The Present-Value Model of the Current Account: Results from Norway

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Abstract

In this thesis I present and test an intertemporal model for the current account. The model predicts a nation that prefers a smooth consumption profile where the current account balance is used as a tool to smooth consumption based on expectations about future changes in net output. The model implies a present-value relationship between the current account and future changes in net output. The present-value model (PVM) is a nested version of a general vector autoregression (VAR). I estimate this general VAR for quarterly Norwegian data for the period 1981 to 2011. The cross-equation restrictions on this VAR that are implied by the PVM is rejected for Norwegian data, but I present some favorable, less formal results from the intertemporal model.

Sammendrag

I denne oppgaven presenteres og testes en intertemporær modell for ett lands betalingsbalanse. I modellen ser vi et land som foretrekker en glatt konsumbane. På grunnlag av forventninger om fremtidig utvikling i inntekt brukes betalingsbalansen til å fordele inntekt mellom perioder for oppnå ett konstant konsumnivå over tid. Modellen implisererer en nåverdi sammenheng der betalingsbalansen er lik neddiskontert sum av fremtidige endringer i inntekt. Denne nåverdi-modellen er et spesialtilfelle av en generell vektor autoregresjons-modell. Jeg estimerer denne generelle modellen for norske kvartalsdata fra perioden 1981–2011. Restriksjonene som gir nåverdi-modellen er forkastet for norske data, men jeg presentrer noen mindre formelle resultater til fordel for den intertemporære modellen.

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Chapter 1

Introduction

In this thesis I present and test a model for a how a nation decides to allocate wealth in international financial markets. The model predicts a country that prefers a smooth consumption path. We will see a nation that allocates wealth among different periods to keep the consumption level constant over time. The measure of the flow of assets in international financial markets is the current account balance, and the theory is called *the intertemporal approach to the current account*. I will test whether an intertemporal model can explain the movements of assets in international markets for Norwegian data.

A country's current account balance is the change in net foreign asset holdings at a given point in time. The nation's net holding of foreign assets is a stock variable and can be positive or negative. If the stock is positive, the nation has a claim to foreign output that is greater than foreign nations claim to domestic output. We can think of this as a way of saving abroad. If the stock is negative, foreign claims to the domestic output is greater than the nation's claim to foreign output and the nation is borrowing money from the rest of the world. The current account can then be thought of as the nation's total one-period saving abroad if the current account is positive and the total one-period borrowing from abroad if the current account is negative. I will model a nation's current account balance, also known as the nation's net lending, where the focus will be on how much the nation decides to save or borrow abroad.

In the last 30 years the globalization process has increased the cross-border financial flows by a large amount.[1] More recently we have seen the emergence of the so-called *global imbalances*, which means unusually large current account surpluses and deficits. The best examples of the global imbalances are the Chinese current account surplus and the United States current account deficit. These patterns lack easy economic explanations, and they cannot be explained in a framework where capital flows to the regions where capital is most productive.[2] Norway is also a contributor to the global imbalances; in the forth quarter of 2011 the current account surplus of Norway was 113 billion; this was over 17

[1]See Kose et al. (2006).
[2]See Gourinchas and Jeanne (2011).

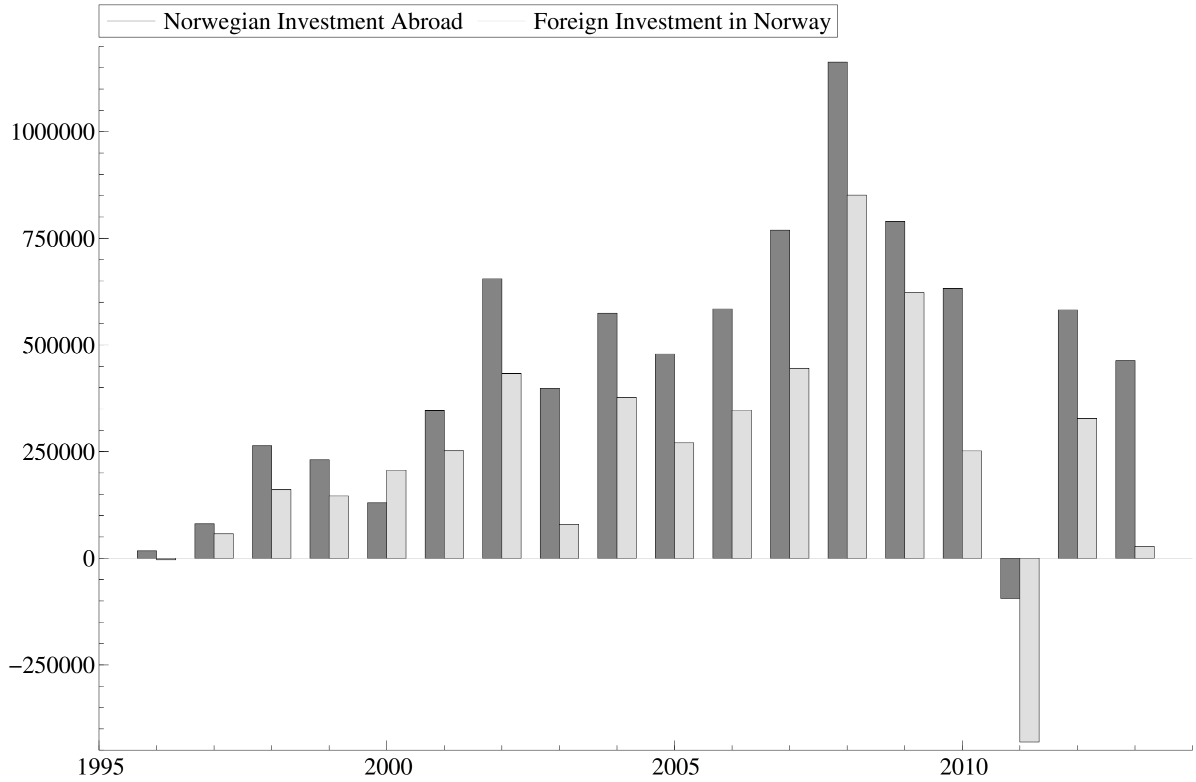

Figure 1.1: Norwegian investment abroad and foreign investment in Norway. The values are in real 2008 million NOK.

percent of Norway's gross domestic product that quarter.[3] The objective of this paper is to explain the current account balance for Norway, where I will focus on the intertemporal consumption decision as the determinant of the current account.

In Figure 1.1 I plot Norwegian investment abroad, which is the change in Norwegian asset holdings abroad, and I plot foreign investment in Norway, which is the change in foreign nations asset holdings in Norway.[4] The change in Norwegian asset holdings abroad is negative in *one* period; that is in 2009. It is plausible to think this has to do with the financial crises, and the reduction in asset holdings is a result from a devaluation of the holdings of foreign assets. The difference between Norwegian investment abroad and foreign investment in Norway is the net lending.[5] Norway's net lending is plotted in Figure 1.2. We can see the increase in both foreign lending and borrowing over time up until the financial crisis in 2007. At the same time we see that net lending has been high and somewhat stable since the beginning of year 2000.

The basis for the theoretical model is the *permanent income hypothesis* developed by

[3]The data is from Statistics Norway; see *http://www.ssb.no/ur_en/*.

[4]The data in Figure 1.1 and Figure 1.2 is from Statistics Norway, and can be found under the financial account in the balance of payments; see *http://www.ssb.no/ur_en/*.

[5]In the data, there are some discrepancies between the difference between Norwegian investment abroad and foreign investment in Norway, and the net lending because of undistributed financial transactions and statistical errors.

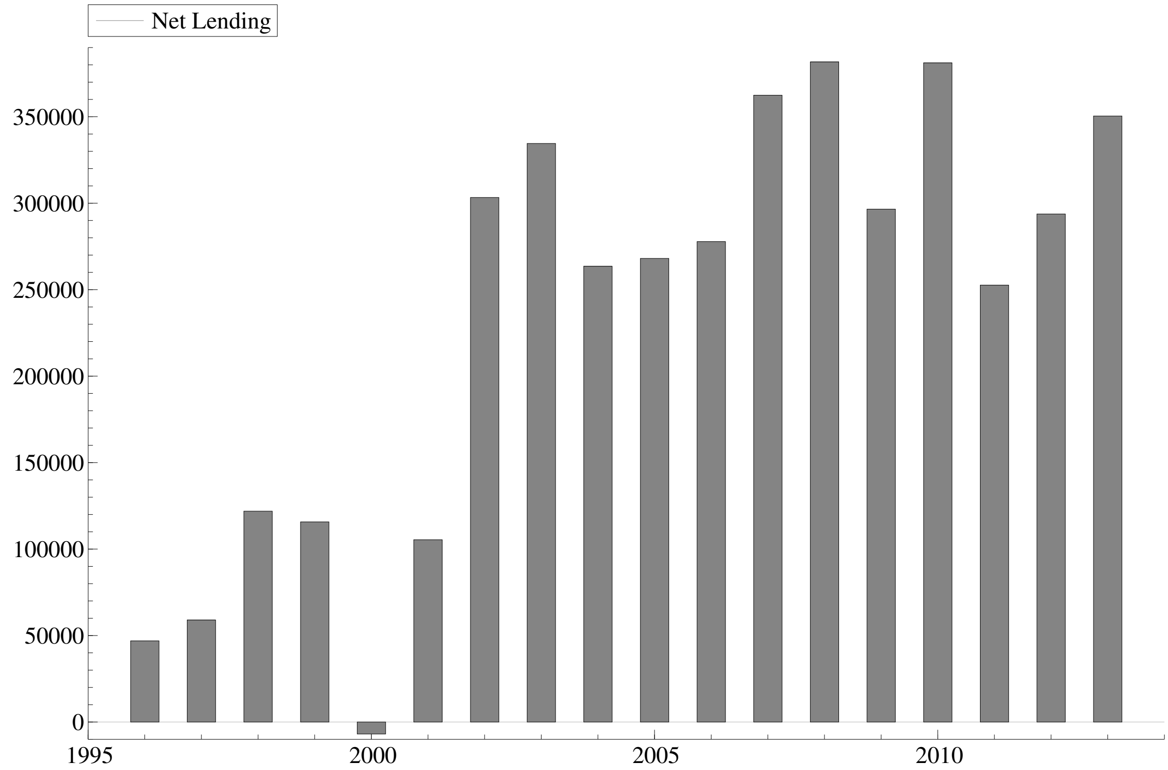

Figure 1.2: Net lending for Norway. The values are in real 2008 million NOK.

Friedman (1957). The permanent income hypothesis is going against the traditional Keynesian view where consumption is a constant share of disposable income in a given period. Instead Friedman theorized that consumption is a share of permanent lifetime income. The theory implies that temporary income shocks do not affect consumption much since temporary shocks has a marginal effect on lifetime income whereas permanent shocks affects consumption one-to-one. Hall (1978) finds evidence in favor of the permanent income hypothesis for postwar data for the United States. In Hall's paper consumption follows a random walk and this theory is often refereed to as *Hall's random walk hypothesis*. This is a result from an Euler equation approach to describe the development of consumption. The Euler equation approach means that economic relations are derived from a mathematical optimization problem. This approach was a response to the Lucas critique where Lucas argued that predicting the effects of economic policy on the basis of historical observed relationships alone was a bad idea.[6] Instead Lucas suggested we should try to model the determinants of individual behavior such as preferences, technology, initial resources etc. This critique encouraged macroeconomic modeling to be based on microeconomic foundations. I will follow this approach and use an Euler equation to model consumption as a random walk.

While Hall is saying something about individual consumption and saving behavior within a country, I am going to analyze the permanent income hypothesis on an interna-

[6]See Lucas (1976).

tional level (from the perspective of one single country) where the current account is used to get the right consumption share out of permanent lifetime income in every period. I use a theoretical model based on Sachs (1982), which emphasizes the intertemporal allocation of consumption as the determinant of the current account balance. Sachs is using a continuous time model while I use a model in discrete time.

I will present a theoretical framework consisting of one representative forward-looking agent that solves a well-behaved optimization problem. The environment is a small open economy with access to international financial markets where the agent can borrow or lend at a constant world real interest rate. It is also important that capital can float freely between the domestic country and the rest of the world. I start off by presenting a highly stylized deterministic model with two periods. I extend this model to a stochastic infinite-period model. The theory implies a present-value relationship between changes in net output and the current account balance. Net output is defined as gross domestic product minus investment and government consumption. The current account balance is a linear function of expected changes in net output. If net output in the future is expected to decline, the model predicts a positive current account balance, and if net output is expected to rise, the model predicts a negative current account balance. Based on expectations of future changes in net output the consumer tries to get as smooth a consumption profile as possible by allocating wealth among periods.

To test the intertemporal approach to the current account I use a methodology from two papers by Campbell and Shiller (1987) and Campbell (1987). They present a way of testing rational expectation present-value models by using cointegrated vector autoregressive (VAR) models. From now on the present-value model is written as PVM. Their method deals with the problem of a non-stationary time series and relevant information that is unobserved by the econometrician. Campbell and Shiller (1987) use the method to test a PVM for the expectation theory of the term structure of bonds, and for the dividend discount model for stocks. Campbell (1987) used the method to test the permanent income theory of consumption; he analyzed the national saving decision in the same way as Hall (1978), and did not consider the current account balance as a way of allocating resources among periods. The first paper to use the Campbell-Shiller procedure to test the intertemporal approach to the current account was Sheffrin and Woo (1990).

I am estimating a bivariate VAR model for the current account and changes in net output for quarterly Norwegian data from the period 1981 to 2011. An implication of the PVM is that when net output is stationary in first differences, the current account balance should be stationary in levels. I can confirm the stationarity for the first difference of net output, but not for the current account in levels. This result is puzzling, but common in related literature.

The model predicts a forecasting relationship from the current account to changes in net output; we get this relationship because the representative agent forms rational expectations about future changes in output and based on these expectations choose the current account that gives a smooth consumption path. The forecasting relationship is confirmed in the data. We can see causality from the current account balance to changes in net output, but not from changes in net output to the current account.

It is possible to evaluate the model graphically by calculating the predicted current account series from the PVM and plotting this against the actual current account series. The result is convincing, the predicted and actual series are close; the correlation coefficient between the actual and predicted series is equal to 0.97. The variance of the actual series divided by the variance of the predicted series is 0.75, so the actual series is more volatile than what the model predicts. The overall graphical picture is a model that tracks the data well.

The PVM is a nested version of a general VAR and I find the restrictions on this general VAR that gives the PVM. I report three formal tests of the PVM. First a log-likelihood ratio test, which evaluates whether the restrictive VAR is a valid simplification of the general VAR. Second I report a test suggested by Campbell (1987), this is a test of the restrictions on the parameter matrix from the VAR based on information at time $t-1$. With information at time $t-1$, the forecast for the present value of changes in net output should be equal on time t and $t-1$. The third test is a Wald test; this is a direct test of whether the estimated parameter values from the VAR satisfy the restrictions implied by theory. All three tests reject the PVM.

The paper proceeds as follows: Chapter 2 presents the theoretical framework for the intertemporal approach to the current account, Chapter 3 gives a background on the empirical methodology and a review of related literature, Chapter 4 presents the empirical approach and my findings for Norwegian data, Chapter 5 discusses causes for why the model is rejected and presents some empirical results unrelated to the theoretical model, while Chapter 6 concludes.

Chapter 2

Theoretical framework

The basis for the model presented in this chapter is the model developed by Sachs (1982). The focus in this model is the intertemporal allocation of wealth among different periods in time. This chapter will present a discrete time theoretical model for *the intertemporal approach to the current account*. We will see a model that implies a *present-value relationship* between the current account and expected changes in net output.

A great advantage in an open economy is the possibility to lend too, and borrow from, the rest of the world. In a particular period a nation can decide how much to spend and how much to save. It is possible to make a consumption plan where consumption deviates from the disposable domestic income in the periods to come. This form of resource exchange across time is called *intertemporal trade*. The size of this intertemporal trade is measured as the current account of the balance of payments. The country's current account balance is the net increase in foreign asset holdings and is defined as

$$CA_t \equiv B_{t+1} - B_t, \tag{2.1}$$

where B_t is the holdings of foreign assets at time t. The current account is a flow variable and is a measure of the change in total holdings of foreign assets. The total amount of foreign assets, B_t, is a stock variable. An alternative formulation for the current account is given by

$$CA_t = Y_t + rB_t - C_t - G_t - I_t, \tag{2.2}$$

where Y_t is gross domestic product, C_t is private consumption, G_t is government consumption, I_t is investment, and r is the word real interest rate. Equation (2.2) can be interpreted as the one-period budget constraint where $Y_t + rB_t$ is the gross national product, which can be thought of as total one-period income for the economy, and $C_t + G_t + I_t$ is total one-period domestic expenditure for the economy. The intertemporal trade is used to fill the potential gap between domestic expenditure and domestic income. When $CA_t > 0$ the current account balance is in surplus and the country is a creditor, and if $CA_t < 0$ the current account balance is in deficit and the country is a debtor.

The standard Keynesian view of the current account is treating it as the nation's net export. When the production in the economy is above national demand, the country is a net exporter. If national output is below national demand the country is a net importer. This view is not in any way contradicting the view in this thesis, but my approach will be to treat the current account balance as the difference between national saving and national investment. In this framework imports and exports are not directly observed. In the net export approach the usual focus is on the determinants of imports and exports such as the relative competitiveness of the nation, the trade determinants does not play any direct role in the model I will use.

To model the current account I use the theory of forward-looking rational agents, which on the basis of expectations of the future decides how much to save and invest. Actually, the model will not include a production side, so investment is treated as an exogenous variable, and the only decision variable is how much to save. If our rational agents are a good approximation to the actual population, we may be able to model the fluctuations in the current account balance based on a model of their behavior. The foundations for the model presented in this chapter are from the book by Obstfeld and Rogoff (1996), Chapter 1 and 2.

2.1 A model with two periods and no uncertainty

Let us consider a small open endowment economy with one representative consumer. In an endowment economy the production side is treated as exogenous. First, let the economy consist of two periods; the first period represents the present and the second period the future. I extend the model to an infinite-period model in the next section. The consumer has access to international financial markets and can borrow or lend at the constant risk-free world real interest rate. There is one good in the economy, and the good lasts for one period only. For now let $G_t = I_t = 0$. There is no uncertainty, so the output in both periods is known for sure to the consumer in the first period. By combining equation (2.1) and (2.2) the one-period budget constraint can be written as

$$C_t + B_{t+1} = (1+r)B_t + Y_t, \quad \text{for} \quad t = 1, 2. \tag{2.3}$$

If we combine the one-period budget constraints in equation (2.3) for both periods we find the intertemporal budget constraint where B_3 is equal to 0[1]

$$C_1 + \frac{C_2}{1+r} = (1+r)B_1 + Y_1 + \frac{Y_2}{1+r}. \tag{2.4}$$

[1]Non-satiation and the no-Ponzi game condition that ensures $B_3 = 0$ is discussed in detail for the infinite-period model in the next section.

This constraint ensures that the present value of *all* consumption equals the present value of *all* income. The term $(1+r)B_t$ is the return from foreign asset holdings obtained before period 1, and can be consumed during period 1 and 2. All consumption sets $\{C_1, C_2\}$ that satisfies equation (2.4) are feasible to the consumer. The variables B_1, r, Y_1, and Y_2 are exogenous and known to the consumer in both periods. The only choice the consumer can make is how to allocate the resources between consumption in the two periods.

To figure out what consumption set the consumer chooses we need to introduce preferences. Let the preferences be time-separable; in period 1 the consumer has the following lifetime utility level

$$U_1 = u(C_1) + \rho u(C_2), \quad 0 < \rho < 1, \tag{2.5}$$

where ρ is the subjective discount factor and $u(\cdot)$ is a one-period utility function where $u' > 0$ and $u'' \leq 0$. The consumer has perfect foresight and the optimal consumption set is the solution to maximizing equation (2.5) subject to equation (2.4) with respect to C_1 and C_2. From equation (2.3) we can see that B_2 determines both C_1 and C_2 where

$$C_1 = (1+r)B_1 + Y_1 - B_2 \ \text{ and } \ C_2 = (1+r)B_2 + Y_2.$$

Since B_1, Y_1, and Y_2 are exogenous, the only decision variable for the consumer is B_2. From equation (2.1) the consumption set $\{C_1, C_2\}$ determines the current account and

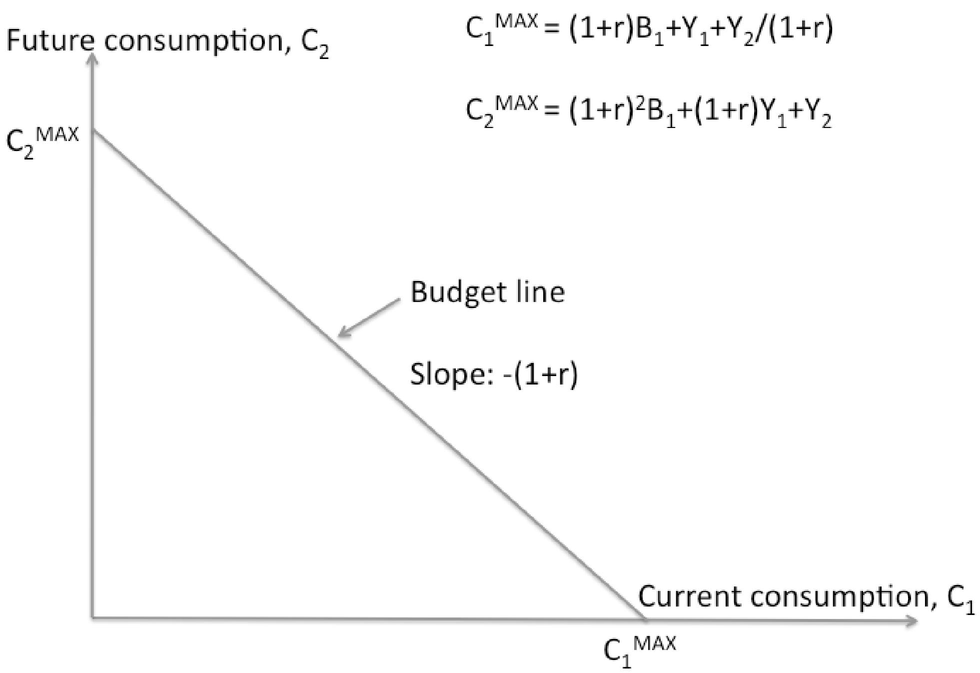

Figure 2.1: The budget line gives the possible combinations of future and current consumption.

this intertemporal allocation of consumption is all that is needed to fully pin down the current account, hence the intertemporal approach to the current account. To solve this simple model graphically, the intertemporal budget constraint in equation (2.4) can be written as

$$C_2 = -(1+r)C_1 + (1+r)^2 B_1 + (1+r)Y_1 + Y_2,$$

and can be plotted as a budget line, see Figure 2.1. The budget line is a representation of all efficient combinations of current and future consumption levels that are available to the consumer. The consumer chooses the achievable consumption set that gives the highest utility level. Utility levels are represented graphically as indifference curves; see an example in Figure 2.2. Along the indifference curve the utility level is constant.

To maximize utility, the consumer chooses the consumption set on the budget line that also lies on the indifference curve farthest away from the origin. An optimal consumption set is the tangency point between an indifference curve and the budget line. The optimality condition can be written as

$$\frac{\rho u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+r}, \tag{2.6}$$

and is depicted in Figure 2.3. The optimal consumption set is given by $\{C_1^*, C_2^*\}$, this set gives the consumer the highest achievable utility level given the budget constraint. Since the output is non-storable the only possible way to save is to lend out some of the output in the first period, and get it back in the second period. If the country cannot trade in international markets, consumption must be equal to income within a period.

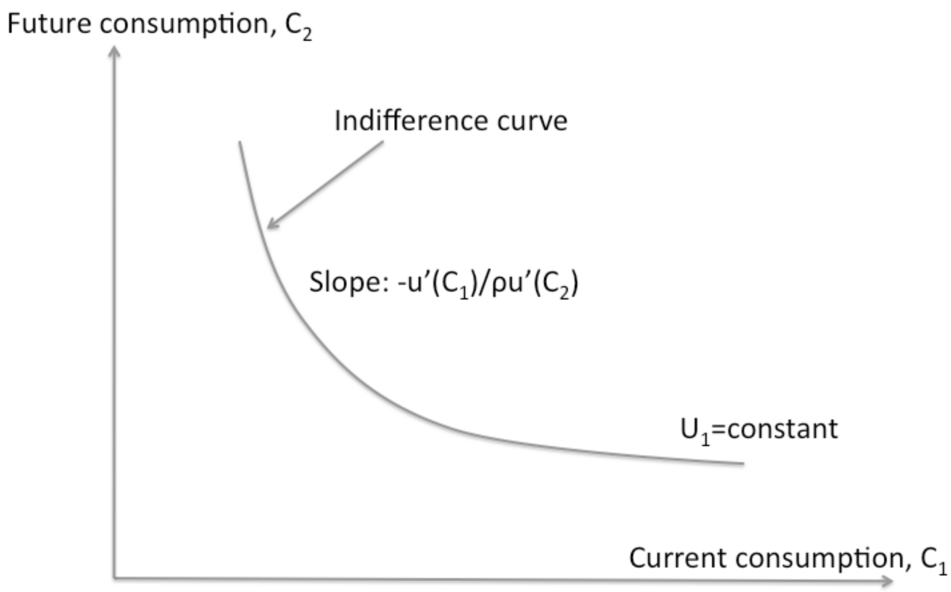

Figure 2.2: The indifference curve gives bundles of current and future consumption plans that give the same utility level.

In Figure 2.3 total autarky income or wealth is given by $\{Y_1, Y_2\}$, and in this particular case the optimal consumption level in the present lies to the right of this point. In this case national wealth in the first period Y_1 is lower than the optimal consumption level C_1^*. To achieve the optimal consumption level the representative consumer runs a current account deficit the first period, $CA_1 < 0$, so consumption in this period is higher than domestic wealth. This borrowed consumption must be repaid in the future, and we can see that consumption in the future, C_2^*, is lower than future national wealth, Y_2. If international markets are closed in all periods we have $Y_t = C_t$ for $t = 1, 2$.

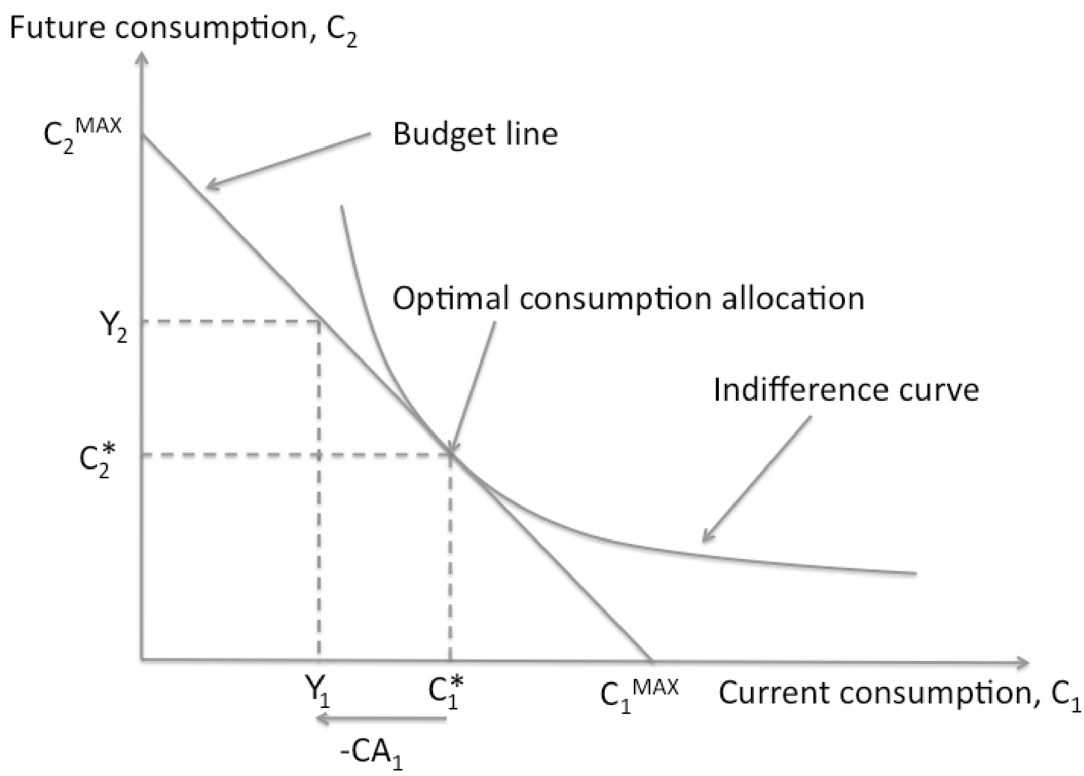

Figure 2.3: This figure gives the optimal consumption set given by $\{C_1^*, C_2^*\}$. The exogenous variables and the consumer's preferences decide the size and the sign of the current account.

An important measure in this model is the elasticity of intertemporal substitution. This is a measure of the responsiveness of the intertemporal allocation of consumption to changes in the real interest rate. A higher interest rate may make the agent want to consume less and save more due to the increased return on savings, this is a substitution effect. It is also possible that the agent want to increase consumption due to the higher return on what he or she already saves, this is an income effect. The total effect on consumption from a change in the interest rate is measured by the elasticity of intertemporal substitution. If the elasticity of intertemporal substitution is constant over time, it can be written as[2]

$$\sigma = -\frac{u'(C)}{Cu''(C)}.$$

Graphically, a change in the real interest rate will rotate the budget line around C_1^{MAX} in Figure 2.3, and the effect of this change on the current account depends on the particular form of the indifference curve.

The model in this section is giving us a way of thinking about how a nation can allocate wealth between the present and the future. Let the subjective discount factor ρ be equal to the real discount factor $1/(1+r)$, so $\rho(1+r) = 1$, then the marginal utility in the two periods must be equal in the optimal allocation (see equation (2.6)), and the model predicts full consumption smoothing. We can look at what happens if the income changes. Assume we are in the first period and we have an optimal consumption allocation, then imagine an increase in the income that can be of two types: First, let the income be permanently higher so the income is equally larger in both periods. Then, to get to the new optimal allocation, consumption in both periods must be increased by the same amount, this can be done without interacting in international financial markets, and a permanent higher income does not affect the current account balance. The second type is an increase in income in one of the two periods. To get an equal consumption level in both periods the consumer must transfer wealth from the period with higher income to the period with unchanged income. The only way to do this is to borrow or lend in international financial markets. The consumer must borrow if income has increased in the second period and lend if it has increased in the first. We see that shocks to the income in one period only, can create large fluctuation in the current account.

[2]The elasticity of intertemporal substitution is defined as the percentage change in consumption growth to a percentage point change in the interest rate, this can be written as

$$\sigma = \frac{d \log \left(\frac{C_2}{C_1}\right)}{dr}.$$

By taking logs of the optimality condition in equation (2.6) we get $\log(1+r) = \log \frac{u'(C_1)}{\rho u'(C_2)}$, by approximating this expression we get $r = \log u'(C_1) - \log u'(C_2)$ and differentiating gives

$$dr = \frac{u''(C_1)C_1}{u'(C_1)} d \log C_1 - \frac{u''(C_2)C_2}{u'(C_2)} d \log C_2.$$

If $u''(C_1)C_1/u'(C_1) = u''(C_2)C_2/u'(C_2) = u''(C)C/u'(C)$, we have

$$\frac{d \log \left(\frac{C_2}{C_1}\right)}{dr} = -\frac{u'(C)}{Cu''(C)}.$$

2.2 A model with an infinite number of periods

The two-period model is good for intuition, but is obviously far from reality. To get a model that can be tested against actual data some changes are needed. The infinite-period model presented in this section consists of the same building blocks as the two-period counterpart. The main elements are the time-separable utility function and the budget constraint. The model still consists of one representative consumer; the consumer is infinitely lived and is getting utility from consumption. By introducing uncertainty, the representative consumer maximizes the expected value of lifetime utility given by

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \rho^{s-t} u(C_s) \right\}, \quad 0 < \rho < 1, \tag{2.7}$$

where E_t is the expectation operator conditioned on the available information set at time t. As before the economy consists of one asset, a risk-free bond B that pays the constant world real interest rate r. If we open up for government spending and investment the one-period budget constraint is given by

$$C_t + G_t + I_t + B_{t+1} = (1+r)B_t + Y_t, \quad \text{for all } t. \tag{2.8}$$

When external borrowing is permitted, the total amount of borrowing needs to be restricted to prevent a potential Ponzi scheme. A Ponzi scheme is when new debt is issued to repay old debt and this process is continued so the debt never gets repaid. In the two-period model all the debt must be repaid in the second period so $B_3 \geq 0$. The no-Ponzi game condition for the infinite-period case can be stated as

$$\lim_{i\to\infty} \left(\frac{1}{1+r}\right)^i B_{t+i+1} \geq 0. \tag{2.9}$$

The concave utility function ensures that the utility level is always increasing in consumption, this is called non-satiation, therefore it is optimal to consume everything that is available to the consumer during the lifetime and leave no resources unused when the life is over. In the two-period case this is satisfied if $B_3 \leq 0$. For the infinite-period case, consuming everything during the lifetime is harder to picture, but both the no-Ponzi condition and the non-satiation condition is satisfied if equation (2.9) holds with equality, that is

$$\lim_{i\to\infty} \left(\frac{1}{1+r}\right)^i B_{t+i+1} = 0. \tag{2.10}$$

The consumer's optimization problem is to maximize equation (2.7) subject to equation (2.8) and (2.9). If we solve equation (2.8) for the consumption level and substitute into equation (2.7) we get

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \rho^{s-t} u((1+r)B_s - B_{s+1} + Y_s - G_s - I_s) \right\}.$$

The representative consumer has one decision variable, which is to choose how much to save or borrow in international markets. This is equivalent to the two-period case where the consumer decided B_2, in this case the decision variable is B_{t+1}. If U_t is maximized with respect to B_{t+1} the first order condition is given by the intertemporal Euler equation

$$E_t\{u'(C_s)\} = (1+r)\rho E_t\{u'(C_{s+1})\} \text{ for } s = 0, 1, 2, \ldots, \tag{2.11}$$

which is the same condition as the optimality condition for the two-period model given by equation (2.6) except we now allow for uncertainty. The Euler equation is a condition that ensures that no intertemporal shifts in consumption can give the consumer a higher utility level. To get a solution for the consumption level, further specifications are needed. First let $(1+r)\rho = 1$. I also use a quadratic utility function given by

$$u(C_t) = C_t + \frac{a}{2}C_t^2,$$

where a is a constant. Then, put the quadratic utility function into the Euler equation given in (2.11) and solve for the consumption level, and we get

$$C_t = E_t[C_{t+s}] \text{ for } s = 0, 1, 2, \ldots. \tag{2.12}$$

With a quadratic utility function, consumption will follow a random walk. The best predictor of next period consumption is the consumption level this period. We get the same consumption level in all periods because the subjective discount rate is equal to the real discount rate and there is no tilting of consumption to the present or to the future. If we allow for consumption tilting, we get a consumption profile that is increasing or decreasing over time in a predictable pattern, relaxing the assumption that the subjective discount rate is equal to the real discount rate will not alter the main conclusions in this model.

To see how this result affects the current account we need to find the intertemporal budget constraint. To find the intertemporal budget constraint for the infinite-period case we are summing the one-period budget constraint, given in equation (2.8), over all periods and discounting to period t values, this gives

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \{C_s + G_s + I_s + B_{s+1}\} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \{(1+r)B_s + Y_s\}. \tag{2.13}$$

If we use equation (2.10), equation (2.13) can be rewritten as[3]

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_s=(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\{Y_s-G_s-I_s\}.$$

If we substitute for the consumption level from equation (2.12) we get

$$C_t\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}=(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\{Y_s-G_s-I_s\}.$$

By taking expectations of this equation we get[4]

$$C_t=\frac{r}{1+r}\left[(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t(Y_s-G_s-I_s)\right]. \tag{2.14}$$

This equation implies *certainty equivalence*. If we define net output as $NO_t \equiv Y_t - G_s - I_s$, consumption is determined as if future net output is known for sure. Certainty equivalence means that the consumer acts as if there was no uncertainty even when there is. This is because the utility function is quadratic so marginal utility is linear, and the consumer is risk neutral.[5] Equation (2.14) is an example of the *permanent income hypothesis*; the consumer want to consume a given share of the present value of net output in every period and the consumption plan is perfectly smooth. In this type of model it is important to distinguish between permanent and transitory shocks. Let a permanent shock be a permanent higher income in future periods, and let the consumer anticipate the shock. Then, the consumer's optimal strategy is to increase consumption in every period to adjust to the higher value of total lifetime income, since the shock is permanent this can be done without using the current account. On the other hand if the shock is temporary, the present value of lifetime income will not change much. Then, to smooth consumption the consumer will choose to save almost all of the temporary higher income and run a current account surplus. The consumption every period will only increase by a small amount. The model implies that transitory shocks to the economy will create large fluctuations in

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[3]Where I use

$$\begin{aligned}&\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\{(1+r)B_s-B_{s+1}\}\\&=[(1+r)B_t-B_{t+1}]+\left(\frac{1}{1+r}\right)[(1+r)B_{t+1}-B_{t+2}]+\left(\frac{1}{1+r}\right)^2[(1+r)B_{t+2}-B_{t+3}]+\dots\\&=(1+r)B_t-\lim_{i\to\infty}\left(\frac{1}{1+r}\right)^i B_{t+i+1}=(1+r)B_t.\end{aligned}$$

[4]Note that $\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}$ is a geometric series such that $\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}=\frac{1+r}{r}$ for $r>0$.

[5]To see why the consumer is risk neutral note that $E[u'(C)]=u'(E[C])$ for a quadratic utility function.

the current account, while permanent shocks have no effect.

In this model Y_t, G_t, and I_t are treated as random variables and their present values are given as permanent values. Let the one-period average permanent value for a random variable at time t be given by $\tilde{X}_t$, and we can write

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tilde{X}_s=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s\Rightarrow\tilde{X}_t=\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s.$$

If we use this notation, equation (2.14) can be rewritten as

$$C_t=rB_t+E_t\tilde{Y}_t+E_t\tilde{G}_t+E_t\tilde{I}_t. \tag{2.15}$$

Then, substitute for C_t from equation (2.15) into equation (2.2), and we get

$$CA_t=(Y_t-E_t\tilde{Y}_t)-(I_t-E_t\tilde{I}_t)-(G_t-E_t\tilde{G}_t),$$

and by using $\widetilde{NO}_t=\tilde{Y}_t-\tilde{G}_t-\tilde{I}_t$ we get

$$CA_t=NO_t-E_t\widetilde{NO}_t. \tag{2.16}$$

We can see from this equation that the current account in period t is determined by the difference between net output and the expected average present value of future net output. If we let the change in net output be defined as $\Delta NO_t\equiv NO_t-NO_{t-1}$, equation (2.16) can be rearranged to the following model

$$CA_t=-\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}E_t\Delta NO_s. \tag{2.17}$$

The derivation of this expression can be found in Appendix A. Equation (2.17) is a PVM for two variables, CA_t and ΔNO_t. The model states that the current account balance is a linear function of the discounted value of expected changes in future income streams. When the discounted value of future changes in net output is positive, the current account balance should be negative. When we expect higher output in the future we should run a current account deficit today. The current account is a linear function of expected future changes in net output, and the model implies that if net output is stationary in first differences the current account must be stationary in levels. Another important point can be found by following Campbell (1987), where a testable implication based on information at time $t-1$ is given by

$$CA_t-\Delta NO_t-(1+r)CA_{t-1}=-r\varepsilon_t, \tag{2.18}$$

where ε_t is given by

$$\varepsilon_t \equiv \left(\frac{1}{1+r}\right) \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[E_t NO_s - E_{t-1} NO_s\right].$$

See Appendix B for the derivation of equation (2.18). At time $t-1$, let $\varepsilon_t = 0$, this is the case if the expected present value of future net output is equal at time t and $t-1$, and this should be true at time $t-1$. Then, we have the following expression for the current account

$$CA_t = \Delta NO_t + (1+r)CA_{t-1}. \tag{2.19}$$

If the agent is rational, equation (2.19) is the best prediction of the current account, and we do not expect this prediction to be systematically wrong. At period $t-1$, before the realized net output next period is known, all the available information on the outcome in the next period is used, and the best model for the next period current account is the change in net output next period plus the gross current account this period. Since we are using period $t-1$ information, and the values of CA_t and ΔNO_t are known only in expectation, a more intuitive formulation of equation (2.19) may be

$$E_{t-1}CA_t = E_{t-1}\Delta NO_t + (1+r)CA_{t-1}.$$

The current account is decided by the rational agent where the objective is to get a smooth consumption path, which is done by adjusting external saving. Let us assume that net output is above its permanent value, so the current account is in surplus. If we expect net output next period (period t) to be at the same level as this period (period $t-1$) the current account should be equal in the two periods except the change in income from the increase in interest income since the total amount of foreign assets has increased. To hold disposable income constant when ΔNO_t is expected to be zero, the current account in period t should equal $(1+r)CA_{t-1}$. If we expect net output to change from this period to the next, we need to compensate for this change by changing the current account to keep consumption constant. If we expect net output to be higher in period t, $\Delta NO_t > 0$, all of this higher net output should be saved and this is what the model predicts. This argument is parallel for a nation with a current account deficit.

The main result from this chapter is the PVM given in equation (2.17). I present equation (2.18) because this formulation can easily be tested, and we will se this result in the Lagrange-multiplier test in section 4.7.2. These observations and other implications of the model in equation (2.17) are examined for Norwegian data in Chapter 4.

Chapter 3

Empirical background

In this chapter I will look at the background for the empirical part of my thesis. I use the empirical methodology developed by Campbell and Shiller (1987) and Campbell (1987), their method is to use vector autoregressive (VAR) models to test present-value relationships. The advantage with the Campbell-Shiller method is that they deal with the problem of non-stationarity in the data and the possibility that the econometrician lack relevant information that the rational agent is assumed to use when predicting the future. A VAR with no contemporaneous terms on the right-hand side can easily be estimated by the ordinary least squares (OLS) procedure.[1] OLS is valid if the model to be estimated satisfies the classical linear regression (CLR) model. In this chapter I present some background for the Campbell-Shiller method, the assumptions behind the CLR model, some principles of forecasting by using a VAR, and some theory for empirical testing of a PVM. I also present some results from other literature that evaluates the validity of the intertemporal approach to the current account for other countries.

3.1 The empirical methodology

Campbell and Shiller (1987) and Campbell (1987) develop a method for testing PVMs by estimating cointegrated variables in a VAR. The first paper by Campbell and Shiller (1987) use the method to test the rational expectations theory of the term structure of bonds and the PVM for stock prices. Campbell (1987) tests the permanent income hypothesis (PIH) for consumption on the national level where he differentiates between labor income and capital income, Campbell is investigating saving as national capital accumulation, while I will look at saving as foreign asset allocation. Campbell's paper lie close to what I will do and I will take a brief look at it in the next paragraph.

If the PIH is true, the theory states that saving, which means buying capital, should

[1] A contemporaneous right-hand side variable is a variable that is varying at the same time as the left-hand side variable; if the system has contemporaneous right-hand side variables, we have a simultaneous equation system.

be equal to the discounted sum of future declines in labor income so consumption is kept constant over the lifetime, and as Campbell writes, "they save for a rainy day." The PIH predicts that saving contains all available information on future developments of labor income. To test this implication from the PIH, Campbell estimates a VAR for saving and the change in labor income, and he writes:[2]

> *If the PIH is true, saving is the optimal forecast of the present value of future declines in labor income, conditional on agents' information; therefore the unrestricted VAR forecast of this present value should equal saving.*

This is the basis for the empirical part in my thesis, the model in Chapter 2 implied that the current account is equal to the discounted sum of future changes in net output, hence the current account balance should be equal to the forecast of the present value of changes in net output. Campbell and Shiller show how this implication can be tested formally, and I follow their approach in Chapter 4.

A VAR system consists of two or more variables, all variables are treated symmetrically and every component depends on own lagged values and lagged values of all the other components in the system. In a VAR all variables are endogenous and we can say that everything depends on everything. A VAR of order p, written as VAR(p), means a system consisting of one equation for each endogenous variable, and the dependent variable depends on p lags of all the components in the VAR. A VAR(p) with two endogenous variables can be formulated in the following way

$$y_{1t} = c_1 + b_{11}y_{1,t-1} + b_{12}y_{1,t-2} + \cdots + b_{1p}y_{1,t-p} + a_{11}y_{2,t-1} + a_{12}y_{2,t-2} + \cdots + a_{1p}y_{2,t-p} + u_{1t}$$

$$y_{2t} = c_2 + b_{21}y_{1,t-1} + b_{22}y_{1,t-2} + \cdots + b_{2p}y_{1,t-p} + a_{21}y_{2,t-1} + a_{22}y_{2,t-2} + \cdots + a_{2p}y_{2,t-p} + u_{2t}.$$

This is called a bivariate system where the endogenous variables are y_{1t} and y_{2t}, the a's, b's and c's are parameters, and u_{1t} and u_{2t} are residuals. In a VAR all endogenous variables depend on predetermined lagged values of all the variables in the VAR, and all the right-hand side variables in the system are exogenous. This is a general strength of a VAR, when all right-hand side variables are exogenous there can be no feedback from the left-hand side to the right-hand side and simultaneity bias is not an issue.

In the intertemporal model the current account is used by the agent to absorb temporary fluctuations in net output; if this theory holds, the agent chooses the current account balance in a given period on the basis of the best predictions of future developments in net income. The current account then contains all available information for future developments in net income and can be used by the econometrician to construct estimates of changes in net output in the future. The rational agent's information, which is only observed through the realized values of the current account, is used to forecast changes

[2]See Campbell (1987), pp. 1251.

in net output. It follows that relevant variables that are omitted from the VAR is no problem as long as lagged values of the current account is included. So at least in theory, omitted variables are not a problem when using this method.

In section 3.3, I demand that the data used for forecasting should be stationary, but this is not always the case. Campbell and Shiller show how to transform a model so the right-hand side variable in a PVM is in first differenced form while the left-hand side variable is in levels. Appendix A shows this transformation for the intertemporal model where net output can be included in first differenced form, this is important since we expect net output to have a unit root.

3.2 The classical linear regression model

A general VAR can be estimated equation by equation using OLS. OLS is a method for estimating the parameters in a model; the method is based on choosing the parameters for the model that minimizes the sum of squared residuals (SSR). In regression analysis some assumptions must be fulfilled for the OLS estimates to be valid. I will present the assumptions for the classical linear regression (CLR) model. The CLR model is only valid if we have access to a sample that has the exact distribution of the population we want to explain; this is usually not satisfied, but can often be solved by using large sample theory. In large sample theory the asymptotic properties of a sample when the number of observations is getting large are approaching the properties of the whole population and this is used to derive the model. I will not go into large sample theory in this thesis.[3] The CLR assumptions are given by:

Assumption 1 (linearity):
The regression model can be written as

$$y_t = b_1 x_{1t} + b_2 x_{2t} + \cdots + b_K x_{Kt} + u_t \quad (t = 1, 2, \ldots, T),$$

where the b's are the parameters to be estimated, y are the variable we are modeling, the x's are the explanatory variables and u is an error term. There are K regressors and T observations.

Assumption 2 (strict exogeneity):
The error has an expected value equal to zero for any value of the explanatory variables, this can be written as

$$E(u_t | x_{11}, x_{12} \ldots, x_{1T}, x_{21}, \ldots, x_{2T}, \ldots, x_{K1}, \ldots, x_{KT}) = 0 \quad (t = 1, 2, \ldots, T).$$

[3]For details on the CLR model and large sample theory see Hayashi (2000), Chapter 1 and 2.

Assumption 3 (no perfect multicollinearity):
There are none exact linear relationships among the explanatory variables.

Assumption 4 (homoskedasticity):
The error terms has a constant variance dependent on any value of the explanatory variables, this can be written as

$$E(u_t^2|x_{11}, x_{12}\ldots, x_{1T}, x_{21}, \ldots, x_{2T}, \ldots, x_{K1}, \ldots, x_{KT}) = \text{constant} > 0 \quad (t = 1, 2, \ldots, T).$$

Assumption 5 (no autocorrelation):
There is no correlation between the residuals, this can be written as

$$E(u_t u_\tau|x_{11}, x_{12}\ldots, x_{1T}, x_{21}, \ldots, x_{2T}, \ldots, x_{K1}, \ldots, x_{KT}) = 0 \quad (t, \tau = 1, 2, \ldots, T; t \neq \tau).$$

Assumption 6 (normality):
The distribution of the error terms conditional on
$\{x_{11}, x_{12}\ldots, x_{1T}, x_{21}, \ldots, x_{2T}, \ldots, x_{K1}, \ldots, x_{KT}\}$ is jointly normal.

The first assumption states that the regression model must be a linear function of the regressors. The second assumption demands the regressors to be exogenous, which means determined outside the model. In a VAR all regressors are predetermined so this assumption holds. Assumption 3 is about multicollinearity, we have perfect multicollinearity if two or more regressors are perfectly correlated. In the case of perfect multicollinearity the model is not identified and cannot be estimated. Assumption 4 is about the absence of heteroskedasticity, which means the variance of the residuals must be constant. Assumption 5 of no autocorrelation states that the correlation between any pair of residuals should be 0 (except for the same two residuals, correlation between the same two residuals is equal to the variance and is covered in assumption 4). Autocorrelation may be a sign of a misspecified model where for example too few lags are included. The last assumption is about the normality of the distribution of the error terms. The normality assumption does not matter for the efficiency of the estimator, but it is important for standard testing procedures to be valid. For a regression where assumptions 1–5 hold, the estimators are both unbiased and consistent, and OLS produces the best linear unbiased estimators (BLUE).[4]

The first three assumptions will not be discussed further, and they are expected to hold. I will investigate the last three assumptions for Norwegian data in section 4.8.

[4]See Hayashi (2000), Chapter 1.

3.3 Forecasting using a VAR

To test the PVM, I need a forecast for changes in net output. In this section I will present some principles of forecasting. Assume we have access to two data processes given by $\{y_{1t}\}_{t=1}^{T}$ and $\{y_{2t}\}_{t=1}^{T}$. To make the arguments simple I will assume that the data processes can be explained by a VAR(1). The model for explaining y_{1t} and y_{2t} can be written as

$$y_{1t} = c_1 + b_1 y_{1,t-1} + a_1 y_{2,t-1} + u_{1t}$$

$$y_{2t} = c_2 + b_2 y_{1,t-1} + a_2 y_{2,t-1} + u_{2t},$$

where y_{1t} and y_{2t} are the endogenous variables, y_{1t-1} and y_{2t-1} are the regressors, the a's, b's and c's are parameters to be estimated and, u_{1t} and u_{2t} are the residuals. If both equations satisfy the CLR assumptions given above, the parameters can be estimated by OLS. An important property for forecasting to make sense is that the variables need to be stationary. For a non-stationary process the mean and variance of the series is varying over time, which makes standard interference invalid. For a process to be covariance-stationary the mean and the autocovariances should be constant and independent of time, this can be written as

$$\begin{aligned} E(y_{it}) = \mu_i & \qquad \text{for all } t \text{ and } i = 1,2 \\ E[(y_{it} - \mu)(y_{it-j} - \mu)] = \gamma_{ij} & \qquad \text{for all } t \text{ and any } j, \text{ and } i = 1,2, \end{aligned}$$

where the E is an expectations operator, μ_i is the mean and γ_{ij} is the covariance between observation t and observation $t - j$.[5] Then, we can estimate the VAR and determine the coefficients. Next, the estimated coefficients can be used to predict the future; a one period ahead forecast is given by

$$E(y_{1,t+1}) = c_1 + b_1 y_{1,t} + a_1 y_{2,t}$$

$$E(y_{2,t+1}) = c_2 + b_2 y_{1,t} + a_2 y_{2,t}.$$

A two period ahead forecast is given by

$$\begin{aligned} E(y_{1,t+2}) &= c_1 + b_1 E(y_{1,t+1}) + a_1 E(y_{2,t+1}) \\ &= c_1 + b_1 c_1 + b_1^2 y_{1,t} + b_1 a_1 y_{2,t} + a_1 c_2 + a_1 b_2 y_{1,t} + a_1 a_2 y_{2,t} \end{aligned}$$

$$\begin{aligned} E(y_{2,t+2}) &= c_2 + b_2 E(y_{1,t+1}) + a_2 E(y_{2,t+1}) \\ &= c_2 + b_2 c_1 + b_2 b_1 y_{1,t} + b_2 a_1 y_{2,t} + a_2 c_2 + a_2 b_2 y_{1,t} + a_2^2 y_{2,t}. \end{aligned}$$

[5] See Hamilton (1994), Chapter 3.

For a VAR(1), we can in principle forecast values as far into the future we want by following the procedure given above, as long as we have reliable estimated parameters and the values y_{1t} and y_{2t}. This result can be generalized to a higher order VAR, but this will get algebraically messy. In Chapter 4 I show a general formulation for forecasting with a higher order VAR by using matrix notation.

3.4 Testing the PVM

The PVM derived in Chapter 2 is a nested version of a general VAR. To get to the PVM we need to impose several restrictions on the parameters in the VAR. The method for testing the theory is to evaluate the validity of the cross-equation restriction on the VAR implied by the theory. Standard approaches for doing this are a Wald test, a log-likelihood ratio test and a Lagrange-multiplier test, which are all asymptotically equal.[6] I will present all three tests because I believe they all give some valuable insights to the intertemporal model.

For *the log-likelihood ratio test*, both the general and the restricted model are estimated and the log-likelihood functions for both models are compared.[7] The restrictions are valid if we fail to reject the restricted model as a valid simplification of the general model. An issue when using the log-likelihood ratio test is how to estimate the restricted model. The restricted model is usually estimated by the maximum likelihood (ML) method. I will not go into ML estimation in this thesis.[8] I present the log-likelihood ratio test in section 4.7.1 where I present results from a ML estimation reported from the software package OxMetrics.

The Lagrange-multiplier test, also known as the Score test, evaluates whether deviations from the derived model is unpredictable at time $t-1$. We are not supposed to make any systematic predictions about the future that deviates from the PVM (see equation (2.19)) and this can be formally tested. The test is easy to implement by testing for significance in the available information and whether we can outperform the theoretical model. I present the test and its result in section 4.7.2.

The last test, *the Wald test* can be calculated using the results from the general VAR alone, no additional regressions are necessary. This test is similar to a standard t-test where the actual parameter values are compared to the hypothesized values. In addition to a standard Wald test, I will report a heteroskedastic robust Wald test. I present the Wald test and its results in section 4.9.

[6] See Engle (1984).
[7] The calculation of the log-likelihood function is given in section 4.7.1.
[8] See e.g. Hamilton (1994), Chapter 11.

3.5 Results from related literature

The first paper to use the Campbell-Shiller methodology to test the intertemporal approach to the current account was a paper by Sheffrin and Woo (1990). They derived the parallel implications from Campbell (1987) for the PIH in the open economy. As noted above, the current account should equal the present value of expected future changes in net output. The paper estimates a VAR for changes in net output and the current account and test the restrictions on this VAR implied by the intertemporal model. I follow the same approach so further details can be found in Chapter 4. Sheffrin and Woo uses data from Canada, Denmark, Belgium, and the United Kingdom for the period 1957 to 1985. The cross-equation restrictions imposed on the VAR are rejected for Canada, Denmark, and the United Kingdom, but the model cannot be rejected for Belgium.

Otto (1992) follows the same approach and test the theory for the United States and Canada for the period 1950 to 1988. The paper reports a fairly good graphical fit for the United States, but not for Canada. Formally the model is rejected for both countries.

Ghosh (1995) is taking a similar approach, but his motive is to examine the capital flows amongst major industrialized countries in the intertemporal framework. The countries he investigates are the United States, Japan, Germany, United Kingdom, and Canada for the period 1960 to 1988. Ghosh finds that the capital flows are larger than what is expected from economic fundamentals for all countries expect the United States. The PVM is rejected for all countries except the United States.

Agénor et al. (1999) test the intertemporal model for France for the period 1970 to 1996; they find evidence both graphically and formally in favor of the PVM.

I summarize the results in Table 3.1. Despite some satisfactory performances, the overall picture does not look well for the basic version of the intertemporal model. For further review of the early literature on the intertemporal approach to the current account, see Obstfeld and Rogoff (1995) and Razin (1995).

Table 3.1: Results from the literature

Country	Sheffrin and Woo (1990)	Otto (1992)	Ghosh (1995)	Agénor et al. (1999)
Canada	Rejected	Rejected	Rejected	-
Denmark	Rejected	-	-	-
Belgium	Not rejected	-	-	-
U.S.	-	Rejected	Not rejected	-
U.K.	Rejected	-	Rejected	-
Japan	-	-	Rejected	-
Germany	-	-	Rejected	-
France	-	-	-	Not rejected

Results from tests of the intertemporal model of the current account in related literature.

Chapter 4

Empirical analysis

In this chapter I evaluate the model presented in Chapter 2 for Norwegian data. We saw in Chapter 3 that the intertemporal approach to the current account was rejected in the majority of previous studies. This chapter is going to show a rejection of the model for Norwegian data, but I will present some less formal results that advocate the usefulness of the model.

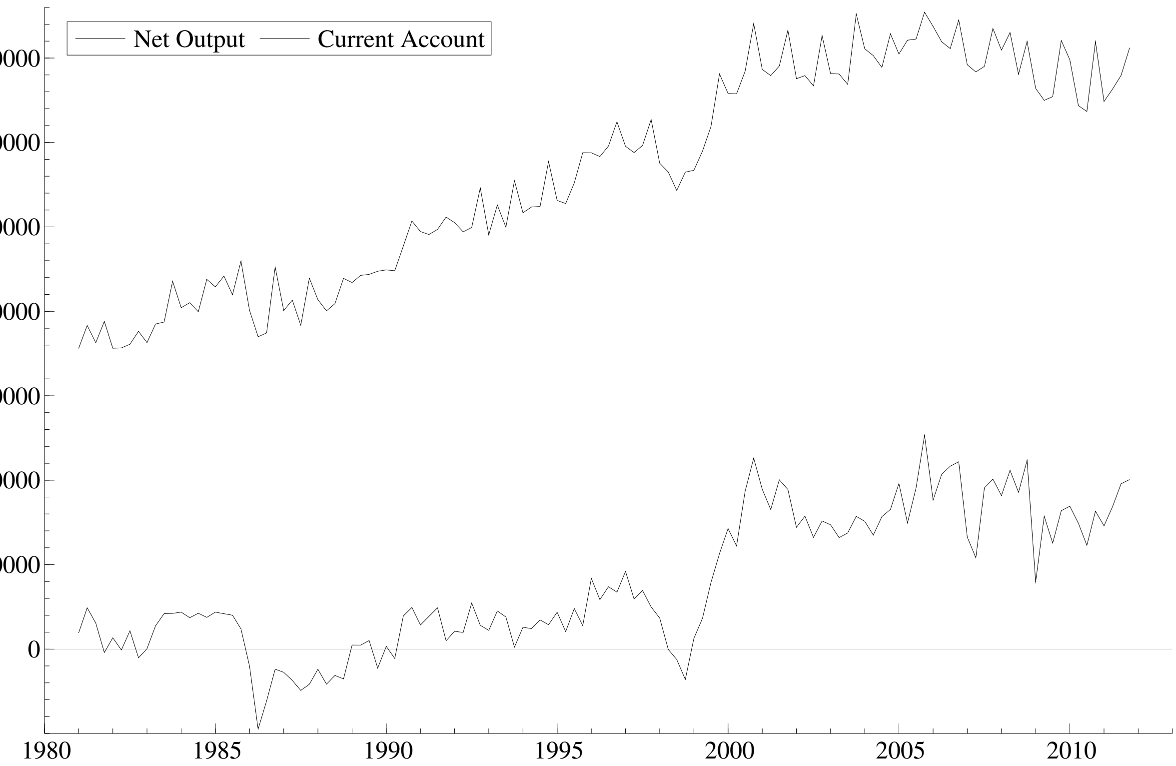

Figure 4.1: The real net output and the real current account plotted for Norwegian data for the period 1981Q1 to 2011Q4.

4.1 Data

The variables of interest are gross domestic product, government spending, investment, and the current account balance. I use quarterly data from 1981Q1 to 2011Q4, all data used in this chapter is obtained from Statistics Norway.[1] The data is deflated to real and per-capita terms. The net output series is constructed by subtracting government consumption and investment from the gross domestic product. The current account balance is equal to total export minus total import plus the balance of income and current transfers. It is also worth noting that the data is not adjusted for seasonally fluctuations, this is because I lack seasonally adjusted data for the current account series. I get some significance for seasonal dummies in my regressions, but this will be ignored in the rest of this chapter. I report the means and standard deviations for the main variables in Table 4.1. The net output and current account series are plotted in Figure 4.1.

Table 4.1: Quarterly moments

	Mean	Standard deviation
Net output	56333	12598
Change in net output	289	3256
Current account	7744	8107

All values are in per-capita terms and denoted in constant 2009 NOK.

4.2 Testing for stationarity

A first step in evaluating the PVM for the current account given in equation (2.17) is to check the stationarity conditions implied by theory: If net output is stationary in first difference the current account should be stationary in levels. We can start off by looking at the correlograms; I plot the correlograms for the current account, net output, and changes in net output in Figure 4.2.[2] We see that both the net output series and the current account series shows strong persistence, which indicates non-stationarity. For a series to be stationary, we want the autocorrelation function to die away fast. The correlogram for net output in first difference shows this property and this indicates stationarity. The correlogram for the net output in levels and the current account indicates non-stationary data. To get a conclusion I need a more formal approach to determine whether the variables are stationary or not.

[1]The gross domestic product, investment, and government consumption are from the national accounts and can be found at *http//:www.ssb.no/knr_en/*. The current account is from current and capital account in the balance of payments, and can be found at *http//:www.ssb.no/ur_en/*.

[2]A correlogram is a plot of the autocorrelation function, which gives the correlation between a variable and its lagged values.

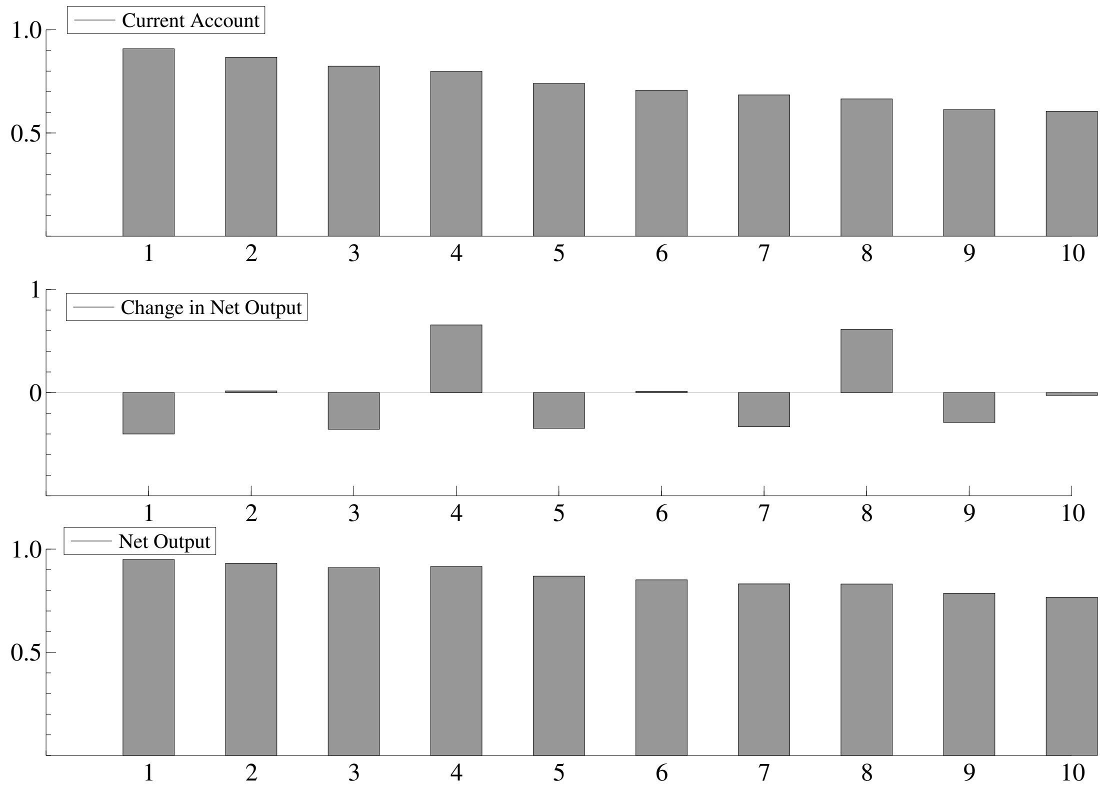

Figure 4.2: Correlograms for net output, change in net output, and the current account.

A standard approach to test whether a time series is stationary is to use the augmented Dickey-Fuller (ADF) test.[3] The Dickey-Fuller test investigates whether a series is reverting back to a mean. To perform the test we run a regression of the form

$$\Delta X_t = \text{constant} + \phi X_{t-1} + \sum_{i=1}^{q} \gamma_i \Delta X_{t-i} + \eta_t, \tag{4.1}$$

where X_t is the variable of interest and, ϕ and the γ_i's are the parameters to be estimated. The operator Δ is defined as standard backward difference. The null-hypothesis is a unit root in the time series X_t, and can be formulated as H_0: $\phi = 0$. If the time series has a unit root the series is non-stationary. The alternative-hypothesis of no unit root is H_A: $\phi < 0$. The test is performed as a t-test on ϕ, but with a different distribution of the critical values. An important practical issue is how to choose the lag length q in equation (4.1). If q is too low, the remaining serial correlation in the residuals can bias the test. A too high q and the degrees of freedom will be low and the power of the test will suffer. To determine q I apply the Akaike information criterion.[4]

The results from a unit root test for NO_t, CA_t, and ΔNO_t are reported in Table 4.2. For the ADF test for a unit root in net output the Akaike information criterion suggests

[3]See Dickey and Fuller (1979).

[4]See section 4.4 for details on lag-selection using information criterion.

Table 4.2: Unit root test for the period 1981–2011

NO_t	CA_t	ΔNO_t
-0.02	-0.05	-1.36
(-1.3)	(-1.3)	(-4.1)

The reported parameter is ϕ. The sample size is 124. The number of lags is 11 in the test of NO_t, 1 in the test of CA_t, and 7 in the test for ΔNO_t. t-values in parentheses.

a lag length of 11. The computed ADF test statistic (-1.3) is higher than the 5 percent critical value (-2.9), and we cannot reject the null-hypothesis of a unit root in net output, net output is non-stationary as expected. For the stationarity test of ΔNO_t, Akaike suggests a lag length of 7, the ADF test statistic (-4.1) is below the 5 percent critical value (-2.9) and we can reject the hypothesis of a unit root in ΔNO_t, and the change in net output is stationary. For the current account balance, the Akaike information criterion suggests a lag length of 1. The computed ADF test statistic (-1.3) is higher than the 5 percent critical value (-2.9) and we cannot reject the hypothesis of a unit root in CA_t. This is not in line with what we expected based on the intertemporal model in the previous section. This is a crucial problem; the model and the statistical tests are not valid if the current account series is non-stationary in levels.

However, the finding of a non-stationary current account is common in the literature when standard Dickey-Fuller tests are applied; in Sheffrin and Woo (1990) the hypothesis of a unit root in the current account cannot be rejected for any of the countries they study. This problem is addressed in detail in a paper by Wu (2000): He finds support for a stationary current account balance for industrialized countries even when standard tests reject stationarity. The rest of this analysis is based on the assumption of a stable current account balance.

4.3 The VAR and the PVM

To test the PVM given in equation (2.17), I follow the method developed by Campbell (1987) and Campbell and Shiller (1987). The PVM for the current account is a nested version of a general VAR. Campbell and Shiller show how to test PVMs by testing a set of restrictions on a general VAR. I will show how to derive the PVM from a VAR. I start off by finding an estimate for the representative consumer's forecast of the change in net output. To do this I treat conditional expectations as linear projections on information. This means that the model for forecasting the current account and changes in net output is a linear function of available information about the past. The information set I use is previous values of the current account and previous changes in net output. The VAR is set up with current dated variables of CA_t and ΔNO_t on the left-hand side, and lagged variables on the right-hand side. In general the system can be formulated as

$$\begin{aligned} \Delta NO_t &= \alpha_{11}\Delta NO_{t-1} + \cdots + \alpha_{1p}\Delta NO_{t-p} + \beta_{11}CA_{t-1} + \cdots + \beta_{1p}CA_{t-p} + \epsilon_{1t} \\ CA_t &= \alpha_{21}\Delta NO_{t-1} + \cdots + \alpha_{2p}\Delta NO_{t-p} + \beta_{21}CA_{t-1} + \cdots + \beta_{2p}CA_{t-p} + \epsilon_{2t}, \end{aligned} \tag{4.2}$$

where the α's and the β's are parameters to be estimated, ϵ_{1t} and ϵ_{2t} are white noise disturbance terms, and p is the number of lagged variables.[5] There are no constant terms in the system; this is because the data I use in the VAR is demeaned. The PVM is not saying anything about the mean values of the variables, only the dynamic relationship between them, so using demeaned values does not seem to affect the results in this analysis. A matrix formulation of the system given in (4.2) can be obtained if we define

$$\mathbf{z}_t \equiv \begin{bmatrix} \Delta NO_t \\ CA_t \end{bmatrix}, \quad \mathbf{A}_p \equiv \begin{bmatrix} \alpha_{1p} & \beta_{1p} \\ \alpha_{2p} & \beta_{2p} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\nu}_t \equiv \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix},$$

then the VAR(p) can be written more compactly as

$$\mathbf{z}_t = \mathbf{A}_1\mathbf{z}_{t-1} + \mathbf{A}_2\mathbf{z}_{t-2} + \cdots + \mathbf{A}_p\mathbf{z}_{t-p} + \boldsymbol{\nu}_t. \tag{4.3}$$

To find a forecast for the change in net output, it is more convenient to work with a model with only one lagged variable. It is possible to rewrite the VAR(p) in equation (4.3) as a VAR(1) if we define

$$\boldsymbol{\Psi}_t \equiv \begin{bmatrix} \mathbf{z}_t \\ \mathbf{z}_{t-1} \\ \mathbf{z}_{t-2} \\ \vdots \\ \mathbf{z}_{t-p+1} \end{bmatrix}, \quad \boldsymbol{\Gamma} \equiv \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_p \\ \mathbf{I}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I}_2 & \mathbf{0} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\xi}_t \equiv \begin{bmatrix} \boldsymbol{\nu}_t \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}.$$

The subscript on the identity matrix $\mathbf{I}$ indicates the matrix dimensions, so $\mathbf{I}_2$ is a (2×2) matrix. Let T be the number of observations and $\mathbb{R}$ the set of real numbers, then $\boldsymbol{\Psi}_t, \boldsymbol{\xi}_t \in \mathbb{R}^{2p \times T}$ and $\boldsymbol{\Gamma} \in \mathbb{R}^{2p \times 2p}$. The bold zeros represent matrices of zeros. The matrix $\boldsymbol{\Gamma}$ is called the companion matrix of the VAR. Then, equation (4.3) can be written as a VAR(1)

$$\boldsymbol{\Psi}_t = \boldsymbol{\Gamma}\boldsymbol{\Psi}_{t-1} + \boldsymbol{\xi}_t. \tag{4.4}$$

[5] A process is called white noise if the mean is zero, the variance is constant and the covariance's between the terms are zero; see Hamilton (1994), Chapter 3. If the residuals satisfy the CLR assumptions presented in Chapter 3 in this thesis, they are white noise.

I use the regression model in equation (4.4) to make forecasts of future changes in net output. If the VAR is stationary we can iterate backwards to get the following expression for the expected value of $\mathbf{\Psi}_s$

$$E[\mathbf{\Psi}_s|\mathbf{\Omega}_t] = \mathbf{\Gamma}^{s-t}\mathbf{\Psi}_t, \tag{4.5}$$

where $\mathbf{\Omega}_t$ is the representative consumer's information set at time t.[6,7] This information set includes more information than what is directly observable to the econometrician. The model predicts a current account balance that reflects the representative consumer's expectations about future changes in net output. If we include the current account in the estimate for changes in net output, all the information available to the consumer is used in our forecast of changes in net output. This is a strength of the Campbell-Shiller procedure, and at least in theory, omitted information is not a problem when applying this method. If we use equation (4.5), an expression for the representative consumer's forecast of future changes in output can be written as $E[\Delta NO_s|\mathbf{\Omega}_t] = \mathbf{e_{\Delta NO}}\mathbf{\Gamma}^{s-t}\mathbf{\Psi}_t$, where $\mathbf{e_{\Delta NO}} \equiv [1 \;\; 0 \;\; 0 \;\; \cdots \;\; 0]$ is a $(1 \times 2p)$ row vector constructed so that $\mathbf{e_{\Delta NO}}\mathbf{\Psi}_t = \Delta NO_t$. If we use the expected value of the change in net output from the VAR in the PVM in equation (2.17) we get

$$CA_t = -\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{e_{\Delta NO}}\mathbf{\Gamma}^{s-t}\mathbf{\Psi}_t.$$

I define the real discount rate as $\tau \equiv 1/(1+r)$. Contingent on a stationary VAR, the model for the current account can be written as[8]

$$CA_t = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}\mathbf{\Psi}_t. \tag{4.6}$$

I omit the subscript on the identity matrix and let $\mathbf{I} = \mathbf{I}_{2p}$. To find the formal restrictions, write $\mathbf{e_{CA}}\mathbf{\Psi}_t = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}\mathbf{\Psi}_t$, where $\mathbf{e_{CA}} \equiv [0 \;\; 1 \;\; 0 \;\; \cdots \;\; 0]$ is a $(1 \times 2p)$ row vector, constructed so that $\mathbf{e_{CA}}\mathbf{\Psi}_t = CA_t$. The model imply the following restriction on the companion matrix from the VAR

$$\mathbf{e_{CA}} = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}. \tag{4.7}$$

The next step is to estimate the parameter matrix $\mathbf{\Gamma}$ and find $\hat{\mathbf{\Gamma}}$. When $\hat{\mathbf{\Gamma}}$ is determined the restriction in equation (4.7) can be formally tested.

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[6] The VAR is stationary if all the roots of the equation $\det(\mathbf{I} - \mathbf{A}_1 x - \mathbf{A}_2 x^2 - \cdots - \mathbf{A}_p x^p) = 0$ lie outside the unit circle, see Appendix D.

[7] I use $E[\mathbf{\Psi}_s|\mathbf{\Omega}_t] = E[(\mathbf{\Gamma}\mathbf{\Psi}_{s-1} + \boldsymbol{\xi}_s)|\mathbf{\Omega}_t] = \mathbf{\Gamma}E[\mathbf{\Psi}_{s-1}|\mathbf{\Omega}_t] + E[\boldsymbol{\xi}_s|\mathbf{\Omega}_t] = \mathbf{\Gamma}^2 E[\mathbf{\Psi}_{s-2}|\mathbf{\Omega}_t] = \ldots$, because $\boldsymbol{\xi}_t$ is white noise where $E[\boldsymbol{\xi}_s|\mathbf{\Omega}_t] = 0$ for all $s > t$.

[8]

$$\text{Because } \sum_{s=t+1}^{\infty}\tau^{s-t}\mathbf{\Gamma}^{s-t} = \tau\mathbf{\Gamma}\left[\mathbf{I} + (\tau\mathbf{\Gamma}) + (\tau\mathbf{\Gamma})^2 + (\tau\mathbf{\Gamma})^3 + \ldots\right] = (\tau\mathbf{\Gamma})(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}.$$

4.4 Deciding the lag length for the VAR

The results from estimating a VAR depends on the lag length, so p needs to be decided. A standard approach to discriminate between models of different lag lengths is to use *information criterion*. To evaluate a model with information criterion, we must calculate the following expression

$$S_p = \log\det(\hat{\boldsymbol{\Sigma}}_\xi) + pg(T), \quad \text{where } \hat{\boldsymbol{\Sigma}}_\xi = \frac{\hat{\boldsymbol{\xi}}_t \cdot \hat{\boldsymbol{\xi}}_t'}{T} \in \mathbb{R}^{2p\times 2p}$$

is the estimated variance-covariance matrix from the VAR and $\hat{\boldsymbol{\xi}}_t$ is the estimated residual values from the VAR in equation (4.4) given by $\hat{\boldsymbol{\xi}}_t = \boldsymbol{\Psi}_t - \hat{\boldsymbol{\Gamma}}\boldsymbol{\Psi}_{t-1}$.[9] The first term $\log\det(\hat{\boldsymbol{\Sigma}}_\xi)$ is a measure of the fit of the model, where $\det(\hat{\boldsymbol{\Sigma}}_\xi)$ is the determinant of the matrix $\hat{\boldsymbol{\Sigma}}_\xi$. The lower this value is the more accurate is the model in explaining the actual data. We prefer a parsimonious model, so the second term $pg(T)$ is a penalty term for including many parameters in the model. When using information criterion, the optimal lag length is the solution to the following problem

$$\hat{p} = \arg\min_p S_p.$$

Table 4.3 reports information criterion for different lag lengths and different forms of $g(T)$.[10] We can see that the three specifications in Table 4.3 favors a lag length of $\hat{p} = 4$.

Table 4.3: Lag selection in the VAR

	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$	$p=8$	$p=10$	$p=12$
Akaike	32.03	31.73	31.14	30.77	30.80	30.78	30.85	30.86	30.92
Schwartz	32.13	31.91	31.42	31.15	31.27	31.34	31.62	31.83	32.09
Hannan-Quinn	31.07	31.80	31.26	30.93	31.00	31.01	31.17	31.26	31.39

All specifications of the information criterion prefer a lag length of 4.

4.5 Results from a general VAR(4)

A VAR where all right-hand side variables are exogenous can be estimated equation by equation using ordinary least squares.[9] The next step is to estimate the parameters in the general VAR given in equation (4.4). Using matrix notation, the parameters are calculated in the following way

$$\hat{\boldsymbol{\Gamma}}^{OLS} = (\boldsymbol{\Psi}_{\mathbf{t-1}} \cdot \boldsymbol{\Psi}_{\mathbf{t-1}}')^{-1}\boldsymbol{\Psi}_{\mathbf{t-1}} \cdot \boldsymbol{\Psi}_{\mathbf{t}}'.$$

[9]See Hayashi (2000), Chapter 6.
[10]Akaike: $g(T) = 2/T$, Schwartz: $g(T) = \log T/T$, and Hannan-Quinn: $g(T) = 2\log(\log T)/T$.

Table 4.4: Results from the VAR(4)

	ΔNO_{t-1}	ΔNO_{t-2}	ΔNO_{t-3}	ΔNO_{t-4}
ΔNO_t	-0.71	-0.65	-0.54	0.26
	(-6.5)[-6.5]	(-5.3)[-4.9]	(-5.0)[-4.6]	(2.9)[3.0]
CA_t	0.31	0.20	0.28	0.30
	(1.9)[1.9]	(1.0)[1.0]	(1.7)[1.5]	(2.2)[1.8]
	CA_{t-1}	CA_{t-2}	CA_{t-3}	CA_{t-4}
ΔNO_t	0.34	-0.01	-0.16	-0.24
	(4.2)[4.3]	(-0.2)[-0.2]	(-2.0)[-2.1]	(-2.8)[-2.5]
CA_t	0.54	0.21	0.01	0.22
	(4.5)[3.6]	(1.7)[1.8]	(0.0)[0.0]	(1.8)[1.5]

The number of observations is 119, the number of lags is 4, t-values in parentheses, and heteroskedasticity-consistent t-values in brackets. The formula for calculating heteroskedastic robust variances is given in equation (4.12).

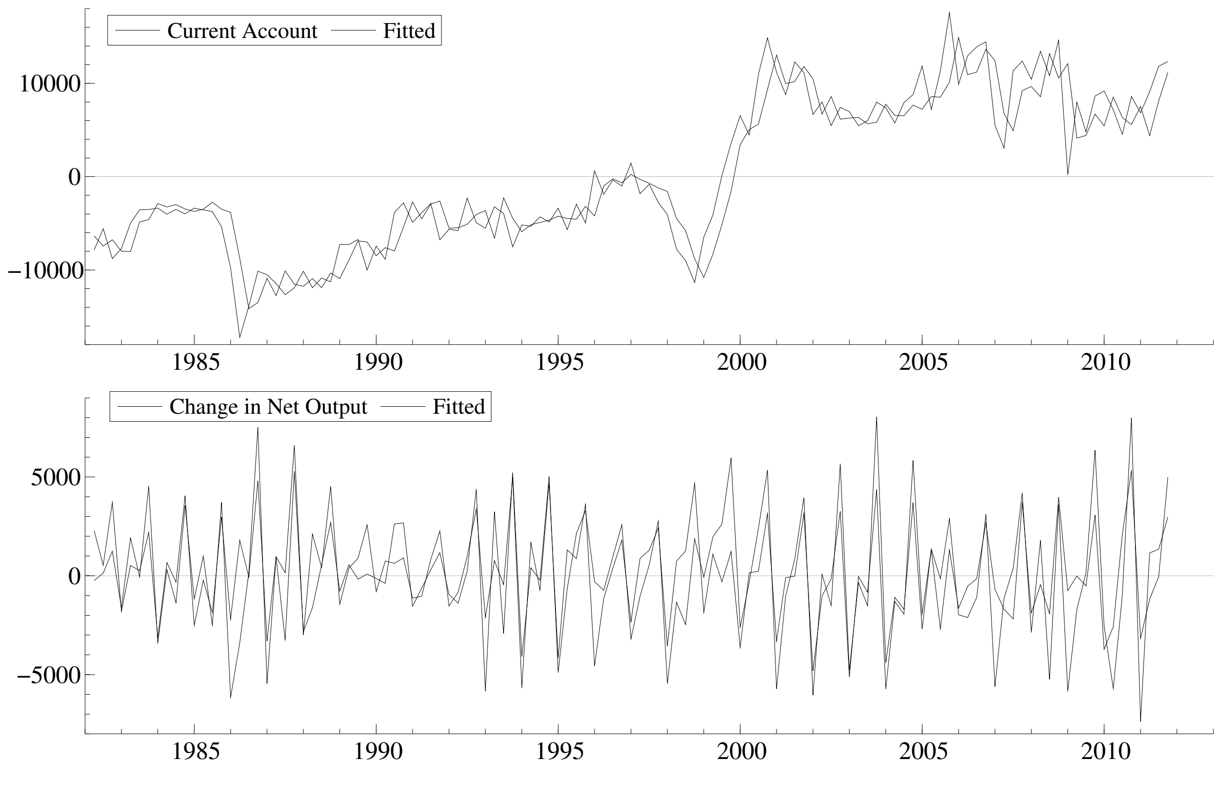

Figure 4.3: Actual values plotted against the fitted values from the unrestricted VAR(4) for the sample period 1982Q2 to 2011Q4.

The results from the VAR(4) is reported in Table 4.4, I also plot the fit of the unrestricted VAR in Figure 4.3. I find that the VAR(4) is stationary, see Appendix D for a proof. The general VAR tracks the current account series pretty good, but with a delay. This general model can be used to investigate the forecasting relationships implied by the intertemporal model.

4.6 Testing for Granger causality in the VAR

In the intertemporal model the agent use the current account to smooth consumption based on the expectations about future changes in net output. If this theory is true the current account contains all available information on changes in future output. In a rational world, the current account is at least as good as any other predictor of changes in net output. This can be examined by testing for Granger causality from CA_t to ΔNO_t. This is a test of forecasting relationship proposed by Granger (1969), the test investigates whether one variable x fails to cause another y. I am going to test for Granger causality from CA_t to ΔNO_t, and from ΔNO_t to CA_t. To perform the test I estimate the equations in the VAR separately and impose restrictions on the parameters. If the notation in the VAR in equation (4.2) is used, the restrictions for the hypothesis that CA_t fails to cause ΔNO_t is given by

$$H_0^{\Delta NO}\colon\ \beta_{1i} = 0 \ \text{ for all } \ i = 1, 2, \ldots, p,$$

and the hypothesis that ΔNO_t fails to cause CA_t is

$$H_0^{CA}\colon\ \alpha_{2j} = 0 \ \text{ for all } \ j = 1, 2, \ldots, p.$$

The results from these tests are reported in Table 4.5. The F-value for $H_0^{\Delta NO}$ is 7.3, and for H_0^{CA} it is 1.6.[11] The critical value for rejecting the null-hypothesis on a 5 percent significance level is 2.4. I cannot reject the hypothesis that ΔNO_t fails to cause CA_t, but I can reject the hypothesis that CA_t fails to cause ΔNO_t. The conclusion is in line with

Table 4.5: Test for Granger causality in the VAR(4)

Hypothesis:	CA fails to Granger cause ΔNO	ΔNO fails to cause CA
F-value	7.3	1.6
P-value	0.00	0.17

The number of observations is 119, the number of restrictions is 4 and the degrees of freedom are 115.

[11] The F-value is calculated as

$$F(\#\text{restrictions},\, T-2p) = \frac{(SSR_{\text{restricted}} - SSR_{\text{unrestricted}})/\#\text{restrictions}}{SSR_{\text{unrestricted}}/(T-2p)}.$$

what the theory predicts; the data indicates a forecasting relationship from the current account to changes in net output, so the current account is useful in predicting changes in net output. We do not see causality the other way, past changes in net output is not useful in predicting the current account.

4.7 Testing the PVM

I have the estimated parameter matrix $\hat{\mathbf{\Gamma}}$ and can use equation (4.6) to find the predicted current account from the intertemporal model; the predicted current account series is given by

$$\mathcal{CA}_t = -\mathbf{e_{\Delta NO}}(\tau\hat{\mathbf{\Gamma}})(\mathbf{I} - \tau\hat{\mathbf{\Gamma}})^{-1}\mathbf{\Psi}_t.$$

It is important to distinguish between CA_t, $\widehat{CA}_t$ and $\mathcal{CA}_t$, the first CA_t is the actual current account, $\widehat{CA}_t$ is the fitted current account from the VAR and $\mathcal{CA}_t$ is the predicted current account from the PVM. If the theory holds, the actual and predicted current account should be equal, that is $CA_t = \mathcal{CA}_t$. We can look at this graphically by plotting the predicted values against the actual series. This is done in Figure 4.4. In this plot the annual world real interest rate is assumed to be 4 percent, which corresponds to $r = 0.01$.

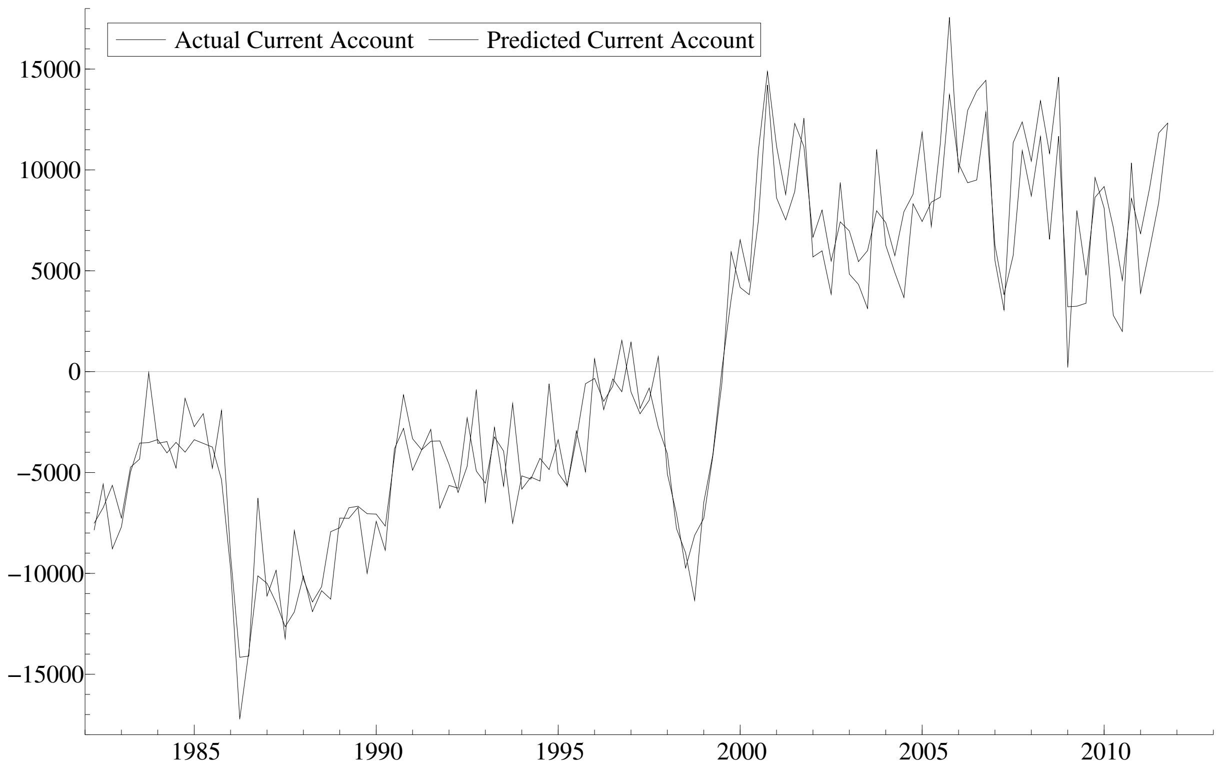

Figure 4.4: Actual current account plotted against predicted current account from 1982Q2 to 2011Q4 where the predicted values are from the intertemporal model.

As we can see, the predicted series tracks the actual series pretty good, the correlation between the actual and predicted current account is 0.97. Obstfeld and Rogoff (1996 pp. 93-95), plots similar results for different countries and state that the nice graphical results are surprising given the simplicity of the model. So at least the graphical results advocate the usefulness of the intertemporal approach for explaining Norwegian data. An even stronger statement is in Otto (1992 pp. 421) where he writes: "...if $\widehat{CA}_t$ tracks CA_t closely, then a statistical rejection of the model may be of little economic importance." Note that $\widehat{CA}_t$ in this sentence is equal to $\mathcal{CA}_t$ in my notation.

Another implication is that the variance of the actual and predicted current account should be equal. In my findings the actual series is more volatile than the predicted series. The ratio of the variance for the predicted series over the variance for the actual series is 0.75, so the model predicts less consumption smoothing than what we observe in the data. This indicates that net capital flow from Norway has been more volatile than what we can expect based on what is necessary for consumption smoothing. At least capital mobility has not been too limited for Norway. In the literature this is called *excess volatility*, which means that the consumption-smoothing component of foreign lending is more volatile than what we expect based on the permanent income theory.[12]

To test whether $\mathcal{CA}_t$ and CA_t are statistical different, I need to test the restrictions that give the predicted series $\mathcal{CA}_t$. To find the specific restrictions on the parameters in the VAR, we can look back to the formal restriction on the parameter matrix $\mathbf{\Gamma}$ given by equation (4.7)

$$\mathbf{e_{CA}} = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}.$$

If the matrix $(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}$ is invertible it is possible to calculate linear cross-equation restrictions on the parameters in the VAR. If we solve $\mathbf{e_{CA}}(\mathbf{I} - \tau\mathbf{\Gamma}) = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})$ for the parameters, we get the following restrictions

$$\alpha_{1i} = \alpha_{2i} \text{ for } i = 1, 2, \ldots, p, \quad \beta_{1j} = \beta_{2j} \text{ for } j = 2, 3, \ldots, p, \text{ and } \tau\beta_{21} = 1 + \tau\beta_{11}, \tag{4.8}$$

see Appendix C for details on the calculation of the restrictions. It is also possible to calculate non-linear restrictions on the parameters that satisfy equation (4.7). To find the non-linear restrictions we need to solve the matrix $(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}$ for the parameters; this will get algebraically messy and I will not pursue these non-linear restrictions in this thesis.

One way to test the validity of the restrictions is to estimate the restricted VAR and compare the general and the restricted VAR and check whether the models are statistically different; this is the log-likelihood ratio test presented bellow. The estimation procedure for a restricted VAR is usually maximum likelihood (ML). I will not go into the details on the ML estimation method.[13] The results from the estimation of the restricted VAR

[12]See examples of excess volatility in Sheffrin and Woo (1990), Ghosh (1995), and Agénor et al. (1999).
[13]See e.g. Hamilton (1994), Chapter 11.

are from the software package OxMetrics. The estimated restricted VAR(4) is given by

$$\begin{aligned}\widehat{\Delta NO_t} = & - 0.73\Delta NO_{t-1} - 0.67\Delta NO_{t-2} - 0.57\Delta NO_{t-3} + 0.30NO_{t-4} \\ & + 0.36CA_{t-1} - 0.02CA_{t-2} - 0.16CA_{t-3} - 0.25CA_{t-4}\end{aligned}$$

$$\begin{aligned}\widehat{CA_t} = & - 0.73\Delta NO_{t-1} - 0.67\Delta NO_{t-2} - 0.57\Delta NO_{t-3} + 0.30\Delta NO_{t-4} \\ & + 1.36CA_{t-1} - 0.02CA_{t-2} - 0.16CA_{t-3} - 0.25CA_{t-4}\end{aligned}$$

I plot the model for the current account where the linear restrictions are imposed in Figure 4.5.

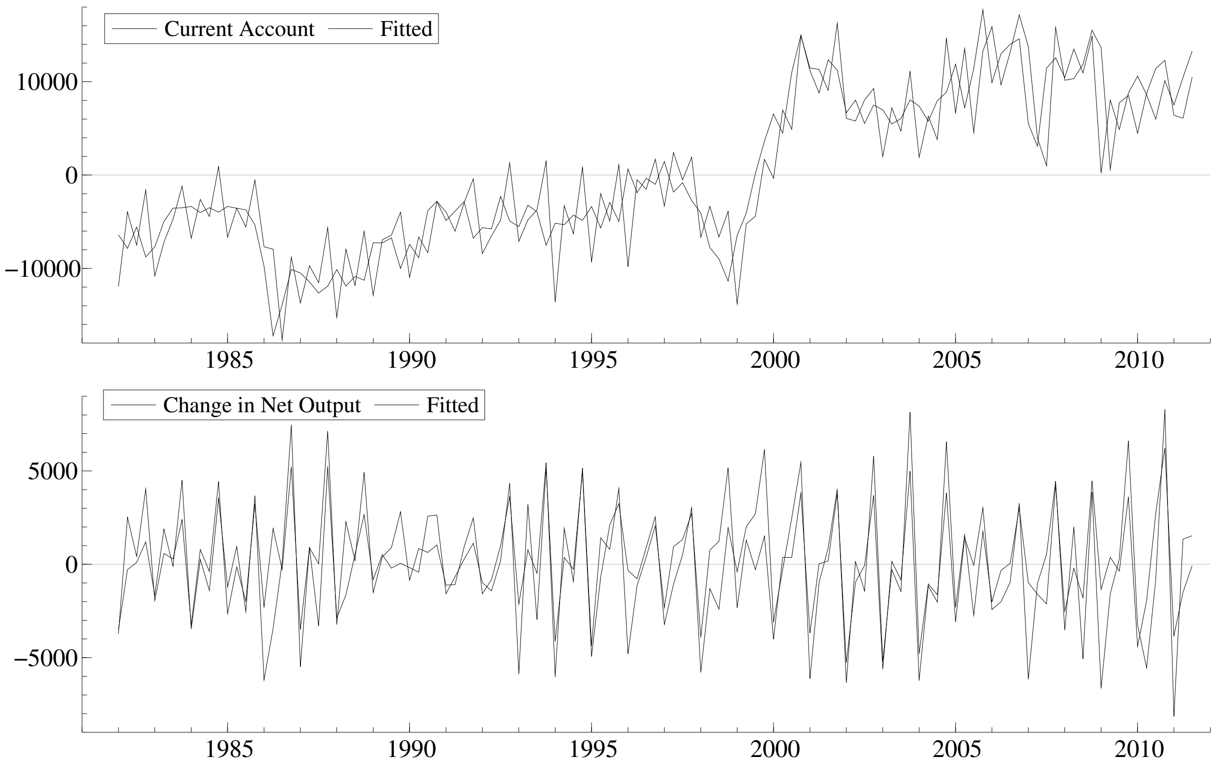

Figure 4.5: The predicted values from the restricted VAR(4) plotted against the actual values for the sample period 1982Q2 to 2011Q4. The restrictions are linear.

For the intertemporal model to be statistical valid, we must be able to confirm that the restrictive VAR presented in Figure 4.5 is a valid simplification of the general VAR presented in Figure 4.3. There are different ways of testing the restrictive VAR against the general VAR. I will present a Wald test, a log-likelihood ratio test, and a Lagrange multiplier test, and as noted in Chapter 3 they are all asymptotically equal. The Wald test and the Lagrange-multiplier test can be performed without estimating the restricted VAR. I present the Wald test after the residual analysis in section 4.9 because the Wald test can easily be made robust to heteroskedasticity, which we will see may be a problem.

The null-hypothesis is that the PVM is valid, and the null is true if we can confirm the statistical validity of the restrictions in (4.8).

4.7.1 The log-likelihood ratio test

The log-likelihood ratio test examines whether the restricted VAR is a valid simplification of the general VAR by comparing the log-likelihood functions for the two models. The log-likelihood function is saying something about how well the model explains the actual data, and can be calculated as

$$LL = -T\log(2\pi) + \frac{T}{2}\log\det(\hat{\mathbf{\Sigma}}_{\xi}^{-1}) - T,$$

where π is the natural constant. The test statistic for a log-likelihood ratio test is given by[14]

$$LR = -2 \cdot (LL_{\text{restricted}} - LL_{\text{unrestricted}}) \sim \chi^2(\#\text{restrictions}).$$

I report test results for different real interest rates, I use 4, 7, and 14 percent because this is the range of what is standard in the literature. We will see that the choice of interest rate does not significantly affect the results. The results from the log-likelihood ratio test are reported in Table 4.6. The 5 percent critical value for the χ^2-distribution with 8 degrees of freedom is given by 15.5, the observed χ^2-statistic is above this critical value for all specifications in Table 4.6. We can reject the hypothesis that the model implied by the restrictions in (4.8) is a valid simplification of the general VAR(4). The log-likelihood ratio test rejects the PVM for Norwegian data.

Table 4.6: The log-likelihood ratio test

	4%	7%	14%
$LL_{\text{unrestricted}}$	-2157.0	-2157.0	-2157.0
$LL_{\text{restricted}}$	-2220.7	-2220.0	-2221.5
χ^2-value	127.4	127.8	128.9
P-value	0.00	0.00	0.00

Interest rates are given as annual values. The sample is 1982Q2 to 2011Q4 and there are 119 observations. The number of restrictions is equal to 8. The 5 percent critical value is 15.5. The value $LL_{restricted}$ is estimated using maximum likelihood and is reported from the software package OxMetrics.

[14] See e.g. Brooks (2008), Chapter 8.

4.7.2 The Lagrange-multiplier test

Campbell (1987) show how a Lagrange-multiplier test can be used to test PVMs. To perform the test, we need to subtract ΔNO_t from CA_t in the VAR system given by the equations in (4.2), and this gives

$$\begin{aligned} CA_t - \Delta NO_t &= (\alpha_{21} - \alpha_{11})\Delta NO_{t-1} + \cdots + (\alpha_{2p} - \alpha_{1p})\Delta NO_{t-p} \\ &+ (\beta_{21} - \beta_{11})CA_{t-1} + \cdots + (\beta_{2p} - \beta_{1p})CA_{t-p} \\ &+ (\epsilon_{2t} - \epsilon_{1t}). \end{aligned}$$

Imposing the restrictions in (4.8) gives

$$CA_t - \Delta NO_t = (1 + r)CA_{t-1} + (\epsilon_{2t} - \epsilon_{1t}). \tag{4.9}$$

Equation (4.9) state that $CA_t - \Delta NO_t - (1 + r)CA_{t-1}$ is unpredictable given lagged values of ΔNO_t and CA_t, which we also saw in equation (2.18) in Chapter 2. To test this implication I follow Sheffrin and Woo (1990) and define $R_t \equiv CA_t - \Delta NO_t - (1+r)CA_{t-1}$. Then, the restrictions in (4.8) imply the testable hypothesis $E(R_t|\mathbf{\Omega}_{t-1}) = 0$, where $\mathbf{\Omega}_{t-1}$ is the information set at $t-1$. If we look back to equation (2.18), we see that we investigate whether the expected value of NO_t is equal on time t and time $t-1$ from the perspective of $t-1$. The reason for using information known at time $t-1$ is because in theory we have $E(E(NO_t|\mathbf{\Omega}_t)|\mathbf{\Omega}_{t-1}) = E(NO_t|\mathbf{\Omega}_{t-1})$.[15] So the theoretical model predicts $\varepsilon_t = 0$, and the right-hand side of equation (2.18) is equal to zero at time $t-1$. Sheffrin and

Table 4.7: The Lagrange-multiplier test

	$E(R_t\|\mathbf{\Omega}_{t-1}) = 0$			$E(R_t\|\mathbf{\Omega}_{t-2}) = 0$		
	4%	7%	14%	4%	7%	14%
F-value	24.4	26.7	27.0	10.9	10.8	10.7
P-value	0.00	0.00	0.00	0.00	0.00	0.00

Interest rates are given as annual values. The sample is 1982Q2 to 2011Q4 and there are 119 observations. I use the same information set used in the VAR(4). For the test on $\mathbf{\Omega}_{t-1}$ the number of restrictions is 10, and the de-numerator degrees of freedom are 109 so the 5 percent critical F-value is 1.9. For the test on $\mathbf{\Omega}_{t-2}$ the number of restrictions is 8, and the de-numerator degrees of freedom are 111 so the 5 percent critical F-value is 2.0.

Woo also report a weaker test where they allow for transitory consumption at time $t-1$, this hypothesis is $E(R_t|\mathbf{\Omega}_{t-2}) = 0$. This is a test of a model that violates the PVM in equation (2.17) and allows for exogenous transitory shocks at time $t-1$. In Table 4.7 I report the F-values for this test for both a regression on an information set dated $t-1$ and $t-2$, I also report results for different interest rates. The test is performed by testing

[15]By using the law of iterated expectations.

for joint significance of the variables in the information set on R_t. The null-hypothesis is no significance in the information set when explaining R_t. The PVM is rejected for all variations of this test.

4.8 Residual analysis

The residuals from the VAR are assumed to be white noise. I will investigate whether this is a plausible assumption for the Norwegian data. I plot the residuals for the VAR(4) in Figure 4.6. It looks like the residuals in the equation for the current account are more volatile in the second part of the sample period. In Chapter 3 I presented the assumptions for the classical linear regression model. If the volatility is higher in the second period of the sample, the homoskedasticity assumption is violated. Now, I investigate whether the last three assumptions of the classical linear regression model presented in Chapter 3 are satisfied for the VAR(4) presented in this chapter.

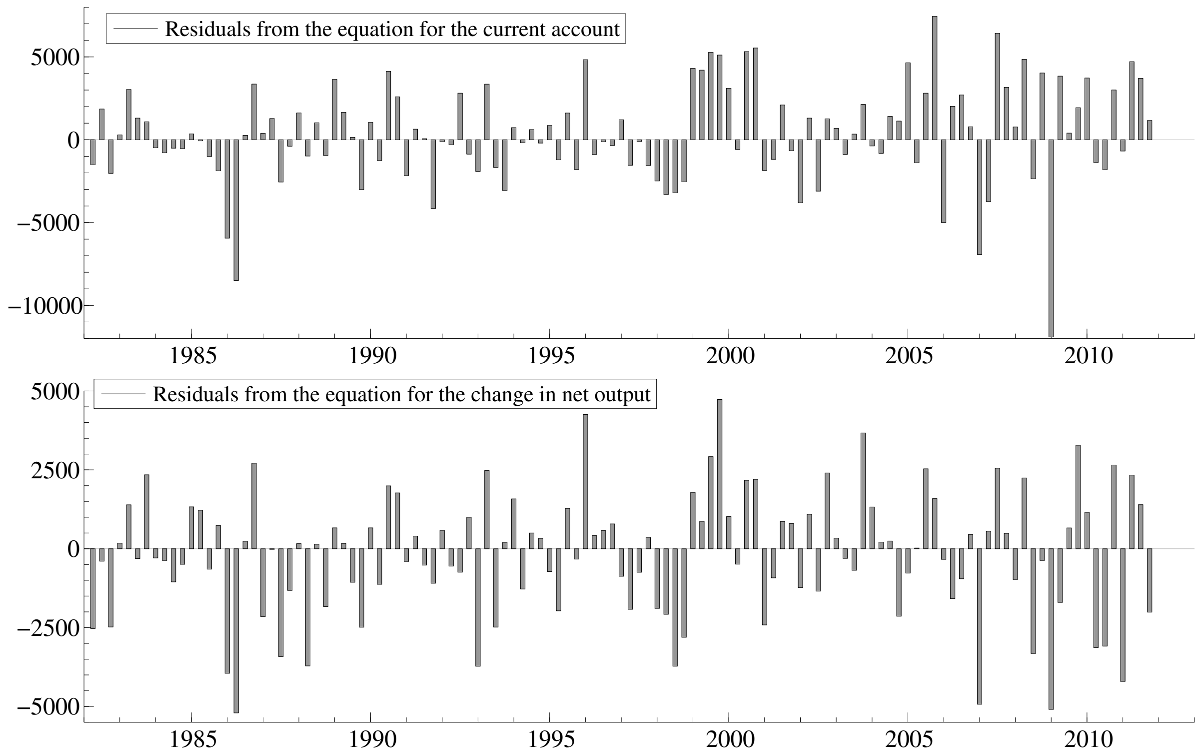

Figure 4.6: The residuals from the VAR(4) for the sample period 1982Q2 to 2011Q4.

4.8.1 A test for homoskedasticity

The assumption of homoskedastic error terms means the variance of the errors is constant and independent of time. If this assumption fails the residuals are heteroskedastic. Heteroskedasticity does not affect the consistency of the estimates, but the standard errors are incorrect and standard testing procedures such as t- and F-tests are invalid. I will now test whether the residuals plotted in Figure 4.6 are homoskedastic as assumed. A popular test for heteroskedasticity is the White test.[16] This test can be performed by regressing the squared residual values on the explanatory variables and squared explanatory variables. The equations to be estimated are given by

$$\begin{aligned}\hat{\epsilon}_{1t}^2 &= \delta_0 + \delta_1 CA_{t-1} + \delta_2 CA_{t-2} + \delta_3 CA_{t-3} + \delta_4 CA_{t-4} + \delta_5 \Delta NO_{t-1} \\ &\quad + \delta_6 \Delta NO_{t-2} + \delta_7 \Delta NO_{t-3} + \delta_8 \Delta NO_{t-4} + \delta_9 CA_{t-1}^2 + \delta_{10} CA_{t-2}^2 \\ &\quad + \delta_{11} CA_{t-3}^2 + \delta_{12} CA_{t-4}^2 + \delta_{13} \Delta NO_{t-1}^2 + \delta_{14} \Delta NO_{t-2}^2 + \delta_{15} \Delta NO_{t-3}^2 \\ &\quad + \delta_{16} \Delta NO_{t-4}^2 + error\end{aligned}$$

$$\begin{aligned}\hat{\epsilon}_{2t}^2 &= \delta_{17} + \delta_{18} CA_{t-1} + \delta_{19} CA_{t-2} + \delta_{20} CA_{t-3} + \delta_{21} CA_{t-4} + \delta_{22} \Delta NO_{t-1} \\ &\quad + \delta_{23} \Delta NO_{t-2} + \delta_{24} \Delta NO_{t-3} + \delta_{25} \Delta NO_{t-4} + \delta_{26} CA_{t-1}^2 + \delta_{27} CA_{t-2}^2 \\ &\quad + \delta_{28} CA_{t-3}^2 + \delta_{29} CA_{t-4}^2 + \delta_{30} \Delta NO_{t-1}^2 + \delta_{31} \Delta NO_{t-2}^2 + \delta_{32} \Delta NO_{t-3}^2 \\ &\quad + \delta_{33} \Delta NO_{t-4}^2 + error,\end{aligned}$$

where $\hat{\epsilon}_{1t}$ and $\hat{\epsilon}_{2t}$ is the estimated residuals from the VAR(4). The null-hypothesis is homoskedasticity and this is the case for both equations if we can confirm statistically that all the δ's except δ_0 and δ_{17} are equal to zero. The test can also include cross-terms of the regressors; results with and without cross-terms are reported in Table 4.8. The results indicate heteroskedastic residuals in the equation for the current account. For the equation for the change in net output we observe mixed results; we cannot reject homoskedasticity in the specification without cross terms, but the null-hypothesis can be rejected on a 5 percent significance level in the specification with cross-terms. It looks like we may have a problem with heteroskedasticity in both equations.

Table 4.8: Testing for homoskedasticity

Test without cross-terms	Equation for ΔNO_t	Equation for CA_t
F-value	1.4	1.8
P-value	0.15	0.04
Test with cross-terms	Equation for ΔNO_t	Equation for CA_t
F-value	1.6	2.5
P-value	0.04	0.00

The sample is 1982Q2 to 2011Q4 and there are 119 observations. The test without cross-terms has 16 restrictions and the degrees of freedom are 102, the test with cross-terms has 44 restrictions and the degrees of freedom are 74.

[16] See White (1980).

4.8.2 A test for serial correlation

The assumption of no serial correlation means that all the residuals need to be independent of one another, and we should not see any correlation between residuals. The presence of serial correlation may indicate a misspecified model. I will test for serial correlation using the Breusch-Godfrey test.[17] This test is performed by regressing the residual values on the explanatory variables and lagged values of the residuals; the regression equations are given by

$$\begin{aligned}\hat{\epsilon}_{1t} &= \delta_0 + \delta_1 CA_{t-1} + \delta_2 CA_{t-2} + \delta_3 CA_{t-3} + \delta_4 CA_{t-4} + \delta_5 \Delta NO_{t-1} \\ &+ \delta_6 \Delta NO_{t-2} + \delta_7 \Delta NO_{t-3} + \delta_8 \Delta NO_{t-4} + \delta_9 \hat{\epsilon}_{1t-1} + \delta_{10} \hat{\epsilon}_{1t-2} \\ &+ \delta_{11} \hat{\epsilon}_{1t-3} + \delta_{12} \hat{\epsilon}_{1t-4} + \delta_{13} \hat{\epsilon}_{1t-5} + error\end{aligned}$$

$$\begin{aligned}\hat{\epsilon}_{1t} &= \delta_{14} + \delta_{15} CA_{t-1} + \delta_{16} CA_{t-2} + \delta_{17} CA_{t-3} + \delta_{18} CA_{t-4} + \delta_{19} \Delta NO_{t-1} \\ &+ \delta_{20} \Delta NO_{t-2} + \delta_{21} \Delta NO_{t-3} + \delta_{22} \Delta NO_{t-4} + \delta_{23} \hat{\epsilon}_{2t-1} + \delta_{24} \hat{\epsilon}_{2t-2} \\ &+ \delta_{25} \hat{\epsilon}_{2t-3} + \delta_{26} \hat{\epsilon}_{2t-4} + \delta_{27} \hat{\epsilon}_{2t-5} + error.\end{aligned}$$

The null-hypothesis is no serial correlation, and can be formally tested by saving the R^2 from the regressions of $\hat{\epsilon}_{1t}$ and $\hat{\epsilon}_{2t}$ and calculating the following test statistic

$$BG = (T-5)R^2 \sim \chi^2(5).$$

The results from this test are reported in Table 4.9. The null-hypothesis of no serial correlation cannot be rejected for any of the equations on the 5 percent significance level. Based on the Breusch-Godfrey test for normality, I can conclude that the assumption of no autocorrelation is satisfied in the VAR(4).

Table 4.9: Testing for serial correlation

	Equation for ΔNO_t	Equation for CA_t
χ^2-value	6.1	9.7
P-value	0.29	0.08

The sample is 1983Q3 to 2011Q4 and there are 114 observations. The degrees of freedom are 5.

[17] See Breusch (1978) and Godfrey (1978).

4.8.3 A test for normality

Normality is a property of the underlying distribution of a variable. If the normality assumption is violated the statistical tests are invalid. For the normality test I use the Jarque-Bera test, which has the following test statistic

$$JB = \frac{T}{6}\left(Sk(\hat{\epsilon}_t)^2 + \frac{1}{4}(Ku(\hat{\epsilon}_t) - 3)^2\right) \sim \chi^2(2),$$

where $Sk(\hat{\epsilon}_t)$ is the skewness and $Ku(\hat{\epsilon}_t)$ is the kurtosis for the sample residuals.[18] The skewness and kurtosis is defined as

$$Sk(\hat{\epsilon}_t) \equiv \frac{1/T\sum_{t=1}^{T}(\hat{\epsilon}_t - \bar{\hat{\epsilon}})^3}{\left(1/T\sum_{t=1}^{T}(\hat{\epsilon}_t - \bar{\hat{\epsilon}})^2\right)^{3/2}} \quad \text{and} \quad Ku(\hat{\epsilon}_t) \equiv \frac{1/T\sum_{t=1}^{T}(\hat{\epsilon}_t - \bar{\hat{\epsilon}})^4}{\left(1/T\sum_{t=1}^{T}(\hat{\epsilon}_t - \bar{\hat{\epsilon}})^2\right)^{2}},$$

where $\bar{\hat{\epsilon}}$ is the average value of the estimated residuals. The null-hypothesis is a normal distribution. If the sample has a normal distribution, we expect the skewness to be 0 and the kurtosis to be 3. If the data deviates from these properties, the JB-statistic will increase and the null-hypothesis gets less probable. The results from this test can be found in Table 4.10. We see the null is rejected for the equation for the current account while it cannot be rejected for the equation for changes in net output. Based on this investigation, I conclude that $\hat{\epsilon}_{1t}$ has a normal distribution while $\hat{\epsilon}_{2t}$ does not.

Table 4.10: Testing for normality

	Equation for ΔNO_t	Equation for CA_t
$Sk(\hat{\epsilon}_t)$	-0.23	-0.60
$Ku(\hat{\epsilon}_t)$	3.00	4.94
χ^2-value	1.1	25.8
P-value	0.59	0.00

The sample is 1982Q2 to 2011Q4 and there are 119 observations. The degrees of freedom are 2.

From the residual analysis we see that the model has a problem with both heteroskedastic residuals and residuals that are not normally distributed. I will not address the normality problem. In the next section I present a heteroskedastic robust test of the PVM.

[18] See Jarque and Bera (1987).

4.9 The Wald test

In this section I present the Wald test, I report the test for both the standard variance-covariance matrix and for a heteroskedasticity robust variance-covariance matrix. Let us again start off by the general restriction given by $\mathbf{e_{CA}} = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})(\mathbf{I}-\tau\mathbf{\Gamma})^{-1}$. If $(\mathbf{I}-\tau\mathbf{\Gamma})$ is invertible we can calculate linear restrictions in the parameters, these restrictions are given in (4.8), presented in matrix notation, they can be written as

$$\mathbf{e_{CA}} = \tau(\mathbf{e_{CA}} - \mathbf{e_{\Delta NO}})\mathbf{\Gamma}.$$

To perform the linear Wald test I follow a paper by Bouakez and Kano (2009). First define

$$\mathcal{I}(\mathbf{\Gamma}) \equiv \mathbf{e_{CA}} - \tau(\mathbf{e_{CA}} - \mathbf{e_{\Delta NO}})\mathbf{\Gamma} \in \mathbb{R}^{1\times 2p}, \tag{4.10}$$

then the null-hypothesis is given by $\mathcal{I}(\mathbf{\Gamma}) = \mathbf{0}$. By using the estimated parameters we can calculate

$$\tau(\mathbf{e_{CA}} - \mathbf{e_{\Delta NO}})\hat{\mathbf{\Gamma}} = [1.01 \;\; 0.20 \;\; 0.85 \;\; 0.22 \;\; 0.81 \;\; 0.16 \;\; 0.04 \;\; 0.46],$$

which should not be statistical different from the vector $\mathbf{e_{CA}} = [0 \;\; 1 \;\; 0 \;\; \cdots \;\; 0]$ for the PVM to be confirmed. By visually comparing the two vectors, the results are not convincing. I use the Wald test to check whether they are statistically different. The Wald statistic is given by

$$\mathcal{W} = \mathcal{I}(\hat{\mathbf{\Gamma}}) \left[\frac{\partial \mathcal{I}(\hat{\mathbf{\Gamma}})}{\partial \hat{\mathbf{\Gamma}}} \hat{\mathbf{\Sigma}}_{\mathbf{\Gamma}} \frac{\partial \mathcal{I}(\hat{\mathbf{\Gamma}})}{\partial \hat{\mathbf{\Gamma}}'} \right]^{-1} \mathcal{I}(\hat{\mathbf{\Gamma}})' \sim \chi^2(\#\text{restrictions}),$$

where $\hat{\mathbf{\Sigma}}_{\mathbf{\Gamma}}$ is the estimated variance-covariance matrix for the parameter matrix, and $\partial\mathcal{I}(\hat{\mathbf{\Gamma}})/\partial\hat{\mathbf{\Gamma}}$ is the derivative of the matrix $\mathcal{I}(\mathbf{\Gamma})$ with respect to $\mathbf{\Gamma}$ evaluated at $\hat{\mathbf{\Gamma}}$. The estimated variance-covariance matrix for the parameters can be calculated as

$$\hat{\mathbf{\Sigma}}_{\mathbf{\Gamma}} = \frac{T}{T-2p}\hat{\mathbf{\Sigma}}_{\xi} \otimes (\mathbf{\Psi}_{t-1} \cdot \mathbf{\Psi}'_{t-1})^{-1} = \frac{\hat{\boldsymbol{\xi}}_t \cdot \hat{\boldsymbol{\xi}}_t{}'}{T-2p} \otimes (\mathbf{\Psi}_{t-1} \cdot \mathbf{\Psi}'_{t-1})^{-1} \in \mathbb{R}^{4p^2 \times 4p^2}, \tag{4.11}$$

where $\otimes$ is the Kronecker product.[19,20] We also need the derivative of equation (4.10) with respect to $\mathbf{\Gamma}$, which is given by

$$\frac{\partial \mathcal{I}(\mathbf{\Gamma})}{\partial \mathbf{\Gamma}} = -\tau(\mathbf{e_{CA}} - \mathbf{e_{\Delta NO}}) \otimes \mathbf{I}_{2p} \in \mathbb{R}^{2p \times 4p^2}.$$

[19] See Hayashi (2000), Chapter 6.
[20] For details on how to calculate a Kronecker product see e.g. Graham (1981).

If the residuals are heteroskedastic, the standard variance-covariance matrix in equation (4.11) is no longer valid. A solution to the problem with heteroskedasticity is to replace the usual variance-covariance matrix in equation (4.11) with

$$\hat{\mathbf{\Sigma}}_{\mathbf{\Gamma}}^{\text{White}} = (\mathbf{\Psi}_{t-1} \cdot \mathbf{\Psi}'_{t-1})^{-1} \cdot \mathbf{\Psi}_{t-1}\mathbf{V}\mathbf{\Psi}'_{t-1} \cdot (\mathbf{\Psi}_{t-1} \cdot \mathbf{\Psi}'_{t-1})^{-1}, \tag{4.12}$$

where $\mathbf{V}$ is a diagonal matrix with squared residuals.[21,22] Results from the Wald test, using both the standard variance-covariance matrix and the White corrected variance-covariance matrix are reported in Table 4.11. The critical 5 percent value for the χ^2-distribution with 8 degrees of freedom is given by 15.5. We can see that the heteroskedastic robust test gives a lower χ^2-value, but the null-hypothesis that the restrictions are valid is still rejected. The Wald test also rejects the PVM.

Table 4.11: The Wald test

	4%	7%	14%
χ^2-value	214.6	215.7	218.9
P-value	0.00	0.00	0.00
White corrected test			
χ^2-value	97.6	97.8	98.7
P-value	0.00	0.00	0.00

Interest rates are given as annual values. The sample is 1982Q2 to 2011Q4 and there are 119 observations. The number of restrictions is 8.

We saw from the Granger causality test that the VAR had the desired forecasting relationships suggested by the intertemporal model. The theory stated that the current account balance should be equal to the discounted sum of changes in net output, and the data confirmed a forecasting relationship from the current account to the changes in net output. Next, we observed a good graphical fit of the model, the result looks convincing with a correlation coefficient between the actual and predicted series of 0.97. However, the model fails when evaluated formally, all three tests: the log-likelihood ratio test, the Lagrange-multiplier, and the Wald test reject the model.

[21] See White (1980).

[22] The variance-covariance matrix is calculated equation by equation, and the results are pooled into one matrix that is used in the test.

Chapter 5

A discussion of the rejection

As Chapter 4 shows, the theoretical model developed in Chapter 2 is rejected for Norwegian data. In this chapter I will discuss the rejection of this simple version of the intertemporal model. I will try to point out some possible problems of the model and suggest possible improvements of the basic framework. Implementing these possible improvements in the theoretical model is beyond the scope of this thesis. I present a brief review of some related papers that try to improve the basic model. I am also going to estimate a general VAR for the current account and the change in net output where I will include the unemployment rate and the price of oil as exogenous explanatory variables. I will test whether these extra variables are helpful in explaining the current account and the change in net output.

5.1 Possible causes for the rejection

The model in Chapter 2 makes many simplifying assumptions such as an exogenous supply side, a one-good economy, and a single representative rational agent. Obviously these are not realistic assumptions, but there are some good arguments for this approach. The purpose of the economic model is to study a specific feature of the economy, which in this thesis is a nation's intertemporal wealth allocation. If we use a more complicated model it may be harder to interpret the effects of this particular feature of the economy. The model also needs to be simple enough so the theory can be tested against actual data.

The exogenous supply side assumption may be justified if the economy is small and open so the production side does not affect prices. The single good can be thought of as an index that represents an aggregate of all available goods. The model in my thesis lack the choice between labor and leisure, it may be reasonable to think that some agents prefer to increase leisure when the wealth increases. It may be relevant for the current account how this leisure allocation is affecting the consumption and saving decision; this may be solved by using a utility function that includes leisure. Next, I look at the rationality assumption and the possibility that the agent dislikes risk.

5.1.1 Rational agents

The rationality assumption carries a heavy load for the intertemporal model. It is assumed that all agents behave symmetrically, and that one representative agent can model their behavior. This agent has preferences that can be explained mathematically by a utility function, and the agent make choices where the motive is to maximize the value of this utility function. All the choices the agent makes are based on rational forward-looking calculations where all publicly available information is used. It is worth noting that rationality does not imply an agent that is always right about the future, but the mistakes are random so the agent does not make systematic errors when predicting the future. We can say the agent is right on average.

One possible break of the rationality assumption is dynamic inconsistency or present bias, which means that the agent has a misperception about own time-preference. The model in Chapter 2 assumes exponential discounting, which implies that events in the far future are valued less than events that are close to the present, and the present is valued the most. Present bias means that the agent's preferences are changing over time, and when the agent actually reach one of the future periods the agent realizes that the present is valued more than what was expected in earlier periods. The effect from present bias is that the agent consumes more every period than what was actually planned. When consumption is higher than planned, saving is lower and consumption get tilted to the present and we may see a lower current account surplus or even a deficit when we expect a large surplus.

5.1.2 Uncertainty and precautionary saving

Another aspect of holding foreign assets that is ignored in the model presented in this thesis is the motive for holding foreign assets as a way of insurance. In a world where future outcomes are uncertain, it may be desirable for the agent to insure against unforeseen events. If the agent is risk averse he or she may want to save more today than what we can explain by consumption smoothing alone. This extra saving is called precautionary saving. The current account will then consist of one intertemporal smoothing part and one precautionary saving part. The reason why precautionary saving is ignored in the model in this thesis is the assumption that the agent's utility can be represented by a quadratic function, a quadratic utility function makes the agent risk neutral and we get the result of certainty equivalence when the agent is calculating the present value of permanent income. Technically precautionary saving is a result of using a utility function where the third derivative is positive.

Norway has large oil reserves and it is reasonable to think that this affects the country's choice on how much to save or borrow from the rest of the world. One factor is that the oil industry is very capital intensive and has a high demand for investment in early stages of development. When the oil industry is starting up it may be an optimal strategy for the country to borrow heavily to invest and repay this as the industry matures. When the oil reserves are large they have a big impact on the nation's wealth; uncertainties about these reserves and the price of oil in the future may be important for the saving decision. Uncertainties about both the size of unexplored reserves and the price of the oil in the future may create incentives for significant levels of precautionary savings.

5.2 Extensions of the basic model in the literature

Several papers extend the basic model presented in Chapter 2. I give a brief review of some of these attempts to improve the empirical fit of the intertemporal model. Bergin and Sheffrin (2000) develop a model that allows for both traded and non-traded goods. The paper presents a model that includes a variable interest rate and a variable exchange rate. The model is tested for Canada, Australia, and the United Kingdom for the period 1961 to 1996. They compare the model to a benchmark model similar to the one I present in Chapter 2 and show an improved fit for Canada and Australia, but not for the United Kingdom. İşcan (2002) extends the basic model to include durable and non-traded goods; the model is tested for Canadian data for the period 1937 to 1994. The model shows improvement when both durable and traded goods are included, but durable goods alone do not improve the model. Ghosh and Ostry (1997) introduce a utility function that makes the consumer risk averse. In this framework precautionary saving is present and is an additional channel to the current account balance besides the intertemporal smoothing motive. Their results suggest increased saving when the macroeconomic uncertainty is heightened in the economy. If the current account is in surplus because of the intertemporal smoothing motive, the precautionary saving motive will add to this surplus.

Nason and Rogers (2006) tries to find the parts of the PVM that is responsible for the broad empirical rejection in the literature. They set up a small open economy real business cycle model, which nests the PVM as a special case. In their framework they investigate the impact of what they call the *usual suspects* for causing the PVM to fail. These usual suspects are non-separable preferences, fiscal policy and real interest rate shocks, external imperfect international capital mobility, and internalized risk premium. When Nason and Rogers talk about the non-separability of the preferences, this is with respect to consumption and leisure, the preferences are still time-separable. Fiscal policy is modeled as demand shocks affecting the current account. Interest rate shocks affect the return on savings. External imperfect capital mobility hinders the flow of capital and make both saving and borrowing more difficult. Internalized risk premium is taking up a point where in a small open economy the supply decision may affect the national risk premium, and may therefore affect the decision on how much to save home and abroad. Nason and Rogers confirm the rejection for Canadian data in a model without any suspects. To investigate the importance of the usual suspects they use a Bayesian Monte Carlo experimental approach. They find an effect from all suspects, but the basic model without any suspects performs best. However, the paper identifies the internalized risk premium and exogenous world real interest rate shocks to be two important factors for the rejection of the model.

5.3 An extended VAR

It may be interesting to see empirically how other factors affect the current account and the change in net output. To do this I will estimate a VAR where I include the unemployment rate and the price of oil as exogenous variables. I use quarterly data for the period 1987Q3 to 2011Q4. The unemployment rate is from the Organisation for Economic Co-operation and Development (OECD).[1] The oil price is the price of Brent Blend and is denoted in real 2008 USD. The data for the price of oil is from Statistics Norway.[2] The regression model is given by the following two equations

$$\begin{aligned} CA_t &= constant + \alpha_{11} CA_{t-1} + \cdots + \alpha_{1p} CA_{t-p} + \beta_{11} \Delta NO_{t-1} + \cdots + \beta_{1p} \Delta NO_{t-p} \\ &\quad + \delta_{11} P^{\text{Oil}}_{t-1} + \cdots + \delta_{1p} P^{\text{Oil}}_{t-p} + \gamma_{11} U_{t-1} + \cdots + \gamma_{1p} U_{t-p} + error \end{aligned} \tag{5.1}$$

$$\begin{aligned} \Delta NO_t &= constant + \alpha_{21} CA_{t-1} + \cdots + \alpha_{2p} CA_{t-p} + \beta_{21} \Delta NO_{t-1} + \cdots + \beta_{2p} \Delta NO_{t-p} \\ &\quad + \delta_{21} P^{\text{Oil}}_{t-1} + \cdots + \delta_{2p} P^{\text{Oil}}_{t-p} + \gamma_{21} U_{t-1} + \cdots + \gamma_{2p} U_{t-p} + error \end{aligned} \tag{5.2}$$

where U_t is the unemployment rate in percent and P^{Oil}_t is the price of oil. In this section the data is not demeaned so I include a constant. The Akaike information criterion suggest a lag length of 5, and I present the results for the VAR(5) in Table 5.1. The model is plotted in Figure 5.1. I will test whether P^{Oil} and U are significant explanatory variables. I test the null-hypothesis that none of the new variables are significant in explaining CA and ΔNO. I use a log-likelihood ratio test to test the null-hypothesis. The log-likelihood for the unrestricted model is -1925.4 and for the restricted model it is given by -1961.9. The test is given by

$$LR = -2 \cdot (-1961.9 + 1925.4) = 73.2 \sim \chi^2(20).$$

The critical 5 percent value for the χ^2 distribution with 20 degrees of freedom is given by 31.4, so we can conclude that the oil price and the unemployment rate are jointly significant in explaining the current account and changes in net output.

[1]The data can be found at *http://stats.oecd.org/*.
[2]The data can be found at *http://www.ssb.no/olje_gass_en/*.

Table 5.1: Results from the extended VAR(5)

	ΔNO_{t-1}	ΔNO_{t-2}	ΔNO_{t-3}	ΔNO_{t-4}	ΔNO_{t-5}
ΔNO_t	-0.81	-0.73	-0.68	0.01	-0.09
	(-4.8)	(-3.4)	(-3.1)	(0.0)	(-0.1)
CA_t	0.36	0.40	0.23	0.35	0.12
	(1.5)	(1.3)	(0.7)	(1.3)	(0.7)
	CA_{t-1}	CA_{t-2}	CA_{t-3}	CA_{t-4}	CA_{t-5}
ΔNO_t	0.28	0.10	-0.05	-0.26	-0.13
	(2.4)	(0.9)	(-0.5)	(-2.5)	(-1.2)
CA_t	0.24	0.23	0.22	0.20	-0.01
	(1.4)	(1.5)	(1.4)	(1.3)	(-0.1)
	P^{Oil}_{t-1}	P^{Oil}_{t-2}	P^{Oil}_{t-3}	P^{Oil}_{t-4}	P^{Oil}_{t-5}
ΔNO_t	344.85	-266.08	-117.17	-260.13	206.88
	(2.6)	(-1.2)	(-0.5)	(-1.0)	(1.2)
CA_t	1022.02	-838.91	180.25	-355.25	9.21
	(5.2)	(-2.5)	(0.5)	(-1.0)	(0.0)
	U_{t-1}	U_{t-2}	U_{t-3}	U_{t-4}	U_{t-5}
ΔNO_t	-2906.61	4796.75	1286.52	-7029.46	3343.06
	(-1.0)	(1.4)	(0.4)	(-1.9)	(1.2)
CA_t	-784.83	4561.65	-6816.19	-2248.41	2539.05
	(-0.2)	(0.9)	(-1.4)	(-0.4)	(0.6)

The sample is 1988Q4 to 2011Q4, there are 93 observations, t-values in parentheses, and the log-likelihood is equal to -1925.4.

5.3.1 Granger causality in the extended VAR

In Chapter 4 we saw a forecasting relationship from the current account balance to the change in net output; at the same time the change in net output did not help to forecast the current account. These results are confirmed when extending the model with the oil price and the unemployment rate. I report results from Granger causality tests for the extended VAR in Table 5.2. The tests are performed as before where the equations in the VAR are estimated separately and restrictions are imposed. I report the restrictions in Table 5.2. In addition to the confirmation of the previous results, we see that the oil price is significant in forecasting both the current account and the change in net output. We cannot reject the hypothesis that unemployment fails to cause both the current account and changes in net output.

This exercise has shown that the price of oil is an important predictor for both the current account balance and the change in net output. The unemployment rate is not significant in explaining neither the nation's current account nor the change in net output.

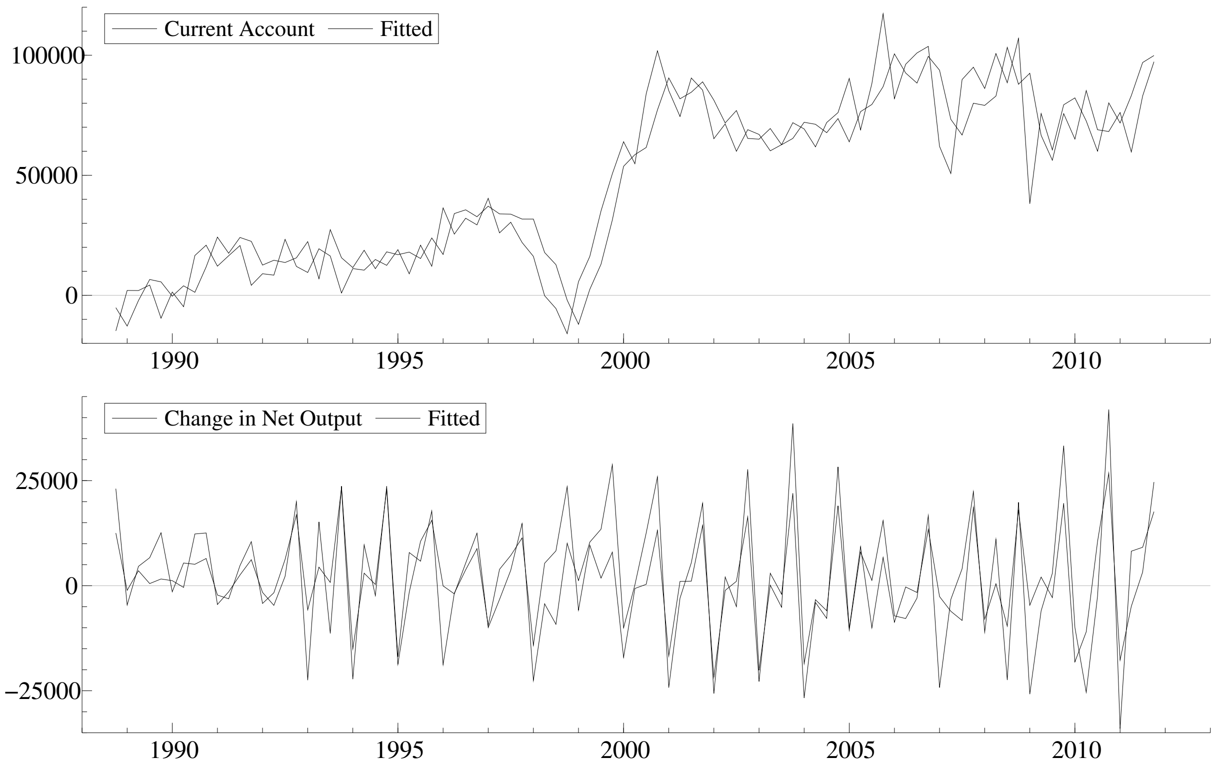

Figure 5.1: The figure plot the graphical fit for a general VAR(5) given in equation (5.1) and (5.2).

Table 5.2: Granger causality in the extended VAR(5)

The equation for the current account (equation (5.1))			
Hypothesis	Restrictions	F-value	P-value
ΔNO fails to cause CA	$\beta_{11} = \cdots = \beta_{15} = 0$	0.8	0.55
P^{Oil} fails to cause CA	$\delta_{11} = \cdots = \delta_{15} = 0$	5.8	0.00
U fails to cause CA	$\gamma_{11} = \cdots = \gamma_{15} = 0$	1.2	0.33
The equation for the change in net output (equation (5.2))			
Hypothesis	Restrictions	F-value	P-value
CA fails to cause ΔNO	$\alpha_{21} = \cdots = \alpha_{25} = 0$	2.5	0.04
P^{Oil} fails to cause ΔNO	$\delta_{21} = \cdots = \delta_{25} = 0$	3.0	0.01
U fails to cause ΔNO	$\gamma_{21} = \cdots = \gamma_{21} = 0$	0.9	0.48

The sample period is 1988Q4 to 2011Q4 and there are 91 observations.

Chapter 6

Concluding remarks

This thesis has examined the intertemporal approach to the current account for quarterly Norwegian data for the period 1981 to 2011. In Chapter 2 I presented a dynamic model for a small open economy where a rational forward-looking agent gets utility from consumption. The model was solved by maximizing the agent's lifetime utility subject to a budget constraint and a no-Ponzi game condition. A key element in this model is the result that the one-period consumption level is a constant share of the permanent value of lifetime income. This result is known as the permanent income hypothesis. The model predicts full consumption smoothing. I have shown that the optimal decision path for the consumer, under a set of assumptions, can be described as a present-value relationship between future changes in net output and the nation's current account balance. If income is expected to decline in the future, the optimal strategy is to save today to keep the consumption level constant, if income is expected to rise; the optimal strategy is to borrow today.

In Chapter 4, I tested the present-value relationship implied by the intertemporal model by using a VAR for Norwegian data. Previous studies on this subject are not very convincing; the intertemporal approach to the current account is frequently rejected in the literature, with some exceptions.

I presented three formal tests of the validity of the PVM. The PVM can be derived from imposing cross-equation restrictions on a general VAR. I test these restrictions where the null-hypothesis is that these restrictions are valid. All three tests reject the restrictions, and therefore also the PVM, on the 1 percent significance level.

On the other hand, my analysis has shown some matching attributes between the model and the actual data. The graphical fit of the model is good; the correlation coefficient between the actual and predicted current account series is equal to 0.97. Since the current account is used to smooth consumption against future changes in net output, the current account should be as good as any predictor of changes in net output. I find a one-way Granger causality from the current account to changes in net output, so I can confirm that the current account contains valuable information for explaining changes in net output.

So, why is the model rejected? First, it is possible that consumption is a function of current income and the permanent income hypothesis is false. Secondly, when testing the model, we are actually testing several hypotheses; the main one is the consumption-smoothing behavior of the consumer, but we are also testing a hypothesis about rational agents, perfect capital mobility, and a quadratic utility function. When rejecting the model we cannot say which of the hypotheses that is the source of the rejection, nor can we know if there is more than one of them who are responsible.

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Appendices

Appendix A: Derivation of the PVM

Let us start with equation (2.16)

$$CA_t = NO_t - E_t\widetilde{NO}_t.$$

Then, use $\widetilde{NO}_t = \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} NO_s$ and we get

$$CA_t = NO_t - E_t\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} NO_s.$$

This expression can be written as

$$\begin{aligned}
CA_t &= NO_t - E_t\frac{r}{1+r}\left[\left(\frac{1}{1+r}\right)^0 NO_t + \left(\frac{1}{1+r}\right)^1 NO_{t+1} + \left(\frac{1}{1+r}\right)^2 NO_{t+2} + \ldots\right] \\
&= E_t\left\{\frac{1}{1+r}NO_t - r\left[\left(\frac{1}{1+r}\right)^2 NO_{t+1} + \left(\frac{1}{1+r}\right)^3 NO_{t+2} + \left(\frac{1}{1+r}\right)^4 NO_{t+3} + \ldots\right]\right\} \\
&= \frac{1}{1+r}NO_t - \frac{r}{(1+r)^2}E_tNO_{t+1} - \frac{r}{(1+r)^3}E_tNO_{t+2} - \frac{r}{(1+r)^4}E_tNO_{t+3} - \ldots \\
&= \frac{1}{1+r}NO_t - \left(\frac{1}{1+r} - \frac{1}{(1+r)^2}\right)E_tNO_{t+1} - \left(\frac{1}{(1+r)^2} - \frac{1}{(1+r)^3}\right)E_tNO_{t+2} - \ldots \\
&= \frac{1}{1+r}NO_t - \frac{1}{1+r}E_tNO_{t+1} - \left(\frac{1}{1+r}\right)^2 E_tNO_{t+1} - \left(\frac{1}{1+r}\right)^2 E_tNO_{t+2} - \ldots \\
&= \frac{1}{1+r}\left(NO_t - E_tNO_{t+1}\right) + \left(\frac{1}{1+r}\right)^2\left(E_tNO_{t+1} - E_tNO_{t+2}\right) + \ldots \\
&= -\frac{1}{1+r}E_t\Delta NO_{t+1} - \left(\frac{1}{1+r}\right)^2 E_t\Delta NO_{t+2} - \left(\frac{1}{1+r}\right)^3 E_t\Delta NO_{t+3} - \ldots.
\end{aligned}$$

And by using the summation operator we get

$$CA_t = -\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t\Delta NO_s.$$

Appendix B: A testable implication of the model

Calculating equation (22) on date $t-1$ gives

$$\begin{aligned} CA_{t-1} &= -\sum_{s=(t-1)+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-(t-1)} E_{t-1}\Delta NO_s \\ &= -\frac{1}{1+r}E_{t-1}\Delta NO_t - \left(\frac{1}{1+r}\right)\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1}\Delta NO_s. \qquad (A2.1) \end{aligned}$$

I set up the following expression for $E_{t-1}\Delta NO_t$

$$E_{t-1}\Delta NO_t = E_t\Delta NO_t + E_{t-1}\Delta NO_t - E_t\Delta NO_t. \qquad (A2.2)$$

For $\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1}\Delta NO_s$ I use

$$\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1}\Delta NO_s = \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1}\Delta NO_s + CA_t - CA_t. \qquad (A2.3)$$

Then, substitute for (A2.2) and (A2.3) into (A2.1)

$$(1+r)CA_{t-1} = -E_t\Delta NO_t - E_{t-1}\Delta NO_t + E_t\Delta NO_t - \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1}\Delta NO_s - CA_t + CA_t.$$

Rearranging gives

$$CA_t - E_t\Delta NO_t - (1+r)CA_{t-1} = E_{t-1}\Delta NO_t - \Delta NO_t + \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_{t-1}\Delta NO_s + CA_t.$$

Substituting from equation (2.17) gives

$$CA_t - \Delta NO_t - (1+r)CA_{t-1} = E_{t-1}\Delta NO_t - \Delta NO_t - \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} [E_t\Delta NO_s - E_{t-1}\Delta NO_s].$$

Rearranging gives

$$\begin{aligned} CA_t - \Delta NO_t - (1+r)CA_{t-1} &= -[NO_t - NO_{t-1} - E_{t-1}NO_t + NO_{t-1}] \\ &\quad -\frac{1}{1+r}[E_tNO_{t+1} - NO_t - E_{t-1}NO_{t+1} + E_{t-1}NO_t] \\ &\quad -\left(\frac{1}{1+r}\right)^2 [E_tNO_{t+2} - E_tNO_{t+1} - E_{t-1}NO_{t+2} + E_{t-1}NO_{t+1}] \\ &\quad -\left(\frac{1}{1+r}\right)^3 [E_tNO_{t+3} - E_tNO_{t+2} - E_{t-1}NO_{t+3} + E_{t-2}NO_{t+2}] \\ &\quad \ldots \end{aligned}$$

$$\begin{aligned}
CA_t - \Delta NO_t - (1+r)CA_{t-1} &= -\{\left(NO_t - \frac{NO_t}{1+r}\right) - \left(E_{t-1}NO_t - \frac{E_{t-1}NO_t}{1+r}\right) \\
&+ \left(\frac{E_tNO_{t+1}}{1+r} - \frac{E_tNO_{t+1}}{(1+r)^2}\right) - \left(\frac{E_{t-1}NO_{t+1}}{1+r} - \frac{E_{t-1}NO_{t+1}}{(1+r)^2}\right) \\
&+ \left(\frac{E_tNO_{t+2}}{(1+r)^2} - \frac{E_tNO_{t+2}}{(1+r)^3}\right) - \left(\frac{E_{t-1}NO_{t+2}}{(1+r)^2} - \frac{E_{t-1}NO_{t+2}}{(1+r)^3}\right) \\
&+ \ldots\} \\
CA_t - \Delta NO_t - (1+r)CA_{t-1} &= -\{\frac{r}{1+r}(NO_t - E_{t-1}NO_t) \\
&+ \frac{r}{(1+r)^2}(E_tNO_{t+1} - E_{t-1}NO_{t+1}) \\
&+ \frac{r}{(1+r)^3}(E_tNO_{t+2} - E_{t-1}NO_{t+2}) \\
&+ \frac{r}{(1+r)^4}(E_tNO_{t+3} - E_{t-1}NO_{t+3}) \\
&+ \ldots\}.
\end{aligned}$$

And by using the summation operator we get

$$CA_t - \Delta NO_t - (1+r)CA_{t-1} = -r\left(\frac{1}{1+r}\right)\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(E_tNO_s - E_{t-1}NO_s).$$

Appendix C: Restrictions on the parameter matrix

The restriction is given by

$$\mathbf{e_{CA}} = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})(\mathbf{I} - \tau\mathbf{\Gamma})^{-1}.$$

The companion matrix from the VAR(4) is given by

$$\mathbf{\Gamma} = \begin{bmatrix}
\alpha_{11} & \beta_{11} & \alpha_{12} & \beta_{12} & \alpha_{13} & \beta_{13} & \alpha_{14} & \beta_{14} \\
\alpha_{21} & \beta_{21} & \alpha_{22} & \beta_{22} & \alpha_{23} & \beta_{23} & \alpha_{24} & \beta_{24} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}.$$

If $\mathbf{I} - \tau\mathbf{\Gamma}$ is an invertible matrix, we can calculate the linear restrictions as linear relations between the coefficients from the VAR. We can write $\mathbf{e_{CA}}(\mathbf{I} - \tau\mathbf{\Gamma}) = -\mathbf{e_{\Delta NO}}(\tau\mathbf{\Gamma})$ which imply the linear restriction $\mathbf{e_{CA}} = \tau(\mathbf{e_{CA}} - \mathbf{e_{\Delta NO}})\mathbf{\Gamma}$. The parameter by parameter restrictions can be calculated if we note that

$$\mathbf{e_{CA}} = [0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0], \ \ \mathbf{e_{\Delta NO}} = [1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],$$

$$
(\mathbf{I}-\tau\boldsymbol{\Gamma}) = \begin{bmatrix}
1-\tau\alpha_{11} & -\tau\beta_{11} & -\tau\alpha_{12} & -\tau\beta_{12} & -\tau\alpha_{13} & -\tau\beta_{13} & -\tau\alpha_{14} & -\tau\beta_{14} \\
-\tau\alpha_{21} & 1-\tau\beta_{21} & -\tau\alpha_{22} & -\tau\beta_{22} & -\tau\alpha_{23} & -\tau\beta_{23} & -\tau\alpha_{24} & -\tau\beta_{24} \\
-\tau & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -\tau & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\tau & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -\tau & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -\tau & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -\tau & 0 & 1
\end{bmatrix},
$$

$$
-\mathbf{e}_{\mathbf{\Delta NO}}(\tau\boldsymbol{\Gamma}) = [-\tau\alpha_{11} \quad -\tau\beta_{11} \quad -\tau\alpha_{12} \quad -\tau\beta_{12} \quad -\tau\alpha_{13} \quad -\tau\beta_{13} \quad -\tau\alpha_{14} \quad -\tau\beta_{14}],
$$

and

$$
\mathbf{e}_{\mathbf{CA}}(\mathbf{I}-\tau\boldsymbol{\Gamma}) = [-\tau\alpha_{21} \quad 1-\tau\beta_{21} \quad -\tau\alpha_{22} \quad -\tau\beta_{22} \quad -\tau\alpha_{23} \quad -\tau\beta_{23} \quad -\tau\alpha_{24} \quad -\tau\beta_{24}].
$$

The restrictions are then given by

$$
\alpha_{11} = \alpha_{21}, \quad \alpha_{12} = \alpha_{22}, \quad \beta_{12} = \beta_{22}, \alpha_{13} = \alpha_{23}, \quad \beta_{13} = \beta_{23},
$$

$$
\alpha_{14} = \alpha_{24}, \quad \beta_{14} = \beta_{24} \text{ and } \tau\beta_{21} = 1 + \tau\beta_{11}.
$$

Appendix D: The stationarity of the VAR(4)

To evaluate the stationarity of the VAR(4), the following equation must be solved[1]

$$
\det(\mathbf{I}_2 - \mathbf{A_1}x - \mathbf{A_2}x^2 - \mathbf{A_3}x^3 - \mathbf{A_4}x^4) = 0,
$$

which can be written as a polynomial of degree 8, the polynomial is known as the characteristic equation of the VAR and can be written as

$$
-0.129x^8 + 0.007x^7 - 0.008x^6 + 0.122x^5 + 0.945x^4 + 0.0232x^3 + 0.044x^2 - 0.175x = 1.
$$

This equation has the following solutions

$$
\begin{aligned}
x_1 &= 1.678, & x_2 &= 1.049, \\
x_3 &= -1.055, & x_4 &= -1.447, \\
x_5 &= -0.069 + 1.568i, & x_6 &= -0.069 - 1.568i, \\
x_7 &= -0.017 + 1.081i, & x_8 &= -0.017 - 1.081i,
\end{aligned}
$$

where i is the square root of -1. All the roots of the characteristic equation lie outside the unit circle, and the VAR(4) is stationary.

[1]See Hamilton (1994), Chapter 10.

Appendix E: Symbol glossary

I have tried to let one letter serve one purpose only, but in some cases a letter may have a different interpretation in different parts of my thesis, when this is the case I explain the different usages. Matrices are in uppercase bold letters while vectors are in lowercase bold letters.

Uppercase English Alphabet

A : Parameter matrix from the VAR.
B : Holdings of foreign assets.
BG: Breusch-Godfrey test statistic.
C : Private consumption.
CA: Current account balance.
E : Expectations operator.
G : Government consumption.
H : A hypothesis.
I : Investment.
$\mathbf{I}$: Identity matrix.
JB: Jarque-Bera test statistic.
K : Number of regressors.
Ku: The kurtosis of a distribution.
LL: Log-likelihood value of a model.
LR: Log-likelihood ratio test statistic.
NO: Net output.
P : The price of oil.
R : Generic variable.
S : Expression for information criterion.
Sk: The skewness of a distribution.
T : Number of observations.
U : Lifetime utility in chapter 2 and unemployment rate in chapter 5.
$\mathbf{V}$: Matrix of squared residuals from the VAR.
W : National wealth.
X : Generic variable.

Lowercase English Alphabet

a: Generic parameter/constant.
b: Generic parameter/constant.
c: Generic parameter/constant.
$\mathbf{e}$: Vector of zeros and one unity element.
g: Penalty function in expression for information criterion.
i: Generic index.
j: Generic index.
p: Number of parameters in the VAR.
q: Limit of a generic summation.
r: Real interest rate.
s: Summation index.
t: Time index.
u: One-period utility function in chapter 2 and residuals from a regression model in chapter 3.

x: Generic variable.

$\mathbf{z}$: Matrix of CA and ΔNO.

Uppercase Greek Alphabet

$\mathbf{\Gamma}$: Companion matrix from the VAR.

Δ: First-difference operator.

$\mathbf{\Sigma_\Gamma}$: The variance-covariance matrix for the parameter matrix Γ.

$\mathbf{\Sigma}_\xi$: White noise variance-covariance matrix for the residuals ξ.

$\mathbf{\Psi}$: Matrix of the variables from the VAR.

$\mathbf{\Omega}$: The agents information set.

Lowercase Greek Alphabet

α: Parameters in the VAR in front of the NO terms.

β: Parameters in the VAR in front of the CA terms.

γ: Generic parameter/constant.

δ: Generic parameter.

η: Residuals in the ADF test.

μ: The mean of a stochastic process.

ν: Residual matrix from the VAR.

ϵ: Residuals from the VAR.

ε: Generic variable.

ξ: Matrix of the residuals from the VAR.

π: Natural constant.

ρ: Subjective discount factor.

σ: The elasticity of intertemporal substitution.

τ: Real discount factor in chapter 4 and time subscript in chapter 3.

ϕ: Parameter of interest in augmented Dickey Fuller test.

χ: Represents a χ^2-distribution.

Other

$\mathcal{I}$: Is a representation of the null-hypothesis for the Wald test.

$\mathbb{R}$: The set of real numbers.

$\mathcal{W}$: Test statistic for Wald test.