



## The Preparation of Teachers of Mathematics: Considerations and Challenges: A Letter Report

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# **THE PREPARATION OF TEACHERS OF MATHEMATICS: CONSIDERATIONS AND CHALLENGES**

**A LETTER REPORT**

**MATHEMATICAL SCIENCES EDUCATION BOARD**



**NATIONAL RESEARCH COUNCIL  
CENTER FOR SCIENCE, MATHEMATICS, AND ENGINEERING EDUCATION**

**MARCH, 1996**

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This report has been reviewed by a group other than the authors according to procedures approved by a Report Review Committee consisting of members of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine.

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# THE PREPARATION OF TEACHERS OF MATHEMATICS: CONSIDERATIONS AND CHALLENGES

**A letter report of the Mathematical Sciences Education Board, prepared for the National Science Foundation Review of Undergraduate Education in Science, Mathematics, Engineering, and Technology**

**March, 1996**

## INTRODUCTION

The National Science Foundation (NSF), through its ongoing Review of Undergraduate Education in Science, Mathematics, Engineering and Technology (SME&T), is seeking input and testimony about the current state and future needs of undergraduate education in the United States in science, mathematics, engineering, and technology. Members and staff of the Mathematical Sciences Education Board (MSEB) have been involved in this process as individuals and in their roles with various professional societies. Given the MSEB's long-standing interest in the preparation of teachers of mathematics, we are pleased to answer NSF's request for a letter report on this topic.

The MSEB has been central to the development of several reports raising issues about mathematics teacher preparation, including *Everybody Counts* (National Research Council [NRC], 1989), *Moving Beyond Myths* (NRC, 1991), and *The Teacher of Mathematics: Issues for Today and Tomorrow* (NRC, 1987). The most recent and major documents prepared by professional societies on teacher preparation, *A Call for Change* (Leitzel, 1991) and the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics [NCTM], 1991), are consistent with MSEB findings, as well as with the sentiments of many MSEB discussions. Recently, MSEB members and staff provided input for the report, *Mathematical Preparation of Elementary School Teachers: Issues and Recommendations* (AMATYC, AMS, MAA, NCTM, and SIAM, 1994), which makes recommendations to the mathematics professional societies.

This MSEB letter report begins with a set of considerations which ground our thinking on mathematics teacher preparation and which the reader should bear in mind throughout the report. We then discuss five challenges, as a way of addressing the questions posed by the NSF for its undergraduate review: "What are the specific innovations, as well as the evidence that their adoption represents a superior practice of education? What are the unresolved requirements for those who are already receiving SME&T instruction? What infrastructure needs must be supported for institutions to implement the best instructional practice?" (Boylan & Yankwich, p. 6). Clearly teacher preparation is a critical SME&T issue. The majority of the future teachers of this nation experience much of their mathematics content preparation as undergraduates, often before they have identified themselves as prospective mathematics teachers. Further, the effectiveness and competence of the nation's K-12 teachers of mathematics directly bear upon the

health of the undergraduate SME&T enterprise. There will not be enough students who choose to, and are adequately prepared to, participate in SME&T, without an effective K-12 teaching force.

This report is about the preparation of teachers of K-12 mathematics. It raises questions about directions for future work in critical areas of mathematics education. It does not address more general topics in teacher preparation and development, such as school reform, the reform of teacher preparation, teacher learning and program philosophy, and the professional education of prospective teachers. In some cases the assumptions and issues raised are equally relevant to the preparation of teachers of science, and to the preparation of the professoriate more generally.

## CONSIDERATIONS

Below we explicitly highlight some considerations to be kept in mind while reading this report.

*Teacher preparation—the formal undergraduate and graduate experience of prospective teachers—is only one part of a continuum of experiences which contribute to the process of learning to teach.* Prospective teachers are deeply influenced by their own background as K-12 students (Lortie, 1975). Their professional development continues through the field experience, the induction years into teaching, and in ongoing formal and informal staff development throughout their careers (Loucks-Horsley et al., 1987). The undergraduate preparation of a teacher provides a bridge from the prospective teacher's precollege experiences to beginning teaching, and a foundation for subsequent professional development. Preservice experiences at the undergraduate level need to be coordinated, in both a practical and theoretical sense, with teachers' initiation into the profession in schools, as well as with their continuing education.

*Educational research, about K-12 mathematics teaching and learning, and about the preparation and continuing education of teachers, is a critical component in the improvement of mathematics teacher preparation.* There is an increasingly coherent body of research about mathematics teaching and learning (Grouws, 1992), and about the preparation and development of teachers of mathematics (Brown & Borko, 1992; Grouws & Schultz, 1996). Reform of K-12 mathematics teacher preparation programs can be based on such investigations. Dewey (1910/91) commented on the limitations of unanalyzed empirical impression, and we are positioned in teacher preparation to move beyond this (Lampert, 1988). Scholarly inquiry in a field not only challenges assumptions and beliefs, it is essential to systematic change.

*Many of the mathematical preparation issues for prospective elementary school teachers, middle school teachers, and secondary school specialists differ considerably.* Preparation programs differ substantially depending on whether they are for generalist teachers in elementary and middle schools, or for specialists in middle and secondary schools (National Center for Education Statistics [NCES], 1993, 1995; Nelson, Weiss & Conaway, 1992). This is partly because of the organization of schools, of universities, and of state credentialing and certification programs. Moreover, K-8 teachers face a need to integrate mathematics with other content areas that their secondary school colleagues do not face to the same degree. There is some evidence that people

entering teaching at these different levels bring different expectations, experiences, and professional goals (Ball, 1988). The trend is that elementary teachers tend to enter the profession more deeply committed to children, and less committed to particular content areas.

### CHALLENGES IN THE PREPARATION OF K-12 TEACHERS OF MATHEMATICS

We identify five challenges to improving the preparation of teachers of K-12 mathematics. Both through applied work in mathematics teacher preparation, and through research, there have been efforts to better understand each. Many practical problems and theoretical dilemmas remain, however. We discuss “where we are now” -- highlighting what is happening in practice, what is known from research, and where there is consensus. We then address “what is needed” by pointing out where consensus does not exist, where movement toward consensus might be possible, and where informed debate among researchers, practitioners and policymakers might be especially productive.

#### Issue I: What Mathematics Should Teachers Know?

The mathematics community has a long history of supporting strong mathematics content preparation for prospective teachers. Current publications of the professional societies continue to make this case, emphasizing that the new K-12 reforms require teachers to have increased mathematical *breadth*. *A Call for Change* (Leitzel, 1991, preface) notes: “The content of collegiate level courses must reflect the changes in emphases and content of the emerging school curriculum and the rapidly broadening scope of mathematics itself. In general, current requirements for certification of teachers of school mathematics, particularly at the elementary and middle school levels, and the learning experiences of prospective teachers within college mathematics classes fall far short of these goals.” With the publication of the NCTM *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the ensuing development of curriculum materials reflecting the mathematical emphases of the *Standards*, teachers face subjects, such as data analysis and discrete mathematics, not traditionally included in preservice preparation programs. Sources which describe contemporary mathematics (Peterson, 1988; Steen, 1990) should be considered in re-thinking content issues.

New curriculum materials and standards also raise issues about the *depth* of mathematical understanding needed by teachers. The NCTM *Professional Standards*, for example, suggest that teachers “orchestrate discourse by deciding what to pursue in depth from among the ideas that students bring up in a discussion” (NCTM, 1991, p. 35). Teachers may also need deeper mathematical understanding in order to promote mathematical sense-making, problem solving, reasoning, and justification. Ball observes “elementary teachers, most of whom experienced school knowledge as given—and who acquired facts and memorized rules—must invent a teaching that engages students in complex reasoning in authentic contexts” (Ball, in press, p. 14). Lampert foreshadowed the current content need in her choice of the single indicator of an ideal mathematics teacher: “whether that teacher could give students at the grade level he or she is teaching a mathematically legitimate and comprehensible explanation for why the procedures students are using are appropriate or not, or why the answers they are giving are correct or not”

(Lampert, 1987, p. 37). The inservice project *Teaching to the Big Ideas* seeks to identify the “central organizing principles of mathematics with which students must wrestle as they confront the limitations of their existing conceptions” (Schifter, Russell, & Bastable, in press, p. 3). Cooney, Brown, Dossey, Schrage, and Whittman (in press) are producing materials for prospective secondary school teachers to bridge the abstraction of typical college mathematics courses with the realities of the secondary curriculum and pedagogy. Differences in need among elementary, middle grades, and secondary teachers may be especially great relative to mathematical depth.

Despite strong mathematics community consensus about the importance of subject matter knowledge, only in recent years has the teacher preparation research community been able to assemble a convincing case that subject knowledge matters in teaching. Begle and Geeslin (1972), for example, found that teachers' mathematical preparation did not seem to affect students' test performance. More recent work has been able to demonstrate a relationship (Chaney, 1995). McDiarmid, Ball, and Anderson (1989) conclude: “Recent research highlights the critical influence of teachers' subject matter understanding on their pedagogical orientations and decisions. . . . Teachers' capacity to pose questions, select tasks, evaluate their pupil's understanding, and make curricular choices all depend on how they themselves understand the subject matter” (pp. 195-196). Research perspectives and methodologies are increasingly useful in helping to confirm and illuminate impressions and beliefs about depth and breadth issues.

### What Is Needed

*Detailed standards about mathematics content knowledge for prospective K-12 teachers do not exist.* Would standards be useful? Who should lead such an effort? What philosophical base (that of mathematics, that of student learning, or both) might ground such an endeavor? How might these balances differ for elementary, middle, and secondary school teachers? Should university mathematics departments offer credit (sometimes graduate credit) for courses about the mathematics content of the K-12 curriculum? How does understanding of advanced mathematical subject matter influence understanding of elementary mathematical concepts? What are the most fruitful ways to think about the integration of mathematics and science? How could such work be informed by practices in other countries?

*Many mathematics teacher educators contend that teachers, in addition to knowing mathematics, also need to know and experience mathematical inquiry and the “practice” of mathematics* (Copes, 1996; Ernest, 1994; NCTM, 1991). Where do prospective teachers acquire mathematical “habits of mind” (Brown, Collins, & Duguid, 1989; Cuoco, Goldenberg, & Mark, in press)? How can study of the history and philosophy of mathematics be a meaningful and appropriate part of the mathematical preparation of teachers? How can teachers learn to appreciate the coherence of mathematics, so that it informs their selection of curriculum materials and their lesson planning? What is meant by mathematical “practice”? What aspects of mathematical practice are most defensibly connected to teaching and learning?

*Preservice programs for elementary teachers are very crowded and allow little time for study of mathematics.* Can preservice teachers learn mathematics outside of their mathematics content courses, such as in their clinical experiences, from their cooperating teachers and supervisors, or

from mathematics methods courses? Can studying the practice of mathematics teaching lead to deeper knowledge of mathematics? Can pre- and post-undergraduate experiences be viewed as part of a teacher's mathematical preparation? How can graduate credit be given for elementary mathematics? In what ways do teachers need to “own” knowledge before teaching?

*There are hard questions concerning whether prospective teachers should learn mathematics in courses specially designed for teachers.* Should we imagine “every student a teacher” (Goroff, in press), and thus provide experience appropriate for prospective teachers through regular departmental mathematics course offerings? Or, do prospective teachers need to “come to know” particular mathematics in particular ways that will be most influential for their subsequent practice in classrooms? Who has the proper authority and expertise to make such judgments?

**Summary.** Subject matter matters. Deciding what subject matter, for whom, and in what depth, is a substantial challenge for mathematicians and mathematics educators.

### **Issue II: How Should Teachers Come to Know Mathematics?**

The MSEB (NRC, 1995) has argued that “To prepare teachers to implement the new vision of mathematics education, colleges and universities need to reflect the same principles in their programs for the preparation of teachers.” Postsecondary institutions provide a variety of structures in which prospective teachers learn mathematics. Influenced by 1983 Mathematical Association of America recommendations (MAA, 1983), many institutions offer special courses with titles such as “Number Systems for Elementary Teachers” and “Geometry for Elementary Teachers” for prospective elementary teachers. Judging from the textbooks, the approach often involves teaching these topics at an elementary level while modeling instructional methods appropriate for use in the elementary classroom. Such experiences can therefore be considered both methods and content courses. In other situations, elementary teachers learn mathematics in general education offerings. Secondary candidates typically elect to major in mathematics, taking a number of standard department offerings. At advanced levels prospective secondary teachers sometimes take courses in abstract algebra, linear algebra, geometry, and real analysis along with other mathematics majors, and sometimes take courses specially designed for prospective teachers.

Reform documents promote learning through active engagement with subject matter, both for students and for prospective teachers. There are various innovative attempts at providing mathematics content preparation for prospective teachers. Projects such as the Middle School Mathematics Project (Stake, 1993) and the Middle School Mathematics Program (Sullivan, 1993) have experimented extensively with the use of “hands-on” materials and discovery approaches toward mathematics instruction for teachers. However, not only is there little national documentation about attempts of these types, little is known from research about how such experiences might differentially affect student learning of mathematics and use of mathematics in teaching practice later on. Studies which describe, synthesize, and compare college and university-based programs of teacher preparation are important to the improvement of teacher preparation (Ball & Wilson, 1990; Goodlad, 1991; Howey & Zimpher, 1989; Raizen

& Michelsohn, 1994; Stake et al., 1993). Additional work specific to mathematics would be useful.

There is growing consensus among mathematics teacher educators and researchers (Ball, in press; Even & Lappan, 1994; Schifter, in press; Thompson & Thompson, 1994) that preparing future teachers to be effective in the standards-based reform climate depends in part upon teachers' experience of "qualitatively different and significantly richer understanding of mathematics than most teachers currently possess" (Schifter & Bastable, 1995, p. 1). But Carpenter (1995) argues that, even when teachers are taught additional content in their undergraduate programs, they do not necessarily apply that knowledge to their teaching, or even retain that knowledge. He claims that the way in which teachers come to understand the content is critical, and its relationship to future teaching practice is not well understood; ". . . teachers need to understand how their content knowledge applies to their teaching . . . [so] that the content is learned in a context that provides some links with how that knowledge is used in teaching" (Carpenter, 1995, p. 23).

### **What Is Needed**

*Research presents little evidence about the connections between how teachers come to know mathematics and their own practice in the mathematics classroom. How can we learn from programs, both inservice and preservice, which are experimenting with helping teachers come to know mathematics in new ways? How can we articulate these approaches and share them within the mathematics teacher education community?*

*Despite recurring calls for blending content and pedagogy in teacher preparation, research tells us little about this area. Are there effective ways of integrating mathematics content and pedagogy in teacher preparation? How do these differ across levels? What skills and knowledge do teacher educators need to enact these approaches? What are the effects of "some content, some pedagogy"?*

**Summary.** It's not just the mathematics. Knowing mathematics does not ensure the effectiveness of prospective teachers. How they come to know their mathematics matters as well.

### **Issue III: How Do Teachers Learn about Teaching Mathematics?**

Most would probably agree that scientists learn to do science by working in laboratories, and that students learn to do mathematics in part by solving problems. What is the appropriate "laboratory" in which the prospective teacher of mathematics learns about teaching mathematics? Many mathematics education methods courses across the country address pedagogical content knowledge, defined by Shulman to include ". . . for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, examples, explanations, and demonstrations . . . an understanding of what makes the learning of specific topics easy or difficult . . ." (Shulman, 1986, p. 9). Prospective mathematics teachers learn about pedagogical content knowledge when their instructors model activities, introduce tools such as manipulatives and technology, and discuss literature about how students learn certain

mathematical concepts and about student misconceptions. Increasingly, prospective K-12 teachers are learning about what is known from research about children's learning of mathematics.

There is increased evidence that prospective teachers can learn about teaching mathematics from studying the “practice of mathematics teaching.” Several projects are underway in which actual mathematics classrooms, or suitable proxies, become fruitful sites for learning about mathematics teaching. Some preservice and inservice teacher education projects are beginning to draw heavily on video excerpts from classrooms as the material from which students can learn about mathematics teaching (Ball, Lampert, & Rosenberg, 1991). The current expanding pool of material in the form of vignettes, scenarios, case studies, teaching cases, and sample student work is consistent with this trend (Merseeth 1991; Barnett, Goldenstein, & Jackson, 1994). Related approaches involve teacher reflection and writing about practice (Schifter, 1996), and, more generally, action research (Oja & Ham, 1994) and inquiry into student learning (Raizen & Michelsohn, 1994). In the case of new and innovative K-12 curriculum, many developers are taking quite seriously their responsibility to educate teachers through the curriculum materials (Russell, 1994). Other possibilities for extending the opportunities to learn about teaching mathematics might be found by studying other countries' inservice mentoring practices.

Little (1993) points out that the reforms in subject matter teaching “represent, on the whole, a substantial departure from teachers' prior experience, established beliefs, and present practice. . . . they hold out an image of conditions of learning for children that their teachers have themselves rarely experienced” (p. 130). Lampert (1985) and Ball (1993) have characterized teaching as the management of dilemmas, and posit that helping teachers prepare to teach is mainly about preparing teachers for handling the uncertainty of their work. It is a great challenge to prepare people for a kind of mathematics teaching that is so unfamiliar, invisible, and unpredictable. Cases, vignettes, and video tapes are beginning to help.

Clinical experiences, such as practica, student teaching, and internships, have long been a part of the preparation of teachers of mathematics. A substantial general literature on field experience examines the role of practice in developing teaching (Buchmann & Schwille, 1983; Feiman-Nemser, 1983; Floden, Buchmann, & Schwille, 1987), giving it mixed reviews. Raizen and Michelsohn (1994) point out that schools of education are requiring increased involvement of faculty in students' practica in schools, but the level of involvement of content-specific faculty in clinical experiences varies widely. A number of studies have found that prospective teachers often have insufficient knowledge of content and pedagogy when they enter their student teaching and internship experiences (Brown, 1985), and in their initial teaching positions (Brown & Borko, 1992), and that there is a tendency for beginning teachers to retreat to teaching styles that conform with the setting in which they are working. The notion of teachers learning from one another is more thoroughly developed in other countries, especially Japan (Stevenson & Stigler, 1992).

The NCTM *Professional Teaching Standards* (NCTM, 1991) make the case that the preservice education of teachers should develop teachers' knowledge of “the influence of students' linguistic, ethnic, racial, and socioeconomic backgrounds and gender on learning mathematics” (p. 144). Preparing teachers to make mathematics instruction work for student groups for whom it is currently highly ineffective, if not failing altogether, is a difficult challenge for mathematics

teacher educators. A number of researchers and practitioners have addressed this area (Secada, Fennema, & Adajian, 1995), consistently making the point that consideration of equity issues is critical in teacher development.

### **What Is Needed**

*There are several well-known inservice efforts which are based on learning through mathematics teaching practice.* What is the potential of such efforts for **preservice** teachers? What kinds of adaptations are necessary? What can preservice teachers who lack classroom experience still learn from the “practice of mathematics teaching”? How promising are classroom-like sites as places to learn about mathematics? How can preservice teachers best learn about how children learn mathematics?

*The induction years and transition into the profession are critical parts of the teacher preparation experience.* What happens when new teachers work in settings where the methods other teachers use are inconsistent with what the new teachers have learned? What is the responsibility of postsecondary institutions in shaping and supporting the induction experience of beginning teachers who are embarking on reformed teaching practices? How can mathematicians be involved?

**Summary.** It's not just “some mathematics and some pedagogy.” There is much to be learned about mathematics teaching by examining the practice of mathematics teaching.

### **Issue IV: How Can We Build Capacity Among Those Who Educate Mathematics Teachers?**

Not only are prospective K-12 teachers faced with teaching mathematics in ways they have never experienced in the reform climate, but mathematics teacher educators are faced with helping teachers learn to teach in a way that they themselves have probably neither experienced nor used much. Often the mathematics faculty members who teach content courses for elementary school teachers are isolated in their departments, without colleagues to consult about new trends and materials. Sometimes mathematics methods courses are taught by education faculty with little expertise or knowledge of current reform trends in mathematics education. Networking and interaction among mathematics teacher education community is only at a fledgling stage. The Association of Mathematics Teacher Educators, founded in the late 1980s, is one example. The professional societies are beginning to take up the issue of professional development for those who teach prospective teachers.

There are occasional faculty enhancement opportunities for mathematics teacher educators, many NSF-sponsored, where faculty can learn about trends, materials, and research directions in mathematics teacher preparation. Examples include Project RADIATE, Project PROMPT, Project NEx T, and the East Carolina University Middle Math project. Additional professional support, access to emerging research developments, and resources for use in the preparation of teachers could be helpful as well. Aside from commercial textbooks used either in methods courses or in content courses especially designed for teachers, few curricular resources are available for mathematics teacher educators to use and adapt in their instruction. A number of innovative

projects and programs are generating materials which might prove useful (Cooney et. al., in press; Graeber & Johnson, 1991; Schifter, in press; Merseth, in preparation). Too often, promising materials are implemented in specific projects only, without being produced or packaged in more widely accessible forms.

“It is difficult to prepare elementary school teachers to teach science (or any subject) well without having them practice with excellent clinical teachers in classrooms.” (Raizen & Michelsohn, 1994). Despite the critical role of the internship and clinical experience, there are few models of mathematics-specific professional development for cooperating and supervising teachers coming from disparate backgrounds. Little mathematics-specific literature or activity exists in this area.

### **What Is Needed**

*Mathematicians, mathematics educators, teacher educators, cooperating and supervising teachers all are involved in mathematics teacher preparation.* What kind of diversity of training and experience do they represent? What professional development experiences for mathematics teacher educators are needed? What are some exemplary opportunities and structures, and how do they work? How can a “community” be encouraged among those involved in the preparation of teachers of mathematics? Could standards be developed for this community?

*Mathematics teacher education faculty need materials to use in the preparation of teachers of mathematics, as well as opportunities to learn about these new materials and their effective use.* How can the development of alternative materials, resources and technological tools for use in the preparation of teachers of mathematics be encouraged? How can such materials be disseminated so that faculty can learn to use them? How can school-university professional development programs for faculty and school personnel involved in mathematics teacher preparation be funded and continued? How can the effectiveness of teacher preparation curriculum materials be assessed?

**Summary.** Teaching teachers is substantive work. We can learn from one another and from research.

### **Issue V: How Can Mathematics Teacher Preparation Be a Coherent Process?**

Learning to teach mathematics occurs through a variety of experiences: study of mathematics content, pedagogical preparation, formal clinical preparation, and experiences as a student and learner of mathematics. Thus “. . . the dispersion of the teacher preparation effort has resulted in teacher education being nobody's clearly defined responsibility” (Goodlad, 1991, p. 6). Rather, it is influenced by a collection of stakeholders with differing expectations, values, and assumptions about what is important in mathematics teacher preparation. These stakeholders include mathematics teacher education faculty, school of education faculty, mathematics faculty, faculty in community colleges, supervisors of clinical experiences, cooperating teachers in clinical experiences, and administrators in clinical settings. State certification and licensure agencies, national accreditation agencies, professional societies, authors of content and methods textbooks,

and, of course, all teachers at all levels who have taught the prospective teachers also play important roles. There is virtually no research that helps clarify the array of structural arrangements, or the impact of various types of coordination, in promoting effective mathematics teacher preparation.

There are policy shifts and trends that could possibly influence preparation programs for mathematics teachers. For example, the American Association of Colleges for Teacher Education (*Teacher Education Policy in the States Survey* (AACTE, 1994) indicates that states are considering portfolio assessments, adaptations for multicultural considerations, and state-developed tests as measures of competency in subject areas as requirements for regular licensure. There is also much discussion of demonstrated ability to teach as part of state and national teacher licensure and credentialing processes. The Interstate New Teacher Assessment and Support Consortium (INTASC, 1992) has developed a new portfolio assessment system for initial certification which is quite ambitious in its demands for high quality mathematics teaching. Faculty with mathematics-specific content and pedagogy interests should be involved in shaping and learning about in these trends.

Some states now require instructors of methods courses to be certified, or to have recent inschool experience. The National Council for Accreditation of Teacher Education (NCATE) also expects K-12 expertise among teacher educators, thus impacting the participation of mathematicians in the teacher preparation enterprise. Decisions and policies made in schools of education, state agencies, national organizations concerned with licensure, accreditation, and standards-setting efforts for teacher credentialing, will affect specifically the mathematics preparation of teachers. Because mathematics teacher educators face very challenging problems in the more limited domain of mathematics, there is little research or codified practice to inform our thinking about how infrastructure shifts might either facilitate or impede mathematics teacher preparation, and about how the mathematics community might engage effectively in policy-related work.

There are alternative programs at the state level (AACTE, 1993) and innovative programs such as Teach for America (Kopp, 1994) and Troops to Teachers (Keltner, 1994). How these relate to the growing base of knowledge about research and practice in mathematics teacher preparation is unclear. Additional information about the nature, extent, and effectiveness of mathematics teacher preparation through all these programs is needed.

It may be important to acquire much deeper understanding of a number of important models for teacher development, and to analyze the implications for the improvement of mathematics teacher preparation. Studies of professional development school models (Abdal-Haqq, 1995), for example, which focus particularly on the preparation of teachers of mathematics might be beneficial. The National Board for Professional Teaching Standards is developing assessments that recognize “accomplished practice” in mathematics teaching of adolescents and young adults (NBPTS, 1994). The Salish I Research Project is a “national collaborative, working with a coherent research design, to establish a national data base on science teacher preparation programs and a study of the teaching abilities as reflected by the national science education standards” (Brunkhorst, 1994, p. 3). The NSF Collaboratives for Excellence in Teacher

Preparation should provide a rich source of evidence about how changes in infrastructure might influence policy shifts, as well as effective mathematics teacher preparation, provided there is adequate support for the longitudinal research that this requires. Although research does exist concerning successful non-mathematics teacher preparation and development programs, such as the National Writing Project (Smith & Ylvisaker, 1993), little of that work has been analyzed or used by the mathematics education community. Comparative study of mathematics teacher preparation activity in other countries might also be rewarding, especially as the results of the Third International Mathematics and Science Study (US National Research Center, 1995) become available.

### **What Is Needed**

*Prospective teachers experience contradictions and inconsistencies among the many components of their programs.* How can prospective teachers make sense of contradictions and differences they might find among their content, mathematics pedagogy, and clinical experiences? Can they incorporate these inconsistencies in some productive way into their own learning, and their own efforts at building working models of mathematics teaching and learning? Should school teachers be equal participants in the planning and redesign of teacher preparation programs? (Certainly accrediting agencies such as NCATE encourage their involvement.) How can the differences in expectations and views between mathematics content faculty and school personnel be mediated?

*The lack of coherence among program elements presents a great challenge for mathematics teacher preparation.* How can cross-department and cross-college arrangements facilitate more coherence? In what ways should mathematics faculty become more involved in the clinical experiences of prospective teachers? How can mathematics department missions and reward structures reflect commitment to such involvement? What administrative structures within colleges and universities will support and encourage collaboration among the many stakeholders in teacher preparation? Are mathematicians sufficiently committed to teacher preparation to provide encouraging advice to promising candidates?

*A significant percentage of the prospective teachers in this country begin their education in the community and two-year college system.* How can the community college system, with its base of experience and commitment to teaching-related matters, become a full partner in the teacher preparation debate?

*Much of the preparation of teachers of mathematics is significantly influenced by state, regional, and national entities.* How can the mathematics community learn more about these processes? How might mathematicians interact with them? How can effective input from the mathematical community to the state and national teacher preparation infrastructure be assured?

*Districts and states have great discretion over the hiring, promotion, and development of teachers.* Can the mathematics community become involved in these policies in order to support teachers' ongoing professional development? Are there organized intervention strategies which might be appropriate?

**Summary.** Mathematics teacher preparation is situated in a much larger arena. Engagement with the larger teacher preparation infrastructure is critical.

## CONCLUSION

We reiterate the summary statements:

- Subject matter matters. Deciding what subject matter, for whom, and in what depth, is a substantial challenge for mathematicians and mathematics educators.
- It's not just the mathematics. Knowing mathematics does not ensure the effectiveness of prospective teachers. How they come to know their mathematics matters as well.
- It's not just "some mathematics and some pedagogy." There is much to be learned about mathematics teaching by examining the practice of mathematics teaching.
- Teaching teachers is substantive work. We can learn from one another and from research.
- Mathematics teacher preparation is situated in a much larger arena. Engagement with the larger teacher preparation infrastructure is critical.

The MSEB believes that the issues raised here must be addressed by those concerned with postsecondary education, jointly with those concerned about mathematics education research, K-12 curriculum, K-12 schools, continuing teacher education, and policy development. Specifically, we recommend continued attention to the research knowledge base in mathematics teacher preparation, a relationship between teacher preparation research and practice which enables both enterprises to be more effective, the creation of diverse sets of resources for mathematics teacher preparation, and heightened emphasis on providing faculty development. We see a need for efforts that will join the many parts of the mathematics teacher preparation enterprise in productive collaborations to address these challenges. The MSEB looks forward to being involved in continued work on this important topic.

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