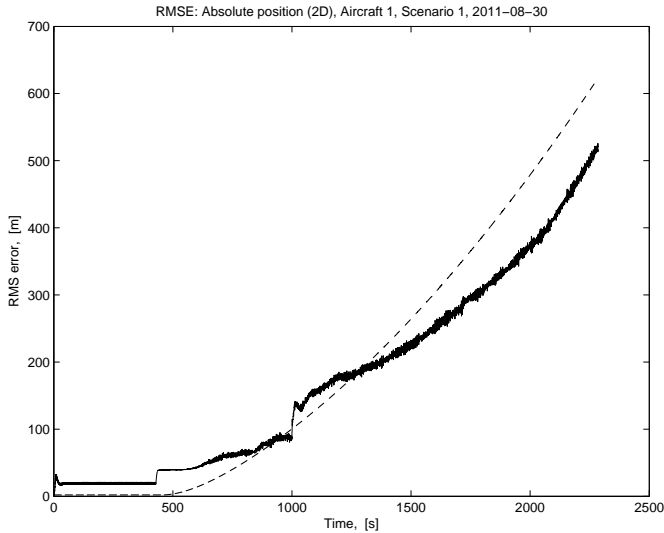
## 8.5 Scenario 1

Some results of the scenario are here presented, showing some of the plots from the simulations. These plots show the absolute position for only one aircraft, aircraft 1, and its relative positions. Since the most interesting in this scenario is the relative positions, mainly the relative positions are presented. It would else be a lot of plots, showing similar results.

### Absolute Position

An example of one of the aircraft absolute position performance is here presented, which is for aircraft 1, and is shown in Figure 8.7. This performance of absolute position is shown since aircraft 1 is chosen to be studied, and it loses its GPS performance first. Its absolute position performance is improved, but this is not

the achieved result for all of the aircraft. When they lose their GPS performance later on, their absolute position performance became affected in a more negative way. But these performances were in the end not degraded more than a couple of hundred meters in RMS error, which can be due to several aspects explained in the thesis.



**Figure 8.7.** Aided and unaided (dashed) absolute position performance

The sudden degradation of the performance, at the instant jump in the absolute position error when the GPS is lost, can be a result of bad estimations of the range biases due to observability and the non-informative modeling of these biases. This of course also affects the relative position performance.

## Relative Position

As the relative position performance is the most interesting among the studied aspects, and it is possible to relate different aircraft here, it is studied a little bit more further. But like already denoted, it is centered around aircraft 1, since the other aircraft gave similar results.

To give an example of how the relative position performance is when not using any aiding information, Figure 8.8 is shown. This performance is similar for all of the relative positions when they are losing their GPS performance, which differs only some.

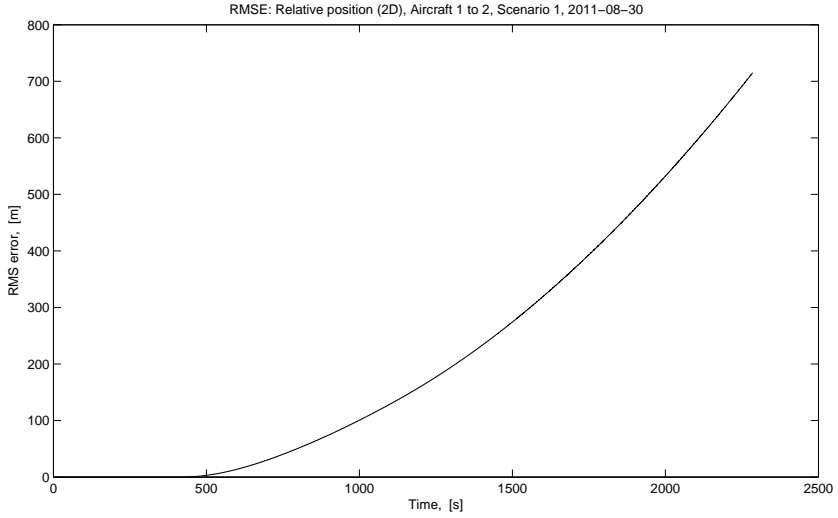


Figure 8.8. An example of unaided relative position performance

Here the corresponding relative position performances of aircraft 1 are studied, which are shown in Figure 8.9-8.11. As seen the performance can be improved, but the results can differ and also be similar to the unaided performance and even degraded as in Figure 8.9.

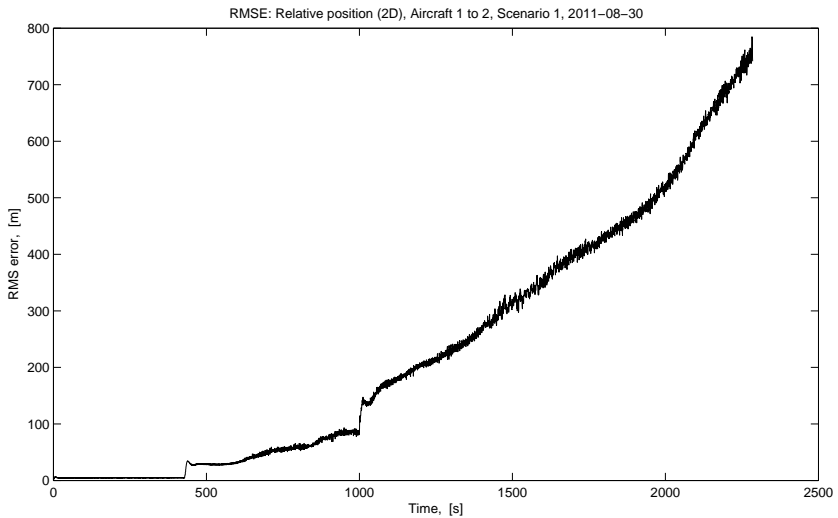


Figure 8.9. Aided relative position performance

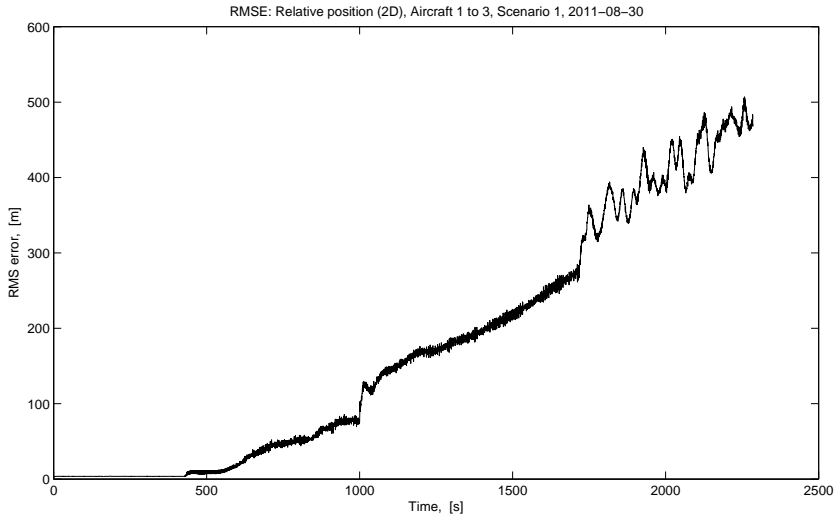


Figure 8.10. Aided relative position performance

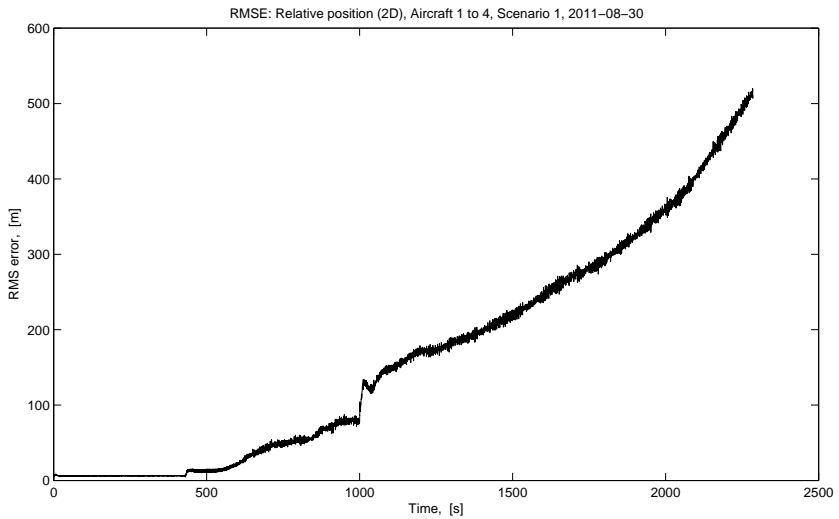


Figure 8.11. Aided relative position performance



## Conclusion

As a conclusion of these results the navigational performance can be improved, even if the geometry is not so beneficial. But it is hard to do any certain conclusions of the absolute position performance, since the aircraft got no reference in the absolute position, where it can also get degraded. For the relative positions the most of the improvements can be achieved, which is an expected result. The aided results behaves strangely and a better result can be expected, why errors in the implementation cannot be excluded completely as an explanation, but no errors were found. Another explanation can be bad filter performance, due to e.g. the chosen tuning parameters.

## 8.6 Scenario 2

As in Scenario A, there is only one aircraft to study, and the positions of the landmarks absolute positions are assumed to be exact. Then the absolute position performance both represents the absolute position performance and the relative position performance to the landmarks.

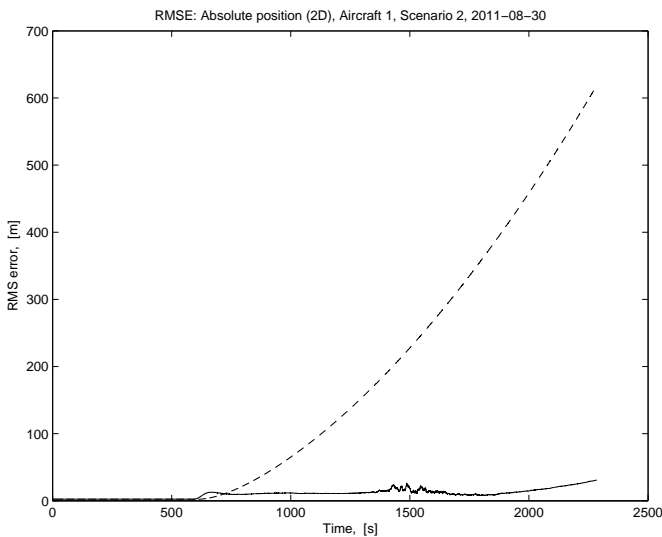


Figure 8.12. Aided and unaided (dashed) absolute position performance

## Absolute Position

The absolute position performance of the aircraft is shown in Figure 8.12. Through time this figure shows that the performance is improved in the aided case, were aiding data are used. When the GPS is not usable, the performance becomes degraded for both of them. But the growth in position error is limited in the

aided case. As landmark  $I$  is lost later on, the absolute position error grows some in the long term.

## Conclusion

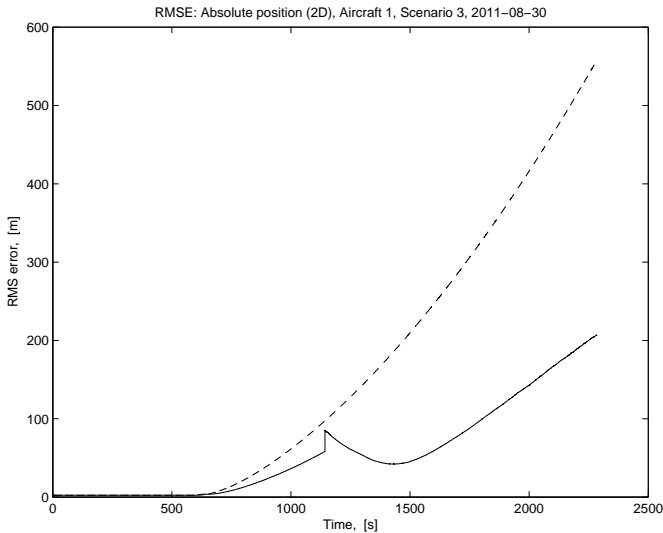
As a conclusion of this scenario, the absolute position performance can be improved in these conditions, were very good absolute references are used.

## 8.7 Scenario 3

Like in previously described Scenario A and 2, there is only one aircraft to study, and the positions of the landmarks absolute positions are assumed to be exact. As before, the absolute position performance both represents the absolute position performance and the relative position performance to the landmarks.

### Absolute Position

The absolute position performance of the aircraft is shown in Figure 8.13. This figure shows that the performance is improved in the aided case, were aiding data are used. When the GPS is not usable, the performance becomes degraded for both of them. But the growth in position error is less in the aided case. As landmark  $II$  is used later on, the performance is improved some, but in the long term the position error grows.



**Figure 8.13.** Aided and unaided (dashed) absolute position performance

The rapid performance degradation, at the sudden jump in absolute position error, can be a result of bad estimations of the range biases due to observability

and the non-informative modeling of these biases. This is because of the GPS is already lost, which makes it hard to do any good estimations of these biases.

### Conclusion

As a conclusion of this scenario, the absolute position performance can be improved in these conditions, where very good absolute references are used.

## 8.8 Scenario 4

Some results of the scenario are here presented, showing some of the plots from the simulations. These plots show the absolute position for only one aircraft, aircraft 1, and its relative positions. Since the most interesting in this scenario is the relative positions, mainly the relative positions are presented. It would else be a lot of plots, showing similar results.

### Absolute Position

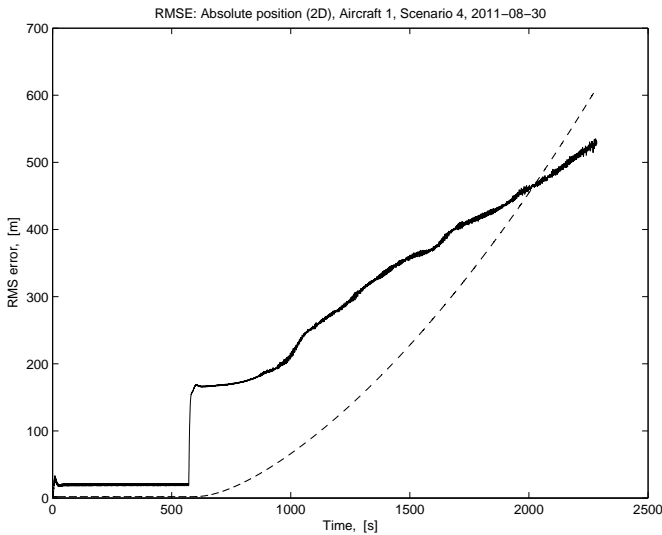


Figure 8.14. Aided and unaided (dashed) absolute position performance

An example of one of the aircraft absolute position performance is here presented, which is for aircraft 1, and is shown in Figure 8.14. This performance of absolute position is shown since aircraft 1 is chosen to be studied, here by only picking one of them. Its absolute position performance when the GPS is not usable seems to be improved, and this is the achieved result for all of the aircraft absolute positions.

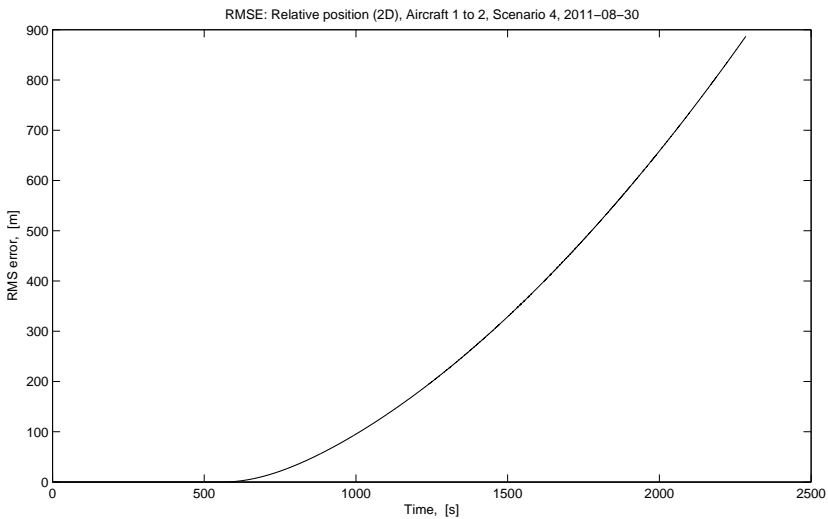
The degradation of the performance, at the rapid jump in absolute position error when the GPS is lost, can be a result of bad estimations of the range biases

due to observability and the non-informative modeling of these biases. This of course also affects the relative position performance.

## Relative Position

As the relative position performance is the most interesting among the studied aspects, and it is possible to relate different aircraft here, it is studied a little bit more further. But like already denoted, it is centered around aircraft 1, since the other aircraft gave similar results.

To give an example of how the corresponding relative position performance is when not using any aiding information, Figure 8.15 is shown. This performance is similar for all of the relative positions when they are losing their GPS performance, which differs only some.



**Figure 8.15.** An example of unaided relative position performance

Here the corresponding relative position performances of aircraft 1 when using the aiding information are studied, which are shown in Figure 8.16-8.18. As seen the performance can be improved, but the results differ and can also be similar to the unaided performance.

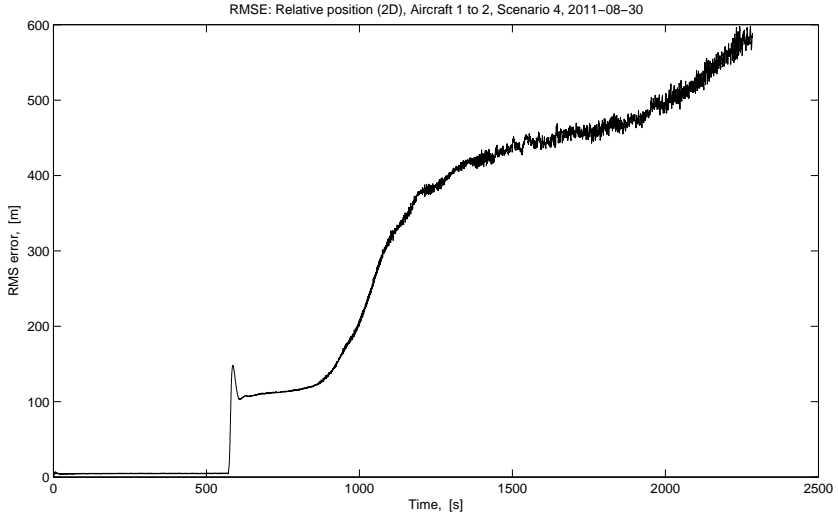


Figure 8.16. Aided relative position performance

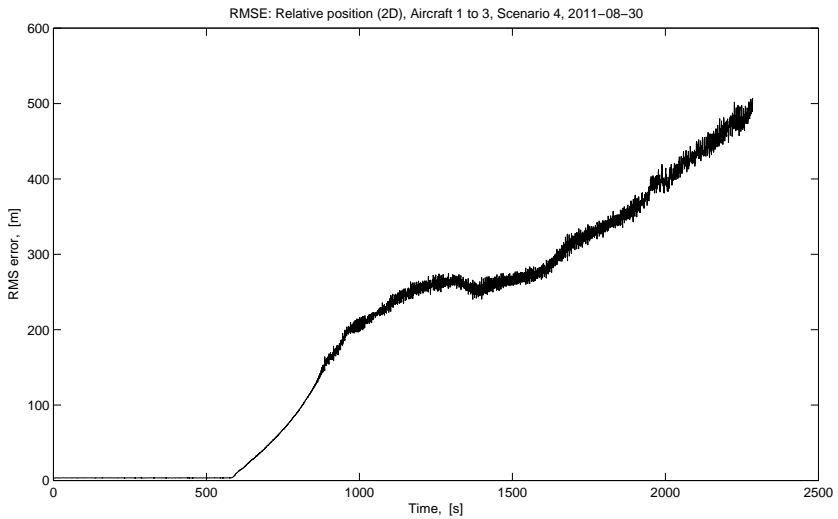
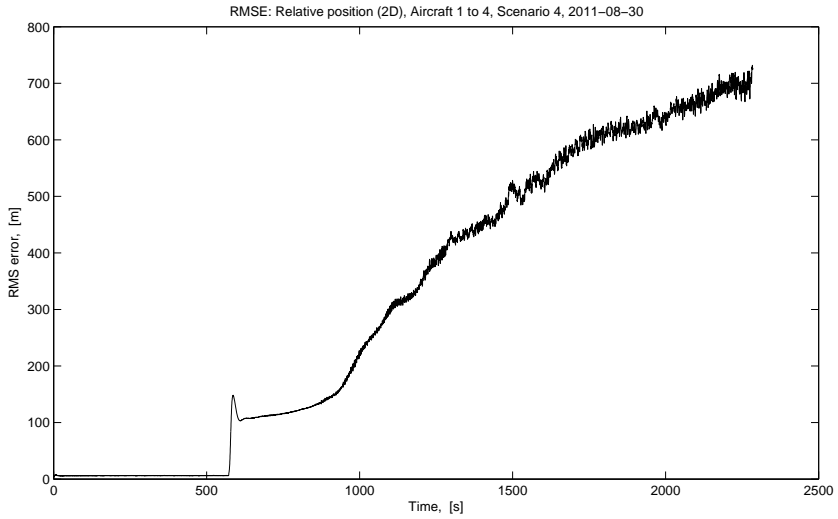


Figure 8.17. Aided relative position performance



**Figure 8.18.** *Aided relative position performance*

## Conclusion

As a conclusion of these results the navigational performance can be improved, even if the geometry is not so beneficial. But it is hard to do any certain conclusions of the absolute position performance, since the aircraft got no reference in the absolute position, where it can also get degraded. For the relative positions the most of the improvements can be achieved, which is an expected result. The aided results behaves strangely and better results can be expected, why errors in the implementation cannot be excluded completely as an explanation, but no errors were found. Another explanation can be bad filter performance, due to the chosen tuning parameters.

## 8.9 Scenario 5

Some results of the scenario are here presented, showing some of the figures from the simulations, where these figures show the absolute positions for all of the aircraft, and some of the relative positions. Here both absolute and relative position performances are considered as interesting to study. But if all results were shown, it would be a lot o plots, showing similar results, why only the relative position performances from aircraft 1 are considered.

### Absolute Position

The aircraft absolute position performances are here presented, and are shown in Figure 8.19 - 8.22. These performances of absolute positions are shown, to be able to see how the usage of the landmarks, given as absolute position references, affects the performance of the aircraft. Their absolute position performances when the GPS is not usable seems to be both improved and degraded. As the aircraft starts to use the landmarks information after the GPS is lost, the performances are improved, but over all the results differs and it can also be reduced.

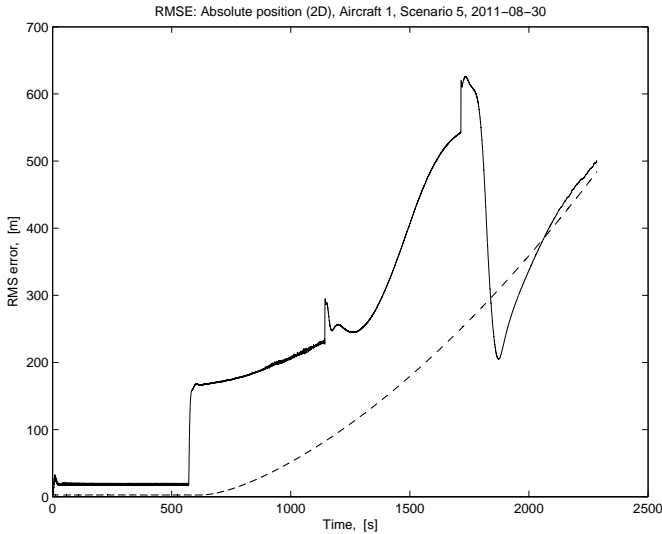


Figure 8.19. Aided and unaided (dashed) absolute position performance

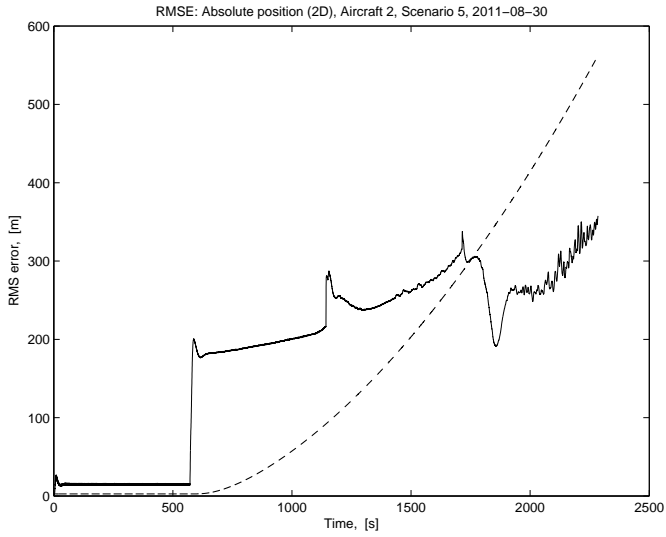


Figure 8.20. Aided and unaided (dashed) absolute position performance

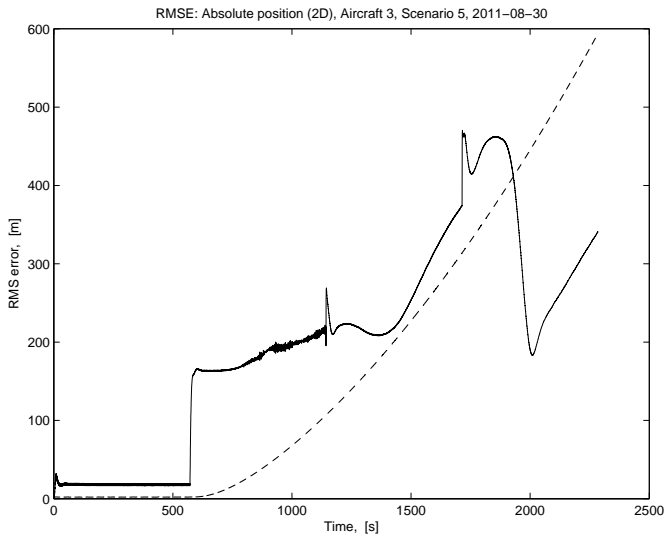
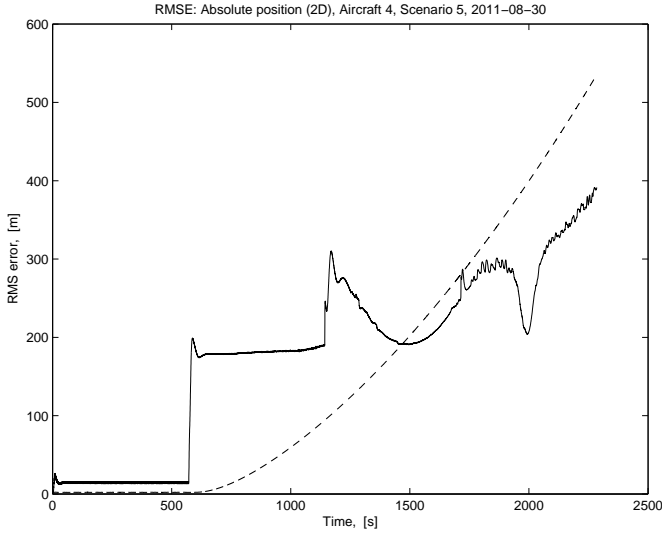


Figure 8.21. Aided and unaided (dashed) absolute position performance





**Figure 8.22.** Aided and unaided (dashed) absolute position performance

The instant performance degradations, at the sudden jumps in absolute position error when the GPS is lost and later on when the landmarks start to signal, can be a result of bad estimations of the range biases due to observability and the non-informative modeling of these biases. Since the GPS is already lost when the landmarks start to signal, it is hard to do any good estimations of these biases. This of course also affects the relative position performance.

## Relative Position

As the relative position performance is the most interesting among the studied aspects, and it is possible to relate different aircraft here, it is studied a little bit more further. But like already denoted, it is centered around aircraft 1, since the other aircraft gave similar results.

To give an example of how the relative position performance is when not using any aiding information, Figure 8.23 is shown. This performance is similar for all of the relative positions when they are losing their GPS performance, which differs only some.

Here the corresponding relative position performances of aircraft 1 are studied when using the aiding information, which are shown in Figure 8.24-8.26. As it can be seen the performance can be improved, even if they differs and the performance can also be reduced.

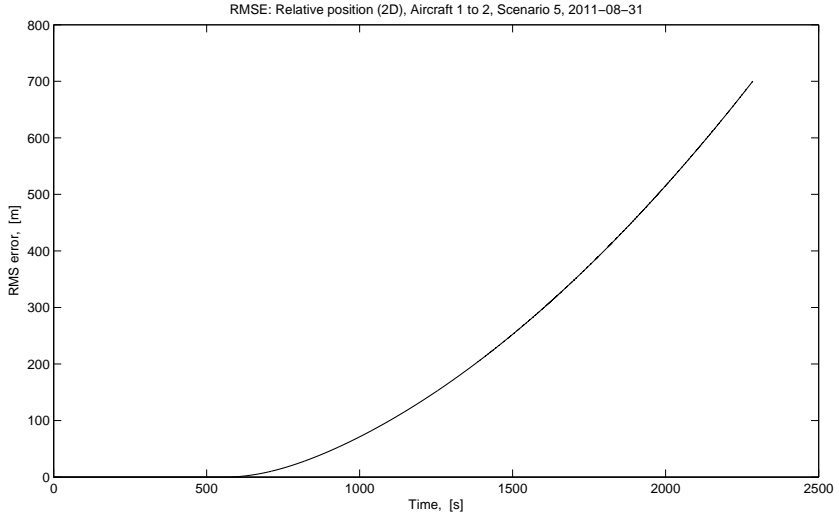


Figure 8.23. An example of unaided relative position performance

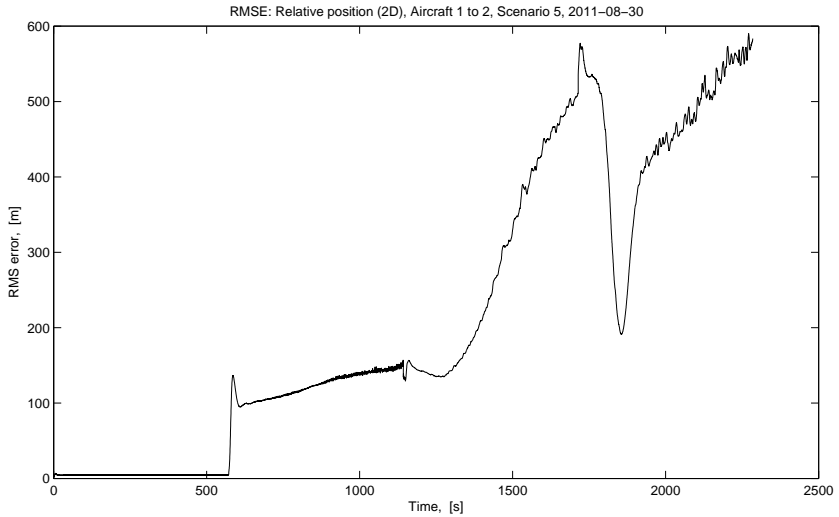


Figure 8.24. Aided relative position performance

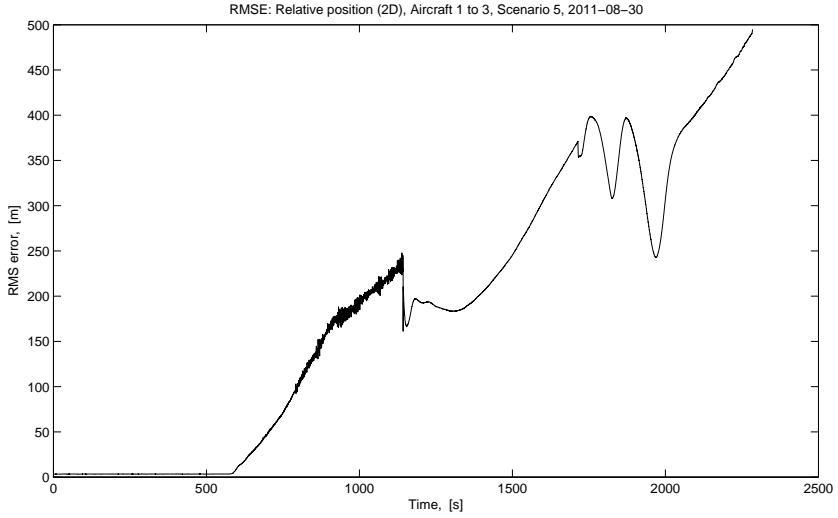


Figure 8.25. Aided relative position performance

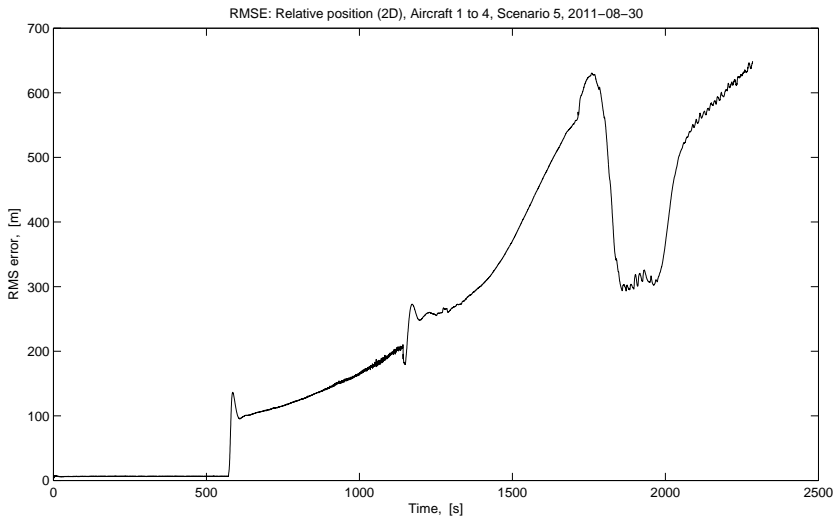


Figure 8.26. Aided relative position performance

## Conclusion

As a conclusion of these results the navigational performance can be improved, even if the geometry is not so beneficial. But it is hard to do any certain conclusions of the absolute position performance, since the aircraft got no reference in the absolute position, where it can also get degraded. For the relative positions the most of the improvements can be achieved, which is an expected result. The aided results behaves strange and a better performance can be expected, why errors in the implementation cannot be excluded completely as an explanation, but no errors were found. Another explanation can be bad filter performance, due to the chosen tuning parameters.

## 8.10 General Conclusion and Expected Result

In many of these scenarios the geometries are not so beneficial, which gives a degraded performance when using the link information. This can be according to the *Cramér-Rao lower bound (CRLB)*, which gives the bounds of the theoretically best performance. It can cause confusing results which can both be improved and degraded, where the results can be dependent on tuning. Another aspect concerning the performance, related some to the geometry issue, is the observability in the filter solution, which for example can mean that not enough information is provided to be able to render correct estimates for all of the states. This is not studied or discussed, but if e.g. an aircraft uses link information where it starts to receive new information from another aircraft it can become harder to estimate the states related to this aircraft. If anything else is known about the other aircraft, and there are for example no information of good GPS quality in either of the aircraft, the range bias is hard to estimate and can become very erroneous.

The tuning of the filter is quite complex and demands a lot of time effort, why the results can be not as satisfying in many of these scenarios as could be expected. This makes the process of tuning more tricky and harder to understand. Since the occurrences in each scenario differs some, it can be harder for this reason too.

But as a general conclusion, the performance can be improved in the long run, if the conditions are beneficial. These beneficial conditions are all of the conditions considered in these scenarios, e.g. the link communication, the geometry and the number of aircraft and landmarks. If, for example, there are several aircraft and landmarks with GPS performance aiding one aircraft, the result of the absolute position estimation can be expected to be improved. This is one of the main conclusions which can be done, which answers the question of it is worth to put any further work in this area positively.

Mainly, the performance in relative position can be expected to be improved in the long run. This is because of the provided link information mainly is relative in its nature, where the range measurements relates the aircraft. It can be suspected in many of the studied scenarios, even if it is not the only case in practice. This is another and the last main conclusion which also answers the same question in the same way. In practice, there are a lot of other practical aspects that has to be considered, but as a theoretical reference a positive answer is given.

As a statistical explanation of the generated results, quite few Monte Carlo simulations has been performed, which might affect the results. Also, nothing about the spread and the uncertainties of the estimations are said or studied, since the position estimation characteristics is the only aspect considered.

Another conclusion that can made, is that the performance can be improved, much by bringing information about the altitude. This is the case in practice, where altitude stabilization is used. Since no altitude stabilization or altitude data is used, the altitude characteristics is the same as in the positional directions. By using other additional information from e.g. cameras even better precision can be achieved.

To notice and already denoted, some of the plotted results from scenario 1, 4 and 5 behaves strangely and better results can be expected, why errors in the implementation cannot be excluded as another reason, but no errors were discovered. This does not affect the general conclusions but can of course affect the results from the scenarios in these cases.

# Chapter 9

## Further Work

A lot of effort can be put in this area. Different improvements and interesting and important studies can be made, both by improving the already existing models and by new extensions, but also by studying these scenarios further and other scenarios where e.g. the GPS performance is degraded. This chapter gives some proposals and examples of improving the performance and the real results, both in terms of internal and external extensions.

### 9.1 Internal Extensions

The internal extensions are improvements of the used models and implementations. Much of the internal improvements are related to the modeling. A lot of assumptions have been made in this work, which instead can be included in the modeling. These used models can be extended to increase the performance, for which the filter tuning parameter was not chosen optimal, performance can certainly be gained by tuning. Also the EKF filter algorithm can be modified and extended, by e.g. applying a distributed Kalman-consensus tracking algorithm, which briefly considers all states representing the same quantities as an optimization problem, which shall be equal. This basically means that another term in the calculation of the states is added, which is a result of the optimization problem. These states are in this thesis the position representations in each aircraft.

### 9.2 External Extensions

The external extensions are improvements of not used models in the implementations. These improvements are more about extension of the information sources. Of course, there are already several different other sensors than the modeled ones used in this work. Extending the already existing implementations with new information, such as e.g. bearing measurements and altitude from barometrical and altitude radar measurements, will certainly improve the performance. Information

about the geographical environment can also be provided. Another information source is using a camera.

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## Appendix A

# Extended Kalman Filter Algorithm

The EKF algorithm based on the discrete-time algebraic Riccati equation (DARE) using the first Taylor expansion term, is presented in Algorithm A.1.

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**Algorithm A.1** The EKF algorithm

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**Require:**  $\hat{x}_{0|-1}$  and  $P_{0|-1}$ , or  $\hat{x}_{0|0}$  and  $P_{0|0}$  if TU first.

$Q$  and  $R$  tuning parameters.

1: Measurement update (MU, estimation step)

$$S_k = H_k P_{k|k-1} H_k^T + G_{e,k} R_k G_{e,k}^T \quad (\text{A.1a})$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1} \quad (\text{A.1b})$$

$$\epsilon_k = y_k - h(\hat{x}_{k|k-1}) \quad (\text{A.1c})$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \epsilon_k \quad (\text{A.1d})$$

$$P_{k|k} = P_{k|k-1} - P_{k|k-1} H_k^T S_k^{-1} H_k P_{k|k-1} \quad (\text{A.1e})$$

$$H_k = \left. \frac{\partial h(x, e)}{\partial x} \right|_{x=\hat{x}_{k|k-1}, e=0} \quad (\text{A.1f})$$

$$G_{e,k} = \left. \frac{\partial h(x, e)}{\partial e} \right|_{x=\hat{x}_{k|k-1}, e=0} \quad (\text{A.1g})$$

2: Time update (TU, prediction step)

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}) \quad (\text{A.2a})$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + G_{w,k} Q_k G_{w,k}^T \quad (\text{A.2b})$$

$$F_k = \left. \frac{\partial f(x, w)}{\partial x} \right|_{x=\hat{x}_{k|k}, w=0} \quad (\text{A.2c})$$

$$G_{w,k} = \left. \frac{\partial f(x, w)}{\partial w} \right|_{x=\hat{x}_{k|k}, w=0} \quad (\text{A.2d})$$


---

## Appendix B

# The Root Mean Squared Error (RMSE)

The RMSE is the estimated standard deviation of each time instance. It can be applied on certain time instances from different simulations, which renders a estimated and time dependent expectation error. A definition of the RMSE can be seen in (B.1), where  $x_{i,k}$  is the error at measurement  $i$  and time instance  $k$ , and  $N$  is the number of simulations.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N x_{i,k}^2}{N}} \quad (\text{B.1})$$

As seen from the definition, the RMSE is a quantity that gives a hint of the mean performance through several simulations. But it does not say anything about the size of the maximum error, and it also assumes a non-biased mean.

# Appendix C

## Parameters

This chapter contains the  $Q$  and  $R$  parameters used by the filter when simulating the scenarios, chosen by ad hoc methods. The measurement noise covariance  $R$ , which is not following considerations of the generated measurement noise, is set to a value and then the state covariance  $Q$  is chosen. These parameters are divided into parameters related to internal and external dynamics and measurements as under the paragraphs below, where the external parameters are the parameters related to the other units.

To be able to improve the performance by tuning or to get a sense of what parameters that can be chosen, the parameters can be helpful to get started.

### Internal Parameters

Table C.1 shows the chosen and used internal parameters, where each scenario corresponds to a row, and each internal parameter corresponds to a column.

| Scenario | $Q^a$                 | $Q^\omega$ | $Q^{b,a}$               | $Q^{b,\omega}$ | $R^a$               | $R^\omega$ | $R^{o,p}$        |
|----------|-----------------------|------------|-------------------------|----------------|---------------------|------------|------------------|
| A        | $0.001I_{3 \times 3}$ | 0.001      | $0.0001I_{3 \times 3}$  | 0.00001        | $100I_{3 \times 3}$ | 100        | $I_{3 \times 3}$ |
| B        | $0.01I_{3 \times 3}$  | $10^{-9}$  | $10^{-8}I_{3 \times 3}$ | $10^{-8}$      | $100I_{3 \times 3}$ | 1          | $I_{3 \times 3}$ |
| 1        | $0.001I_{3 \times 3}$ | 0.001      | $0.0001I_{3 \times 3}$  | 0.00001        | $100I_{3 \times 3}$ | 100        | $I_{3 \times 3}$ |
| 2        | $0.001I_{3 \times 3}$ | 0.001      | $0.0001I_{3 \times 3}$  | 0.00001        | $100I_{3 \times 3}$ | 100        | $I_{3 \times 3}$ |
| 3        | $0.001I_{3 \times 3}$ | 0.001      | $0.0001I_{3 \times 3}$  | 0.00001        | $100I_{3 \times 3}$ | 100        | $I_{3 \times 3}$ |
| 4        | $0.001I_{3 \times 3}$ | 0.001      | $0.0001I_{3 \times 3}$  | 0.00001        | $100I_{3 \times 3}$ | 100        | $I_{3 \times 3}$ |
| 5        | $0.001I_{3 \times 3}$ | 0.001      | $0.0001I_{3 \times 3}$  | 0.00001        | $100I_{3 \times 3}$ | 100        | $I_{3 \times 3}$ |

**Table C.1.** *The internal parameters used in the simulations*

### External Parameters

Table C.2 shows the chosen and used external parameters, where each scenario corresponds to a row, and each external parameter corresponds to a column. If  $i$

is a landmark, then  $Q^{i,p} = 0_{3 \times 3}$ , else the parameter is in this table. And depending on if  $i$  has lost its GPS or not,

$$R_k^{i,p} = \begin{cases} R^{i,GPS} & \text{if GPS} \\ R^{i,-GPS} & \text{if no GPS} \end{cases}$$

are shown parameters in this table. A parameter written in parentheses was not needed in the simulations for the corresponding scenario.

| Scenario | $Q^{i,p}$             | $Q^{i,b}$ | $R^{i,GPS}$        | $R^{i,-GPS}$         | $R^{i,r}$ |
|----------|-----------------------|-----------|--------------------|----------------------|-----------|
| A        | $0_{3 \times 3}$      | 0.001     | $10I_{3 \times 3}$ | $(50I_{3 \times 3})$ | 50000     |
| B        | $50000I_{3 \times 3}$ | 0.0001    | $I_{3 \times 3}$   | $100I_{3 \times 3}$  | 1         |
| 1        | $100I_{3 \times 3}$   | 0.001     | $10I_{3 \times 3}$ | $50I_{3 \times 3}$   | 50        |
| 2        | $0_{3 \times 3}$      | 0.001     | $10I_{3 \times 3}$ | $(50I_{3 \times 3})$ | 50000     |
| 3        | $0_{3 \times 3}$      | 0.001     | $10I_{3 \times 3}$ | $(50I_{3 \times 3})$ | 30000     |
| 4        | $100I_{3 \times 3}$   | 0.001     | $10I_{3 \times 3}$ | $50I_{3 \times 3}$   | 50        |
| 5        | $100I_{3 \times 3}$   | 0.1       | $10I_{3 \times 3}$ | $50I_{3 \times 3}$   | 50        |

**Table C.2.** *The external parameters used in the simulations*