U.S. Research Institutes in the Mathematical Sciences: Assessment and Perspectives

Committee on U.S. Mathematical Sciences Research Institutes, National Research Council

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Assessment and Perspectives

Committee on U.S. Mathematical Sciences Research Institutes
Board on Mathematical Sciences
Commission on Physical Sciences, Mathematics, and Applications
National Research Council

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Preface

This report is the result of a fast-track study of U.S. mathematical sciences research institutes done in response to a request from the National Science Foundation (NSF). The task of the Committee on U.S. Mathematical Sciences Research Institutes was to address the following three questions:

1. What are the characteristic features of effective mathematical sciences research institutes in the ways that they further mathematical research in the United States, and are there ways that the current configuration can be improved?
2. What kinds of institutes should there be in the United States, and how many does the nation need?
3. How should U.S. mathematical sciences research institutes be configured (with regard to, for example, diversity of operating formats, distribution of mathematical fields, and interinstitute cooperation or coordination) in order to have the nation's mathematical research enterprise continue to be most productive and successful?

To address these questions, data that could be assembled and input that could be received from the community in the available time were obtained. In mid-May of 1998, the committee circulated a call for comments (see the appendix) to all PhD-granting department chairs, to officers and board or council members of the major professional societies devoted to mathematical sciences research, and to approximately 35 managers or heads of business or industry mathematical sciences research groups. The committee also held an open information-gathering session in which, at the committee's invitation, the director of the NSF's Division of Mathematical Sciences and executive officers of the Mathematical Sciences Research Institute, the Institute for Mathematics and Its Applications, and the National Institute of Statistical Sciences provided background information and responded to committee members' questions. The committee then held a three-day meeting, had two extended teleconferences, and exchanged electronic mail in which it deliberated extensively on the assigned topics, framed its conclusions and recommendations, and drafted and revised its report.

Chapter 1 of this report gives a brief historical view of mathematical research institutes and summarizes community input to the committee, Chapter 2 reviews the impact of current U.S. mathematical research institutes and offers views on continuing value, and Chapter 3 outlines new challenges and a recommended approach for addressing them. Chapter 4 provides closing comments.

The committee is grateful for the comments of the following individuals who reviewed this report: John Ball, University of Oxford; Spencer Bloch, University of Chicago; William Browder, Princeton University; Jill Mesirov, Whitehead Institute for Biomedical Research; Harrison Shull, U.S. Naval Postgraduate School (retired); I.M. Singer, Massachusetts Institute of Technology; and Frank Stillinger, Lucent Technologies. Responsibility for the report's final content rests solely with the authoring committee and the National Research Council.

Contents

Executive Summary

The Committee on U.S. Mathematical Sciences Research Institutes was charged with characterizing the role that such institutes currently play in furthering research in the mathematical sciences[1] and with evaluating the advisability of changing or expanding that role.

The committee was favorably impressed by the breadth and depth of the contributions of research institutes in the mathematical sciences that have appeared throughout the world. These institutions have catalyzed the development of important new results, nurtured continued growth of the mathematical enterprise, and initiated exciting new applications of mathematics. Existing broadly based mathematical research institutes in the United States have served the scientific community well and will continue to do so by contributing in the following important ways to the vitality of the mathematical sciences:

1. Decisively advancing mathematical research, and ensuring that its progress is robust;
2. Catalyzing group and team interactions focused on topics of noteworthy potential;
3. Supplying high-quality outreach to and interaction with industry and the scientific community;
4. Providing first-class postdoctoral programs in both core and interdisciplinary mathematics;
5. Enabling renowned senior researchers to direct, influence, and mentor younger scientists at crucially beneficial points in those younger researchers' careers;
6. Sharpening, both in core and applied areas, mathematical research's focus via quick-response workshops on key, cutting-edge issues and fast-breaking "hot topics";
7. Enriching and invigorating mathematical education at every level; and
8. Being a hub for mathematical resources, archives, and tools.

Of course some university departments also can and do address at least some of these needs.

In spite of the accomplishments and positive attributes of existing institutes, the committee identified fundamental needs and challenges unlikely to be met by broadly based institutes. A primary reason for considering new types of institutes is the growing U.S. dependence on scientific and technological advances, an increasing number of which rely on progress in mathematical sciences research. New research in the mathematical sciences is essential to address theoretical and practical questions, often radically new and increasingly difficult, that underlie progress in many fields of science and technology. Therefore, the nation will benefit from rapid expansion and robust development of mathematical sciences research, as well as from increases in the level of interaction between core mathematicians, other mathematical sciences researchers, and researchers in areas of science, engineering, medicine, and technology that are ripe for investigation via mathematical techniques. Institutes are a significant mechanism for enabling these sorts of interactions.

Another driver for new types of institutes is that advances in computer technology and software development have changed dramatically the ways in which mathematical scientists interact and do research and have also made possible the creation of completely new mechanisms for experimental research that are playing a rapidly increasing role in the mathematical sciences. Inadequate access to existing research resources and software tools developed by mathematical scientists working in different areas points to an opportunity to make better use of modern technology. Specifically, a systematic effort is needed to make all such information and computer resources visible and accessible to researchers in the mathematical sciences, especially in view of the fact that mathematical researchers are increasingly more geographically dispersed in the

[1] The mathematical sciences include mathematics and its applications, statistics, operations research, and scientific computing (COSEPUP, 1997; NRC, 1992).

United States. Such an effort would promote the development of new tools capable of addressing important open questions that cannot be pursued today.

To address these critical needs, the committee makes two recommendations:

- The committee recommends that the National Science Foundation's Division of Mathematical Sciences embark on a process of cooperation with other funding units or agencies to establish a well-chosen and critically focused research institute for mathematical sciences devoted to intensive research in a field whose emerging interface with the mathematical sciences shows great potential for the incorporation of mathematical ideas to achieve important societal advances.
- The committee recommends that the National Science Foundation's Division of Mathematical Sciences establish a research institute for experimental mathematics and electronic tools in the mathematical sciences. Its mission would be to promote the experimental component of mathematical sciences research, to facilitate the development of new computer-based tools, and to provide visibility and accessibility to existing tools and to existing research resources that are scattered throughout the world.

In making these recommendations, the committee emphasizes that mathematical research institutes must be based on a strong and vigorous mathematical community. With this in mind, *the committee strongly believes that it would not be in the best interest of either the mathematical sciences community or society as a whole to transfer funding from existing NSF mathematical sciences individual (principal investigator) research grant programs to funding for existing or additional mathematical sciences institutes*. The strategic and increasingly important role played by mathematical sciences research in every segment of society constitutes a strong justification for new federal funding for the proposed new ventures.

1

Perspective on and Approach to Characterizing Institutes' Roles in the Mathematical Sciences

Throughout the 20th century, diverse strategies and circumstances have led to the development of a variety of mathematical research institutes, each designed to fit particular needs. Mathematical sciences research institutes of many kinds now play important and varied roles internationally. The committee considered U.S. mathematical research institutes in the context of this worldwide set, not only because of the complementary nature of various institutes, but also because a primary function of mathematical institutes is facilitating communication of mathematical developments worldwide, across a research enterprise that today is thoroughly international.

A BRIEF HISTORICAL VIEW OF MATHEMATICAL SCIENCES INSTITUTES

Initial Mathematical Research Institutes

The concept of a mathematical research institute to host visiting mathematicians was realized early in the 20th century by the Swedish mathematician Gösta Mittag-Leffler and his wife, who founded the Mittag-Leffler Institute in 1916. Incorporated into the Swedish Royal Academy of Sciences in 1919, the Mittag-Leffler Institute fostered individual mathematical research. In the United States, the first mathematical research institute was created in 1930 as part of the Institute for Advanced Study (IAS) near Princeton University. Offering preeminent scholars (among them, Albert Einstein and John von Neumann) the opportunity to conduct long-term research in a quiet multidisciplinary atmosphere, the IAS today has several outstanding permanent members and a large number of long-term visitors.

Interactions at the IAS have fostered the development of deep mathematical ideas. An example is the Atiyah-Singer Index Theorem, which has repeatedly brought together analysis and geometry and provides an important link between mathematics and modern theoretical physics. As noted in the National Science Foundation's 1978 announcement calling for the establishment of new research institutes in the mathematical sciences, "The IAS has been a major force in mathematical research since its very inception, and its effect on the development of U.S. mathematics has been decisive" (AMS, 1978; pp. 484–485).

Institutes Based on the IAS Model

Following the Second World War, several institutes were created along the IAS model. At the Institut des Hautes Études Scientifiques (IHÉS), founded in 1958 at the initiative of a private entrepreneur who wanted to create an IAS counterpart in Europe, activities reshaped algebraic geometry and led to the birth of the first models of turbulence and, thereby, recent theories of chaos. The Instituto de Mathematica Pura e Aplicada (IMPA), founded in Rio de Janeiro in 1957 with the IAS model in mind, became a focal point worldwide for the emerging field of dynamical systems and played a major role in establishing this new area of mathematical research by hosting research-oriented as well as instructional conferences. The Tata Institute, founded in 1945 in Bombay as an affiliated institution of the Indian Atomic Energy Commission, has played an important role in augmenting the international visibility of Indian mathematics and establishing in India an outstanding strength in algebra and algebraic geometry.

Mathematical Sciences in Other Institutes and Research Centers

The early results of mathematical sciences research conducted at Bell Laboratories (now part of Lucent Technologies), founded in 1925, included Walter Shewhart's statistical process control charts of the 1920s, Claude Shannon's information theory and Richard Hamming's error-correcting codes in the 1940s, and the pioneering work of David Slepian on algebraic coding theory in the 1950s.

From its creation in 1931 to today, the Indian Statistical Institute has had an important influence on the development of statistics as a field. The world's first statistical institute when founded by P.C. Mahalanobis, then working part-time in a single room, it currently has more than 250 faculty members and over 1,000 support staff in four major Indian cities. It promotes and pursues all aspects of statistical research, applications, and education.

An example of a different model of mathematical institute, one with a large permanent staff, is the Steklov Mathematical Institute of the Russian Academy of Sciences in Moscow, established as a distinct entity in 1937. It is also noteworthy that many of the former Soviet Union's mathematicians have worked in institutes devoted to other scientific disciplines or applied technologies.

With its launching of the Centre National de la Recherche Scientifique (CNRS) in the late 1940s, France put in place another research structure whereby approximately 10 percent of that nation's mathematical researchers are funded by the CNRS. These research faculty work in university departments rather than in independent institutes.

Conference Centers

Also inaugurated in the 1940s was a completely new type of institute, the Oberwolfach Mathematisches Forschungsinstitut in Germany. This conference center holds 50 week-long meetings annually, each on a specific theme. In the Second World War's aftermath, Oberwolfach was remarkably effective in reestablishing Germany as a major force in mathematics worldwide. Its success has been attributed to attracting top specialists to serve as session organizers, using a simple scheme for workshops (strictly limited in size), providing superb facilities (including an excellent library and good housing), and encouraging interactions among German researchers and between German mathematicians and the many visiting foreign researchers.

In 1981, France created a similar conference center, the Centre International de Rencontres Mathématiques (CIRM), near Marseilles. Supported by the Ministry of Research and the CNRS, CIRM hosts week-long conferences at a well-designed facility.

Worldwide Growth After Mid-Century

The worldwide technology and information revolution in the last half-century has been stimulated and sustained by a global strengthening of and growth in scientific research capability. As part of their efforts to strengthen and nurture their strategic scientific and technological resources, a number of nations created mathematical research institutes.

- Located at Kyoto University, the Research Institute for Mathematical Sciences was established as a cooperative research institute in 1964 with research in the traditional major areas of pure and applied mathematics, as well as in mathematical physics, fluid mechanics, mathematical engineering, and theoretical computer science.

- The Centre de Recherches Mathématiques, founded in 1968 in affiliation with the Université de Montréal, represented an institute of a new type in that positions there are temporary and are filled in close association with the host university.
- The International Stephan Banach Center, established in 1972 in close physical proximity to the Mathematical Institute of the Polish Academy of Sciences, organizes workshops and featured periods devoted to exploration of special themes. During the Cold War, it played an important role in bringing together mathematicians from the East and the West.
- As a result of NSF's 1980 call for proposals for new mathematical research institutes, the Mathematical Sciences Research Institute (MSRI) in Berkeley, California, and the Institute for Mathematics and Its Applications (IMA) in Minneapolis, Minnesota, were created in 1982. Neither has a permanent faculty, and both have a number of universities as sponsors. Each has developed independently and serves different segments of the mathematical community. Both institutes have succeeded in attracting world-class researchers to lead their thematic programs and thus have become focal points of world mathematical research life. Their impacts on the various segments of the mathematical community are discussed in Chapter 2.
- Inspired by the Institute for Advanced Studies and the Institut des Hautes Études Scientifiques, Bonn's Max-Planck-Institut für Mathematik was founded in 1982, further enhancing the city's prominent role in mathematical research.
- More recently, additional mathematical sciences institutes have sprung up around the world, each one differing slightly from its predecessors. The International Center for Theoretical Physics (ICTP) in Trieste, Italy, opened a mathematical section in 1986; the Euler International Mathematical Institute was founded in 1988 in Saint Petersburg; the Centre Émile Borel commenced in 1990 in Paris; the International Centre for Mathematical Sciences was founded in 1990 by Edinburgh University, Heriot-Watt University, the ICTP, and government agencies; the Fields Institute, now headquartered in Toronto, started in 1992, as did the Isaac Newton Institute for the Mathematical Sciences, in Cambridge, England; and the Erwin Schrödinger Institut opened in 1993 in Vienna.

Recent Trends

Most of the institutes created in the past 10 years for mathematical sciences research have more innovative features than did earlier institutes. For example, the NSF-funded Center for Discrete Mathematics and Theoretical Computer Science (DIMACS), founded in 1989 as a center for the advancement of science and technology with a national scope, is a collaborative effort involving academic participants from Rutgers and Princeton universities and corporate researchers from AT&T Labs-Research, Bell Labs, SAIC/Bellcore, and the NEC Research Institute. As another example, the Park City (Utah) Mathematical Institute, now affiliated with the IAS, was established in 1991 to stimulate vertical integration through interactions among researchers, graduate students, undergraduate students, high school teachers, and undergraduate faculty.

The NSF Science and Technology Center (STC) for Computation and Visualization of Geometric Structures at the University of Minnesota, generally known as the Geometry Center, was founded to use visualization as a tool in mathematical research and to communicate mathematical research developments. It was established in 1989 as part of the first round of NSF's STC awards. Although the STCs were envisioned as having 11-year lifetimes (subject to triennial reviews), NSF announced in mid-1996 that the Geometry Center would be phased out; its NSF funding ceased on August 31, 1998. Certain aspects of that center's experience could be worth considering in efforts to plan future mathematical sciences institutes, including: What attention does an institute pay to reaching out and including the larger mathematical research community in the institute's efforts? How does an institute maintain awareness of potentially changing expectations when various changes in its operating environment occur, such as changes within funding agencies? How can an institute ensure that what it does is considered to be valuable by the mathematical research community?

The Max-Planck-Institut für Mathematik in den Naturwissenschaften, which opened in 1996 in Leipzig (Germany), develops interactions with other disciplines via longer-term visits (typically 5 years). A collaborative initiative, the Pacific Institute in the Mathematical Sciences, emerged in 1997 in Alberta and British Columbia (Canada) to establish new types of contacts with industry and with other disciplines, as had been pioneered at the Institute for Mathematics and Its Applications, through special training sessions and seminars.

Supported by U.S. statistical societies as well as universities near its location in Research Triangle Park, North Carolina, the National Institute of Statistical Sciences (NISS) was created in 1991 to focus on project-oriented research while emphasizing feedback to improve statistical methodology. A similar entity devoted to stochastic problems connected to industrial and societal applications, Eurandom, is developing today in the Netherlands.

In addition to collaborative institute efforts targeted to industries or other science, engineering, and technology users, there seems to be growing interest in the establishment of mathematical research units within high-technology companies. Hewlett-Packard opened a mathematical sciences basic research institute in 1993, and Microsoft recently established a fundamental research group dedicated to fairly unfettered pursuit of mathematical questions at the foundations of computer science. As recognition expands of the mathematical nature of problems that arise in engineering and scientific fields, more mathematical institutes and industry research organizations will likely be established around the world to link the industrial, engineering, technological, and academic arenas.

A ROUGH CLASSIFICATION OF EXISTING MATHEMATICAL SCIENCES INSTITUTES

While realizing the variety and plurality of types of mathematical institutes, the committee offers five basic categories that provide a rough taxonomy of most existing institutes:

1. IAS-like institutes that principally host large numbers of rigorously selected visitors for significant periods (for instance, a term or a year), and that have a small group of permanent members of exceptional mathematical stature;
2. Broadly based institutes, such as the Mathematical Sciences Research Institute and the Institute for Mathematics and Its Applications, which have charters to promote diverse areas of mathematical research and mathematical applications and which annually offer scientific programs and workshops of periods from a few months to a year, selected by an advisory committee to emphasize specific topics;
3. Conference centers, which host short meetings of intermediate size aimed at maximizing interactions among participants;
4. Institutes that serve as gateways to other entities, such as an industry, a country as a whole, or an entire subdiscipline of the mathematical sciences; and
5. Institutes designed to promote vertical integration and interaction among the mathematical enterprise's different cohorts (such as researchers, postdoctoral students, undergraduate faculty, high school teachers, graduate students, and undergraduate students).

This rough classification merely isolates certain common aspects of various institutes and should not be interpreted as implying what might be fundamental parameters. In characterizing institutes, it is of course also necessary to take into consideration the methods by which programs are selected, the time scales for decision making on programs and applications, and the nature and importance of the mathematical issues and questions being investigated. The committee did not attempt to define a priori what constitutes a mathematical sciences research institute. For the purposes of this study, however, the committee limited its consideration to research institutes that have public service as part of their mission in the sense that they are constituted to serve a significantly broader community than just that within their host institution(s). The institutes considered in this report are those that are qualitatively different from a collection of individually funded investigators and which purposely create conditions that encourage a mixing of mathematical scientists with different backgrounds.

BRIEF SUMMARY OF INPUT TO THE COMMITTEE

In the course of its study, the committee requested input from a large number of leaders in the mathematical sciences. This section summarizes some of the thoughts received that are relevant to current and potential roles for mathematical sciences research institutes in the United States. More detail on the call for input and the results received is contained in the appendix.

The input received by the committee suggests that the general mathematical sciences community is extremely well disposed toward existing research institutes in the mathematical sciences. However, there was also overwhelming common sentiment expressed by respondents that neither continued funding of existing institutes nor the creation of new institutes should occur at the expense of individual investigator research grants. It was the near-unanimous opinion of respondents that there was no optimal number of institutes.

The prevailing opinion of respondents was that a collection of mathematical institutes, properly constituted, would be well positioned to help the mathematical sciences meet new challenges and take better advantage of the opportunities now facing the community. Among the new challenges identified by some respondents was the need for interdisciplinary programs that link cutting-edge mathematical sciences research to the broader physical, social, and engineering sciences communities; among the opportunities, many respondents noted that institutes, in addition to their role in research, can additionally serve as training grounds that broaden the horizons of young researchers.

There was no enthusiasm among respondents for a U.S. counterpart to an Oberwolfach conference-oriented institute. One concern expressed was the cost, including the cost of bringing geographically dispersed U.S. mathematical sciences researchers to one conference center. Others observed that there are already a great many conferences, and some respondents saw Oberwolfach as a model better suited to intradisciplinary, rather than interdisciplinary, research.

The prevailing sentiment of respondents was that there has been a continual and dramatic decrease in funding for basic research in the mathematical sciences, with one consequence being the virtual disappearance of fully funded sabbatical periods. Both applied and core mathematicians noted a need for increased financial support for senior mathematical scientists to spend sabbatical periods at an institute. The replies reflected a general feeling that new funding for new institutes might help alleviate the decrease in basic research funding in the mathematical sciences at all levels (from postdoctoral programs to research opportunities for senior researchers).

Concerning how institutes might address such needs or evolve in changing times, some individuals expressed the opinion that an institute would be a good mechanism for fostering new and significant advances in mathematical modeling for experimental sciences, while others believed that now is an appropriate time to consider a new type of institute devoted in large part to the dramatically increased role of computers in computation, large-scale modeling and simulation, industrial engineering and manufacturing, and so on. Also, new institutes could help address what now have become significant barriers to professional growth and achievement resulting from the greater than ever generation of research mathematicians by PhD-granting institutions.

2

Current U.S. Research Institutes in the Mathematical Sciences —Impact and Continuing Need

IMPACT OF EXISTING U.S. RESEARCH INSTITUTES IN THE MATHEMATICAL SCIENCES

This chapter considers the impact of the current configuration of U.S. mathematical sciences research institutes in facilitating the development of the mathematical sciences and shaping the U.S. mathematical sciences community. It is based both on the collective experience of the committee members and on the inputs received through the committee's call for comments (see the appendix). Overall, the committee was favorably impressed by the breadth and depth of contributions of the many mathematical research institutes both within and outside the United States.

Impact on Research

Perhaps their most important contribution has been that the U.S. mathematical sciences research institutes provide an especially favorable atmosphere for high-level mathematical sciences researchers to focus with considerable concentration on their investigations. In exceptional cases, a few of the world's best mathematicians have had a lifetime opportunity to achieve their goals in a mathematical institute. More typically, established mathematical scientists visiting these institutes have carried out important projects and developed new ideas during their stays.

The senior mathematical scientists working at institutes have helped set the direction of the mathematical sciences and emphasized the importance of current mathematical discoveries and developments. Institutes have served as a forum for the mathematical sciences through their postdoctoral programs, conference and lecture series, and pre-publications of new mathematical research results. Because short-term visitors from both academia and industry have the opportunity to pose problems and enter into collaborations with longer-term institute visitors, the effect on the mathematical sciences is far broader than might be suggested by the relatively small numbers of long-term visitors.

By sponsoring special year-long programs that highlight specific mathematical areas and applications, mathematical institutes have provided encouragement and support for targeted areas. Frequently, a particular field experiences considerable stimulation from such a program, and the resulting mathematical progress continues to fuel developments in succeeding years. New and important areas of research have emerged and evolved as a result. For example, the mathematics of materials science, the linkage of operator algebras and knot theory, and the applications of statistics to problems of transportation and drug design have all emerged from special-year programs at U.S. mathematical research institutes (see Box 2.1).

Impact on Mathematical Quality and Culture

Most mathematical research institutes host mathematical scientists from many nations. The opportunity for mathematical scientists from different countries to work together has been a considerable boon for the mathematical sciences both in the United States and abroad. Frequently, U.S. mathematical scientists form a large cohort in foreign institutes just as foreign mathematical scientists are frequent visitors to U.S. mathematical institutes. It is vital to preserve these opportunities for mixing different mathematical cultures. Fundamental mathematical research in the United States will continue to profit by having a significant proportion of the most active U.S. mathematical researchers exposed to the best experts worldwide.

BOX 2.1 SOME NOTEWORTHY IMPACTS OF U.S. MATHEMATICAL SCIENCES INSTITUTES

Advances in Materials Science

The Institute for Mathematics and Its Applications (IMA) recognized early that materials science, a critical technology area for the nation, also represented an important opportunity for the mathematical sciences. The IMA helped build a mathematical research community in this area more than five times the size of the community 10 years earlier, and research advances at the IMA were significant. Based on work done at the 1995–1996 IMA Year on Mathematics in Materials Science (which in turn built on a 1984–1985 IMA program focusing on continuum physics and partial differential equations), microstructure theories of martensitic materials were developed to the point that the behavior of new materials could be predicted. A particular advance was the new concept for a hypothetical material combining two types of transformations: a ferromagnetic transition and a martensitic transformation. This research led to a successfully implemented strategy for developing this class of materials. As a result, new alloys have been produced that exhibit this magnetostrictive effect to a degree some 50 times greater than what had been observed in the previous record holders, the so-called giant magnetostrictive materials. Today, nearly every area of active materials science research includes research mathematicians, and the mathematical sciences are playing a crucial role in several large federal thrusts in materials research. The IMA had a major influence in coalescing and nurturing this development.

Knots and Protein Folding

In the mid-1980s a deliberately planned juxtaposition of two research programs at the Mathematical Sciences Research Institute was the catalyst for interaction between Vaughan Jones, whose research concerned operator algebras, and researchers in low-dimensional topology. That interaction led Jones to notice that a sequence of algebraic relations discussed was similar to those that define a mathematical object called the braid group. Jones then found that this correspondence was more than just an analogy, and he obtained a general invariant for characterizing knots and links that was different from and more useful than a previously known classical one. This insight rapidly led to many advances and, ultimately, to a deep pairing of these ideas with another mathematical area, algebraic K-theory. Jones received a Fields Medal (the mathematical equivalent of a Nobel Prize, but which is awarded only once every 4 years) in part for this development.

Pharmaceutical Design

A new research approach to drug discovery was opened through sequential drug design strategies developed at the National Institute for Statistical Sciences. As a consequence, the high-throughput screening of hundreds of thousands of drug compounds is now feasible, in a unique combination of computational chemistry, computer science, and statistics.

Increasingly, mathematical institutes are providing a bridge linking academic mathematical scientists and industries keen on finding new mathematical tools for their work. For instance, the industrial postdoctoral research fellow positions developed by the Institute for Mathematics and Its Applications have given considerable encouragement to mathematical areas that are directly applicable to industry and have also heightened recognition in the marketplace of the relevance and importance of mathematical sciences research. Interactions between mathematical scientists and those industrial researchers are typically not one-way transfers of mathematical techniques. Many mathematical science ideas—for example, finite elements, posipolynomials, splines, and spectral partitioning—have arisen in some heuristic form during contacts between mathematical researchers and applications-oriented researchers with questions to be addressed. Mathematical research institutes have helped to foster this important interaction.

Vitality of the U.S. Mathematical Enterprise

Of course, mathematical research institutes not only contribute to the development of the mathematical sciences and their applications, but also enhance the well-being of the U.S. mathematical community. Senior mathematical scientists visiting a mathematical institute are provided with the opportunity to interact at some length with their peers, resulting in collaborative efforts among like-minded researchers and also leading to unexpected interplay between seemingly disparate areas. This cross-fertilization contributes to the coherence of the mathematical community as well as to the advancement of scientific knowledge (see Box 2.2).

Moreover, when junior mathematical scientists attend one of the mathematical research institutes, they often enjoy their first extended period of uninterrupted research. This period of relative calm has frequently made the critical difference in enabling young mathematical researchers to develop their most creative ideas. Mathematical institutes also provide younger mathematical researchers with opportunities to encounter for the first time other researchers working on similar problems. Frequently, younger researchers are captivated by new developments discussed at these institutes and may find that they can profitably shift the focus of their own research.

For postdoctoral fellows, an appointment at a mathematical research institute is an opportunity to deepen and broaden their doctoral education. The opportunity to associate with important figures in their area of research has often opened vistas to postdoctoral fellows that have led to dramatic advances in their research careers. The mentoring such postdoctoral fellows received from their PhD thesis advisors is often then carried on by other senior mathematicians.

Many others in the mathematical community derive considerable benefit from short visits to research institutes, typically for conferences and workshops. For mathematicians who are somewhat isolated, a visit to a mathematical institute can provide an especially important opportunity to connect with others working on similar problems, and to learn what areas of research are being studied most intensively at mathematical centers.

Benefits to Mathematical Education and Other Areas

The postdoctoral programs of mathematical institutes constitute their major educational component. However, in addition to holding workshops and conferences that educate all levels of the mathematical sciences community (including university faculty and researchers from government laboratories and industry), increasingly institutes organize research conferences that include special lectures, and sometimes entire courses, for graduate students. Mathematical institutes now sponsor summer programs (each focused on a specific research area) targeted to graduate students and summer programs for undergraduate students and pre-college mathematical sciences teachers. They also occasionally sponsor workshops devoted to mathematical education.

Beyond benefiting the mathematical sciences and the community of mathematical scientists, mathematical research institutes benefit industry and commerce through the aforementioned industrial postdoctoral programs and workshops on applications of the mathematical sciences to real-world problems. Scientists and engineers outside academia can visit these institutes as well as interact through participation in problem-solving forums. Moreover, basic scientific research in other scientific, technological, and engineering areas, especially mathematical physics, has traditionally been an important beneficiary of the programs and concerted efforts of mathematical research institutes. Indeed, some of today's institutes consist of a roughly equal mix of mathematical scientists and physicists.

BOX 2.2 PERSONAL EXPERIENCES AT U.S. MATHEMATICAL INSTITUTES (EXTRACTED FROM THE COMMITTEE'S CALL FOR COMMENTS)

[A] short visit at IMA (and MCIM) ... expose[d] me to the people and resources of IMA, which ... eventually ... [led] to several interesting collaborations (one extraordinarily successful).

The interactions with the IMA have been extremely beneficial to me and my company.

I spent about 25 days, in three installments, at the MSRI .... Two weeks were workshops, the rest just participation in the regular activities. It had a tremendous impact on me. In addition to being informed about recent progress and controversies in my subfield proper in one of the workshops, I learned about main current directions in the related field of algebraic geometry in the other workshop. I also had a chance to discuss with colleagues my then emerging interest in a new field ... [and received encouragement rather than discouragement]. Both helped me shape my program. Those discussions taught me about results and open problems ... that might be related to my new interests. In a number of cases those issues indeed proved to be related and had a major impact on the first paper I wrote on the subject. MSRI ... has enriched my research —or shall I say, my mathematical existence—in a way that no other institutional framework has ever done.

My first job after getting my PhD was at the IMA.... For me, it was exactly the right place to be. I found the mix of pure and applied topics exciting, and the opportunity to mix with the leaders of the field on a regular basis invaluable. I rank my stay at the IMA as the pivotal event in my career—I'm still working on some of the research projects I started in Minneapolis.

I visited MSRI for a semester. Without a doubt this was the most exciting, stimulating, challenging period I have experienced since my PhD.... No university in the world can offer the stimulation and challenge that simultaneous contact with so many top researchers offered.... One might think that a good conference offers much the same stimulation; but this is erroneous. I needed the continuing contact over a period of time for the issues to really become clear.

I spent 10 months at the IMA as the main principal organizer of a year's program. Besides postdocs who were at the IMA for a year, there were several people who came for about one quarter, some who came for two quarters, and a couple more who spent the whole academic year. Then of course there were people who came just for a workshop and others who were there for 2 to 4 weeks. There was considerable interaction among the visitors and the postdocs who were selected for the specific program. Many joint papers were written that would not have been written otherwise ... most of them were very good to excellent. The experience at the IMA was very valuable for the postdocs not only for the mathematics learned and created but also for the contacts they made. Daily interaction was the norm rather than the exception.

Finally, some mathematical research institutes have commenced activities to make the results of research in the mathematical sciences more accessible to the general public. Their experimentation with making materials available on the Internet suggests that this aspect of their work might expand.

THE CONTINUING VALUE OF BROADLY BASED RESEARCH INSTITUTES IN THE MATHEMATICAL SCIENCES

The committee believes that broadly based mathematical research institutes (labeled as category 2 in Chapter 1) are of critical importance to the continuing success of mathematical sciences research in the United States. Such institutes are now indispensable, strategic components for the health, strength, and world preeminence (COSEPUP, 1997) of the nation's mathematical research enterprise. Among the specific needs addressed successfully by such mathematical research institutes are the following:

1. Decisively advancing mathematical research, and ensuring that its progress is robust;
2. Catalyzing group and team interactions focused on topics of noteworthy potential;
3. Supplying high-quality outreach to and interaction with industry and the scientific community;
4. Providing first-class postdoctoral programs in both core and interdisciplinary mathematics;
5. Enabling renowned senior researchers to direct, influence, and mentor younger scientists at crucially beneficial points in those younger researchers' careers;
6. Sharpening, both in core and applied areas, mathematical research's focus via quick-response workshops on key, cutting-edge issues and fast-breaking "hot topics";
7. Enriching and invigorating mathematical education at every level; and
8. Being a hub for mathematical resources, archives, and tools.

Of course some university departments also can and do address at least some of these needs.

One of the main purposes of broadly based mathematical research institutes is to bring together internationally recognized scientists to work on fundamental problems and forge new directions for research. A successful program at such an institute can have an impact on an entire mathematical field by setting common standards and establishing research agendas, thereby communicating beyond the leading centers of research an understanding of what is difficult, what is key, and what is exciting. The broadly based institutes strongly emphasize innovation and provide a national forum for research on both core mathematics and mathematical sciences areas at the interface of fields beyond a discipline's traditional boundaries.

Mathematical scientists at all levels of experience who participate in broadly based institute programs return to their home institutions with a new perspective on important research problems and with contacts to a network of other researchers that can influence their future research collaborations. In particular, the 1-year institute programs serve as an important training ground for postdoctoral fellows. By providing mentors who are internationally recognized researchers, such programs give young researchers a chance to immerse themselves in their own and related areas to a depth unmatched at many universities. Institutes also help foster graduate student involvement in the mathematical research enterprise.

Some broadly based mathematical institute programs bring together researchers from industry, government, and academia in order to address in a concentrated time period more facets of mathematical problems from a given area than would be feasible at most individual universities. These initiatives build partnerships among the mathematical sciences, other scientific disciplines, and the industrial sector. They also foster creative research that can lead to innovative applications of the mathematical sciences in business and industry. The broadly based institutes are now also contributing to the enrichment of mathematical education by providing special events and resources for high school teachers. At the same time, their public outreach activities are helping to enhance the image of the mathematical sciences in society.

In short, broadly based mathematical research institutes play a major international role in the development of improved communication of major developments in the sciences. The proliferation of mathematics research institutes worldwide, many modeled on the existing facilities in the United States, testifies to the impact and success of these broadly based institutes.

While many sorts of other mathematical institutes now exist, there nevertheless continues to be a vital need for broadly based mathematical research institutes in the United States along the lines of, but not necessarily identical to, the ones now in operation.

3

New Challenges and Two New Types of Research Institutes in the Mathematical Sciences

EVOLUTION IN THE MATHEMATICAL SCIENCES

The mathematical sciences have evolved significantly during the last two decades—particularly in regard to the role of mathematics in rapidly advancing areas of science, technology, and engineering, and to the research environment for U.S. mathematical scientists. These changes have contributed to an acceleration of developments in the mathematical sciences and to an enormous increase in the amount of mathematical research being pursued. Growth internationally in the number of mathematical conferences and workshops, sponsored by mathematical institutes, universities, and professional societies, has added greatly to the pace of mathematical activity worldwide.

Evolution of Science and Technology: Increasing Need for Mathematical Applications

Fundamental changes in many areas of science and technology—especially in biology and medicine, and in communications and computation—are presenting important new problems that require the expertise of mathematical scientists and also pose deep challenges for the field. Examples of such problems include:

- The protein-folding problem;
- Developments toward a unified field theory;
- Techniques for dealing with uncertainty in large-scale computer simulations;
- Compression, storage, enhancement, and reconstruction of visual images;
- The study of large, complex systems; and
- The mining and analysis of enormous data sets.

Some general areas of growth include financial mathematics, which is developing rapidly because of the need for improved decision making in an increasingly global marketplace; the modeling of very complex problems, such as the impact of human activities on the environment; and new and deep mathematical and statistical advances that address computational issues associated with the Human Genome Project.

In computer science, which traditionally has relied mainly on such branches of mathematics as logic, discrete mathematics, and number theory, areas such as topology, differential and conformal geometries, dynamical systems, and exploratory data analysis and mining have become increasingly relevant. The emergence of the Internet and the astonishing growth in its use have led to new applications of queuing theory, combinatorial optimization, and statistical modeling.

Reflecting an unprecedented resurgence of interplay between core mathematics and physics, a flow of mutually important ideas has been established between the areas of string theory and quantum field theory in physics and the mathematical areas of geometry, topology, and the theory of Lie groups and algebras. Also remarkably successful have been the development and application of wavelet analysis to data compression and image processing.

Engineering in all its sub-areas requires sophisticated mathematics and statistics to formulate, analyze, and solve today's problems. Successful mathematical models have saved billions of dollars by replacing expensive, and sometimes very risky, scaled-down or full-scale experimentation.

Effects of Technology Development on Research in the Mathematical Sciences

Dramatic technological changes, especially in computation and communications, have revolutionized mathematical research itself, even in its core and more traditional areas. The growing power of computers now often enables large-scale experimentation to verify hypotheses and provides tools for the visualization of sophisticated geometric objects and for the manipulation or analysis of algebraic expressions. The result is the expanding field of experimental mathematics. The revolution in communications has also dramatically changed the way in which mathematical research is carried out. Electronic mail and facsimile machines have drastically reduced communication turn-around time, thus facilitating collaborations between mathematical scientists who are physically located at different sites around the world. Indeed, from 1981 to 1993, the percentage of papers written by U.S. mathematicians involved in an international co-authorship almost doubled, from 13% to nearly 25% (NSF, 1998, p. 11). The development of the Internet has also made mathematical information much more accessible to the research community in the form of electronic archives of preprints, electronic journals, and so forth.

However, good communications technology and videoconferencing cannot replace the extended person-to-person experience that is of crucial importance in mathematical research. Nor can it substitute for a stimulating significant immersion in a new and different, supportive, and idea-charged research environment.

Growth and Change in the U.S. Mathematical Sciences Research Community

With the end of the Cold War, many highly qualified mathematicians, including leaders in their fields, relocated from the former Soviet Union and countries in Eastern Europe, as well as from China, to the United States. The influx of talented foreign-born graduate students to U.S. university mathematical sciences departments has increased as well. The best among these foreign students often find jobs and stay in the United States upon completing their degree programs. (For average stay rates, see Figure 7, p. 40, in COSEPUP, 1997). In 1996, non-U.S. citizens earned nearly 57% of the total doctoral degrees awarded in mathematical and computer sciences (AMS, 1996).

The size of the mathematical research community has increased substantially since the competition that resulted in the establishment of the Mathematical Sciences Research Institute and the Institute for Mathematics and Its Applications. In 1993, 22,820 PhD mathematicians were employed in the United States (NSF, 1998, p. 3), compared with about 13,000 in 1979 (COSEPUP, 1997, Figure B-7, p. 62). Of those mathematicians employed in 1993, 14,670 were employed by universities and 4-year colleges, and more than 9,500 were active in research. The number of PhDs awarded annually by U.S. mathematics departments has grown from 800 in 1986 to 1,240 in 1995 (COSEPUP, 1997, Figure 3, p. 35). A more limited job market in academia has tended to cause more PhD mathematicians to seek jobs in industry or finance, which argues for the need for more varied research opportunities for graduate students and younger mathematicians. (For data on employment trends for PhD mathematicians, see Table B-1, p. 63, in COSEPUP, 1997.)

The growth of the international mathematical community (according to the *World Directory of Mathematicians, 1998*, some 50,000 mathematicians in 69 countries are now involved in research activities worldwide, a figure that does not include the global community of statisticians), and the need for the research enterprise to move ahead more quickly to contribute needed expertise, have added pressure to develop centrally collected resources and tools for mathematical scientists. This need has been felt in all mathematically developed countries.

Another change that affects the design of institutes' programs is that more and more researchers now have a working spouse, which can limit researchers' availability for long-term visits at the crucial early stages of their career, before they have secured a permanent position. This factor can also make it more difficult for senior mathematical scientists to relocate during their sabbatical years. In response, most broadly based mathematical research institutes now offer shorter-term workshops or visits that, to some degree, are intended to accommodate this change.

HOW TO ADDRESS NEW CHALLENGES

New Requirements

The technological and logistical problems now facing society are increasingly complex, requiring joint and sustained efforts by researchers from multiple disciplines. The mathematical sciences are almost always basic to, and often pivotal in, the search for ways to address these problems. For example, to maintain their position in a highly competitive international economy, U.S. industry and business must be continuously innovative in product development while minimizing the time from concept to product. Equally pressing needs for technical and scientific research in the environmental, social, medical, and biological sciences can be met only if the United States maintains a world-class science and technology research base. That in turn requires the vigorous participation of the mathematical sciences.

Need for Focused Exploration of Topics That Are Becoming Mathematical

New mathematical and statistical results must be developed constantly to address ever more complicated practical problems and applications and their associated increasingly difficult theoretical questions. Researchers and engineers often find that existing mathematical theories and techniques are not sufficient to completely analyze their mathematical models, to fully substantiate proposed methods of approximation, solution, or simulation, or to deal with very large and complex bodies of data. In practice, non-classical conditions or constraints are often encountered that have not been covered by existing mathematical theories.

Increased interaction in addition to improved communication between core mathematicians and other mathematical sciences researchers, and between mathematical scientists and researchers in quantitative areas of science, engineering, medicine, and technology, is needed to identify and answer important theoretical questions that are basic to realizing the full promise of emerging new knowledge.

Realizing this promise requires concentrated and dedicated support for and nurturing of highly promising theoretical and mathematical foundations once their potential contribution is recognized. Researchers in an area where mathematical promise is emerging must have the opportunity for intense, prolonged communication and cooperative work with mathematical sciences researchers. A concentration of such cooperative efforts has great potential to yield a bountiful harvest of far-reaching developments if stimulated at the right point of scientific development.

Need for an Infrastructure for Mathematical Sciences Experimentation and for Sharing of Tools

The mathematical sciences have always had experimental components. Historically, experimentation has often been (and is increasingly today) an important approach to mathematical discovery. Experimental mathematics does not replace rigorous mathematics; rather, it is a tool for mathematical scientists to test new phenomena or conjectures, so that they can later be proved rigorously.

Thanks to the increasing accessibility of powerful computers, mathematical experimentation has developed rapidly in recent years and will likely increase further. However, even powerful existing commercial software tools for computation frequently cannot handle many of the necessary calculations when the mathematical examples involve massive amounts of data, and a mathematical scientist must then create pseudo-code for the requisite algorithms or even do the programming. This generally reduces the time that can be devoted to the main thrust of the research. Inefficient use of talent is exacerbated when two or more groups write code for the same or very similar calculations.[1] In addition, many of the admirable specialized software tools developed all over the world are at best available for the short period during which the creator is actively engaged in the specific research that motivated the development of the software. A central place is needed where mathematical scientists can learn about specialized and general software that has already been developed in various research areas and explore how well it fits a particular research need. Such a center should also support the use of this software.

Also needed are better and more readily available opportunities for computer-based mathematical experimentation. Further, development of computational software and other computer-based resources needs to be systematically encouraged, and accessibility to them also markedly improved. A systematic effort to encourage development would contribute not only to fundamental mathematical research but also to applications in medicine, science, and technology. A collateral benefit would be the increased visibility of the role of the mathematical sciences in the development of computer-based resources.

A Proposal for Two New Types of Institutes

The committee proposes two new types of mathematical institutes to help address the needs identified above. The current U.S. research institutes in the mathematical sciences are chartered to support and stimulate a broad cross section of mathematical research and applications. The two proposed new types of institutes are viewed as differing in nature from the class of broadly based institutes, and would play quite a different role.

The first new type of institute, described in more detail below, would facilitate the engagement of the mathematical sciences, and particularly core mathematics, with emerging scientific, engineering, medical, and technological fields of national importance. These "emerging-fields" research institutes (there could be more than one based on this model) would each be devoted to long-term, intensive research in a single field whose interface with the mathematical sciences is emerging and shows great promise, but for which the interface has not yet reached the critical mass needed to have a profound, far-reaching impact.

The second type of institute would be a research institute for experimental mathematics and the use of electronic tools in the mathematical sciences, abbreviated in the discussion below as "e-MSI." This institute would provide the beginnings of an infrastructure for the experimental component of mathematical sciences research, including in particular core mathematics research. As such, it would offer access to and support for mathematical software tools used in such

[1] For example, at least four groups now have coded fast algorithms for Gröbner bases so as to be able to handle calculations beyond the scope of standard computer algebra packages.

research, as well as access to mathematical resources needed to meet the ever increasing demands from the other sciences, engineering, medicine, and technology for mathematical sciences research.

The committee recognizes that some in the community will interpret its recommendations as suggesting new uses of scarce resources that might otherwise be dedicated to mathematics pursued in the traditional individual investigator manner. The committee believes, however, that its recommendations offer the promise not only of strengthening the mathematical sciences research enterprise, but also of attracting new partners to the mathematical research community and possibly expanding the funding base.

RESEARCH INSTITUTE FOR MATHEMATICAL SCIENCES IN EMERGING FIELDS

The mission of a research institute for mathematical sciences in an emerging field would be to accelerate the pace of already-begun highly promising interactions and developments between the mathematical sciences and scientific, technological, medical, or engineering fields of national importance.

Envisioning an Emerging-Field Mathematical Research Institute

The committee envisions an emerging-field mathematical research institute as having integrated teams consisting of mathematical sciences researchers and researchers in what might be referred to as a "companion field"—a scientific, technological, medical, or engineering area in which promising mathematical developments are emerging. The teams would devote concentrated long-term, dedicated effort to the challenging fundamental mathematical and statistical questions arising in the companion field. The institute would be co-directed by a mathematical scientist and a scientific researcher in the companion field, both highly regarded in their respective fields. The mission of the institute would include, at a minimum:

- The development of mathematical theories, statistical methods, and computational algorithms for the solution of fundamental problems in the companion field; and
- The education and involvement of mathematical scientists and companion-field scientists in the solution of these problems.

The effectiveness of such institutes would depend critically on creating communication links at the broadest level between mathematical and companion-field scientists, both within the institute and also involving the mathematical sciences community at large. The most effective operational framework for a particular emerging-field mathematical institute should be decided by a competition soliciting proposals to establish such an institute addressing a specific emerging focus area. Some examples of possible focus areas would be the mathematical sciences and the biomedical sciences, the mathematical sciences and the environmental sciences, and the mathematical sciences and the information sciences.

The committee envisions that each such institute would have a flexible but limited life span (depending on the companion field) of perhaps 5 to 10 years, and that several such institutes would be established over a period of time, each addressing one focus area.

The committee also anticipates that concentrated interactions, and the subsequent increased integration of the mathematical sciences with other fields, would increase the visibility of the mathematical sciences, expand their scope, and help make mathematical sciences a more attractive career choice. Furthermore, since mathematical sciences PhDs are now seeking jobs in business and industry in greater numbers than in the past, providing opportunities for future mathematical sciences PhD students to become well versed in both the culture and the emerging mathematical research challenges of one or more companion fields would likely provide them with more career options upon completion of their doctoral degrees.

While it is true that for more than a decade various efforts have been made to enhance the connections of the mathematical sciences with other areas, they have met with varying degrees of success. Perhaps the most common shortcoming has been that a connection was either too brief or relatively superficial. Those potential weaknesses could be obviated by an emerging-field institute emphasizing substantive, long-term interactions.

U.S. research institutes for mathematical sciences in emerging fields would be similar in some respects to the United Kingdom's Interdisciplinary Research Centres (IRCs), which were initiated under the aegis of the U.K. Engineering and Physical Sciences Research Council and also through the Biotechnology and Biological Science Research Council. IRCs were conceived as new forms of research organizations that would enable a concentration of expensive equipment and facilities, and that would draw together a range of disciplines to tackle cutting-edge science and technology problems. However, it was intended from the start that IRCs would seek industrial support and become self-supporting (via industrial funding) within 10 years. Expressions of industry interest are the basis for establishing IRCs, and future IRCs must seek substantial initial industry investment, with success in securing that support taken as a measure of an IRC's value.

The proposed emerging-fields institutes would share with the IRCs the characteristic of being true partnerships between collaborating fields, aiming to maximize research potential and to have a substantial effect on both fields. However, because an emerging-field institute would emphasize long-term, intensive work on fundamental, core mathematical issues that arise in connection with a single area of focus that has a promising interface with the mathematical sciences, the impetus for the research performed there would be thoroughly different from that at an IRC. Nevertheless, given the few similarities, it would be prudent to take note of the United Kingdom's experience with IRCs when establishing U.S. institutes for mathematical sciences in emerging fields, while keeping very clearly in mind their crucial and substantial differences.

Illustration of a Potential Emerging-Field Mathematical Institute

The scope of an emerging-field mathematical institute might be illustrated by, for example, a hypothetical mathematical sciences and biomedical sciences institute. Areas of activities might include human physiology, where mathematical modeling is beginning to make meaningful contributions. For instance, such tools as stochastic processes, dynamical systems, and partial differential equations are used in models of cell biology to better understand how cells perform tasks in response to chemicals that the immune system produces (or to drugs), how blood vessels grow within tumors, or how cell membranes in neurons receive or transmit electrical signals. Fluid dynamics is used in analyzing blood flow or the flow of hormones in the human body. Control theory is a mathematical tool that provides insight into how brain subsystems receive messages and convert them into action through the nervous system and muscle contractions, and it also has useful applications in the development of drug therapies. In genomics, mathematics and statistics play an increasingly important role in addressing issues that involve massive data sets, pattern recognition, large-scale optimization, and topological questions about the conformation of long molecules.

A substantial community of mathematical biologists and biomedical scientists, in mathematical sciences departments, medical schools, national laboratories, and industry, now employ mathematical theory. Although scattered, this community appears well positioned to soon reach a critical mass and to have a significant impact in the biomedical sciences. A research institute for mathematical sciences in emerging fields with this focus would noticeably enlarge the community of mathematical researchers, particularly young researchers, working in this area.

It would also greatly influence and contribute to research progress in areas of the biomedical sciences such as human physiology, cell biology, and genomics.

Recommendation: Start a Process to Establish Emerging-Field Institutes

The committee recommends that the National Science Foundation's Division of Mathematical Sciences embark on a process of cooperation with other funding units or agencies to establish a well-chosen and critically focused research institute for mathematical sciences devoted to intensive research in a field whose emerging interface with the mathematical sciences shows great potential for the incorporation of mathematical ideas to achieve important societal advances.

RESEARCH INSTITUTE FOR EXPERIMENTAL MATHEMATICS AND ELECTRONIC TOOLS IN THE MATHEMATICAL SCIENCES

The mission of a research institute for experimental mathematics and electronic tools in the mathematical sciences, or e-MSI, would be to collect, maintain, and utilize advances in computer technology and software to stimulate and support the experimental and computational aspects of the mathematical sciences, and to reduce infrastructural barriers to breakthroughs and progress in mathematical sciences research. Examples of such barriers are the lack of access to some appropriate software tools and resources for mathematical experimentation, the physical isolation of many mathematical researchers dispersed throughout the United States, and inadequate access to activities and recent developments in the mathematical sciences research community.

An institute is the proper venue for an e-MSI because the effort and resources needed to effectively identify, assess, acquire, and support the rapidly growing number of tools for mathematical experimentation are too great for individual investigators or groups. A dedicated, substantial investment—which an institute would provide, but which individual or group investigator programs would not—will be required to effectively address the need for a proper infrastructure for mathematical sciences experimentation, tools, and research. An institute also offers the prospect of continuance and survivability that, by their nature, individual or group investigator programs cannot.

Envisioning an Institute for Experimental Mathematics and Electronic Tools in the Mathematical Sciences

An e-MSI would:

- Be a national laboratory for mathematical sciences experimentation, playing a role for the mathematical sciences similar to that played by the major facilities for other experimental sciences. It would promote the use of experimentation in the development of new mathematical theories, statistical methods, and computational algorithms for the solution of difficult problems in the mathematical sciences; support the development and maintenance of novel software tools for mathematical experimentation; and make such tools available to the extended mathematical sciences research community.
- Provide additional infrastructure to the mathematical sciences community by serving as the focal point for access to experimental and computational resources produced by the worldwide community for research in the mathematical sciences. These resources are dispersed throughout the world but are needed to meet the ever increasing demands for

mathematical sciences research from the other sciences, engineering, medicine, and technology.

An e-MSI would emphasize providing tools for individual mathematical sciences researchers through the most effective use of electronic resources. Its structure could take any of several possible forms. For example, it might have one director, a technical staff with several members, and visiting research mathematicians. Or it might be a virtual entity, cooperatively overseen by a consortium of institutions with its directorship periodically rotating among designated representatives from the institutions. A competition for proposals would undoubtedly elicit a wide variety of innovative alternatives.

Research at an e-MSI would focus on the role of experimentation in the development of new mathematical sciences results and on the use of mathematical experiments to construct and test conjectures and to generate counter-examples. Such an institute should serve as a forum where experiments can be described, conjectures posed, techniques debated, and standards set. Many of the researchers involved with an e-MSI would be individuals who already pursue some of these goals but seek opportunities to share others' tools and pool their experience. The e-MSI is envisioned as being an entity whose establishment would support and facilitate curiosity-driven mathematical sciences research.

An outgrowth of these activities would be software tools, numerical and non-numerical algorithms, and visualization software that would both support this type of mathematical sciences research and be made available to the mathematical sciences research community at large. A primary mission of an e-MSI would be to collect and maintain a mathematical software library of tools for mathematical experimentation and to produce new software in support of research in the mathematical sciences.

Currently, there exist various electronic research resources that are not easily accessible. It is increasingly difficult for researchers to keep track of this information or to even know where to look for it. No concerted effort has been made to coordinate it and make all of it readily available to all members of the mathematical research community. Anticipated advances in network communications would enable an e-MSI to provide additional access to such resources to aid in the development of mathematical researchers' capabilities and stimulate new mathematical sciences research. The institute would solicit and accumulate cutting-edge resources developed in various fields in the mathematical sciences.

The committee envisions that an e-MSI would be initiated by the National Science Foundation for a minimum operating period of 5 years. Whatever its ultimate form, an e-MSI's design should reflect careful attention to the experiences (including successes, difficulties, and any failures) of previous or continuing institutes or centers that could be viewed as being similar in some way. Examples might include the Geometry Center and the center for Discrete -Mathematics and Theoretical Computer Science, among others.

Recommendation: Establish a Research Institute for Experimental Mathematics and Electronic Tools in the Mathematical Sciences

The committee recommends that the National Science Foundation's Division of Mathematical Sciences establish a research institute for experimental mathematics and electronic tools in the mathematical sciences. Its mission would be to promote the experimental component of mathematical sciences research, to facilitate the development of new computer-based tools, and to provide visibility and accessibility to existing tools and to existing research resources that are scattered throughout the world.

4

Recap and Closing Comments

Research institutes in the mathematical sciences have fostered and can continue to accelerate both the central role that the mathematical sciences are playing in our rapidly evolving society and the growth of core and applied mathematics and statistics to meet ever increasing demands for mathematical sciences knowledge. These institutes are catalysts stimulating far-reaching mathematical developments while serving the strategic and critical needs of science and society.

The mathematical sciences community's responses to the committee's call for comments clearly indicated that the existing broadly based mathematical research institutes are serving well, and should continue to fulfill, very important needs of both the mathematical community and its applications partners.

Considering new needs in a rapidly changing environment, this report presents sketches of two new types of research institutes proposed for the mathematical sciences. These new institutes would address needs of the mathematical sciences community as it strives to meet the challenges presented by other sciences and society. The sketches offered are intended only as guidelines. These two new types of research institutes would enable the mathematical sciences to increase their contributions to U.S science, engineering, and technology, and would strengthen the nation's position in science and technology.

The committee believes that the needs (identified in Chapter 3) that the two new types of institutes would be designed to address, and the contributions they would make in addressing those needs, constitute a strong justification for the new funding that would be necessary to establish them. It is hoped that a competition for proposals for such institutes would encourage many innovative approaches to their design.

The committee emphasizes that mathematical research institutes must be based on a strong and vigorous mathematical research community. With this in mind, *the committee strongly believes that it would not be in the best interest of either the mathematical sciences community or society as a whole to transfer funding from existing mathematical sciences individual (principal investigator) research grant programs to funding for existing or additional mathematical sciences institutes*.

At the same time, the committee notes that the modest increase by 53% in federal funding for U.S. mathematical sciences research during the past 15 years has not kept pace even with the growth of the U.S. mathematical research community and does not correct imbalances noted in the David I report.[1] Changes in national and federal agency priorities have also occurred in that time frame, leading to decreases in support for curiosity-driven research. For example, sponsors such as the Air Force Office of Scientific Research, the Office of Naval Research, and the Army

[1] In its 1984 report, *Renewing U.S. Mathematics: Critical Resource for the Future* (David I report; NRC, 1984), a committee of the National Research Council found that although the mathematical sciences in the United States were thriving intellectually, funding had deteriorated to the dangerously low level of $78 million in constant 1984 dollars. To enable the mathematical sciences to continue to play their synergistic role with the broader science and engineering community, which had expanded, the committee called for an increase in federal support to the level of $180 million (in 1984 dollars) over the 5 years from 1985 to 1989. However, that level of funding was never achieved. In 1998 the total federal support for mathematics was $180.8 million ($119.1 million in 1984 dollars), 57% of which was concentrated in the National Science Foundation (Thompson, 1998).

Research Office have grown more mission oriented in recent years. This trend is cause for concern because many, and perhaps most, significant innovations in scientific or engineering fields began with one or two people, on a small scale, and in an unglamorous but curiosity-driven way. For example, it was not anticipated that purely mathematical, number-theoretic basic research on elliptic curves would become of key importance in secure encryption procedures that are now widely used in (to give just one context) banking.

The mathematical sciences are a comparatively low-cost, high-leverage contributor to the nation's research enterprise. In many instances, mathematical developments have been springboards for the launching of new industries, medical imaging and image compression being just two. The ever increasing role that the mathematical sciences play in science, technology, engineering, business, and industry warrants increased government funding for mathematical sciences programs and, in particular, new funding for new research institutes in the mathematical sciences. New funds should be made available so that serious consideration can be given to establishing research institutes for mathematical sciences in emerging fields and a research institute for experimental and electronic tools in the mathematical sciences. The establishment of these institutes would be a good investment involving minimal risk and offering a solid likelihood of great impact and benefit to the nation.

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Appendix

Description of Input from the Mathematical Sciences Community

The Committee on U.S. Mathematical Sciences Research Institutes circulated a call for comments (Box A.1) to all PhD-granting department chairs, to officers and board or council members of the major mathematical sciences research professional societies, and to approximately 35 managers or heads of business or industry mathematical sciences research groups. Although, due to the pressure of time, the committee was unable to obtain input from a large number of individuals, there were a fair number of responses (82) to the committee's request for comments. The caveat that these results are most likely not robust should be kept in mind.

Due to the fast-track nature of the committee's study, the request was circulated internationally by electronic mail and through an Internet home page. Although this was not a formal survey, it is the committee's view that electronic circulation of the call for comments resulted in contact with a reasonably wide cross section of mathematical scientists, perhaps a more diverse class than might have been reached in a more traditional circulation of printed queries.

BOX A.1 REQUEST FOR COMMENTS

Dear********:

The National Research Council Committee on U.S. Mathematical Sciences Research Institutes has started a fast-track study to advise the National Science Foundation (NSF). This committee is considering several questions such as those listed below. The committee would appreciate your thoughts or opinions on these issues, and is especially interested in reasons that explain why you answer as you do.

The four general topics on which the committee seeks input from the community are:

1. What in your view are the U.S. mathematical sciences community needs that you believe mathematical sciences research institutes can best or should address?
2. In what creative ways, either tried or as yet untried, do you believe that mathematical sciences research institutes can or should address the needs you noted in 1?
3. Given that the mathematical sciences community is constantly changing as are its boundaries and its interconnections, how should mathematical sciences research institutes evolve or reflect such change to best contribute to the continued health of the U.S. mathematical enterprise?
4. What role or roles might the mathematical community play through its mathematical sciences research institutes that would address your discipline's/business's/industry's/organization's needs (for instance, might they help spotlight areas of research need of concern to you)?

In addition, the committee seeks your input on particular questions such as:

5. Whether you have ever been at an institute, and if so, what was the impact or result of that experience for you?
6. What would be the optimal number of NSF-supported mathematical sciences research institutes in the United States?
7. Do the current institutes satisfy the needs of mathematical scientists in providing free research time?
8. Do special years at institutes on specific mathematical fields occur frequently enough to sustain the needs of the fields?
9. If you think more American mathematical sciences research institutes would be desirable in principle, would you favor their creation if it took money away from individual research grants?
10. Would you favor their creation if the money came as an add-on to the mathematical budget, similar to add-ons for large experimental or research laboratory facilities in other sciences (such as in physics or biomedical research)?
11. What specific kinds of institutes does the U.S. need? Should there be an American "Institute" facility analogous to that at Oberwolfach whose purpose would be to host short conferences?
12. Should there be a regular program in the mathematical sciences that supports special years at individual universities?
13. Should there be mathematical sciences research institutes that run instructional conferences for graduate students, as there are in Europe?
14. Does the U.S. need another institute similar to the Institute for Mathematics and Its Applications that would promote interaction between mathematics and other fields (rather than mainly with industry)?

Respondents to the request included individuals in academia, industry, and government, as well as mathematical sciences professional society leaders, with the bulk of the respondents being academics. The majority of respondents (77) were U.S.-based mathematical scientists; a few comments were also received from European scientists.

The research backgrounds of respondents represented a broad spectrum of the mathematical sciences, including what is traditionally considered pure mathematics (32), classical applied mathematics (13), theoretical and applied statistics (12), computational modeling and simulation (11), and several closely connected other disciplines (9). For academic respondents, the institutions represented ranged from Group I through Group III universities (as defined in AMS, 1996, pp. 1501–1502).

The general community that responded was extremely well disposed toward the concept of research institutes in the mathematical sciences.

The prevailing opinion of respondents was that a collection of mathematical institutes, properly constituted, would be well positioned to help the mathematical sciences meet new challenges and take better advantage of the opportunities now facing the community. A number of those who responded noted that the U.S. mathematical sciences community has an outstanding record of research advances based in academic departments, and some of those achievements can be traced directly to the existence and influence of mathematical sciences research institutes. Such comments were received from both junior and senior academic researchers, as well as from individuals in industry and government.

Many respondents provided extensive commentary on their highly positive professional interactions with existing mathematical sciences research institutes inside and outside the United States. Several junior researchers noted that a dramatic improvement in their early research career could be traced directly to a critical involvement with an institute. A number of senior researchers reported on the highly positive effect that a sabbatical period spent at a research institute had on their careers, or on the careers of mathematical scientists known to them.

Some respondents commented on the personal growth they experienced as the direct consequence of contact with one or more mathematical sciences research institutes. They noted that they had benefited greatly from the opportunity to meet other mathematical scientists with dissimilar backgrounds; sometimes there was the added effect of a broadening and, in some cases, a rejuvenation of the respondent's research interests.

A small group of respondents commented that the list of questions circulated seemed to be somewhat narrowly drawn, focusing mainly on the needs and health of the mathematical sciences community rather than on the progress of U.S. science as a whole. This group was of the opinion that an application for substantial additional funding for existing or new institutes would be most strongly supported if a clear benefit for science and the wider society were demonstrated. Nevertheless, this group also expressed strong support for research institutes dedicated to advancing the mathematical sciences.

There was overwhelming agreement among respondents that neither the continued funding of existing institutes, nor the creation of new institutes, should take place at the expense of research grants for individual investigators. Moreover, nearly all considered that there was no "optimal number" of institutes. These views were a dominant theme among reactions to the committee's questions.

COMMENTS FROM INDUSTRY RESPONDENTS (12)

Industry respondents pointed out that institutes that make a strong effort to interact with the physical sciences, engineering, and computer science communities have a superb opportunity to increase awareness of the importance of the mathematical sciences to the nation's economic activities. These institutes can display instances of the mathematical sciences at work and demonstrate the usefulness of the field to a public that often does not understand or appreciate the roles that the mathematical sciences play in everyday life.

There is great need for a variety of interdisciplinary programs to communicate cutting-edge mathematical sciences research to the broader physical and social sciences and engineering communities.

Because of tightening budgets and escalating competition from foreign-based corporations, U.S. businesses are under great pressure to improve their efficiency. Mathematical scientists and institutes willing to interact with corporate scientists in a way that helps the latter's efficiency are in a position to benefit both scientifically and economically.

COMMENTS FROM GOVERNMENT RESPONDENTS (5)

Government respondents commented that the high visibility of institutes provides the mathematical sciences community with an opportunity to capture the attention of policymakers. Institutes should make it a priority to facilitate cross-disciplinary interactions that expose academic mathematical scientists to non-academic problems. Institutes can also serve as a training ground for young researchers to broaden their horizons.

Institutes can be a highly effective vehicle for the mathematical sciences community to respond to federal or state educational initiatives. They can also help in publicizing mathematical sciences' contributions and in conducting outreach to the general public.

COMMENTS FROM ACADEMIC RESPONDENTS (61)

All of the academic respondents said that funding from individual research investigator grants should not be redirected to institutes, and the overwhelming majority of academics felt that the number of mathematical sciences institutes should not be increased at the expense of individual investigator grants.

Overwhelmingly favorable comments were received from academics who had attended an institute. These comments were positive about virtually all aspects of the visit, ranging from interactions between researchers with differing expertise, to rejuvenation of slowing research careers, to the physical surroundings of the institute.

Academic respondents felt strongly that institutes help to fill the need for greater interaction within the broader mathematical community. Given the importance of interdisciplinary research both within disparate branches of the mathematical sciences and between the mathematical sciences and other fields, the view was expressed that institutes help fill a gap that universities may find more difficult to bridge.

There was no enthusiasm among academic respondents for the United States to establish an Oberwolfach-type conference-oriented institute, one concern being the cost, and another being that Oberwolfach was seen as an operation better suited to intradisciplinary, rather than interdisciplinary, research.

COMMENTS FROM APPLIED (13) AND CORE (32) MATHEMATICS RESPONDENTS

The prevailing sentiment among the group of respondents identifying themselves as applied and core mathematicians was that there has been a continual and dramatic decrease in funding for basic research in the mathematical sciences, with one consequence being the virtual disappearance of fully funded sabbatical periods. Both applied and core mathematicians who responded noted the need for increased financial support for senior mathematical scientists to spend sabbatical periods at an institute.

Some of the applied mathematicians who responded expressed the opinion that institutes would be well placed to foster new and significant advances in mathematical modeling for experimental sciences. Some also indicated that now is an appropriate time to consider a new type of institute devoted in large part to the dramatically increased role of computers in computation, large-scale modeling and simulation, industrial engineering and manufacturing, and so on.

Both core academic and industry-based respondents pointed to the resurgence of the strong interaction between core mathematics and theoretical physics.

Respondents felt that new institutes would alleviate the continual and dramatic decrease in funding for basic research in the mathematical sciences, at all levels (from postdoctoral programs to research opportunities for senior researchers). They also felt that new institutes would help counter the great dispersion of research mathematicians from PhD-granting institutions.

COMMENTS FROM STATISTICAL SCIENCES RESPONDENTS (12)

Statisticians who responded expressed the view that significant advances in mathematical modeling for the experimental sciences should be a priority for institutes.

It was pointed out that statistics (and probability) play major roles in significant problems involving interdisciplinary research. Statistics is notable in its contact with the empirical side of science. As a discipline, it has undergone a culture shift, returning to its roots in data analysis and scientific inference. Statisticians would benefit from institutes that could facilitate interdisciplinary work between statisticians and, for instance, neuroscientists, geophysicists, and molecular geneticists, as well as social scientists, financial modelers, and engineers.

Among the statisticians who responded, extremely strong support was expressed for institutes modeled after the existing National Institute for Statistical Sciences (NISS). Significant advantages to the mathematical sciences from NISS-type institutes were noted in that they are generically very well suited for outreach to the public, making linkages to industry, and influencing public policy decisions.