Nonlinear Science

Panel on Mathematics (Nonlinear Science and the Navy), Commission on Physical Sciences, Mathematics, and Applications, National Research Council

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Panel on Mathematics (Nonlinear Science and the Navy)
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Navy Liaison Representatives

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Ronald N. Kostoff, Office of Naval Research

Consultants

Sidney G. Reed, Jr.
James G. Wilson

Commission on Physical Sciences, Mathematics, and Applications

Preface

To assist with its long-term strategic planning, the Naval Research Laboratory (NRL) requested that the Naval Studies Board (NSB) of the National Research Council (NRC) form a panel on nonlinear science. NRL's request for independent advice acknowledged the importance of this area of science to a broad range of applications. Specifically, the topic of nonlinear dynamics has generated a great deal of interest during the past several years. A number of opportunities appear to be presented by appropriately mining the basic research developments that have been the subject of recent intense activity. This field promises impact in diverse areas such as fluid turbulence, plasma behavior, optical bistability, molecular dynamics, semiconducting and superconducting devices, magnetic properties, particle aggregation, crystal growth, and even possibly quantum mechanics.

In response to NRL's request, the Panel on Mathematics (Nonlinear Science and the Navy) was formed and asked to examine the following areas and consider the following questions:

1. Chaos has become a popular subject for examination in a number of research areas. A number of important principles have been discovered over the past 10 years in this active field. Principles leading to a prediction of bifurcation appear to be developing. Inherently, if one can ascribe some degree of order to what was previously thought to be noise, the extraction of signals from noisy background could be enhanced. Chaotic behavior should be useful in predicting power spectral densities. This could lead to the development of nonlinear algorithms for low signal-to-noise signal processing systems. With the recent advances in this field, which areas are most likely to lead to applications? Where are the most likely opportunities (scientific or procedural) for demonstrating the utility of the elegant mathematics that has been developed?
2. Fractals represent a relatively new method of representing signals, images, and information that appears to have a regularity in an otherwise irregular pattern. Can matched filters be designed for fractal noise that will be able to extract signals that might otherwise go undetected?

Although of considerable interest scientifically and importance technologically, signal processing is in fact only one of the areas in which recent developments in nonlinear dynamics offer promise for significant improvements in naval technologies. Accordingly, after the panel's initial meeting and discussions with NRL Director of Research Timothy Coffey, the above task was expanded to include a broader range of emerging topics of nonlinear science: the scope of the deliberations should range from general concepts to specific potential applications, and the panel's goal should be to identify areas in which the combination of the Navy's needs, NRL capabilities, and recent developments in the field could together increase the potential for impact on the Navy of the future. It is important to note that this report does not claim to be comprehensive in this regard. While the panel did examine the thrust of NRL's efforts in nonlinear science, it did not attempt a comprehensive survey of the laboratory's expertise, staff, or organizational structure. The panel did not limit its discussion to those areas of nonlinear science that it judged to be within the reach of NRL's current programs, but it did limit its specific recommendations to those areas.

The panel held four meetings during the course of the study—on March 23-24, May 8-9, and November 13-14, 1992, in Washington, D.C. (at NRL and at the facilities of the National Research Council) and on December 10-11, 1993, in Urbana, Illinois. In addition, panel members interacted regularly via electronic mail and telephone. Despite the fact that several years have passed since its last formal meeting, the panel believes that the conclusions and recommendations contained in this report will still be of value to the NRL.

Contents

Executive Summary

Nonlinear phenomena are pervasive in problems relevant to the Navy. Potential novel nonlinear approaches to key Navy technologies in areas such as signal processing, turbulence, and drag and wake reduction are largely locked in predevelopment phases, lacking the mathematical and physical tools needed to understand and exploit nonlinear phenomena. The Naval Research Laboratory (NRL) staff can play a critical role in developing these tools or in importing them from the external scientific community. The Panel on Mathematics (Nonlinear Science and the Navy) perceives a significant impact of nonlinear science in several areas relevant to naval needs, including signal processing and sensors (e.g., sonar), robotics and adaptive control, turbulent and reacting flows, lasers and optics, and materials science.

In addition to documenting in many specific instances how nonlinear science is crucial to other areas of research and development at the NRL and to future Navy technologies, in this report the panel maintains the following regarding nonlinear science (NLS):

- NLS is inherently interdisciplinary and gains dramatically when researchers in different fields can recognize and exploit common nonlinear phenomena;
- NLS requires a balanced, quadripartite methodology, including modeling, analytic, computational, and experimental investigations; and
- NLS does not evolve along a straightforward, linear path from basic research to technological application but instead follows a convoluted (nonlinear) path and flourishes best in environments in which basic researchers and applied technologists are encouraged to interact and are rewarded for doing so.

The interdisciplinary nature of nonlinear science requires special administrative and financial mechanisms to ensure its success. The panel recommends that the NRL continue its recent support for a core effort in nonlinear science that is evaluated on the basis of its success in pursuing the following guidelines:

- Maintenance of an internationally recognized experimental, theoretical, and computational research effort in several specific areas of nonlinear science;
- Fostering of nonlinear science within NRL, by informing the staff of relevant recent developments in nonlinear science and by germinating effective collaborations among NRL researchers in both basic and applied areas;
- Ongoing identification of specific developments in nonlinear science, such as controlling chaos, adaptive pattern recognition, and others, which have a potential impact on future Navy needs and technologies, and the coordination of initial efforts to provide proof of principle for the applications of these developments; and
- Enhancement of contacts by NRL and its researchers with external centers of excellence in nonlinear science.

In reviewing the balance of nonlinear science research at NRL, as presented to the panel at the time of its visits to the laboratory, the panel felt that the computational and theoretical science and engineering efforts were of appropriate size and strength but that the experimental nonlinear science and applied mathematics efforts, although containing very strong individuals, were perhaps subcritical in staffing. Efforts to enhance these latter two areas, by selective hiring of outstanding, interactive individuals interested in contributing to the nonlinear science effort across NRL, could be of great benefit. Both to foster interactions within NRL and to enhance contacts with the external nonlinear community, it is important for the core nonlinear effort to maintain the following:

- A high-visibility, intellectually exciting program of seminars, colloquia, and conferences;
- An active program of support for joint postdoctoral research fellows, graduate student interns, and summer students; and
- An attractive environment for visits by university faculty and other non-NRL researchers, with suitable arrangements for visits ranging from a few days to a full year-long sabbatical.

Allocation of sufficient resources to enable these programs should be a priority for NRL.

By focusing and expanding slightly its sound existing base of expertise in nonlinear science and by leveraging its investment through interactions with external centers of excellence in this area, the NRL leadership can ensure that all of its researchers and ultimately the Navy will continue to receive the full intellectual and technological benefits of the ongoing revolution in our understanding of inherently nonlinear phenomena.

Chapter 1

Introduction

Many of the important technological problems confronting the Navy cannot be analyzed using the linear techniques that have dominated engineering development over the past hundred years. The fundamental assumption of linear methods, that response is proportional to stimulus, fails utterly in far-from-equilibrium systems such as turbulent wakes, rocket and jet engine combustion, or detonating explosives, in which a small stimulus can produce an enormous response. Similarly, the linear assumption that a large system is simply the sum of its individual parts is invalid for the complex systems found in automated defense technologies where subsystems interact strongly and the behavior of the whole cannot be anticipated or controlled by linear algorithms.

For both far-from-equilibrium and complex systems, the emerging concepts of nonlinear science are essential. Nonlinear science (NLS) represents, to some extent, a reorganization of the traditional categories of analysis. These categories are connected across, rather than along, the traditional lines between the physical sciences. Further, NLS relies on some qualitatively distinct methods that have the potential to make significant contributions to the next generation of engineering analysis.

The greatest potential for NLS is the creation of new technologies by introducing fundamentally different capabilities and by integrating ideas across the normal disciplinary lines. In this regard, the recent nonlinear revolution has been compared to the early twentieth century quantum revolution. Quantum mechanics did not change the classical world. It did not reduce friction or make engines run more efficiently. Instead, it provided qualitatively new tools for use in the microscopic world. Similarly, nonlinear science will not change the linear world or undo the successes of linear approaches. However, it will provide the essential techniques for understanding, manipulating, and controlling complex systems, processes occurring far from equilibrium, and other important phenomena that underlie the naval technologies of the future.

The central thesis of this report is that the nurturing of a coordinated experimental, computational, and theoretical effort in nonlinear science at the Naval Research Laboratory (NRL) is crucial to emerging Navy technologies. The elements of nonlinear science permeate naval needs from antisubmarine warfare to targeting for strategic defense.

In the remainder of the report, this thesis is developed and supported in some detail. Chapter 2 provides relevant administrative and scientific background information on nonlinear science. Chapter 3 provides several detailed examples of successful transitions from basic concepts of nonlinear science to important technological applications.

In Chapter 4, the heart of this report, six broad areas are examined in which Navy needs, NRL capabilities, and nonlinear science opportunities coincide. In each of these six general areas, specific problems are identified whose solutions will provide critical and in some cases rapid progress on significant naval needs.

Recommendations are discussed in Chapter 5. It is argued that the continued support of a core NLS group is necessary to maintain the excellent research effort in nonlinear science and to provide technical support for, and stimulate interactions among, the many science and engineering efforts at NRL in which NLS is essential for genuine progress.

Chapter 2

Background

WHAT IS NONLINEAR SCIENCE?

Basic Concepts and Definitions

At the end of the last century, scientists discovered that the conventional understanding of classical mechanics did not capture the surprisingly irregular motions of systems of only a few interacting particles or the surprisingly organized collective behavior of systems of many interacting particles. The advent of new mathematical and computational techniques during the last quarter of this century has rendered these phenomena more amenable to analysis and has thus engendered renewed interest in this area. The ideas and techniques of nonlinear science have been developed to describe, categorize, and understand these surprising behaviors.

In the past two decades, the new science known popularly as chaos has given us deep insights into previously intractable, inherently nonlinear, phenomena. Chaos has caused a fundamental reassessment of the way in which we view aspects of the physical world. For instance, certain seemingly simple natural nonlinear processes, for which the laws of motion are known and completely deterministic, can exhibit enormously complex behavior, often appearing as if they were evolving under random forces rather than deterministic laws. One consequence is the remarkable result that these processes, although completely deterministic, are essentially unpredictable for long times.

Practitioners of nonlinear science, as chaos has become known among experts, also recognize that nonlinear phenomena can exhibit equally surprising orderliness. For example, certain a priori very complex nonlinear systems, involving many interacting components, can exhibit great regularity in their motion, and coherent structures, such as the Red Spot of Jupiter, can emerge from a highly disordered background.

Paradigms

Researchers in this new nonlinear science have learned to accept the seemingly contradictory manifestations of chaos and order as two fundamental features of inherently nonlinear phenomena. Indeed, deterministic chaos and coherent structures are often referred to as two paradigms of nonlinear science, in the sense that they represent archetypal aspects of nonlinear phenomena, independent of the conventional discipline in which they are observed. Two other paradigms that have emerged from recent studies of nonlinear phenomena can be termed (1) pattern formation, competition, and selection and (2) adaptation, evolution, and learning. It is perhaps most convincing to clarify the impact of these paradigms by presenting examples of their interdisciplinary relevance.

- Deterministic chaos can be observed in electrical activity from biological systems, in the transition of a fluid to turbulent motion, and in the motion of the moons of the giant planets.
- *Coherent structures* arise in the turbulent atmosphere of Jupiter, in giant Earth ocean waves (e.g., tsunamis), in the spatial spread of certain epidemics, and, on a microscopic scale, in the behavior of certain unusual solid-state materials.
- *Pattern formation, competition, and selection* occur in very similar ways in such seemingly disparate phenomena as instabilities in secondary oil recovery techniques, laser-plasma interactions designed to control fusion energy, and biological morphogenesis.
- Recent attempts to isolate the conceptual, as opposed to the biological, essence of life have identified and clarified the paradigm of adaptation, evolution, and learning and have led to extensive studies of mathematical

models of neural networks and to creation of the field of artificial life (mathematical models that simulate the dynamics of living systems).

Interdisciplinary Nature of Nonlinear Science

Nonlinear science is inherently interdisciplinary, impacting traditional sciences, mathematics, and engineering, as well as the social sciences, notably economics and demographics. Any attempt to circumscribe artificially the scope of nonlinear science inevitably limits the insights it can provide.

The successful pursuit of nonlinear science requires the blending of four distinct methodological approaches:

1. Modeling, which seeks to improve the analytical foundation of the problem under consideration—improved models are particularly necessary for the emerging areas of nonlinear science that enable some of the new technologies discussed in this report;
2. Experimental mathematics, which involves the use of cleverly conceived computer-based numerical simulations to give qualitative insights into problems that are analytically intractable;
3. Novel analytical mathematical methods to treat functional recursion relations, to solve nonlinear partial differential equations, or to describe complex structures arising in chaotic systems; and
4. Experimental observations of similar nonlinear phenomena in natural and man-made systems arising in a variety of conventional disciplines.

An Example: Period Doubling

One specific example that illustrates the interdisciplinary applicability of the paradigms of nonlinear science as well as its multicomponent methodology is the discovery, made in the late 1970s by Feigenbaum,[1] that the particular type of transition to chaos—a sequence of period doublings—observed in a very simple mathematical equation was universal in the sense that it was independent of the specific equation and should in fact arise in a wide class of physical, chemical, and biological systems. Feigenbaum's studies were initially computational, but he soon developed an analytic scaling theory. Nonetheless, skeptics considered his results to be a mere theoretical curiosity of no clear experimental significance until Libchaber[2] and others observed exactly the same period doubling dynamics in laboratory experiments on fluids and electric circuits. The ensuing efforts to prove various aspects of the theory rigorously stimulated large segments of the pure mathematics community. Conversely, in other cases involving new nonlinear phenomena, laboratory observations have stimulated and guided the development of theory and mathematical modeling. This close interaction among experimenters, theorists, and pure mathematicians—rare and refreshing in the recent age of increasingly specialized science—has also proved to be extremely powerful in aiding the rapid advance of nonlinear science.

The Challenges of Pursuing Nonlinear Science

Researchers in nonlinear science typically face particular difficulties with regard to obtaining support. Funding agencies are organized to evaluate proposals in the traditional scientific categories. Interdisciplinary funding of the sort likely to have an impact in nonlinear science is very difficult to obtain, especially with the post-Cold War cutbacks in scientific funding. Further, as a relatively new field, nonlinear science seems to pale in direct comparison to more mature fields (e.g., solid-state physics, space physics, or nuclear physics)

[1] Mitchell J. Feigenbaum, "Universal Behavior in Nonlinear Systems," in *Los Alamos Science*, Summer 1980, pp. 4-27, reprinted in *Physics D*, Vol. 7, 1983, pp. 16-39.

[2] Albert Libchaber and J. Maurer, "Effect of the Prandtl Number on the Onset of Turbulence in Liquid He^4," *J. Phys. Lett. (France)*, Vol. 41, No. 21, 1 Nov. 1980, pp. 515-518.

in areas such as number of publications or (claimed) proximity to applications. It is important to remember that such comparisons are appropriate only at similar stages of development.

Finally, even well-established researchers often face considerable intellectual opposition in venturing into new interdisciplinary areas. In an application discussed below (solitons in telecommunications), a leading researcher in the field had considerable difficulty in convincing engineers that solitons were not just some mathematical construct.

Thus, the interdisciplinary nature of nonlinear science requires special administrative and financial mechanisms to ensure that advances in basic research can be transferred to applied areas. Managers responsible for guiding research and development must pay close attention to improving mechanisms for facilitating this intellectual technology transfer from basic research in nonlinear science to the programmatic areas of their organizations.

Chapter 3

From Paradigms to Practicalities: Successes of Nonlinear Science

Nonlinear dynamics has had proven impact on emerging and commercialized technologies, from which useful lessons can be extracted on how best to facilitate the transfer from basic research, to advanced development, to novel applied technologies. Two specific examples are presented.

SOLITONS IN TELECOMMUNICATIONS

A contending technology for transoceanic communication for both AT&T and NTT is based on soliton transport, a nonlinear phenomena first suggested in 1973. In that year, Hasegawa and Tappert[1,2] predicted that an optical fiber could support both bright and dark temporal soliton pulses, an intrinsically nonlinear phenomenon. At the time, their result was, for the most part, regarded as the satisfaction of an idle mathematical curiosity with few applications possibilities.

In the 1970s, the entire linear fiber optics research community was facing the same challenge of reducing the 20 dB/km losses of the then-best available fibers. The great potential seen in *linear* fiber optics technology was the driving force behind the huge investment in developing low-loss single-mode fibers.

Ironically, it was the availability of the low-loss fibers developed for linear optics that in 1980 enabled Mollenauer, Stolen, and Gordon[3] to test experimentally the earlier prediction of Hasegawa and Tappert for bright solitons. This experiment catalyzed the explosive growth of the field of nonlinear fiber optics.

The key to stable soliton pulses is the balance between weak material or waveguide dispersion and the intrinsic (very fast but weak) optical nonlinearity of the glass core. By operating at wavelengths in the anomalous region, close to the zero dispersion of glass, low-power soliton pulses could be generated over distances of the order of a kilometer. Dispersion-shifted fibers with lower losses were developed rapidly, allowing experimentalists to tune the carrier frequency of the optical soliton pulse and utilize tiny semiconductor laser diodes as light sources. The finite loss in the fiber, although very small, still posed a major impediment to long-haul telecommunications.

Remarkably, another nonlinear phenomenon, called stimulated Raman scattering, provided the critical breakthrough. It was known from bulk nonlinear optics that stimulated scattering involving nonlinear (three-wave) interactions was a common occurrence above some threshold. It was found that the inverse Raman effect could be exploited to provide a regular boost in energy to compensate loss. The idea was to inject a weak continuous wave signal at a frequency corresponding to the Stokes-downshifted Raman frequency to restore the depleted soliton pulse energy. Although the soliton interaction properties could be inferred from inverse scattering theory (IST) or analytical soliton perturbation methods, the actual demonstration relied heavily on computer simulations.

An equally remarkable breakthrough followed from the discovery of the remarkable

[1] A. Hasegawa and F. Tappert, "Transmission of Stationary Nonlinear Optical Physics in Dispersive Dielectric Fibers I: Anomalous Dispersion," *Appl. Phys. Lett.*, Vol. 23, No. 3, 1973, pp. 142-144.

[2] A. Hasegawa and F. Tappert, "Transmission of Stationary Nonlinear Optical Physics in Dispersive Dielectric Fibers II: Normal Dispersion," *Appl. Phys. Lett.*, Vol. 23, No. 4, 1973, pp. 171-172.

[3] L.F. Mollenauer, R.H. Stolen, and J.P. Gordon, "Experimental Observation of Picosecond Pulse Narrowing and Solitons in Optical Fibers," *Phys. Rev. Letters*, Vol. 45, No. 13, 1980, p. 1095.

amplifying property of erbium-doped fibers. By splicing short lengths of such fiber at space intervals significantly shorter than the soliton period, it was possible to transmit soliton pulse trains at multigigabit rates over a distance of 12,000 km. Further increases in transmission rates by factors of 100 may be possible by using sliding frequency filters and multiplexing. The last chapter in this collection of technological breakthroughs may occur in the next decade, with the laying of a soliton-based transoceanic cable.

The potential applications of soliton signal processing appear almost unlimited. Following rapidly on the success in fiber optics, the possibility arose of soliton-based nonlinear switches, the key element of an all-optical digital computer. Further, fiber lasers and extended cavities utilizing fibers are being developed to produce mode-locked pulse trains (soliton laser, modulation instability laser, fiber Raman laser, additive pulse mode locking).

The focused goals of the research on soliton telecommunications in the United States, Europe, and Japan have had remarkable feedback in basic nonlinear science as well. The optical fiber became the ultimate analog computer for testing and verifying nonlinear theories (e.g., dark-bright soliton interactions, soliton self-frequency shift, soliton couplers, and polarization switches). The fiber produced the first quantitatively verifiable testbed for nonlinear theories by allowing a clean separation of space and time scales for nonlinear interactions.

CONTROLLING CHAOS IN HIGH-POWERED LASERS

The hallmark of deterministic chaos is extreme sensitivity to initial conditions. This characteristic, sometimes called the butterfly effect, makes long-range prediction impossible. In the past few years, a variety of techniques, particularly those of Huebler and of Ott, Grebogi, and Yorke (OGY),[4] that build on classical control theory have been developed to exploit this sensitivity in order to control chaos. The OGY technique, for instance, uses small feedback perturbations based on the observed dynamics of a chaotic trajectory and therefore does not require any a priori knowledge of the underlying system structure. Intuitively speaking, a chaotic trajectory can be viewed as comprising an infinite number of (unstable) periodic orbits, each of which is visited repeatedly. Since the butterfly effect causes the chaotic trajectory to depart quickly from any periodic orbit it visits, the time that a trajectory spends near a given orbit tends to be short. However, eventually the system's trajectory returns (for another short visit) to a given orbit along a characteristic path known as the orbit's stable manifold. The OGY technique stabilizes a chaotic system about a chosen periodic orbit by making small, time-dependent perturbations to an accessible system parameter such that the trajectory is directed toward the orbit's stable manifold. The result is that the system is maintained close to the previously unstable periodic orbit—that is, the periodic orbit is effectively stabilized. The OGY technique and its derivatives have demonstrated some success in controlling magnetoelastic ribbons, electronic circuits, thermal convection, lasers, and chemical reactions.

Recently, additional improvements in chaotic control have made further applications possible. Targeting overcomes one important limitation of the OGY technique: namely, because OGY control is not effective until the system's trajectory comes close to the desired unstable periodic orbit, initiation of control sometimes requires a long period. By utilizing the exponential growth of small parameter perturbations to steer the system into the neighborhood of the desired orbit, targeting can reduce this waiting period significantly. It is important to note, however, that targeting can be difficult to apply in certain experimental settings since it requires a global model of the system. Nonetheless, it has proved very effective in a variety of cases. A second modification of the OGY technique, proportional perturbation feedback (PPF), can be applied to systems that lack accessible systemwide parameters. The PPF method perturbs a system variable (not a

[4] E. Ott, C. Grebogi, and J.A. Yorke, "Controlling Chaos," *Phys. Rev. Lett.*, Vol. 64, No. 111, 12 March 1990, pp. 1196-1199.

parameter) so as to force the system trajectory directly onto the stable manifold of a desired unstable periodic orbit. Consequently, the system walks toward the unstable periodic orbit along the stable manifold. Eventually, it is repelled from that orbit, at which point PPF control again intervenes to constrain the system. The PPF technique has been used, for example, to control the interspike intervals of arrhythmic cardiac tissue and spontaneously bursting neuronal networks.

Chapter 4

N^3: Naval Needs, NRL Capabilities, and Nonlinear Science Research Opportunities

This chapter provides a detailed set of specific areas in which naval needs, NRL capabilities (existing or potential), and nonlinear science research concepts and techniques dovetail in such a manner that significant and, in some cases, rapid progress can be made in areas of vital interest to the future of the Navy. These specific problem areas are grouped into six sections: (1) signal and image processing, (2) robotics and adaptive control, (3) turbulent and reacting flows, (4) sonar: shallow-water acoustics, (5) lasers and nonlinear optics, and (6) materials science.

SIGNAL AND IMAGE PROCESSING

Image Processing and Target Recognition

Image processing to recognize patterns is important for a wide variety of tasks in vision and signal processing, including recognition of typed and handwritten text, sound identification, and target recognition. Training machines to recognize patterns from a series of examples is equally important. Both problems are very difficult and have consumed the efforts of researchers in academia, industry, and the military for many years. Aided by an exponential increase in computer power, we are now in a position to see some of the first interesting applications of these techniques.

Pattern recognition is important for a variety of problems of interest to the Navy, including the recognition of visual patterns in images, as well as the recognition of temporal patterns in sonar signals and communications. These techniques are important both for reconnaissance, pattern-change recognition over long time scales and for target recognition and acquisition on much shorter time scales. The most commonly used techniques are intelligent signal processing to emphasize characteristic features such as edges or narrow band noise. Nonlinear dynamics is already proving useful in some of these applications. Techniques developed to recognize and characterize chaotic attractors can be used to gain information about simple chaotic noise sources and to predict the future of simple chaotic signals. This class of techniques could potentially allow recognition of certain types of sonar signals that do not have sharp spectral features. Neural networks constitute a class of nonlinear adaptive systems that can be used for problems in temporal signal processing and spatial pattern recognition. Demonstrated applications include the recognition of simple speech and of handwritten numerals. The advantages of neural networks are that they are fully parallel and potentially very fast, and so can provide possible improvements for some of these applications.

Visual pattern recognition is essentially a problem in nonlinear optimization: a noisy and distorted image is correlated with a stored template, which is scaled, rotated, and shifted to obtain the best match. The error function is typically highly nonlinear with many subsidiary minima in addition to the global best fit, and the design of efficient fitting routines is a complex nonlinear problem. Machine learning from examples is an equally difficult problem. Each training pattern typically is characterized by a vector in a high-dimensional space of possible attributes; clusters of vectors are associated with like patterns. In order to generalize to patterns outside the training set, it is necessary to develop an effective technique to draw surfaces around these clusters. The geometry is typically complicated, and the choice of surfaces is again a difficult problem in nonlinear optimization theory.

Most pattern recognition is currently accomplished via a collection of ad hoc techniques, and it is often not clear how well these techniques really work or how well they generalize to other problems. In handwriting recognition, new systems have claimed efficiencies greater than 90 percent since research began, but few have been adopted for general use, because they do not work for many users. Only a few general theoretical facts are known about pattern recognition and machine learning. These include Bayes theorem, PAC (probably almost correct) learning developed by Leslie Valiant[1] and others, and the Vapnik-Chervonenkis dimension for distribution-free learning.[2] A general theory of pattern recognition would be highly desirable but is probably out of reach at present. In the absence of a general theory, the development of techniques to evaluate rationally and efficiently the effectiveness of ad hoc approaches should be a priority.

Data Compression or Encryption

Chaos is a behavior that is typically associated with unpredictable and undesirable phenomena. For example, similar but separate chaotic systems have motion that is never correlated and that is noise-like with broadband spectral densities. In the last five years, research at NRL by Pecora and coworkers[3,4] has demonstrated new ways to configure coupled chaotic systems to give unexpected, but useful, behavior and has shown that the unique properties of chaotic systems permit them to accomplish things that better-behaved systems cannot.

The earliest finding was that chaotic systems could be coupled to synchronize exactly their behavior in a stable, reproducible fashion. The overall motion was chaotic, but the linked systems moved in lockstep. The method for accomplishing such synchronization is remarkably simple and immediately suggests techniques to synthesize such setups. A signal (the *drive*) is taken from a specific part of an autonomous chaotic system and transmitted to another system (the *response*). The transmitted signal takes the place of that part of the response, and the remaining parts see only each other and the incoming signal as though the whole system were still in place. The remaining signals in the response synchronize with their counterparts back in the drive. Theory subsequently developed at NRL establishes the conditions under which this arrangement is stable.

The synchronization approach demonstrates that a chaotic system can be broken into parts and that, by duplicating of some of those parts and linking various parts with chaotic signals, new behavior, such as synchronization, is possible. The main theme here is that the synthesis of new chaotic systems leads to interesting and potentially useful results, including the synthesis of new signals and signal processing hardware.

The first drive-response approach to synchronous chaos was patented along with several elementary approaches to using such behavior in communications with chaotic drives and carriers. Amplifying the theme of synthesis, new drive-response constructs have been pursued. Several drive-response subsystems have been cascaded to give a system that can reproduce the input, chaotic drive signal, provided all the identical subsystem parts have the same parameters. This arrangement allows one to identify incoming signals uniquely. Only signals that come from the same type of chaotic circuit with the same parameters will yield matching synchronous behavior in the response receiver. This discovery of the cascading of electronic components has been patented and currently is being investigated for potential use in identifying friend-or-foe (IFF) situations.

Nonautonomous, chaotic systems have been constructed using approaches similar to drive-response synchronization that allow the chaotic drive to carry phase information about the sinusoidal forcing of the transmitter and receiver. This configuration allows the receiver to follow phase changes similar to FM in the transmitter even though only chaos is broadcast.

[1] Leslie Valiant, "Computational Learning Theory," *Proceedings of the Fourth Annual Workshop on Computational Learning Theory*, Morgan-Kaufmann, San Mateo, Calif., 1991.
[2] Vladimir N. Vapnik, *The Nature of Statistical Learning Theory*, Springer, New York, 1995.
[3] T.L. Carroll and L.M. Pecora, eds. *Nonlinear Dynamics in Circuits,* World Scientific, River Edge, N.J., 1995.
[4] W.M. Ditto and L.M. Pecora, "Mastering Chaos," *Sci. American*, Vol. 269, No. 2, Aug. 1993, p. 78.

The phase-following technique combines the use of return maps with signal averaging to accomplish the phase lock. The averaging makes the technique particularly robust to noise or other contamination in the drive signal.

Finally, the drive signal can be filtered or transformed, both of which can be undone at the response receiver to keep the transmitter and receiver acting chaotically but in synchronization with each other. This ability to mangle a chaotic signal and recover it has potential uses in private communications. A patent has been disclosed for filtering and unfiltering synchronous, chaotic signals. Several other approaches to control and exploit chaos have been found, and two other patents are pending.

Wavelets and Image Processing

Approaches to image processing using traditional Fourier methods characteristically have difficulty with edges because the Fourier basis fractions are spatially extended. Recently, an approach involving basis functions of finite extent, called wavelets, has been applied successfully to image processing. Although wavelets are finite, they need not all be of the same length. It has been proven that wavelets form a mathematically complete orthogonal basis set with which to represent a two-dimensional image with arbitrary accuracy. Further, a hierarchical set of wavelets, similar to geometric fractals, can be employed to cover an area. Wavelets can form a representation of fractals. Wavelets lend themselves to very rapid image processing using parallel computers. Images containing edges can be represented using wavelets, with acceptable loss of detail, with data compressed 100:1. As mentioned above, the efficiency of image fitting is a complex nonlinear problem. Commercial ventures are using the wavelet approach encoded on a chip as a contender in the high-definition television (HDTV) compression competition.

Sensors

Image sensors are the first step in a system for visual pattern recognition. To reduce the communication load and improve speed, it is desirable to perform image processing in the sensor itself, before the image is read out to the rest of the system. The development of smart sensors that use distributed processing for this purpose has been actively pursued at a number of laboratories. The principles of operation make use of developments in electronic neural networks, cellular automata, and related fields.

The highly parallel architectures associated with neural network approaches are well suited to rapid signal processing for pattern recognition. For example, the Caltech group has produced a silicon retina that uses on-chip processing to do spatial and temporal derivatives for moving edge detection. In more recent work, an associated company has produced a commercial chip to optically read account numbers from checks. The vision chip project at the Massachusetts Institute of Technology has developed novel charge-coupled techniques for charge-coupled device (CCD) camera chips to do on-chip smoothing and recognition of orientation.

ROBOTICS AND ADAPTIVE CONTROL

Robots

Robots will be increasingly important for a wide range of applications in the military. The optimal control of robotic arms and actuators is an interesting problem in nonlinear dynamics. Robot arms typically consist of a number of rotary joints connected together, each with its own sensors and actuators; the choice of the most efficient individual motions of these joints to accomplish a single collective motion can be a surprisingly complex problem in nonlinear optimization. Training robots to accomplish specific tasks via a series of procedures or examples is also an important and active area. For example, researchers at MIT and the University of Michigan have developed systems to train robots to maintain their balance while hopping or running and to bounce balls on a paddle, and researchers at the University of Utah have developed sophisticated techniques to program body motions of robots in human form for use in Disneyworld.

Nonlinear dynamics is important for problems in control of machines and target acquisition. Recent progress has been demonstrated in the stabilization of chaotically vibrating machines via relatively small inputs computed by means of nonlinear algorithms. Training robot arms to perform certain tasks is also important. Because most target acquisition systems operate via a deterministic mechanism or algorithm, they are also examples of nonlinear control. This subject may be particularly well suited to work at the Naval Research Laboratory. From the point of view of the seeker, one would like to develop a system that locks onto the strongest target and is difficult to spoof. However, most target acquisition systems in the field are actually relatively simple dynamical systems. A detailed study of their properties as dynamical systems may prove useful in developing effective decoy and spoofing techniques.

Controlling Chaos

Exciting recent developments, both in basic theory and in technological applications, of controlling chaos are discussed in Chapter 3. Most of this work has focused on chaotic systems in which there are only a few system variables. Although it is essential to establish that such low-dimensional chaotic systems can be controlled, most real-world chaotic systems—including turbulent flows and wide-aperture solid-state lasers—are not low dimensional, and controlling the high-dimensional spatio-temporal chaos they exhibit is much more challenging. Fortunately, there has been significant recent progress in this area. For example, feedback control techniques that use external observations of a few system variables to determine stabilizing parameter perturbations have been developed for controlling high-dimensional systems. Further, spatially extended systems of chaotic elements have been controlled via perturbations made to an external parameter at several spatial locations. Controlling spatio-temporal chaos is certainly an area of great importance to the Navy, and it is also one area in which in-house expertise at NRL, combined with judicious external interactions, can make a significant contribution.

Interactive Numerical and Visualization Environments

Numerical methods for finding approximate solutions to dynamical systems have undergone development for centuries. However, we frequently ask questions about dynamical systems that are not easily answered by the integration of individual trajectories. Rather, we want to obtain a qualitative picture that allows us to see key features of all the trajectories in a system. For example, we would like to know the size of a basin of attraction associated with a given operating point in a control system since this tells us how robust the controller is to large disturbances. Answering such questions is a computationally intensive task, one that has required constant interaction between expert and machine.

Creating effective problem-solving environments for dynamical systems is an interdisciplinary challenge. One would like to make available to engineers an environment that combines several features:

1. A *simulation language* (perhaps complete with graphical icons) that translates a high-level description of a device in terms of rods, joints, springs, and so forth, into machine executable code within the analytic environment described below;
2. A *computational environment* for the simulation and analysis of dynamical systems—this environment should provide the user with a graphical interface that allows access to a comprehensive set of numerical algorithms for the analysis of dynamical systems; several good computational tools are available now, but standardization and further attention to software stability and maintenance are required;
3 A *scientific database* manager that allows for storage and retrieval of the information generated in a study of a system;
4. *Expert systems* that check the results of the analysis for consistency with

underlying theory and give interpretations of the observed behavior; and

5. *Visualization tools* that allow one to develop a better understanding of the dynamics of multidegree-of-freedom systems.

The benefits of creating an environment with all of these features would be widespread. It would help especially with the design process for vehicles, spacecraft, robots, controllers, and machines of many kinds by providing the means for rapidly testing whether a proposed design meets desired specifications. There are attractive research opportunities in the development of both general computational tools for exploring nonlinear dynamical systems and tools aimed at specifying and visualizing systems within more restricted domains. The NRL is a natural environment in which to encourage and coordinate the development of these tools.

TURBULENT AND REACTING FLOWS

Turbulence and Drag Reduction

Turbulence is the most difficult problem in all of nonlinear science and one of central importance to the Navy. The techniques of nonlinear dynamics have not evolved sufficiently to address the general problem; however, there has been significant progress in the description of certain limiting cases in which a relatively few degrees of freedom are excited.

Combustion and Detonation

From jet engines and rocket motors to propellants and explosive devices, combustion and detonation are crucial to the Navy's applied warfare technologies. The propagation of detonations and deflagrations involves highly nonlinear processes. From a nonlinear science perspective, premixed flames are ideal dynamical systems. The intensity of the emitted light, which is monotonic with temperature, is a dynamical variable that defines the flame front and can be used to characterize the system. Standard optical and video techniques can be used to measure the complete spatial and temporal characteristics of the dynamics. Measurements with similar spatial resolution for fluid flows would require a large number of probes, making such experiments prohibitively complicated and expensive.

The theoretical research on reacting flows and combustion at NRL is competitive with the best work in this area anywhere in the world. Oran and Boris, together with others, have compiled an excellent research record.[5] The principal work of this group, entitled *Numerical Simulations of Reactive Flows*,[6] is a major contribution to the field of computational methods in combustion science. Their research topics have spanned the full range of combustion systems. They have examined the effects of gravity and heat loss on diffusion flames and have performed numerical simulations on the cellular structure of detonations. In addition, they have studied the effects of gravity in the thermodiffusive instability of both pulsating and cellular flames. They have state-of-the-art computational facilities that are equipped with graphics capabilities to simulate both the spatial and the temporal evolution of the flame front. Most importantly for the purposes of this report, they have the knowledge and the capabilities to undertake any problem in combustion dynamics.

There is extensive theoretical literature on the instabilities of premixed flames, especially those arising from thermodiffusive instability. Most of this work has been analytical, but there are both ample opportunity and need for numerical simulations. Recent experimental results in burner-stabilized premixed flames have shown that these dynamical modes can be observed. A wide array of interesting modes has been found in experiments, including rotating, spinning, pulsating, and intermittent dynamics.

The current theoretical effort in computational combustion at NRL is ideally suited to be a principal contributor to the study of nonlinear dynamics as this subject evolves

[5] E.S. Oran and J.P. Boris, *Numerical Approaches to Combustion Modeling,"* American Institute of Aeronautics and Astronautics, Washington, D.C., 1991.

[6] E.S. Oran and J.P. Boris, *Numerical Simulation of Reactive Flow*, Elsevier, New York, 1987.

toward a study of chaotic dynamics with both spatial and temporal characteristics. Combustion phenomena represent an important and interesting class of such dynamical systems.

Many of the combustion problems of interest to the Navy involve instabilities that result from nonlinear processes in combustion systems. The smooth firing of rocket motors and the proper functioning of propellants rely on the ability to operate in safe parameter ranges or to suppress or control these instabilities. The problem of spikes in the operation of shuttle engines is a manifestation of this kind of problem that has plagued rocket-powered systems since their inception.

For the past 10 years, nonlinear theories of premixed flame propagation have been developed. Recently they have been shown to describe much of the dynamics observed in laboratory experiments. The optical emissions associated with a combustion front can be used as a dynamical variable that characterizes both the spatial and the temporal characteristics of its dynamics. This situation creates possibilities for the development of advanced diagnostic techniques for combustion systems to detect and analyze the onset of instabilities.

SONAR: SHALLOW WATER ACOUSTICS

The use of sonar in shallow water is difficult, because sound waves do not propagate freely for long distances. Scattering can produce complicated paths for sound rays, as well as speckle similar to that observed with optical lasers. Speckle patterns for sound waves are due to the coherent interference of sound scattered to the same point through multiple paths, because of either objects and variation in sound speed within the water or scattering from surfaces. The effect is to distort the apparent position of sonar returns and to obscure them with clutter. The physical problem of multiply scattered waves in complex media is difficult but has been studied for years in the nonlinear dynamics and statistical physics communities. Much of what has been learned about the propagation of light and electron waves can be used to analyze analogous problems with sound.

The propagation of sound rays in shallow water is analogous to the paths of classical particles in a scattering potential. This problem has been studied for many years in connection with ergodic theory, chaos, and basic statistical mechanics, and much is known about specific examples such as stadium-shaped billiards, as well as randomly located scatterers. The corresponding quantum problem has also been studied extensively by theorists in connection with the topic of quantum chaos, which has been explored experimentally in chemical dynamics and, recently, in semiconductor nanostructures.

Because mesoscopic quantum effects are essentially wave phenomena, many of the ideas developed for mesoscopic systems can be applied to the propagation of sound waves in disordered media and in complex geometries. This area is a natural one for NRL. Enhanced backscattering analogous to weak localization should occur for sound waves emitted into random media as well as for sound waves emitted into homogeneous media, but scattered from rough surfaces. Theorists S. Feng and Patrick Lee,[7] working on mesoscopic phenomena, have found unexpected correlations speckle patterns from multiply scattered waves that allow information to be gained about an object moving through a random medium, even when the object itself is not directly visible. An investigation of the applicability of these ideas to sonar is clearly worthwhile.

LASERS AND NONLINEAR OPTICS

Coherent High-Power Semiconductor Laser Sources

The invention of the semiconductor laser diode in 1962 followed rapidly on the first observation of lasing from ruby, but early versions suffered from poor materials growth techniques and required relatively high drive currents to operate. Modern materials growth techniques (e.g., molecular beam epitaxy) revolutionized semiconductor laser design by making it possible to lay down ultrapure layers of GaAs and GaAlAs (or InP and InAsP) with

[7] S. Feng and P.A. Lee, "Mesoscopic Conductors and Correlations in Laser Speech Patterns," *Science*, Vol. 251, No. 4994, 8 Feb. 1991, pp. 633-639.

layer thicknesses ranging from atomic-scale dimensions to many thousands of angstroms. The new heterojunction lasers, although highly efficient, were limited to relatively low output power due to facet damage. A potentially useful way to increase output power without encountering facet damage is to increase the effective aperture of the laser. This approach also had the advantage that a wider aperture would mean less beam divergence at the laser output; typical diode laser transverse dimensions range from 3 to 8 μm.

However, increasing the aperture was soon found to be a recipe for disaster. Broad-area lasers tend to undergo strong filamentation, a highly nonlinear phenomenon. This is a manifestation of a modulational instability in which an initially smooth laser field profile breaks up into intense, chaotically spiking filaments. These filaments are nonlinear coherent structures (solitons) developing in the device and can lead to facet damage by generating localized high field intensities. Efforts to tame nonlinear filamentation have been going on for more than 10 years with very limited success. These have included the fabrication of multistripe gain and index-guided evanescently coupled semiconductor laser arrays, mixed evanescent and Bragg grating surface-emitting lasers, strongly coupled index antiguided arrays, vertical cavity surface-emitting lasers, and many more. Index-guided evanescently coupled, edge-emitting arrays are nearest-neighbor coupled nonlinear oscillators and, not surprisingly, exhibit a rich phenomenology of chaotic behaviors. Experimental investigations have been carried out on various geometries of laser arrays including multistripe gain-guided, Y-coupled index-guided laser arrays and two-dimensional grating surface-emitting lasers. The output intensity in each case was observed to show highly erratic intensity fluctuations and, in some devices, to sample several different dynamics regimes during the course of a single electrical pulse.

The experimental observation of these spatio-temporal instabilities was made using a streak camera to capture the extremely rapid intensity fluctuations (200 picoseconds). It was emphasized in these experiments that either slow detection or detection of the total integrated output power of the laser can be very misleading due to an effective averaging process that smoothes the intensity-time traces. Since the goal was to build wide-aperture, high-power coherent laser sources capable of delivering their energy in a single far-field lobe, it was even more disturbing to find that most of the energy was distributed in side lobes because of the tendency of these lasers to operate in an out-of-phase mode.

Despite their importance in industrial and military applications, understanding of these laser devices has not improved much over the years. They obviously are marvelously rich nonlinear dynamical systems whose full potential can be realized only by discarding oversimplified theoretical models using ad hoc parameterization. A proper theoretical framework that has useful predictive power will require full understanding of the microscopic physics of the interaction of light with semiconductor media and consideration of longitudinal and transverse inhomogeneities in the laser structure. The many spatiotemporal events occurring in these structures on widely different space and time scales—carrier diffusion, carrier-carrier scattering (50 femtoseconds), carrier-hole recombination (nanoseconds), electromagnetic field damping (picoseconds), and thermal effects (milliseconds)—characterize this as a "grand challenge" problem, requiring the full arsenal of novel high-performance computational tools and the application of sophisticated modeling and mathematical techniques of nonlinear analysis.

Experiments on wide-aperture amplifiers indicate that the most efficient amplification process occurs if the external signal is off-axis by a few degrees. Similarly, injection locking of broad-area semiconductor lasers is most efficient for off-axis injection. Preliminary unpublished theoretical evidence suggests that wide-aperture lasers and amplifiers tend to support nonlinear traveling wave solutions as their natural modes. These traveling waves manifest themselves as an off-axis emission in the far field of the laser.

The whole development of high-power, wide-aperture, coherent laser sources has been driven to such an extent by short-term technological demands that the many issues preventing their actual realization have never been brought to the attention of the nonlinear science community. This is an area of tremendous potential for nonlinear scientists at NRL and elsewhere. In particular, developing novel methods for controlling the spatiotemporal chaos in these lasers is a topic ripe for exploitation by NRL experts and their external collaborators.

Bright X-Ray Sources

Bright x-ray sources offer many important applications, including x-ray lithography to create structures on submicron scales; x-ray holography for time-resolved, three-dimensional images of living cells; operation within the water window for high-contrast live biological studies; surface studies (surface dislocation cell structure and crack formation); and diagnostics of high-density fusion plasmas, to name just a few. Hot, dense plasmas produced by either electrical discharge (pulsed power drivers) or laser beams, ($I > 10^{14}$ W/cm^2) emit x-rays efficiently. A common mode of operation involves single-pass amplified spontaneous emission (ASE), which requires large gain-length products and leads to poor spatial coherence. True lasing would require multipass configurations, and many challenges remain with regard to designing efficient reflectors that avoid damage from hot, gaseous x-ray laser media.

The potential impact of nonlinear science on this emerging field is both broad and deep. Beginning with pump sources, if tabletop x-ray lasers are to be realized, challenges lie in developing subpicosecond, high-repetition-rate, short-wavelength lasers. Significant developments in higher harmonic generation in recent years (up to the sixty-fifth harmonic of 825-nm laser light in neon at an irradiance of 1×10^{15} W/cm^2 has been reported) bring reasonably accessible, tunable sources closer to implementation. Understanding the subtle interplay among nonequilibrium plasma generation (in particular, the possible role of plasma turbulence, vortex generation, and mixing in limiting the length of the gain region), x-ray emission (strong refraction due to high-density gradients in plasma has a deleterious defocusing effect), and propagation (ray or wave optics) is essential. Current state-of-the-art modeling techniques utilize plasma codes (e.g., LASNEX), gain medium modeling (rate equation kinetics) including plasma density and temperature effects, and ray tracing or wave optics propagation codes. These large-scale computational approaches offer little fundamental insight into the myriad of nonlinear interactions that contribute to or detract from efficient x-ray lasing. Finally, a challenge to materials scientists is the discovery and growth of robust materials for efficient reflection of x-rays down to very short wavelengths (~5 Å) that approach the spatial periods of naturally occurring crystals. Multilayer stacks appear to offer the best solution for normal-incidence reflectors covering the 10-Å to 50-Å wavelength range.

Blue-Green Lasers

The Navy has particular interest in blue-green lasers because of the good optical transmission properties of the sea in that wavelength region. A major challenge, up to a few years ago, was satellite-to-submarine communication in the deep ocean. More recently, Navy concerns have emphasized operations in shallower coastal waters where the optical properties are variable and generally better toward the green. Navy desiderata, besides the wavelength region, include high power, controllability/high pulse repetition frequency (prf), modest turbidity, high efficiency, long lifetime, compact size, and low cost. Recent industrial interest in short-wavelength lasers for compact disc recording and medical applications has some of the same desiderata, but at low power. The Fibertek medium-power device operates at 532-nm wavelength and appears to be the state of the art in meeting many of these desiderata. However, Roy and coworkers[8] have demonstrated that an

[8] R. Roy, Z. Gills, and K. Scott Thornburg, "Controlling Chaotic Lasers," *Optics and Photonics News*, Vol. 5, No. 5, 1994, p. 8.

active approach to control of laser stability, if extensible to high-power lasers (which have many modes excited) and arrays, might offer attractive possibilities for flexibility.

MATERIALS SCIENCE

Novel Electronic Materials

In the past two decades, technological developments in materials synthesis and nanofabrication have led to the creation of whole new classes of man-made materials with exotic and highly nonlinear electronic properties. These novel electronic materials fall into two broad classes: (1) chemically engineered materials, including conducting polymers, charge-transfer solids, organic superconductors, and high-temperature superconductors, which are produced by creative chemical synthesis or preparation; and (2) physically engineered materials, including semiconductor superlattices, quantum wells, quantum wires, and quantum dots, which are produced by molecular beam epitaxy (MBE) and electron beam lithography (EBL). Often these materials are sufficiently anisotropic in their electronic properties as to be considered quasi-two dimensional (i.e., layered, such as copper oxide-based high-T_c superconductors), quasi-one dimensional (i.e., chain-like, such as conducting polymers or quantum wires), or even so-called zero dimensional (i.e., like artificial atoms, such as quantum dots). This reduced dimensionality has several important physical consequences on the microscopic scale, including most prominently the enhancement of nonlinear effects and of the importance of residual electron-electron interactions. On the macroscopic scale, consequences include a tremendous range of effects of potential use in future electronic devices, including negative differential conductivity (and even negative absolute conductivity in response to certain time-dependent fields) and the existence of superconductivity at temperatures above that of liquid nitrogen. The possible implications for applications of interest to the Navy strongly suggest that the nonlinear science and materials efforts at NRL should be involved actively in these developments.

Organic Electronic Materials: Flexible Polymer LEDs and Nonlinear Optics

As one important class of novel electronic materials, consider the conducting polymers and related organics. These materials are typically quasi-one dimensional (i.e., chain-like) and consequently exhibit larger anisotropies in their conductivity and optical properties. Their quasi-one-dimensional nature also strongly enhances the role that coherent nonlinear structures are expected to play in their transport and response properties, and experimental evidence for solitons (in *trans*-polyacetylene), polarons and bipolarons (in many polymers), and excitons and biexcitons (also in many polymers) confirms this expectation. Understanding the interplay of these nonlinear excitations with other degrees of freedom and modeling their effects on the optical and magnetic response and transport are crucial challenges for fundamental science and are also important for technology. Several laboratories have demonstrated that polymers, such as PPV (poly(*para* -phenylene)vinylene), can be made into large-format, flexible, color-tunable light-emitting diodes (LEDs). In addition, several spinoff companies are preparing this technology for the marketplace. Similarly, certain conducting polymers show some of the largest nonlinear optical responses measured to date, and the ability to tune these responses further by altering the microchemistry promises still better responses. It is important for the NRL to remain abreast of developments in this field and related areas in which the nonlinear properties of electronic materials are being studied.

Semiconductor Nanostructures: Chaotic Scattering and Nonlinear Electronic Devices

As noted above, modern semiconductor fabrication techniques such as MBE and EBL make possible the construction of devices approaching the size of the electron wavelength. Electrons move in these mesoscopic systems as waves and exhibit a wide variety of quantum mechanical phenomena. For structures with dimensions on the order of 0.1 mm, very low temperatures (T < 10 K) are required to reach this regime. Considerable experimental and theoretical research in the past decade has been devoted to understanding how electrons move as waves through complex structures. Phenomena typically found in this mesoscopic regime include reproducible conductance fluctuations associated with electron wave interference analogous to laser speckle patterns. Weak localization occurs when time-reversed pairs of electron paths constructively interfere and cause enhanced backscattering and an associated decrease in conductance.

Typically, the motion of electrons in quantum wires and boxes has been thought to be relatively simple. However, scattering due simply to the shape of the box or wire can produce quantum chaos, which has a strong effect on the motion of electrons. In this context, quantum chaos is simply the behavior of quantum systems that are chaotic in the classical limit. A good example is the motion of an electron through a disordered conductor, which corresponds to a pinball scattering problem in the classical limit but gives universal conductance fluctuations in mesoscopic systems. Quantum chaos has been studied theoretically for many years in connection with the foundations of statistical mechanics and chemical dynamics and, recently, in connection with electronic conduction in metals and semiconductors. Experiments have begun on actual semiconductor nanostructures made in controlled geometries. Although this work is currently done at dilution refrigerator temperatures, as the sizes of the devices become smaller, similar phenomena will eventually occur at liquid nitrogen temperatures and higher. However, in future devices with dimensions on the order of 100 Å, wave phenomena should be observable near room temperature. Although research in this area is currently basic and seemingly removed from near-term applications, a good understanding of these inherently nonlinear phenomena will be essential to the design of ultrasmall devices in the future.

In high-speed electronic devices, very large electric fields are applied over short distances to move charge carriers as rapidly as possible. In this hot carrier regime, the charge carriers acquire energies much greater than in thermal equilibrium, and their velocity distribution can change dramatically. A variety of inherently nonlinear phenomena are known to occur in this nonequilibrium regime. Carriers in confined geometries, such as quantum wells, leave the well when they attain sufficient energy, causing spatial instabilities similar to the Gunn effect in GaAs. Impact ionization of carriers bound to charge centers or confined in quantum wells can also occur, as can band-to-band impact ionization. Reliable computations of these nonlinear phenomena are important for device design and characterization. Knowledge of the hot carrier distribution is essential but cannot be computed by purely analytic methods and is difficult to obtain from experiments. Fast numerical methods to compute the hot carrier distribution in actual device geometries are needed. Good design tools can be used to avoid unwanted nonlinear phenomena as well as to exploit nonlinear effects for device operation (e.g., in microwave oscillators).

Structured Materials

Materials Synthesis and Processing

The tools needed to understand growth processes that lead to complex materials have emerged only in the last decade. These tools include nonlinear growth models, fractal analysis, and kinetic models. Application of these nonlinear concepts to materials synthesis and processing can be expected to yield new materials that will enable novel manufacturing technologies. In particular, polymer alloys, toughened ceramics, smart gels, self-reinforced composites, and porous materials can be tailored to achieve specific properties. Research aimed at correcting these properties by manipulating new, far-from-equilibrium synthetic pathways should receive high priority.

The description of colloidal aggregates and branched polymers is one of the recent successes of fractal analysis. These important commodity materials have defied characterization (let alone understanding) for decades. Fractal analysis not only quantified the structure of these materials, but also led immediately to identification of kinetic growth processes as the active process leading to fractal morphologies. The panel notes parenthetically that these same kinetic growth processes must be controlled in the synthesis of optical fibers, which in turn support nonlinear soliton communications technology.

The most promising concepts for high-performance materials involve composites. Current technology typically uses two-pot synthesis of constituents with complementary properties (reinforcing filler and matrix) followed by mechanical mixing, often involving hand lay-up. In several demonstrated cases, however, composite properties have been achieved through in situ filling via chemical reactions. These new synthetic strategies enable new manufacturing concepts including net shape processing and low-temperature synthesis of ceramics.

Self-assembling materials seem to apply local rules of assembly to generate rather complicated long-range structures. These materials, as found for example in natural living organisms, would offer outstanding properties in synthetic materials if the rules of assembly could be deciphered and manipulated. Since biological systems are adaptable (local conditions modify growth rules, leading to variability), the exploitation of self-assembly could open pathways to novel agile materials whose properties are easily tuned. This is an area in which recent discoveries in fundamental nonlinear science—studies of cellular automata, neural nets, and other adaptive complex systems—should provide valuable insights.

Fracture

Despite the importance of fracture to almost every technology, from ancient to modern, the fundamentals of crack growth have eluded materials scientists. Although linear regions of crack growth exist, failure usually involves nonlinear processes. Classical theories introduce a velocity-dependent fracture energy to parameterize experimental data. This approach, however, has proven inadequate to account for even the qualitative aspects of crack propagation.

In brittle materials, thresholds separate regions of nonpropagation, steady crack growth, and unstable growth. Unstable growth occurs at about 40 percent of the speed of sound, where the acceleration of cracks slows sharply and they emit high-frequency acoustic waves. At this point, the fracture surface shows periodic structure correlated with velocity oscillations. The time scale of the velocity oscillations is greater than the time scale on which bonds break and remains unexplained. These basic features are found in materials from network glasses to linear polymers.

Recent work employing nonlinear mathematics and computer simulation accounts for the qualitative features of crack growth but still fails to account for the observed rates of crack propagation. Experiments designed to elucidate the nonlinear phenomena underlying crack growth are nearly nonexistent.

Rubbery materials fail by complex processes that are also poorly understood. Crack propagation in such materials is related closely to adhesion. As opposed to brittle materials, bond breaking is a minor contributor to fracture energy. Rather, nonlinear phenomena associated with the pullout of polymer chains dominate crack propagation. Because of the complexity of polymer dynamics, the viscoelastic processes occurring over 10 decades in time are active in crack propagation. Once again, experiments designed to probe the fundamental processes underlying crack propagation are limited.

Numerous attempts have been made to use fractal analysis to describe the fracture interface. This exercise has yet to lead either to conclusive evidence for fractal roughness or to insights into the fracture process. Interfaces arising from diffusion processes, on the other hand, do display fractal characteristics, but the universal nature of the diffusion process guarantees similar geometric structures despite radical differences in interface strength.

Smart Materials

Smart materials have duties beyond simple structural support and containment. Such materials, although nearly nonexistent in man-made structures, are abundant in living structures. Biomaterials not only heal but also perform a variety of sensing and control functions. It is not surprising that biomimetic materials are recognized as a major emerging route to new-generation materials. Although numerous programs are starting up, little progress has been made in imitating even the most primitive aspects of smart biomaterials. Substantial opportunities exist to model not only the nonlinear response of these complex systems but also the self-assembly processes by which they form.

Currently, adaptive control is achieved through integration of structures, sensors, and actuators. All the armed services, including the Navy, have extensive programs devoted to extending these concepts to smart structures capable of responding to their environment. Sensors remain the limiting element. Nonlinear science is essential to the design of adaptive control systems.

Passive-control systems based on smart materials are a logical extension of current active-control systems. Self-healing materials, for example, could extend the life of critical components. Materials that simply provide a prefailure signature would improve reliability and safety. A material that acts as its own corrosion sensors is an example. Materials with tailored sound absorption characteristics would be of obvious interest to the Navy.

The ultimate passive material system would act as its own sensor. It would convert an incoming disturbance to a control signal and then respond appropriately, for example, with a shape change. Speculative systems based on magnetostrictive alloys, shape memory alloys, and piezoelectric ceramics are under study at several laboratories. Primitive examples (e.g., foam packing) of these concepts already exist.

Chapter 5

Summary

This report documents the following regarding nonlinear science:

- NLS is important to a variety of future Navy technologies and related research and development efforts at NRL;
- NLS is inherently interdisciplinary and gains dramatically when researchers in different fields can recognize and exploit common nonlinear phenomena;
- NLS requires a balanced, quadripartite methodology, including modeling, analytic, computational, and experimental investigations; and
- NLS does not evolve along a straightforward, linear path from basic research to technological application but instead follows a convoluted (nonlinear) path and flourishes best in environments in which basic researchers and applied technologists are encouraged to interact and are rewarded for doing so.

The panel is pleased to note that in recent years, NRL has supported nonlinear science through the Special Project in Nonlinear Science (SPNLS). This investment has paid off handsomely, with notable successes from NRL researchers in the areas of controlling chaos, signal recognition and encryption, combustion and reactive flow studies, advanced orbital dynamics, development of novel sensors based on nonlinear materials, and many others.

It is the panel's view that the continued existence of SPNLS, or the creation of a similar repository of nonlinear expertise, is crucial to maintenance of the interactive, interdisciplinary environment required for success in nonlinear science. Although it need not be large, this core effort must involve excellent researchers and should be funded in a stable, multiyear manner. Stable support is particularly critical in nonlinear science because its inherently interdisciplinary nature makes it especially vulnerable to the give and take of internal budget battles among large, well-established, discipline-oriented groups. Well-funded and highly developed fields, such as solid-state science, can, by their very nature, generate a much larger number of publications on topics that incrementally advance their disciplines. The goal in supporting nonlinear science should be of longer range and greater scope, with the hope of repeating successes such as that of controlling chaos.

It is essential that clear guidelines be given this group regarding its mission and the measures of success. Although this report stresses the need for NRL to recognize the long-term nature of its investment in nonlinear science, it is equally important that SPNLS or its successor be evaluated and reviewed on a regular basis. The panel's suggestions for the guidelines to form both the mandate and the basis for evaluation of SPNLS follow:

- Maintenance of an internationally recognized experimental, theoretical, and computational research effort in several specific areas of nonlinear science;
- Fostering of nonlinear science within NRL, by informing the staff of relevant recent developments in nonlinear science and by germinating effective collaborations among NRL researchers in both basic and applied areas;
- Ongoing identification of specific developments in nonlinear science, such as controlling chaos, adaptive pattern recognition, and others mentioned in the previous chapters, which have a potential impact on future Navy needs and technologies, and the coordination of initial efforts to provide proof of principle for the applications of these developments; and
- Enhancement of contacts by NRL and its researchers with external centers of excellence in nonlinear science.

In reviewing the current balance of nonlinear science research at NRL, the panel felt that the computational and theoretical science and engineering efforts were of appropriate size and strength but that the experimental nonlinear science and applied mathematics efforts, although containing very strong individuals, were perhaps subcritical in staffing. Efforts to enhance these latter two areas, by selective hiring of outstanding, interactive individuals interested in contributing to the nonlinear science effort across NRL, could be of great benefit. Both to foster interactions within NRL and to enhance contacts with the external nonlinear community, it is important for SPNLS to maintain the following:

- A high-visibility, intellectually exciting program of seminars, colloquia, and conferences;
- An active program of support for joint postdoctoral research fellows, graduate student interns, and summer students; and
- An attractive environment for visits by university faculty and other non-NRL researchers, with suitable arrangements for visits ranging from a few days to a full yearlong sabbatical.

Allocation of sufficient resources to enable these programs should be a priority for NRL.

In summary, by focusing and expanding slightly its sound existing base of expertise in nonlinear science and by leveraging its investment through interactions with external centers of excellence in this area, NRL can ensure that all its researchers and ultimately the Navy will continue to receive the full intellectual and technological benefits of the ongoing revolution in our understanding of inherently nonlinear phenomena.