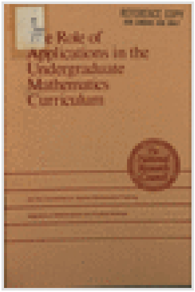

The Role of Applications in the Undergraduate Mathematics Curriculum (1979)

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The Role of Applications in the Undergraduate Mathematics Curriculum

Ad Hoc Committee on Applied Mathematics Training
Assembly of Mathematical and Physical Sciences
National Research Council

NATIONAL ACADEMY OF SCIENCES
Washington, D.C. 1979

NOTICE The project that is the subject of this report was approved by the Governing Board of the National Research Council, whose members are drawn from the Councils of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine. The members of the Committee responsible for the report were chosen for their special competences and with regard for appropriate balance.

This report has been reviewed by a group other than the authors according to procedures approved by a Report Review Committee consisting of members of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine.

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Washington, D.C. 20418

Preface

In every high-school classroom across our land there is a student who is "best in the class" in mathematics. From this large group of students, many go on to major in mathematics in college. In the past, a teaching career, whether in high school or college, was the frequent choice of such students. Today, by contrast, many of our most mathematically gifted students are being dissuaded from pursuing the subject they love best by the advice that "there are very few career opportunities in mathematics."

What a glaring paradox! The slightest acquaintance with contemporary science reveals that mathematics is permeating more and more of our knowledge. Practitioners of the physical, biological, and social sciences need more and more mathematics. Yet college majors are studying mathematics less and less.

This perplexing problem was analyzed by the Advisory Board to the Office of Mathematical Sciences of the National Research Council and the conclusion that emerged, stated bluntly, is the following:

Many of our mathematics departments are doing their mathematically gifted undergraduates a grave injustice by forcing an unnecessarily harsh choice: either undertake a major designed for an academic career in pure mathematics or forego all the upper-level mathematics courses completely. Missing from the options is the mathematics curriculum for the mathematically gifted youngster who plans a career in engineering, economics, epidemiology, or biology. Our mathematics departments must acknowledge that the constituency of potential mathematics majors has changed. The future mathematics producers are vastly outnumbered by the future mathematics users (i.e., appliers).

What must be changed if the present need for mathematical competency is to be met?

In order to solve the problem, the National Research Council formed the *ad hoc* Committee on Applied Mathematics Training (AMTRAC) consisting of 12 mathematical scientists whose collective activities cover the full range of pure and applied mathematics in present-day science and technology. The Chairman of AMTRAC, Peter Hilton of Case Western Reserve University and Battelle-Seattle Research Institute, brings to the committee not only distinguished achievements in algebraic topology but also a deep understanding of the problems of college teaching and an interest in the needs of industry.

I take this opportunity to express the appreciation of the mathematical-sciences community to the Sloan Foundation, whose support made possible the work of the *ad hoc* Committee on Applied Mathematics Training and the preparation of this report.

April 3, 1979
New Haven, Connecticut

G. D. Mostow, *Past Chairman*
Office of Mathematical Sciences

Ad Hoc *Committee on Applied Mathematics Training*

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Shmuel Winograd, IBM Thomas J. Watson Research Center

Jacob K. Goldhaber, *Executive Secretary*
Fern G. Steininger, *Administrative Assistant*

Acknowledgments

On January 8, 1978, during the annual meeting of the Mathematical Association of America in Atlanta, Georgia, a panel consisting of Peter Hilton, G. D. Mostow, Stephen Smale, Jean Taylor, and Shmuel Winograd presented to the Association an outline of the problems that the *ad hoc* Committee on Applied Mathematics Training was addressing and the Committee's initial views on these problems. On May 24, 1978, at the annual meeting of the Society for Industrial and Applied Mathematics in Madison, Wisconsin, a panel consisting of Peter Hilton, G. D. Mostow, Michael Reed, and Shmuel Winograd gave a similar presentation.

The presentations of both panels engendered substantial audience discussions. These discussions were helpful to the Committee in its subsequent deliberations and in the formulation of its report.

The Committee solicited comments and suggestions from the following people. Their remarks were also helpful to the Committee.

Lenore Blum, Mills College
R. Creighton Buck, University of Wisconsin
Richard C. DiPrima, Rensselaer Polytechnic Institute
Alan J. Goldman, National Bureau of Standards
Robert Herman, General Motors Research Laboratories
Peter D. Lax, Courant Institute of Mathematical Sciences, New York University
Cathleen S. Morawetz, Courant Institute of Mathematical Sciences, New York University
Werner Rheinboldt, University of Pittsburgh
Alan C. Tucker, State University of New York at Stony Brook

The Committee wishes to set on record its grateful appreciation of the invaluable cooperation of J. K. Goldhaber, Executive Secretary, Office of Mathematical Sciences, National Research Council, with regard both to its substantive deliberations and to the facilitation of its activities.

The work in this study was conducted with support from the Alfred P. Sloan Foundation.

Overview

The *ad hoc* Committee on Applied Mathematics Training was formed to consider the specific problem of reshaping the teaching of mathematics in our colleges and universities to meet the needs and purposes of today's students. Much evidence attests to the need for such reshaping. Computers are having a profound effect on mathematics itself, both pure and applied. Many new fields of mathematics are being brought into prominence. Mathematics is playing an increasing role in the physical and management sciences and in engineering, in both academic and nonacademic spheres. Yet enrollments are declining drastically in the mathematics major and in many of the traditional postcalculus courses. Also, a gap is opening between the mathematics that is traditionally regarded as proper to "pure mathematics" and the mathematics emphasized in the rest of the mathematical sciences. Plainly the problem is one of major proportions, whose importance extends well beyond the community of academic mathematicians.

Nevertheless the Committee, in making its recommendations (see page 20 of this report), has confined itself to proposing measures that lie within the competence of the mathematics community, avoiding futile exhortation to a broader audience. In this report we discuss what we ourselves can do to improve the situation.

The proposal to the Alfred P. Sloan Foundation (see Appendix) explicitly raised five questions for the Committee to consider. However, the deliberations of the Committee have, in fact, been principally guided by the first two paragraphs of the summary of the proposal to the Sloan Foundation.

There is widespread concern in the mathematics community about the mismatch between college mathematics curricula and opportunities for employment of mathematicians. Mathematical skills are an asset in many activities not specifically identified as mathematics, yet mathematical education does not adequately take account of this fact; it is clear that the mathematical talent of the youth in this country may very well remain untapped unless there is a better match between the instruction in our colleges on the one hand and the opportunities for applying mathematics in laboratories, industry, and business on the other hand.

There is more at stake here than mere reaction to economic pressures. In the last thirty years mathematics has been undergoing syntheses, with methods developed in some areas finding spectacular applications in seemingly totally unrelated areas. There have been spinoffs in the applied areas as well. Thus the time is ripe for a closing of the gap between abstract and applied mathematics; the move towards incorporating more training in applications in the standard college mathematics curriculum is appropriate intellectually as well as pragmatically.

We have given special but by no means exclusive attention in this report to the mathematics major, since the design of courses for the major and the teaching of those courses must be a prime responsibility of the community of mathematicians. However, we have not sought to develop guidelines for an "applied mathematics" major, to be contrasted with what might be called a "pure mathematics" major. On the contrary, as our thinking developed and as we pursued our contacts with members of the academic mathematics community and with industry, we became increasingly convinced that we should be thinking of a broad major in the mathematical sciences, offering sufficient flexibility of choice to suit the needs of very different types of students and enabling *all* students to have the experience of applying mathematics.

We must emphasize that in speaking of flexible programs, meeting the needs of students of varying interest and background, we by no means have in mind any weakening of the major. On the contrary, our recommendations, if implemented, will make considerable demands on both students and faculties. The faculties will need to widen and deepen their knowledge of the mathematical sciences, and this is a major problem in itself. The students will be expected to acquire a substantial body of knowledge and to gain experience in applying that knowledge. They will be given choices within the program; but no program of undergraduate studies can be exhaustive, so they will be expected to acquire in addition the maturity and confidence enabling them to meet new and changing situations.

It became increasingly clear to us from our consultations that the principal problem is one of attitude. We do not say that the attitudes of all academic mathematicians should undergo change. However, we have been made aware of the widespread existence of attitudes that must be substantially modified if students are to be encouraged to view positively the prospect of a career devoted to the use of mathematics in the solution of problems that arise in science, in industry, or in government service.

The attitudes to which we refer are not the only attitudes that are damaging to the mathematical community—we list others two paragraphs below. However, many mathematicians do, in fact, adopt a very withdrawn attitude toward any kind of extramathematical activity, and this may well manifest itself in an apparent lack of interest in applications and even in the difficulties of those students whose motivation for studying mathematics lies outside mathematics itself. The effect on the attitudes and competences of students will be very serious, even where, as is often the case, the behavior of the mathematicians with regard to applications of their subject springs from diffidence and modesty rather than from arrogance. Moreover, by withdrawing from "applications," a mathematics teacher may well be withdrawing from a source of strength.

There can be dispute, on many grounds, as to the respective merits of different parts of mathematics. However, we do not subscribe to the superficial view that only those parts of mathematics for which applications are currently known should be taught to students planning to pursue applications of mathematics in their careers. The choice of areas to be studied should not be related too closely to some picture that the student or the professor (or the future employer) may have of the techniques likely to be used in the next ten years. Techniques will change; but if the mathematics is taught to stress understanding, then this will reinforce the mastery of the techniques and facilitate the creation of, acquisition of, and adaptation to new ones. We are recommending that, where appropriate, subjects should be treated to emphasize their relation to scientific problems and to encourage the student to think in terms of the applicability of the particular mathematical topic under discussion.

Thus we have to seek positive attitudes toward applications of mathematics among students and faculty. We must overcome the distrust of applied mathematics to be found among some "purists." But there are certainly other attitudes that need to be dealt with if mathematics is to occupy its rightful place in the life of the community. Although it would take us beyond our charge to endeavor to influence all those attitudes, it is worth mentioning them here. We are faced, frequently, with a negative attitude on the part of many people—some of them within our universities and colleges—toward any sort of academic mathematics. We are also faced with a negative attitude on the part of many of our best students toward any form of nonacademic employment. Finally, and perhaps here we can do something to put our own house in order, we are faced with attitudes of suspicion, sometimes amounting even to distrust, within the various parts of the mathematics community itself. It is common knowledge that the hostility between pure and applied mathematics, where it exists, is usually mutual!

The Committee has seen its function to be that of making recommendations to alleviate these attitudinal problems, and other problems germane to undergraduate mathematics education, without in any way being prescriptive. We hope that our recommendations will be discussed wherever mathematics is done. Primarily, of course, we expect those discussions to take place within the mathematics departments of our universities and colleges and in conjunction with our colleagues in allied disciplines. Arising from those discussions there should emerge concrete realizations, within the context of particular educational institutions, of those of our recommendations that our colleagues in those institutions regard as relevant and helpful.

It is natural that we have devoted much time and attention in our discussions to the curriculum, both for the major and for the student in a scientific discipline taking upper-level mathematics courses. However, in our recommendations we specify curricula only to the extent that it is necessary to illustrate our position. We have enjoyed the collaboration of Alan Tucker, chairman of a panel of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America; that panel is studying viable curricula in the mathematical sciences. We believe that the principles that have guided our thinking will also be reflected in the report of the Tucker panel.

It is important to point out that today's applied mathematics cannot be regarded as a single entity in any sense. Most particularly it exhibits a pluralism that requires a broadly based attitude toward the design of a sound mathematics curriculum and that leads us, again, to favor a plurality of choice for the student with regard to the elective components of the major. We recommend that the mathematics major be accompanied by a strong minor requiring a substantial understanding of a field to which mathematics is significantly applied, together with the experience of carrying out such applications. Indeed, we attach great importance to the recommendation that students of the mathematical sciences be given the opportunity, as part of their major programs, to acquire experience of applications (see Theme D): such experience need not be exclusively related to the choice of minor.

The core program in mathematics should, in the Committee's view, give due emphasis to the basic unity of mathematics itself, so that mathematics does not appear to students as a collection of barely related disciplines. Interconnections should be stressed, both within mathematics and between mathematics and science; and the history of mathematical ideas will then naturally find its proper place in the syllabus. In particular, this integrated viewpoint would lead naturally to the restoration of the role of differential equations in the core curriculum and to far greater stress being laid on the relationship to linear algebra and real and complex analysis.

For each student, the overall choice of program must, of course, be coherent. The student's program should, in its entire structure, reflect the potential importance of mathematics, the strong interrelations of various parts of mathematics, and the relations of particular parts to particular areas of application. However, the internal richness, the inevitability, and the beauty of mathematics should also be experienced by students of mathematics, and these are features of mathematics that are often stressed by teachers today in their courses. In recommending changes in the style and content of mathematical instruction, the Committee recognizes that much of the present style and content deserves to be retained. We believe that our recommendations build on this firm basis but that their implementation would lead to a greater capacity to meet changing situations. Our students must be ready to meet the future; they will be required not simply to adapt to change, which is hard enough, but to be responsible for it.

The consensus that has emerged in the Committee falls naturally into five themes: attitudes, a new integration of the mathematical sciences, curriculum, programs giving experience in applications, and societal aspects. We elaborate these themes below and then follow with a list of recommendations designed to achieve our objectives. Each recommendation is followed by one or more letters (from A through E), indicating the themes to which the recommendation relates.

A few words may be in order here with respect to Theme D, applications-experience programs. First, we should make it plain that we are recommending that such programs—for example, internships where possible, senior independent projects, the use of the case-study method—should form an *integral* part of the student's program. Thus room should be made in the curriculum for this important additional component. This may well mean that the number of credit hours for the mathematics major would have to be increased to reach comparability with other sciences. This would not be inappropriate since what we are recommending is highly analogous to laboratory experience in the experimental sciences. However, we recognize that such an increase would pose problems that would have to be dealt with by each institution.

Second, we have documented our sources in discussing this theme in order to provide explicit models of the type of program that we have in mind; of course, we have not provided an exhaustive list of such models and recognize that there are several other excellent programs in the country. It is appropriate to remark here that we have not provided any comparable documentation in the rest of the report, since we have based our discussion and our recommendations on facts that are common knowledge within the mathematics community.

This report, of course, cannot be regarded as the last word, not even for the present generation of students, much less for the future. It is, however, perhaps not too much to hope that the spirit of its recommendations may help to usher in a new era of cooperation among all those concerned with the content, nature, quality, and delivery of mathematics education—teachers, administrators, industry, government, the general public, and our students.

Theme A
Attitudes

The nature and history of mathematics are such that the mathematics department in any university should, in our opinion, be one of the most outward-looking of all departments. Instead, it is often one of the most inward-looking. We believe that, unless this is changed, mathematics departments may in the future occupy positions that do not reflect the great importance of the mathematical sciences in our culture.

We believe that, to serve the interests of the majority of their students, to serve the future of mathematics as a well-supported intellectual activity, and to serve the intellectual and economic health of the departments themselves, mathematics departments should (a) pay more attention, both in teaching and research, to the contact of mathematics with the rest of the world and (b) make an effort to publicize those contacts. This certainly does not imply that research or training in pure mathematics should be abandoned or even seriously diminished, nor that we should pay less attention to the unity, beauty, and power of mathematics. It does imply that each department should make a conscious effort to support a broad range of mathematical activity on the part of its students and faculty members. This can range from the development of mathematics for its own sake, without any thought of applications or even of contact with other areas of mathematics, through the development of new mathematical results that are stimulated by possible applicability, to the development of new applications of existing mathematics (in cooperation with members of other departments where appropriate). In the presence of such a range of activities, students will gain a clearer appreciation of the interrelationships among various areas of mathematics and of the role that mathematics can play as a tool of scientific investigation.

The following specific changes indicate the nature of our thinking.

In teaching at all levels, time should be taken to describe the historical development of the mathematical subject being discussed and, to the extent that this is practical, how it relates to other areas of mathematics.

In teaching at all levels, attention should be paid to the computational aspects of the subject being discussed.

All students should take at least some mathematics courses that involve applications, and they should take courses in other areas that use mathematics to a significant extent.

Encouragement should be given to joint appointments that may serve to establish bridges between mathematics and other departments.

Visiting speakers should be invited from nearby industries to give the students an idea of the nonacademic uses of mathematics and from other departments to demonstrate the uses of mathematics in other academic fields.

Many other specific steps could be suggested. Most of them would follow from a new answer to the question, "What is mathematics?" In U.S. mathematics departments since World War II, this question has often been answered in a very narrow way. An alternative procedure for answering the question would be as follows: Ask, "Does the activity in question have enough intellectual content and enough intrinsic interest or importance so that it makes sense for *someone* at the university or college to be doing it?" If the answer to this question is affirmative, ask, "Are there some aspects of the activity for which considerable knowledge of mathematics (in the narrow sense) is required?" If the answer is again affirmative, ask, "Is there, in the university or college, some other department for which the activity would obviously be more appropriate than for the mathematics department?" The answer to this question would, typically, be affirmative if the mathematical knowledge required were already clearly established both as a mathematical discipline and as an effective model for the given object of scientific enquiry. But if the answer is negative, then the activity is, by definition, mathematics. The resulting, admittedly broad, definition of mathematics would, in our opinion, be more useful than the too-narrow definition that has been used in the past.

Theme B
The Unification of the Mathematical Sciences

Looking back from today's standpoint, the organization of mathematics in the United States 40 years ago appears simple and unified. Most university mathematics took place in departments of mathematics. There were some major exceptions such as departments of engineering mathematics; and there were often strong mathematical activities in physics departments. Some hints of specialized interests that would change the organization were beginning to appear, for example, statistics; but there was, in large part, one mathematics department that competed with other departments to obtain students with mathematical talent. The same department represented the interests of mathematics to the university administration and helped search out the meager research support. All of those who thought themselves mathematicians were housed together, had many common day-to-day interests, and shared much common expertise. In 1938, there were applications of mathematics in divers areas, for example, genetics and economics, but "applied mathematics" had a universally accepted meaning, as embodied, for example, in the well-known text by Courant and Hilbert.

Thus in 1938 there was a unification of mathematics in organization and mutual interests. It is true that, even at that time, professional mathematicians were specialists in terms of their research interests and abilities to teach advanced courses. There was, however, still much talk of being "well read" or "mathematically literate." When *Mathematical Reviews* started to appear, some could (and did) read it from cover to cover.

Now the situation is much changed. The causes of the change include (a) the advent of modern computing technology, (b) the development of special areas of mathematics that receive extensive support because of their apparent usefulness, (c) the increase in the number of areas of science that involve applications of mathematics, and (d) the increasing complexity of the organization of mathematics as its size and specializations increase.

Today a major U.S. university is likely to have at least a half dozen departments where mathematics is a major activity. The list includes mathematics, engineering mathematics, computer science, combinatorics, geophysics, fluid dynamics, statistics, biometry, applied or experimental statistics, quantitative methods in . . . (education and the business school, for example), operations research, system analysis, and communication science. Further there is likely to be a nucleus of mathematical activity in biology, chemistry, economics, psychology, physics, and other major disciplines. "Applied mathematics" does not mean in 1979 what it meant in 1938 except for those whose interests have followed classical physics and classical applied mathematics.

Although this complex structure in large part reflects growth, it must be understood that the growth has taken place in an intensively competitive situation.

Many departments and programs now compete for the student with mathematical abilities. Thus departments in the mathematical areas must present attractive programs for both undergraduates and graduates. In 1938, if a student did not find his mathematics department attractive, his only remedies were either to change universities or to move out of mathematics, both major steps. Now a student who is not fully satisfied can shop around several departments closely related to his own. Often he can change programs without loss of time or other penalties. The situation imposes responsibilities on the mathematics departments. To survive, they must attract students. To remain intellectually honest, they must continue to teach the important skills even when those skills are hard to teach.

Within a university there may now be several deans who are responsible for the mathematics activity. The possibility for lack of coordination and poor scheduling of activities is extreme. Inefficiencies in all aspects of the programs are likely to occur. Competition for university funds to support the various areas of mathematical activity is severe.

Within a university the mathematical activities are usually housed in several areas, and no all-encompassing community of mathematicians exists; often the members of the mathematics faculty have little or no personal contact and no intellectual awareness of their colleagues. The Renaissance man or the cover-to-cover reader (or even scanner) of *Mathematical Reviews* does not exist.

Not only has the organization of the mathematical sciences become complicated in the university, but the outside world has changed. The sources of funding for research in the mathematical sciences are greater in number and more specialized in interest than ever before. The competition for these funds among mathematicians is no longer based primarily on quality since the field of research, with an eye on applications, is often the controlling feature. Further, and most importantly, graduate mathematics students are no longer being trained as much as before exclusively for potential employment in education. Now government, research institutes, industry, and commerce require substantial numbers of new employees from mathematics. In fact, the mature mathematician now has opportunities for consulting and changing between academic and nonacademic employment.

The transition from 1938 to 1979 in mathematics has been complex. Trial and error is common in locating a particular activity, such as numerical analysis. The universities have tried quite a large number of organizational schemes including highly centralized programs such as that at Clemson University, more diffuse organizational structures as at the University of Waterloo, and the placing of all mathematical activity within traditional individual departments.

Certainly the kinds of changes being discussed are not over. It is hard to foretell the future, but the continued expansion of computing is certain. Relatively minor activities could easily grow in importance; artificial intelligence or developmental biology are possible candidates for much greater mathematical activity. The transition taking place has met some real demands, but it has often left in its wake bitterness and hostility between parts of the mathematical community.

Is there then a unity within mathematics? The answer is yes, but it is a different unity from that of 1938. From the subject viewpoint, analysis and linear models are a necessary part of the training of all mathematicians and often a substantial part of their work activity. Computing is becoming a centralizing influence in applications. But the problem remains of actually achieving the organic unification.

Thus a recurrent theme in the Committee's detailed thinking has been the manner of moving toward an integrated concept of the mathematical sciences. It is necessary to break down the sharp separation of courses and disciplines into pure mathematics, applied mathematics, statistics, computer science, etc. Where this separation is institutionalized in the departmental structure of the university or college, it is not proposed that such a structure be dismantled. In some institutions a Division of Mathematical Sciences has been established, and this could well serve the purposes that we have in mind. In any case, there must be full cooperation between the faculties of the various departments concerned with the mathematical sciences, in the interests of devising and conducting appropriate courses for students. In this way, the content of the courses can be substantially enriched and artificial divisions eliminated.

The spirit behind this proposal goes further than that of the integration of the mathematical sciences themselves. We must end the isolation of mathematics from the other sciences, and, in the teaching of mathematics, we must lay stress on how mathematics is done and why mathematics is done, as well as on the true nature of mathematics itself. We must deal with the actual way in which mathematics is used so that the curriculum should include substantial parts devoted to computational procedures—for example, in linear algebra and differential equations—and should serve to develop the relationship both between different parts of mathematics and between mathematics and the sciences. Moreover, the responsibility for the design of courses dealing with the applications of mathematics to a given scientific discipline should devolve jointly on mathematicians and faculty representing that discipline, however difficult it may be to achieve this in practice.

It is clear that these considerations apply to the teaching of mathematics to all students and not simply to those who may wish to emphasize applied mathematics in their undergraduate program. The core courses in mathematics for all students must reflect this integrated point of view; students particularly concerned with applied mathematics may then, through their choice of electives, pay special attention to particular sciences and particular parts of mathematics relevant to the study of those sciences.

These recommendations, if carried out, would have the desirable effect of broadening the mathematical education of all students. We believe that this broadening can—and must—be achieved without paying any price in terms of loss of depth. Since one cannot speak meaningfully of a totality of mathematical knowledge necessary (or sufficient) for all undergraduate students, it should be possible to study a mathematical discipline in depth while devoting considerable and effective attention to the role played by that mathematics. Indeed, the understanding of the mathematics may well be deepened and reinforced by such attention.

Moreover, by such a change of emphasis, we would be offering students better motivation and a wider choice of career outlets than are at present available and attractive to them. Thus the nature of mathematics itself and its natural role in science dictate an emphasis in our teaching that is entirely consistent with our obligation to enhance the career opportunities of our students.

A further advantage that might accrue from the implementation of this recommendation is that it could serve to overcome certain latent hostilities and suspicions that exist within the community of mathematicians. Currently that community is divided according to labels that the Committee believes to be on the whole extremely unhelpful. Applied mathematicians often charge pure mathematicians with self-interest and irrelevance; pure mathematicians often charge applied mathematicians with opportunism and insensitivity. Far better that we should all be mathematicians working toward a common goal and having mutual respect for each other's professional accomplishments.

Theme C
Curriculum

In trying to deal with the issue of making undergraduate mathematics majors aware of the possible applications of mathematics and able through their training to enter directly and naturally into these applications, the Committee has concluded that the notion of trying to do so by adding one or a few courses in something called "applied mathematics" is futile and beside the essential point. Unless the basic curriculum that these mathematics majors take already fulfills these objectives to a reasonable degree, no amount of decorative trimming on the edges is going to have much effect. However, many of us have the strong conviction that the sort of adjustment and transformation that might be useful in achieving this awareness of, and involvement in, applications is, at the same time, very close to the kind of development in terms of the undergraduate mathematics curriculum that would be the most valuable in helping the next generation of students to go on as research mathematicians in the most classical sense of the word. There has been a strong and pervasive tendency toward scholasticism in undergraduate mathematics education during the recent past that has not been beneficial for the preparation of students for the most vital and active areas of contemporary mathematical applications or research. If a reform based on the thrust toward the greater applicability of mathematical training in the sciences helps in facing the problem of the content of the mathematics curriculum in the large, it is even more strongly to be welcomed.

The Committee has never conceived of itself as having either the right or the responsibility to prescribe the requirements for the undergraduate major in mathematics; in each institution the prerogative for laying down the requirements for the major rests firmly with the faculty of that institution.

However, there is another reason for not being prescriptive, which lies perhaps at a deeper level and which the Committee wishes to set on record explicitly. We believe that there should be a flexibility in the design of the undergraduate major in mathematics. Where the resources of the institution make it possible, there should be different paths that students can take through the collection of course offerings, such that successful completion of any one of those programs would qualify the student for the degree. The Committee has been particularly concerned with the case of the student who terminates his formal education with the B.S. degree, and we will next address ourselves to the needs of that student. However, the case for flexibility is, in our view, quite as strong for the student who proposes to go on to do graduate work either in the mathematical sciences or in some other scientific discipline in which mathematics plays a key role.

Those B.S. graduates who seek employment on completion of their undergraduate degree are probably not going to be making much use of the actual mathematical *techniques* learned at the junior and senior level. They are probably going to be using mainly elementary computer programming, sophomore calculus, and elementary linear algebra. Of course this does not mean that the advanced work is not important; however, the primary importance of the advanced work will be to enable graduates to understand more about the elementary subjects studied in the freshman and sophomore years and which they will be developing further in their employment. Thus, as long as the junior- and senior-level courses do build on the earlier mathematical experience, they will serve the student in good stead.

Certainly we would say that every student should take courses that lead to an understanding of the role and significance of the theory of differential equations. For example, he should see eigenvalue problems as arising out of systems of linear differential equations and not simply as an algebraic concept.* We would also recommend that the basic calculus sequence should incorporate not only some computer programming but also, where possible, applications to probability and statistics. (We have already argued that all students should become familiar with the computational aspects of any theory studied.) Further, students should be made aware of the historical origins of a given subject and of some, at least, of its principal scientific applications. In their choice of courses the students should have the greatest possible diversity of opportunity. To achieve these objectives, it will, however, almost certainly be necessary to provide new text material having the desired orientation.

With regard to the student going on to graduate school, the case for diversity is surely quite as strong. For since there are many different programs that he may follow at graduate school, involving emphasis on very different parts of mathematics, it would plainly be undesirable to restrict unnecessarily the

*There was not unanimity in the Committee on the role of differential equations in the analysis sequence, but a strong majority supported the recommendation on this issue.

pattern of the undergraduate major. What any graduate school should require of its incoming students is a level of mathematical maturity that enables them to proceed to a deeper study of the topics appropriate to the discipline being pursued. As long as the undergraduate program is conducive to that maturity —and we have indicated above what we regard as essential features of a program that would be conducive to such a maturity—then the actual choice of courses and their content should rather reflect the interests of the individual undergraduate student and the particular competences of the faculty.

There is one current pattern, in some of our best undergraduate programs, on which we wish to comment. We refer to the practice whereby many undergraduates in their senior years act more in the role of beginning graduate students, taking graduate courses and generally studying certain mathematical topics beyond the undergraduate level. We do not oppose this; but we do want to point out that "pure acceleration" has limited value, and that gifted students might well benefit also from taking courses, perhaps at a less advanced level, in related disciplines.

Our curriculum recommendations offer means to help realize the objectives implicit in the above discussion. The recommendations should certainly not be taken to imply that the sequential structure and rigor of the mathematics curriculum should be abandoned or diminished. Rather, their purpose is to give students a sense of the coherent structure of mathematics and of how it relates to other areas, together with a sense of its history, its scholarly tradition, its precise nature, and its standards.

Theme D
Applications-Experience Programs

An important ingredient of education in mathematics is the exposure of the student to applications. As has already been mentioned, applications should be incorporated in the regular course material in the mathematics major in order to motivate the development of the mathematics. However, this is not enough to give the students a significant competence in applying mathematics. They need more realistic exposure to applications than can be conveniently given in a classroom. A good way to provide the students with experience in applying mathematics, which will complement their classroom instruction, is by offering them nontraditional programs wherein the student participates in the applications of mathematics in industry, business, government, or, perhaps, academic research outside the mathematics department. We will call such programs *applications-experience* programs.

These programs, providing actual experience in applications, have a dual purpose: to place the students in an environment in which their mathematical skills and point of view can be applied; and to expose the students to a broad range of activities,[1] which mathematicians encounter while applying their knowledge. This first-hand experience in the application of mathematics, combined with the interaction with workers in other disciplines, will greatly strengthen the students' education.

The benefits of such an experience are numerous. The students will learn to model a physical or operational situation in quantitative terms amenable to analysis (with or without computers). Further, the use of mathematics will give the students a different perspective on some of the material already learned; seeing where mathematics can be applied will guide them in the selection of the mathematical topics that they will study in the future; and, most importantly, the act of direct participation will give the students a feeling of accomplishment during their mathematical development.

It should be emphasized that the mathematical skills employed in the work experience may constitute only a small part of what the student is actually learning in regular courses. The Committee is not disturbed by this, since the student will be acquiring experience in the practice of applying mathematics to "real" problems and in bringing to bear a mathematical viewpoint when confronted with such problems. Nor, of course, should curriculum topics be selected purely on the basis of the students' anticipated work experience, either during their student days or in their subsequent careers. The maturity and breadth provided by a well-designed mathematical education have a lasting value that does not depend on the immediate applicability of skills and techniques acquired.

Several schools already have programs of the sort we have in mind as part of their curricula. Drexel University and Antioch College, for example, have for several years carried on cooperative education programs wherein students alternate between academic terms and internships of equal duration in industry or government. The University of Waterloo also has such an ongoing internship program,[2] in which the students spend a part of their school year working for some business, industrial, or government laboratory. Claremont College has started a mathematical clinic,[3] in which students work on problems brought to the clinic by nearby business and industry. In Hunter College, the nontraditional part of the curriculum takes the form of a case seminar,[4] which has two parts. The first involves lectures by academic and industrial applied mathematicians describing an application of mathematics from its origin to the development of the mathematics needed for a solution; the second part involves the participation of the students in a project. Many university programs in science and engineering departments provide the sort of experience in applications that the Committee has in mind. Other successful examples of applications-experience programs could be cited.

All these programs expose the students to the work of a mathematical scientist in a nonmathematical environment and provide them with the opportunity to use their mathematical output and skills in the solution of "real" problems. Thus these programs help the students better to understand the role of mathematics in the solution of problems that arise outside mathematics. Although the main interest of such programs is to improve the education of the students, the added understanding of the problems of their potential employer cannot but help the future employment possibilities of the mathematics students and serve to bridge the gap that today often separates the industrial and academic communities.

If such programs are to be introduced on a considerable scale, they must be accompanied by programs to retrain members of the faculty in applications of mathematics. Unfortunately, many mathematicians are not well informed about the great scope of contemporary mathematical science. A "guided tour," providing a sampling of important branches of mathematical science would be helpful to both pure and applied mathematicians in breaking down the barriers imposed by specialization. Another form of retraining would consist of a deeper study of one or more particular areas of application.

A suitable vehicle for such retraining would take the form of summer institutes for faculty members. This would, however, require some measure of federal funding, and the Committee recommends that such funding be made available.

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Theme E
Societal Aspects

Under this heading we contemplate a variety of problems. We have already discussed the problem of improving relations within the mathematical community. We turn now to the rest of the academic community and to broader societal problems. For it is plainly necessary to correct prejudiced attitudes within the academic community and to improve relations between the mathematical community on the one hand and industry and the public on the other.

The Committee believes that the mathematics profession and mathematics training are seen in a false light by many people outside the mathematical community and that these misconceptions have a deleterious effect on the profession and on society.

Mathematics is undervalued by many in government and business; consequently, support for mathematics and job prospects for our graduates are curtailed. We believe that the withdrawn attitude of many mathematicians and their unwillingness to seek out opportunities to talk to laymen or devise other means to explain their work is partially responsible for this situation. Although it is admittedly difficult, a substantial effort to communicate with the general public would be beneficial both to the profession and to society. We should not be ashamed to advertise mathematics.

Further, it is well documented that women and minorities continue to be grossly underrepresented in most fields of mathematical endeavor; the lack of adequate mathematical education among those groups, by the end of high school, is particularly notable and detrimental. To what extent this avoidance of mathematics is freely chosen, subtly influenced, or actually imposed is beyond the scope of this Committee to discuss. The net effects are that mathematics continues to be perceived as a profession where barriers exist and that talented women and minorities are thereby lost to the profession. It is also beyond the scope of this Committee to make any effective recommendations to redress the imbalance. However, greater involvement of mathematicians in the early training of mathematics students–by lectures, articles in school publications, informal discussions with students and their teachers, or other means–should help to encourage young people to keep their options more open.

Recommendations

Note that, following each recommendation, the designations A, B, C, D, and E indicate the themes in the report to which the recommendations are related. As emphasized in the Overview, we are particularly concerned to propose steps that will counter certain unhelpful attitudes that we find in the mathematics community. The recommendations referring, not necessarily exclusively, to *attitude* (Theme A) have been collected together to form the first part (Recommendations 1-6) of our list.

1. *That* appropriate recognition be given to faculty interest and accomplishment in applications and applicable aspects of the mathematical sciences. This could take the form of opportunities for promotion and tenure, leaves of absence, or lighter teaching loads for faculty members engaged in the development of new courses, for example, those involving applications, or in applications-experience programs. [A, D]
2. *That*, in consultation with appropriate departments, upper-level courses in mathematics be designed to take account of the needs of students majoring in other disciplines. [A, B, C]
3. *That* mathematical scientists in industry and government be brought to university campuses in faculty-industry exchange programs or be invited to participate in lecture programs for students. [A, D]
4. *That* summer institutes for college faculty, in some area of applied mathematics, be made available to enable faculty members to teach courses in those areas and, in the most favorable cases, to cooperate with other faculty members in research projects related to those areas; and that federal funding be sought to finance such institutes. [A, D]

5. *That* due account be taken of the possible types of future employment of the students in designing curricula, it being understood that details of the curricula should not be determined by considerations of the techniques likely to be used by students in the first few years of their industrial employment. [A, C, E]

6. *That* members of the mathematical-sciences community take cognizance of the extent of that community and help to overcome barriers within the community and barriers to entry into the community. [A, E]

7. *That* mathematics departments undertake, in consultation with science and other concerned departments, to redefine the mathematics major with a view to (a) increasing the flexibility of the programs (without in any way weakening those programs); (b) doing justice to the many-faceted nature of mathematics itself and its ever-widening area of application; (c) taking account of different student objectives; (d) taking account of the strengths within the faculty in the mathematical sciences (not only in the departments of mathematics). [B, C]

8. *That* there be incorporated in the programs of mathematics majors strong minors in mathematics-related disciplines. [B, C]

9. *That* a course in linear algebra, at least as far as the study of eigenvalues and eigenvectors, and including computations and applications to diverse areas (e.g., linear differential equations, operations research, statistics), precede abstract algebra in the algebra sequence. [B, C]

10. *That* the treatment of analysis be designed to take account of as many as possible of the following criteria: it should be concrete as well as abstract; it should emphasize the historical and the actual role of differential equations; it should pay special attention to the study of certain classical differential equations; and it should emphasize computational aspects. Further that, in studying integration theory, explicit attention be paid to applications to probability. [C]

11. *That* other aspects of applicable mathematics (e.g., numerical analysis, combinatorics) be available in elective courses in the junior and senior years, the electives also to include those mathematics courses of a more traditional nature that are currently available. [B, C]

12. *That* every mathematics major acquire early in his or her college career a basic competence in computer programming so that he or she can design algorithms and write programs of some complexity. This competence may often best be gained by requiring a course in computer science; where this option is used, the required course should be designed so that many of the examples and exercises relate to the material of the student's mathematics courses. Further that, to the extent possible, the student's computing knowledge be utilized in appropriate mathematics courses. [C]

13. *That* the number of course credit units required in the mathematics major be increased in order to be consistent with the greater breadth of under-

standing and maturity to be expected of a student on completing the bachelor's program. [C]

14. *That* experience in applications (including internships, where feasible; courses of case studies; and senior research projects) be an integral part of the program for the major in the mathematical sciences. [C, D]

Appendix

Proposal to the Alfred P. Sloan Foundation
for a
Committee on Applied Mathematics Training

SUMMARY

There is widespread concern in the mathematics community about the mismatch between college mathematics curricula and opportunities for employment of mathematicians. Mathematical skills are an asset in many activities not specifically identified as mathematics, yet mathematical education does not adequately take account of this fact; it is clear that the mathematical talent of the youth in this country may very well remain untapped unless there is a better match between the instruction in our colleges on the one hand and the opportunities for applying mathematics in laboratories, industry, and business on the other hand.

There is more at stake here than mere reaction to economic pressures. In the last thirty years mathematics has been undergoing syntheses, with methods developed in some areas finding spectacular applications in seemingly totally unrelated areas. There have been spinoffs in the applied areas as well. Thus the time is ripe for a closing of the gap between abstract and applied mathematics; the move towards incorporating more training in applications in the standard college mathematics curriculum is appropriate intellecutally as well as pragmatically.

It is proposed that, under the auspices of the National Research Council, a committee be formed to make specific and detailed recommendations on the

expansion of the applied mathematics curriculum in the undergraduate mathematics programs of U.S. colleges and universities. A revised and expanded mathematics curriculum that offers mathematically gifted undergraduates training related to applications in laboratories, industry, and business will provide superior students with alternatives to an academic career and better equip those who remain in academia to teach applied mathematics courses. The importance of applied mathematics is now commonly recognized, but undergraduate programs in this field are not generally available and the curricula that do exist differ markedly in timeliness and quality. Although total uniformity in these curricula would be neither desirable nor attainable, an answer is needed to the question: What constitutes adequate undergraduate applied mathematics training in the many diverse areas?

The committee will consider a related problem. Most mathematics faculty members of many colleges have had little training in applied mathematics; there is some doubt that current staff in many colleges can teach applied mathematics effectively. It seems clear that the initiation of some programs to train existing faculties to teach applied mathematics courses would be beneficial. The nature and format of such programs could be quite varied and should be determined after some study.

Another obstacle to change is the reluctance of some colleges to modify the present curriculum for fear that students who do not offer evidence of outstanding performance in the traditional advanced undergraduate courses will impair their chances for admission to high-quality graduate schools. The prestige of a committee representing the National Research Council may help overcome this resistance.

The presidents of the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics have warmly endorsed the formation of such a committee and offered the cooperation of their societies. The project will be carried out in close coordination with any related activities of these societies.

PROPOSAL

A committee of some 12 members would be formed and meet five times during the period November 15, 1976, through May 15, 1978; a staff officer would be employed. The staff officer would make an analysis of the current applied mathematics curricula and faculty orientation programs as a preliminary step in the study. The committee would attempt to answer the following questions:

1. What constitutes suitable applied mathematics training: For an undergraduate mathematics major? For students interested in the physical, life, or