



Analysis of Life Prediction Methods for Time-Dependent Fatigue Crack Initiation in Nickel-Base Superalloys (1980)

Pages
96

Size
8.5 x 10

ISBN
0309331641

Committee on Fatigue Crack Initiation at Elevated Temperatures; National Materials Advisory Board; Commission on Sociotechnical Systems; National Research Council

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|--|-----------------------|--|
| 1. REPORT NUMBER NMAB-347 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) Analysis of Life Prediction Methods for Time-Dependent Fatigue Crack Initiation in Nickel-Base Superalloys | | 5. TYPE OF REPORT & PERIOD COVERED Final Report |
| | | 6. PERFORMING ORG. REPORT NUMBER NMAB-347 |
| 7. AUTHOR(s) | | 8. CONTRACT OR GRANT NUMBER(s) MDA 903-74-C-0167 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS National Materials Advisory Board National Academy of Sciences 2101 Constitution Ave., NW, Wash. DC 20418 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS Department of Defense -- ODDR&E Washington, DC 20301 | | 12. REPORT DATE 1980 |
| | | 13. NUMBER OF PAGES 91 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report) UNCLASSIFIED |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report) This report has been approved for public release and sale; its distribution is unlimited. | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| 18. SUPPLEMENTARY NOTES | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Crack Growth Nondestructive Evaluation Fatigue Crack Initiation Nondestructive Testing Fracture Mechanics Superalloys Life Prediction Methods Time-Dependent Fatigue Crack Nickel-Base Superalloys Initiation | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Design of turbine disks for high-performance aircraft gas turbine engines requires an elevated temperature, time-dependent fatigue lifetime prediction method for advanced nickel-base super- alloys. Prediction of crack initiation -- defined by the committee to be a surface crack 0.8 mm long -- requires knowledge of the effects of local strain range, mean stress, temperature, and mission profile on the formation, the coalescence (generally | | |

20. Abstract (continued)

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Engineering considerations dictate that the lifetime prediction method be based on mean stress and total, rather than inelastic, strain range, and that it must be able to predict initiation at a strain concentration from smooth specimen test results, including the statistical variation. An incremental approach to damage accumulation is preferred for evaluating complex duty cycles.

Two prominent empirical methods, strain-range partitioning and frequency-modified fatigue life, contain elements that are suited to representing viscoplastic and oxidation-assisted cracking but require modification to meet the requirements of turbine disk design in the early stages of use. In addition, these methods have not yet been applied successfully to the formation and link-up of microcracks and thus are not suitable where this process constitutes the major portion of the initiation lifetime. Fracture mechanics (crack growth) methods offer certain advantages including some consistency with microstructural mechanisms. Pursuit of a conceptual model of this inherently statistical process is recommended for the longer term, as is the development of a constitutive model for cycle- and time-dependent analysis of the mean stress.

ANALYSIS OF LIFE PREDICTION METHODS FOR
TIME-DEPENDENT FATIGUE CRACK INITIATION
IN NICKEL-BASE SUPERALLOYS

Report of

The Committee on Fatigue Crack Initiation
at Elevated Temperatures

NATIONAL MATERIALS ADVISORY BOARD
Commission on Sociotechnical Systems
National Research Council

Publication NMAB-347
National Academy of Sciences
Washington, D.C.

1980

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The project that is the subject of this report was approved by the Governing Board of the National Research Council, whose members are drawn from the Councils of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine. The members of the panel responsible for the report were chosen for their special competence and with regard to appropriate balance.

This report has been reviewed by a group other than the authors according to procedures approved by a Report Review Committee consisting of members of the National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine.

This study by the National Materials Advisory Board was conducted under Contract No. MDA 903-74-C-0167 with the Department of Defense and the National Aeronautics and Space Administration.

Printed in the United States of America.

Order from
National Technical
Information Service,
Springfield, Va.

ii

22161
Order No. PB80-158116

ABSTRACT

Design of turbine disks for high-performance aircraft gas turbine engines requires an elevated temperature, time-dependent fatigue lifetime prediction method for advanced nickel-base superalloys. Prediction of crack initiation -- defined by the committee to be a surface crack 0.8 mm long -- requires knowledge of the effects of local strain range, mean stress, temperature, and mission profile on the formation, the coalescence (generally referred to as link-up), and the propagation of slip band or grain boundary microcracks or of a crack from a processing defect. The principal time-dependent mechanisms are viscoplastic strain and grain boundary oxidation depending on the upper limit of service temperature. Cavitation and grain boundary triple-point cracking are expected to be of minor significance.

Engineering considerations dictate that the lifetime prediction method be based on mean stress and total, rather than inelastic, strain range, and that it must be able to predict initiation at a strain concentration from smooth specimen test results, including the statistical variation. An incremental approach to damage accumulation is preferred for evaluating complex duty cycles.

Two prominent empirical methods, strain-range partitioning and frequency-modified fatigue life, contain elements that are suited to representing viscoplastic and oxidation-assisted cracking but require modification to meet the requirements of turbine disk design in the early stages of use. In addition, these methods have not yet been applied successfully to the formation and link-up of microcracks and thus are not suitable where this process constitutes the major portion of the initiation lifetime. Fracture mechanics (crack growth) methods offer certain advantages including some consistency with microstructural mechanisms. Pursuit of a conceptual model of this inherently statistical process is recommended for the longer term, as is the development of a constitutive model for cycle- and time-dependent analysis of the mean stress.

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PREFACE

The National Materials Advisory Board (NMAB) Committee on Fatigue Crack Initiation at Elevated Temperatures was established in response to a request from the Air Force Materials Laboratory ". . . to assess the approaches for predicting crack initiation time in aircraft engine disks of nickel-base superalloys operating at elevated temperatures and lifetimes of greater than 10,000 duty cycles, particularly where time-dependent effects must be considered."

The basis for the study was the concern of the U.S. Air Force with the current limited capability of predicting the number of cycles required to initiate fatigue cracks in components operating at elevated temperatures. Jet engine component life limits are conservatively established today and, as a result, many components are being retired with significant potential life remaining.

Rapidly escalating maintenance costs for existing and next-generation engines operating at even higher temperatures than used now dictate that this conservatism be reexamined. One objective of this examination is to determine whether existing approaches to predicting initiation times in the temperature regime where interactions of creep and fatigue exist, justify extending the component use life.

In recent years numerous theories and techniques have evolved that give insight into fatigue-initiation mechanisms at elevated temperature. This insight has led to the development of prediction methods that have achieved some degree of success in a wide variety of applications. No one method appears to fit the anticipated requirements of advanced superalloys.

It was agreed that a detailed examination of the status of the prediction methods would permit a better understanding of current limitations. From this analysis, the committee

sought to ascertain which research activities would advance the capability in prediction, especially for the more aggressive operating conditions expected of next-generation engines.

The report includes data collected through January 1979.

Clifford H. Wells
Chairman

ACKNOWLEDGMENTS

Each chapter or section was written by one or more members, expert in the topic under discussion. The summary, conclusions, and recommendations of the overall report represent the consensus of the committee. Under chairman Clifford H. Wells, chapter coordinators responsible for consolidating the material from the committee members and guest contributors were: Mr. Anton Coles (Chapter 3) and Dr. A. Eugene Carden (Chapter 4).

Special thanks are due the guest experts who made oral presentations and submitted written material from which the text has been drawn extensively. The contributions, presented on two separate meetings with the committee, significantly aided the deliberations of the committee. Contributing guest speakers were: Louis F. Coffin, Jr., General Electric Company; Thomas A. Cruse, Pratt & Whitney Aircraft Group, United Technologies; G. R. Leverant, Southwest Research Institute; S. Majumdar, Argonne National Laboratory; S. S. Manson, Case-Western Reserve University; Donald Mowbray, General Electric Company; Warren J. Ostergren, General Electric Company; Richi Raj, Cornell University; Brian Tomkins, United Kingdom Atomic Energy Authority, Reactor Fuel Element Laboratories; and R. Wallace, Pratt & Whitney Aircraft Group, United Technologies.

Captain J. Hyzak, Air Force Materials Laboratories, provided a survey of microstructural mechanisms and typical engine disk test data, and Dr. G. R. Leverant supplied an extensive bibliography on the metallurgical aspects of fatigue of superalloys. The committee thanks Lt. Roclevich, Base Protocol Officer, for a briefing and tour of the San Antonio Air Logistics Command engine overhaul facility. Finally, the generous participation of the liaison representatives in the committee discussions is gratefully acknowledged.

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Chapter One

SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

The committee first surveyed the special problems of fatigue lifetime prediction of wrought nickel-base superalloys in application to the turbine disks of advanced aircraft engines. It was agreed at the outset that the practical lifetime of concern would be that required to form a surface crack 0.8 mm (0.031 in.) in length, in order to avoid the problem of the subjective definition of fatigue initiation.

The committee concluded that the most general requirement for a lifetime prediction method should be minimization of uncertainty: first, in predicting the lifetime at a notch or other strain concentration based on laboratory tests of simple specimens; and second, in evaluating complex mission histories. Although the study was not directed towards "retirement for cause" (lifetime extension based upon nondestructive flaw evaluation and fracture mechanics), crack growth approaches were considered in addition to crack initiation methods. The committee further concluded that the prediction method should be consistent with observed microstructural crack initiation mechanisms. The available observations of these mechanisms in wrought superalloys were reviewed by the committee. While most of the experience with conventional superalloy disks has been in the regime of transgranular, slip-band crack formation and coalescence (link-up), it was assumed that intergranular cracking in advanced alloys, such as IN-100, René-95, and AF2-1DA might be tolerated in the absence of specific information to the contrary. It was concluded that the most significant effects of temperature and time would be viscoplasticity (time-dependent inelastic strain) in the case of transgranular cracking, and oxidation-assisted intergranular cracking. It was thought that grain boundary cavitation would be less important than surface oxidation in the temperature

range contemplated for advanced disks (approximately 700°-800°C [1292°-1472°F]); resolution of the dominant mechanism of crack initiation was considered important to the future development of more accurate damage prediction methods.

One of the most significant microstructural variables identified was the initiation criterion grain size relative to the 0.8-mm (0.031-in.) criterion for initiation of a surface crack. In fact, different lifetime prediction methods might be optimum for different values of this ratio. For example, microcrack formation and link-up are known to dominate the initiation lifetime in older, coarse-grained disks, whereas crack propagation may dominate in modern fine-grained, powder metallurgy alloy disks, especially in the presence of microscopic processing flaws. The committee concluded that greater prediction accuracy probably could be obtained by treating the link-up and propagation phases separately. Linear elastic fracture mechanics methods appear to apply to the propagation of a crack, provided it spans several grains along its front. Modification of fracture mechanics to include viscoplastic crack tip opening in low-alloy steels has progressed to the point where application to nickel-base superalloys is warranted.

The prediction of microcrack formation and link-up presents a difficult, essentially statistical, problem. While the slip-band cracking process is better understood than surface intergranular cracking, neither has been quantitatively modeled to date, and pursuit of such a statistical model is recommended. Pending such a successful development, empirical crack initiation methods must be relied upon. The committee thus surveyed the currently available methods in the light of the special requirements of turbine disk analysis.

One principal requirement was identified to be the explicit dependence on the mean or peak value of stress in a cycle, since laboratory tests have shown the fatigue lives of nickel-base superalloys to be highly sensitive to mean stress. Incorporation of mean stress effects was not considered to be a basic limitation of any of the methods surveyed. The committee also concluded that the lifetime prediction method should be based upon total, rather than inelastic, strain, since the inelastic strains experienced by turbine disks are too small to be calculated or measured accurately. Fracture mechanics and frequency-based methods

such as the frequency-modified fatigue life method, are easily formulated in terms of total strain. However, the strain-range partitioning method at present requires separation of inelastic strains into time-dependent and time-independent components (e.g., should be modified if it is to be applied to nearby elastic conditions). Both methods were considered capable of predicting lifetimes in the presence of oxidation-assisted intergranular crack formation as well as transgranular cracking. Incremental damage accumulation approaches, such as embodied in one version of the frequency-modified fatigue life method, appear to offer some advantages in representing the nonlinear effects of complex mission cycles since they are based upon the definition of damage as crack growth. The committee concluded that both the frequency-modified fatigue life and the strain-range partitioning methods contained concepts that were attractive from the standpoint of predicting time-dependent damage in superalloys and recommended that their extension to turbine disk analysis be undertaken.

While not a specific objective of this study, the accuracy of calculating the time- and cycle-dependent redistribution of mean stress around a strain concentration was considered a significant factor in lifetime prediction. A critical evaluation of the capability of current constitutive behavior models to predict the asymptotic analysis of mean stress was recommended. The form of the optimum constitutive relation for the prediction of mean stress might differ from that of the current models, which are primarily directed towards predicting the range of inelastic strain in the cycle. Closely allied with this problem is the prediction of the time- and cycle-dependent changes in surface residual stress and its gradient, which are known to affect significantly the initiation of surface cracks. The committee recommended that work be undertaken to define the uncertainty in the initiation lifetime associated with residual stresses and their changes during service.

Chapter Two

INTRODUCTION

2.1 OBJECTIVES

This study was requested by the U.S. Air Force Materials Laboratory (AFML) as part of an extended Department of Defense (DoD) and National Aeronautics and Space Administration (NASA) study program by the National Materials Advisory Board (NMAB), to address the problem of fatigue lifetime prediction in turbine disks of advanced aircraft gas turbine engines. During the last decade dramatic increases in the cost of advanced turbine disks have been caused primarily by the use of complex designs combined with advanced materials and processing techniques. At the same time, the cyclic lives of these components have tended to decrease because of the additional performance requirements (higher thrust/weight). Consequently, the replacement costs of these components over the useful service life of the engines are anticipated to rise over the next 10-20 years. It is widely believed that the current life limits may be overly conservative and that significant cost savings could be achieved through a more accurate assessment of the real lifetime of the components.

While this study is not directed toward the prediction of the reliability or safe life of a turbine disk on the basis of nondestructive evaluation or in-service inspection, some overlap is unavoidable. The distinction between the objectives of the current study and the so-called retirement-for-cause methodology is that the latter is based upon specific measurements of an individual disk and possibly upon tracking of its service history. This study is concerned with the design lifetime prediction of a population of disks. Whatever local quality control measurements are made during manufacture, they primarily involve metrology and bulk material properties and are not intended to grade the

disks according to lifetime. Within a population of disks, the individual lifetimes may vary significantly and, since the design lifetime must be based upon an acceptably low probability of crack initiation in service, the average lifetime may greatly exceed the design lifetime. The retirement-for-cause approach attempts to effectively eliminate the lowest-life parts from the population, thus providing the economic benefits afforded by the average lifetime. Similarly, one very important aspect of design lifetime prediction is the narrowing of the statistical distribution of lifetimes in addition to providing greater accuracy in predicting the mean lifetime.

The overall objective of this study is perhaps best summarized as the minimization of uncertainty in fatigue lifetime prediction at elevated temperature. Specific objectives were to:

- Evaluate available lifetime prediction methods in terms of applicability to advanced disk designs and alloys;
- Determine the principal parameters in lifetime prediction;
- Identify critical experiments for validating existing approaches; and
- Identify, where appropriate, alternative procedures for lifetime prediction.

2.2 SCOPE

The scope of the study was restricted to high-strength nickel-base superalloys such as powder metallurgy IN-100, AF2-1DA, and thermomechanically processed René-95. Maximum anticipated disk rim temperatures were specified to be in the range of 650°-815°C (1202°-1500°F), where time-dependent effects must be considered. The cyclic lifetime of interest was in excess of 10,000 cycles.

The definition of disk lifetime was taken to be the number of cycles required to develop a large enough crack to be reliably detected during shop inspection with suitable automated nondestructive evaluation (NDE) methods. This

surface crack length was chosen arbitrarily to be 0.8 mm (0.031 in.) which represents the current limit of the state of the art in production inspection environments. No attempt was made to define this lifetime as crack "initiation" or "propagation." It was recognized that some portion of the cycles to produce an 0.8-mm (0.031-in.) crack could be treated by a crack-growth law, while some of the life might have to be handled empirically or by another type of micro-mechanical model. The potential improvements in flaw resolution afforded by automated electromagnetic and radiographic methods were also considered.

With these constraints, the committee reviewed a wide range of lifetime prediction methods from the simplest representation of fatigue data (elastic stress analysis with modified Goodman diagram) to the most advanced (partitioning of inelastic strain, inelastic small flaw fracture mechanics). Available information on the microstructural mechanisms of "damage" in the alloys and service environment of concern was reviewed in order to assess the consistency of the methods with physical reality. At the same time, the practicality of currently available methods was evaluated from the standpoint of engineering requirements and limitations. Inevitably, the conclusions and recommendations of the committee represent a compromise between the minimization of uncertainty in lifetime prediction, and the time and cost of analysis and testing. Extension of current prediction methods may overcome any apparent limitations.

2.3 BACKGROUND

The subject of time-dependent fatigue has been reviewed extensively in recent years (Oak Ridge National Laboratory, 1977), particularly under the liquid-metal fast-breeder reactor (LMFBR) program. Surveys of elevated-temperature fatigue lifetime prediction of low-alloy steels have been recently published. On the other hand, the problem of aircraft engine disk fatigue is distinguished from that of stationary power generation equipment, such as steam turbine rotors and nuclear reactor pressure boundary components, in the following ways:

- Duty cycles are much shorter, although the same number of cycles (10,000) may be involved in power peaking

installations (one cycle per day for 30 years). The duty is more varied, and the effects of mission sequence must be included.

- Turbine disks are not operated at high net stresses in the creep range. Rather, the effects of time-dependent deformation are limited to local regions of strain concentration, primarily involving short-range stress level redistributions from relaxation.
- Effects of time, stress and temperature on the turbine disk fracture process differ from the cavitation* and grain boundary sliding experienced in stainless steels, at least in degree. Bulk damage is frequently developed in the latter, whereas the physical changes in nickel-base superalloys usually are confined to the surface.
- The high yield strength and creep resistance of nickel-base superalloys at elevated temperature, combined with the decrease of Young's modulus with temperature, result in local inelastic strain ranges that are small relative to the total strain range.
- In the temperature range of concern, most nickel-base superalloys exhibit a heterogeneous, crystallographic, planar form of slip which, in the absence of intergranular fracture, results in Stage I fatigue cracking.

For the above reasons, and since no comprehensive reviews of fatigue of these alloys have been made since 1971, a study of the applicability of recent methods of lifetime prediction for the turbine disk was judged to be in order. Particularly questioned were the validity of fracture mechanics in the range of crack length below 0.8 mm (0.031 in.) and the ability of the methods to predict reliably the major portion of disk lifetime, since the concept of a unique initial flaw size had been found by the committee members and others to be at variance with the results of smooth specimen low-cycle fatigue tests. The committee also addressed the material- and process-dependent sources of data scatter. In

* Term common to metallurgists; see Section 4.1.1 for the definition of cavitation as employed here.

particular, the prediction of mean surface stress and its gradient were considered in connection with the measurement and control of material properties needed to reduce this aspect of uncertainty in lifetime prediction. The measurement and calculation of mean surface stress and its gradient were considered extensively in connection with lifetime correlations between smooth and notched specimens.

The committee was extensively aided in its deliberations by presentations of research in progress as well as many diverse views and ideas from the proponents of the leading lifetime prediction methods. We recognize that not all these approaches have been reviewed with equal emphasis and that certain omissions may be evident. While in no way intending to slight any of the available published material, the committee was compelled to sort out the approach, or combination of approaches, that appeared to present the greatest potential for meeting the specific objectives of this study. Therefore, this report is not presented as an exhaustive review of the subject.

Chapter Three

ENGINEERING CONSIDERATIONS

The ability to predict accurately the low-cycle fatigue (LCF) lifetimes of engine components requires a basic understanding of the gas turbine engine operating environment, the local stress-strain state, and the ways in which the material sustains and accumulates damage. In addition, appropriate mathematical models that are capable of simulating this behavior must be employed in order to predict analytically the lifetimes of components. One of the basic elements of the technology is the accurate modeling of the response of the material to an applied stress-strain state and environment. The best analytical design method may be useless if the failure mode of the material is unknown or poorly defined.

The objective of this chapter is to provide an overview of general engineering requirements that an engine designer uses. It also considers the applicability and limitations of analytical methods currently available. The processes of stress analysis, materials response analysis behavior, life estimation and verification testing are discussed. In addition, the future need to address time-dependent mechanisms in higher temperature application is identified.

Rotating structures are designed primarily in the elastic range. Any yielding of the material is confined to localized regions of high strain concentrations at geometric discontinuities and/or large temperature gradients. For compressor and turbine disks, the failure mode is primarily LCF and is controlled by localized yielding and stress redistribution at such regions of high strain concentration.

Past experience indicates that for most compressor and turbine disk applications, a time-independent LCF life prediction analysis is quite appropriate. However, for the next generation of advanced superalloy turbine disks in high performance U.S. Air Force engines, which are expected to

operate at higher temperatures than present engines, the time-dependent effects may become an important aspect of LCF life prediction analyses. It is expected that the development of an advanced lifetime prediction methodology for future disks will have the capability of analyzing local time-dependent, viscoplastic stress-strain response as well as incorporating the time-dependent material property data for crack propagation.

Although the present study is primarily concerned with the advanced nickel-base powder metallurgy turbine disk alloys, the results of the study are expected to be equally applicable to other product forms (e.g., cast nickel-base superalloys for advanced turbine blades, and integrally cast turbine wheel applications). Future ceramic turbine components may require other considerations, but these are not within the scope of this evaluation.

3.1 COMPONENT/SPECIMEN CORRELATION

The basic approach used in current life prediction methods is to measure the lifetime of simple specimens under known environmental conditions and then to develop analytical models that accurately describe their behavior. The suitability of the analytical models is then tested by comparing the predicted lives with the observed (measured) lives of simulated hardware (or model) specimens, which are designed to have appropriate complex geometries and stress-strain states. Upon successful completion at this level, the analytical models then are used to predict the LCF lifetimes of actual engine components with verification by rig testing.

3.1.1 Specimen Testing and Modeling

The current LCF life prediction methodologies are based on the assumption that a smooth, strain-controlled LCF specimen simulates the local behavior of the material at a geometric discontinuity in a component. Since rotating structures can tolerate only very small dimensional changes, they are designed in the elastic range. Any yielding or creep is confined to localized regions of high strain concentrations (e.g., areas around bolt holes, rim attachments). On unloading (at shutdown), the large elastic volume of

material forces the local inelastic zone into compression. In extreme cases, compressive yielding may occur locally which induces residual tensile stresses in the material.

The strain at unloading may differ somewhat from the initial value (usually zero) before loading because of stress redistribution by inelastic deformation. Because the flow stress of the material varies with repeated cycling, several cycles may be required before the material shakes down to a relatively constant state. Larger numbers of cycles may be required at elevated temperature, and the level of compressive stress may be much greater. However, because of the constraint of surrounding elastic material, the behavior at the geometric discontinuities can be viewed essentially as one of localized strain-controlled deformation.

Consequently, strain-controlled LCF specimen testing is extensively utilized in characterizing the LCF behavior of materials for life analysis. For turbine disk materials, the variables generally included in such an investigation are:

- Stress (strain range),
- Mean stress (mean strain),
- Temperature, and
- Surface integrity
- Frequency of loading and hold-time.

In the absence of any time-dependent effects, the primary variables controlling the LCF lifetime of a material (at a given temperature and surface condition) are considered to be the mean (or peak) stress and the total strain range. To fully characterize the mean stress (or peak stress) and strain range cyclic life dependence, tests are conducted over a range of crack-initiation lives at a given temperature, using several strain ratios, defined as the ratio of minimum to maximum strain for the selected cycle. Sufficient numbers of specimens are tested so that the dispersion (standard deviation) of fatigue life distribution is established. These tests are generally conducted using a triangular strain-versus-time profile at cyclic frequencies low enough (e.g., 5-20 cpm) so that any time-dependent environmental or

metallurgical effects are allowed to take place. The data thus obtained establish the baseline behavior at a given temperature for the time-dependent LCF analysis.

In the modeling of the time-independent LCF data, several approaches can be used. One approach is the representation of the data in terms of the Coffin-Manson formulation (see Chapter 4). The mean (or peak) stress effects in this case are determined through the construction of interaction diagrams based on the specimen data base. In another approach, the mean (or peak) stress (strain) range interaction is represented directly using an analytical expression. A number of methods are available for this purpose (see Jaske et al., 1973; Cruse et al., 1977; Cruse and Meyer, 1977).

The tests mentioned in the preceding paragraphs generally are carried out at several temperatures. Interpolations between temperatures are based on empirical relationships. Another variable known to affect the LCF behavior is the surface integrity of the material (e.g., residual stress distribution, surface finish). Generally, the baseline LCF data (described above) are generated using specimens that have been manufactured using a standardized metal removal technique. Translation to other surface conditions and finishes is then made through the use of correction factors established on the basis of selective specimen testing. Effects of cycle frequency (or hold-time) are handled in a manner similar to that described above. Again, selective specimen testing is used (generally within a limited range of cycle frequencies and hold-times) to establish correction factors for application to the baseline LCF data.

3.1.2 Complex Geometries

In addition to constant-strain amplitude LCF testing to develop a material data base for a range of temperature and strain (stress) ratios of interest, complex geometry specimen tests frequently are used to study component geometric details. The motivation behind complex geometry testing is that it frequently provides verification for stress analyses, material behavior, and processing-induced material condition effects (e.g., microstructural features, residual stress); and forms a valuable interim testing, prior to actual component testing. Complex geometry specimens are either machined from hardware to include critical regions or

designed to model such a feature and tested in a manner to induce multi-axial strain (stress). Simple triangular strain-time profiles or a simulated flight strain-temperature-time wave-form can be applied, depending on the test objective. Turbine disk details such as bolt holes, cooling holes, rabbet radii, dovetail slots can be evaluated in this manner (Coles and Popp, 1977) and the data used to modify life prediction models based on uniaxial behavior.

Specifically, with the testing of such specimens, material response to multiaxial loading and particular stress-time-temperature history can be properly evaluated. In addition, any surface conditioning effects that occur can also be evaluated and can be used to modify or calibrate the life prediction models. The final step in the LCF life prediction process is verification by full-scale component testing (e.g., cyclic spin testing of engine disks) under controlled conditions of temperature and stress.

3.1.3 Stress Analysis

To determine accurately the stress, deflection, vibratory modes and thermal response of both rotating and static engine components, the engine manufacturers have at their disposal a complete library of computerized analysis methods. The detailed structural analysis techniques employed for final design evaluation are based on finite element modeling. The finite element computer programs utilize plate, shell, ring and solid elements. The primary tool for the analyses of disks is a two-dimensional axisymmetric finite element program. Two-dimensional finite element programs are also available for a time-independent cyclic plasticity analysis of disks. However, whenever necessary, a more comprehensive stress analysis of critical locations is performed using a three-dimensional finite element model.

In addition to the time-independent stress analysis techniques described above, time-dependent models also are available to account for the effects of creep on stress (strain) redistribution at geometrical discontinuities.

3.2 BASIC CONSIDERATIONS IN LCF MODEL DEVELOPMENT

An essential part of material characterization for life analysis is that the data base developed and the models

generated be compatible with available structural analysis techniques. In the development of realistic models for crack initiation in gas turbine materials, the following considerations should be kept in mind:

- Experimental conditions should be established by the parameters of the engine operating environment.
- Ideally, models should account for various interactive effects (e.g., environment and mission profile correlations).
- Material property scatter should be accounted for.
- The use of models in structural life prediction analyses should not require parameters (such as plastic strain rate or strain energy) that can neither be reliably calculated by the available analytical tools nor directly measured.
- Models should be kept as simple as possible and should rely on a minimum amount of specimen testing.
- Interpolative and extrapolative capabilities of the models should be assessed carefully by selective testing of simple and/or complex geometry specimens.

The first of the above considerations is basic to any materials characterization program intended for a specific application. What is important here, however, is the determination, early in the characterization program, of those variables that have a primary effect on lifetime. The characterization program therefore should concentrate on evaluating the effects of these variables. The need for a model based on physical principles arises because of the possibility that a model inconsistent with metallurgical mechanisms may give erroneous results when extrapolations are made outside the range of the data used in its construction.

The need for the consideration of scatter in LCF life stems from the fact that crack initiation lives of components are determined using statistical methods. Unless the material-induced scatter for the operative LCF mechanism is properly established, the applicability of structural life

prediction analytical results becomes questionable. The model that is developed to represent the crack initiation behavior of the material should accurately reflect the influence of the populations of the materials parameters upon the statistical distributions of fatigue lifetime.

In the development of models based on specimen testing, an important consideration is the facility with which empirical models can be applied to the component. The application is accomplished through the local mechanical variables that characterize the behavior of the material at the peak stress (strain) location of a geometrical discontinuity in a component. If the local mechanical variables cannot be calculated reliably with the available analytical methods, then the utility of the model becomes questionable. For example, for most compressor and turbine disk applications, the plastic strain range (associated with cyclic loading) at LCF-critical locations may be too small to be reliably predicted (or even measured) by inelastic stress analyses.

The time-dependent cyclic plasticity analysis capabilities that can be used in the study of disks are not yet fully developed. Consequently, the separation of the local inelastic strains (associated with cyclic loading) into time-independent and time-dependent components, calculation of local inelastic strain rates, etc., may not be accurately accomplished at the present time on a cost-effective basis. Development of such capabilities will first require the development of constitutive relations that accurately describe materials behavior under cyclic plasticity.

Another important aspect of time-dependent LCF model development is that an adequate data base be obtained at a reasonable cost. The introduction of time as an additional parameter in LCF data generation can easily increase testing costs to considerably more than those required for time-independent LCF analysis. Consequently, in the final analysis, a trade-off has to be made between accuracy and cost. It is also important that the models developed be convenient to use so that they can be efficiently employed.

Finally, the interpolative and the extrapolative capabilities of the models as well as the ability to describe the local conditions at geometric discontinuities must be thoroughly assessed. This can be done by selective testing of simulated complex geometry specimens under various stress (strain)-time-temperature combinations. As described previously, specimens may be machined from hardware to include

critical regions, or be designed to include such a feature, and arranged for testing in a manner to induce multiaxial strain (stress). For such evaluations, obviously, the best available analytical methods must be used to describe the local stress-strain conditions since both the material LCF model itself and the entire life prediction methodology based on this model will be evaluated.

3.3 STATISTICAL TREATMENT

Since fatigue crack initiation is a highly variable process exhibiting large scatter at a given stress (or strain) level and environment, the structural life prediction analysis is based on probabilistic methods. The life of a component is defined as the number of cycles to crack initiation for a specified probability of occurrence (e.g., one in a thousand).

The LCF life prediction analysis is based on consideration of the dispersion of fatigue properties and the dispersion in stress level, both of which are assessed using relevant statistical techniques. The LCF data for a given material (under a specified loading condition and environment) generally are represented in terms of statistical properties, such as the kind of distribution (e.g., Weibull and log-normal), the central measure of the dispersion (i.e., mean, median), and the dispersion (i.e., standard deviation) of the data. Once the aforementioned properties are defined for the material under a specified set of conditions, it is possible to generate appropriate plots that relate fatigue life to probability.

An accurate estimate of the standard deviation is one of the primary requirements in LCF data generation, since it is used directly in the construction of material survivability plots. It is known, for example, that in addition to the "basic" scatter which may be related to the fundamental fatigue damage process operative (for a given material and environment), there are a number of other variables that contribute to the apparent scatter in LCF data. These include, for example, heat-to-heat differences in chemistry, microstructural properties, and the state of the material's surface integrity (i.e., the surface residual stress levels and distribution and surface finish). In design of fatigue experiments aimed at generating LCF data for life prediction

analysis, attention should be given to these variables, to the extent that each occurs in the specific application.

Another source of scatter which should be addressed in LCF life prediction analysis is the stress level variability. This arises from physical variations such as dimensional tolerances, or blade weights. Appropriate statistical methods are used in determining and combining this type of scatter with the variability in material LCF properties described above.

3.4 MISSION CYCLE ANALYSIS

The LCF life of a disk is determined in terms of a given design mission profile (or a family of profiles). In order for the life of a component to be predicted accurately, it is essential that the mission profile(s) be defined as accurately as possible with respect to the known (or anticipated) engine service. For a given aircraft, a composite engine duty cycle can be defined by an appropriate mix of a family of mission profiles, such as combat, training, functional checks and ground operation. The composite engine duty cycle then establishes the frequency, ordering and magnitude of thrust level changes as a function of time. This in turn involves their effect on magnitudes and changes of such engine parameters as rotor speed, turbine inlet pressure and temperature, which are needed to define the detailed boundary conditions on each component.

State-of-the-art heat transfer and stress analysis computer programs are used to define the structural state of a rotor system for the composite duty cycle. The composite duty cycle is examined for selected steady-state operating conditions and transients.

Given the calculated time-dependent local stresses and time-dependent temperatures (for each portion of the composite duty cycle) at critical disk locations, an LCF life analysis is performed. The incremental LCF damage occurring in each of the subcycles of the composite duty cycle is determined, then summed up using an appropriate damage-accumulation rule. However, in order to define the damage associated with each subcycle, a cycle counting method must be used. This is readily accomplished by the use of the "rain flow" cycle counting method, for example.

A number of cumulative damage rules have been proposed, the simplest being Miner's linear cumulative damage rule, which does not account for load interaction effects (for a partial listing of these rules, see Cruse and Meyer, 1977). Analyses of existing cumulative damage rules seem to indicate, however, that no universal rule exists that is distinctly superior to Miner's simple rule (Coles and Popp, 1977; Sataar and Sundt, 1975). Consequently, current LCF life prediction methods primarily utilize Miner's rule for damage summation. In the event that a more efficient damage rule has been developed for a given material under the appropriate loading conditions, it is a simple matter to introduce such a model into the existing life prediction methodologies. An example of how Miner's rule can be modified has been provided by Cruse and Meyer (1977).

All the preceding discussion is readily applicable to the case of time-independent LCF analysis of components. The extension of these procedures to the time-dependent LCF analysis necessitates the inclusion of a time-related parameter such as a function of cycle period or hold-time. The basic mission properties required for time-dependent analysis are:

- The total strain-stress-time trajectory over each rainflow cycle (i.e., damage cycle); and
- The "apparent hold-time" or "apparent frequency" associated with each rainflow cycle.

A time-dependent finite element inelastic stress (strain) analysis and an appropriate model or models describing materials time-dependent LCF properties may have to be employed in this case. All other aspects of damage summation (over the mission at hand), however, are essentially the same as for time-independent LCF analysis.

The nucleation of fatigue cracks at stress concentration areas is of particular interest in that microcracks form in the plastically deformed region at the "notch detail." The growth of small cracks embedded in plastically strained material is perhaps best addressed by nonlinear fracture mechanics methods and is reviewed in the following chapter. The problem of life prediction for such geometric configurations is essentially one of estimating cycles to crack initiation to some appropriate microcrack size. From this point,

the use of crack growth theory is employed to calculate crack extension process to the designated "initiation" length. Modeling of crack growth behavior also is described in the next chapter.

A detailed review of emerging life prediction methods is pertinent in order to examine their adaptability to accommodating time-dependent materials behavior and to point out the degree of complexity involved in characterizing such failure modes. Selected life prediction approaches are reviewed and evaluated in the next chapter.

3.5 SUMMARY OF MODEL REQUIREMENTS

In the previous discussion, analytical LCF model requirements were noted to have the ability to:

- Predict the life at a disk stress concentration based upon smooth specimen data.
- Predict life under thermal-mechanical cycling with hold time.
- Incorporate scatter in material properties.
- Recognize differences in surface conditions.
- Predict stress ratio effects.
- Predict cumulative damage effects (mission cycles).
- Be practical in terms of cost and analytical capability.
- Have ability to extrapolate and interpolate.

Chapter Four

REVIEW OF LIFE PREDICTION METHODS

This chapter reviews current methods of predicting fatigue crack initiation lifetime in the light of the design considerations discussed in the preceding chapter. Since one of the major requirements for a lifetime prediction method was concluded to be a physically based algorithm for computing damage, or at least a procedure that is consistent with microstructural observations, a brief summary of crack initiation mechanisms in nickel-base superalloys is included.

In the course of this review the question of the importance of crack "initiation" relative to crack growth could not be resolved at the outset. This necessitated a survey of a considerable body of fracture mechanics literature along with methods for predicting "initiation," which is the primary object of this study. Fatigue crack initiation, of course, is viewed differently by physicists, metallurgists, design engineers, and by overhaul and maintenance inspectors. A basic assumption made in this survey is the existence of a practical limit to the detectability of a surface-connected fatigue crack, i.e., one that might reliably be detected in an advanced turbine disk by automated nondestructive examination methods such as eddy current testing or the krypton exposure technique (KET). The questions of relative performance of such methods, or of the critical crack sizes in turbine disks, lie outside the scope of this study. Guided by industrial experience and trends in advanced disk design, the decision was made to specify an 0.8-mm (0.031-in.) surface crack length as the criterion for crack "initiation" and as the degree of damage that must be predicted with a minimum of uncertainty. Given this definition of initiation, the process may involve several stages which can be described in terms of the microstructural scale on which they take place. The 0.8-mm (0.031-in.) length may, in some cases,

be the result of crack propagation alone. Clearly, in the case of very large-grained materials which might exhibit grain diameters on the order of 0.8 mm (0.031 in.), current crack growth methods would be inappropriate. Intermediate situations might suggest a combination of methods. Thus apparent in this review are the relative roles of the current fatigue initiation prediction methods, which do not involve the specification of crack length, and of fracture mechanics methods, which are explicitly based upon crack length.

4.1 REVIEW OF CRACK INITIATION MECHANISMS

The last extensive reviews of the mechanisms of fatigue in nickel-base superalloys were conducted about seven years ago (Wells et al., 1971; Gell and Leverant, 1973). More recent reviews of fatigue mechanisms at elevated temperature are available (Wells, 1979) but do not deal specifically with nickel-base superalloys. The process of fatigue crack initiation can, in general, be viewed as a sequence of three processes: (1) microcrack formation, or the first microscopically detectable cracks, usually on the scale of the microstructure; (2) microcrack link-up, whereby adjacent independently formed microcracks join to produce a distinct crack front; (3) and finally, for the purposes of this study, propagation of such a distinct crack to 0.8 mm (0.031 in.) surface length.

4.1.1 Microcrack Formation

The mechanism of high temperature microcrack formation in nickel-base superalloys generally can be thought of as an interaction between the deformation processes and the microstructure of the alloy (Gell et al., 1970; Laird and Feltner, 1967; Wells et al., 1971). Deformation concentrates at different rates depending on temperature and strain rate and exploits the weakest feature of the microstructure, which is frequently some defect (second-phase particles, porosity, inclusions). It is for this reason that crack initiation must be examined under specific conditions and that few absolutes apply for these types of alloys. Some of the scatter in elevated temperature LCF tests can be traced

to specific features of the microstructure such as grain size, grain boundary morphology, second phase morphology, or chemistry. Thus microstructure not only establishes the base line for the average behavior, but the variety of microstructural characteristics also contributes to the scatter.

In rationalizing individual microcrack formation modes, it is the slip intensity that seems to control the overall process (Gell, 1973). At lower temperatures, nickel-base superalloys deform by planar slip which is heterogeneous in nature. Dislocations remain in planar arrays and produce shear offsets on polished surfaces. Under such conditions, microcracking is crystallographic and is termed Stage I. Planar slip is at least partially reversible, with surface slip displacements disappearing and reversing direction during a strain cycle. Ideally, no strain hardening accompanies this process, except to the extent that slip bands intersect one another. The repeated cyclic slip displacement results in the accumulation of dislocation dipoles which lower the fracture energy and result in the appearance of cleavage facets along the slip bands. Gell et al. (1970) attribute the fracture to a peak value of tensile stress across the slip plane and is one reason suggested for the dependence of fatigue crack initiation lifetime on the mean or peak stress in the cycle. At a free surface, the fatigue fracture process may simply result from the progressive oxidation of a slip band when there is a component of slip normal to the free surface (Fujita, 1963). This concept appears to apply well to the superalloys below 800°C (1472°F), provided intergranular fracture does not intervene. This condition may explain the tendency for microcrack formation at free surfaces. Since slip-band cracking usually occurs on the scale of the grain diameter, it follows that large variations in grain size will be reflected in a large scatter in microcrack formation lifetime. Also the impetus to reduce grain size (as accomplished by powder metallurgy) results from the smaller individual slip displacements and slower microcrack formation.

Merrick (1974) studied LCF crack initiation in three nickel-base superalloy disk materials: INCO-901, Waspaloy, and Inconel-718. In most cases he found Stage I cracking at room temperature and 537°C (1000°F) associated with intense planar slip bands termed persistent slip bands. Crack nucleation at favorably oriented twin boundaries also

occurred. The LCF life of each alloy also was expressed in terms of a Coffin-Manson type equation relating the total strain range to cycles to failure. This agreed with the conclusion of Wells and Sullivan (1968) who found that crack formation in highly planar-slip materials was controlled by the total strain range. They reasoned that the total strain range reflected the intensity of shear displacements while the plastic strain range represented the integration of all the slip-band displacements throughout the gage length of the specimen without regard for individual intensities. Merrick also associated the development of persistent slip bands with the progression of cyclic softening. The lack of hardening on initial slip planes leads to continued deformation associated with shearing of the gamma prime precipitates. Hardening was accompanied by dispersal of slip onto neighboring planes, thus reducing the slip intensity on any one slip plane. Softening on the other hand, implied increasing concentration of cyclic slip into the active band which produces increased slip intensity usually associated with decreased initiation life (Gell and Leverant, 1970).

Defects and hard second phases can be the controlling unit features in the initiation process. The effect of MC carbide size on the cyclic life of single crystals of the superalloy blade material MAR-M200 has been studied (Gell et al., 1970). The MC carbides in this alloy are in platelet form and contain pre-existing cracks in the plane of the plate. Material containing large carbides (0.30 mm or 0.012 in.) failed upon loading above the proportional limit whereas the alloy with the smaller carbides (0.15 mm or 0.006 in.) exhibited much longer lifetimes. Cracking in material with no MC carbides showed further improvement in life: fracture originated at a casting pore. As with the carbides, when the size of the casting pores was reduced by a factor of five in the same alloy, a commensurate increase in fatigue life was obtained. In both studies, fatigue life improvement was directly related to identifying the critical unit (carbide size, pore diameter), then altering the processing to reduce the size of the defect.

With increased temperature or time, the deformation character becomes more homogeneous. Thermal activation leads to a much more uniform distribution of dislocations due to enhanced recovery processes (Gell and Leverant, 1970). In addition, there is some indication that slip on (100) planes

may occur (Kear and Copley, 1968). There is also evidence that the deformation mode is a function of strain rate at elevated temperature. At low strain rates, the dislocations may be generated initially in coarse, planar bands followed by rapid dispersal of dislocations out of the bands due to thermally activated cross slip (Gell and Leverant, 1970). Fatigue cracking under these conditions can be transgranular and normal to the principal stress axis, termed Stage II cracking, or the mode can be intergranular.

Along with changes in slip character, the strength of various microstructural features and the interaction of these features with surface oxidation are also temperature dependent. It is the combined effect of these parameters that determines the mechanism of cracking. A study by Wells and Sullivan described the crack initiation process in Udimet-700 over a range of temperatures (Wells and Sullivan, 1965, 1967, 1968). At room temperature and up to 650°C (1202°F) cyclic deformation was shown to be markedly heterogeneous. Localization of slip in intense bands led to slip-band cracking at the free surface. At 760°C (1400°F) surface cracking became predominantly intergranular.

McMahon and Coffin have described the intergranular cracking process in the superalloy Udimet-500 fatigued at 815°C (1500°F) (McMahon and Coffin, 1970.) For the case of fatigue in an air environment, the authors observed the development of surface ridging. These ridges formed at selective grain boundaries and actually were wedge-shaped oxide intrusions along grain boundaries. Later, these oxide intrusions cracked, and the cracks, surrounded by oxide and a denuded zone, propagated in a plane roughly normal to the maximum stress.

In a study on Udimet-700, Organ and Gell (1971) investigated the effect of cyclic frequency on the fatigue life at 760°C (1400°F). They reported a hundredfold increase with a frequency change from 2 to 600 cpm. At the lowest frequencies, crack initiation occurred at surface-connected grain boundaries and propagation was initially intergranular. With an increase in frequency, intergranular notches produced by oxidation and creep deformation became less important. Also, there was a transition to subsurface crack initiation at hard phases located at grain boundaries or coherent annealing twin boundary intersections and to Stage I crack propagation.⁴ With a further increase in frequency from 600 to 6×10^4 cpm, the fatigue life was reduced by a

factor of seven, and Stage I remained the predominant mode of cracking. The reduction in life was attributed to the concentration of inelastic deformation in fewer and fewer planar bands.

The situation in the case of current advanced alloys such as IN-100 and René-95, appears to be that a small number of instances of an intergranular cracking has been observed and has been traced to operating temperature in excess of the design temperature. These instances of cracking have also been premature, but it would not be reasonable to conclude that intergranular cracking is always responsible for sharply reduced lifetime. On the other hand, specimen tests generally show that the LCF initiation life of nickel-base superalloys does fall off accompanying a transition from transgranular to intergranular fracture. Obviously, there is no way to predict the behavior of new disk materials in this regard, but the most reasonable assumption would be that future disks will not be designed to operate in the regime of intergranular fracture. However, they must be expected to operate as close to this limit as possible, and thus it behooves us to understand the sensitivity of crack initiation lifetime to the intergranular mechanism.

Intergranular cracking presents a variety of possible damage mechanisms (Wells, 1979), which is the major reason few microstructural models of fatigue crack initiation have been attempted. One of the most probable mechanisms of intergranular failure involves the nucleation, growth, and coalescence of grain boundary cavities. Both constant and cyclic loading can lead to cavity formation, although the latter seems to be more effectual. The phenomenon of grain boundary cavitation* is extremely complex which makes it difficult to generalize. However, deformation at higher temperatures and low stress levels tends to produce rounded cavities distributed throughout the grain boundaries. However, deformation at lower temperatures and high stresses creates cavities at the triple junctions of grain boundaries and can lead to triple-point fracture.

Both Boettner and Robertson (1961) and Scaife and James (1968) observed an enhancement in cavity formation near the free surface of specimens. Scaife and James measured the number and size of cavities as a function of distance from

* Cavitation is the development and growth of voids at multiple locations within the structure. The dilatational stress ($\sigma_{ii}/3$); the hydrostatic component is the dominant governing variable.

the surface in a stainless steel undergoing creep at 800°C (1472°F) in air and in vacuum. They found little change in cavitation levels near the surface in specimens tested in vacuum (2×10^{-5} torr), but a substantial increase in both the size and the number of voids up to a distance of several millimeters from the surface in the air-tested steel. Surface cracking was prevalent in specimens tested in air, whereas a vacuum promoted general grain boundary cavitation. Further study certainly is needed of the relationship between intergranular oxidation and cavitation.

Dyson and McLean (1972) proposed that the service life remaining in a part subjected to high temperature stress can be estimated from the state of development of grain boundary cavitation. They based this statement on the results of experiments performed on a Nimonic alloy. More recently, Raj and coworkers (Min and Raj, 1978; Pavinich and Raj, 1977; Raj, 1975, 1977, 1978) have carried out extensive modeling of intergranular failure resulting from the nucleation, growth, and coalescence of grain boundary cavities. They have attempted to explain, on the basis of cavitation, the influence on intergranular crack propagation of hold-time, cycle shape, strain rate, and area fraction of second phase particles. Since it is, at the present time, a reasonable assumption that cavitation may be a prerequisite for fracture at grain boundaries, the processes that govern cavity nucleation and growth must be considered. However, it was the judgment of the committee that bulk cavitation (i.e., remote from free surfaces) probably would not be encountered in a turbine disk. This judgment is based upon experience with the current generation of disks, which are surface LCF-limited but not creep-limited. It is observed that surface intergranular cracking is dominated by environmental attack and occurs much more rapidly than bulk internal cavitation at practical disk service temperatures. For this reason, a discussion of the mechanisms and analytical models for bulk cavitation is included in Appendix A. The possible significance of cavitation, which has been observed in all the conventional disk alloys, is not denied but to date it has been found only at temperatures that exceed the upper limit of disk operation.

4.1.2 Transgranular Microcrack Link-up

Incongruously, one of the most important stages of LCF crack initiation has received the least attention. Whereas

individual microcracks may form after only a small percentage of life, the link-up of microcracks at the surface may occupy most of the initiation lifetime -- until the onset of macrocrack propagation. The process has often been described pictorially (e.g., Wells and Sullivan, 1964, 1965), but has not been quantitatively studied. Perhaps the major impediment has been the probabilistic nature of surface cracking. This involves, simultaneously, an increase in microcrack density and a developing interaction between neighboring microcracks, resulting in two or more cracks propagating toward each other. This process appears to involve many variables -- grain size, grain orientation, grain boundary mismatch, and their correlations with respect to the location of a potential crack path. Yet LCF crack initiation lives of smooth, polished specimens do not exhibit significant scatter. Thus the indication is that the link-up process within a large aggregate of surface grains is reproducible, although one cannot predict the location or the sequence of the microcracking.

Qualitatively, it is known that slip band microcracking is most probable in the largest diameter surface grains and on slip systems that have the highest resolved shear stress. Except for the possibility of small defects raising the local stress, the grain orientation and diameter and the total strain range govern the shear displacements on slip systems. The rate of microcracking is generally believed to increase with the range of slip band displacement. In a random aggregate of surface grains, the increase in density of microcracks with fatigue cycling should be predictable, as well as the effect of strain range. The role of mean stress in this stage of crack initiation is not known.

Once a microcrack has formed, it will eventually initiate slip and cracking in the adjacent grains. This microcrack extension process is known to be strongly dependent upon mean stress (Burck et al., 1970), although here again the physical explanation is not available. Recent microcrack propagation observations by Cook et al. (1978) appear relevant to the extension process. Miniature crack propagation specimens of IN-100 and René-95 were observed at high magnification with initial crack lengths as small as 0.05 mm (0.002 in.). Observations at high magnification were made continuously at room temperature and at 650°C (1202°F). Transgranular propagation of microcracks was sporadic, with lengthy periods of apparent arrest at grain boundaries

followed by periods of constant crack growth rate. However, on average the crack growth rates agreed well with results of macrocrack propagation tests, wherein the crack tip encompassed several grains, and it appeared that linear elastic fracture mechanics was applicable to this process. The major deviation of the microcrack behavior from conventional fracture mechanics data is the appearance of a constant microcrack growth rate near the threshold range of stress intensity defined by larger specimens. Dynamic observations of Stage I crack growth in a scanning electron microscope demonstrated the discontinuous nature of crack growth in IN-100. Upon encountering a grain boundary, crack growth was arrested for several cycles during which the crack tip was blunted by plastic deformation in the blocking grain. Eventually the crack propagated along one of these shear bands, although in some cases a microcrack nucleated ahead of and linked with the major crack. These observations appear consistent with the behavior of surface microcracks.

4.1.3 Intergranular Microcrack Link-up

The process of intergranular microcrack formation and link-up exhibits some characteristic differences from the transgranular process. Intergranular cracking often occurs preferentially on grain boundaries oriented normally to the maximum tensile stress. The crack arrest and reinitiation observed by Cook et al. (1978) at room temperature was absent at 650 °C (1202 °F) (which partly accounts for the increase in crack growth rate). Grain boundary oxidation appeared to precede cracking, but the dependence on strain or stress was not known. Preoxidation in the absence of stress may influence the tendency towards intergranular cracking. The strain to fracture of surface grain boundaries has been found to correlate with LCF initiation data (Wells and Sullivan, 1965), but the detailed mechanism has not been identified. The analogy with intergranular stress-corrosion cracking in aqueous environments has been suggested (Speidel et al., 1972; Wells, 1979). If this view is correct, the kinetics and properties of intergranular oxide films may provide the mechanistic explanation of this process.

4.2 REVIEW OF LIFE PREDICTION METHODS

Historically, the empirical study of the fatigue phenomenon began with the study of fatigue of railroad car axles. The "S-N" diagram resulted, and much of the subsequent fatigue design approach has been stress based. In the design of equipment operating at temperature levels where creep was a significant design parameter, methods were devised to sum the "damage" from the creep loading with that from the cyclic loading. A modification of this method is still incorporated in the American Society for Mechanical Engineers (ASME) Boiler and Pressure Vessel Code (1977). Twenty-five years ago, Coffin (1954) and Manson (1953) observed that inelastic strain was the dominant variable in LCF and devised prediction methods based on strain range. A slight modification of the strain-based method is to use ductility as a measure of fatigue resistance which is reduced by time and temperature; each fatigue cycle is treated as though it consumes a portion of that ductility. At temperatures where creep is significant, void coalescence and triple point cracking can be involved in initiating cracks that are later propagated by cyclic loading. A multiplicity of creep mechanisms depend upon strain rate and temperature. Consequently, some approaches assume that the inelastic strain rate is the significant variable modifying strain range. Still others emphasize the separation of time-dependent from time-independent inelastic strain. Finally, the flaw growth approach focuses upon the local processes at the crack tip. In this view, the bulk strain is the driving force but the rate of growth depends upon the present size of the crack, that is, a nonlinear damage function. Early fracture mechanics approaches were based on strain energy release rate concepts and, therefore, were restricted to nearly elastic applications. Modifications of this view have focused on plastic strain energy and J-integral concepts. The early work in fatigue revealed the statistical nature of fatigue crack initiation. In crack growth, data scatter is more associated with measurement irregularities and material properties scatter. Insofar as possible, engineers prefer a simple relationship to use in design. If the number of independent variables is large, simple algorithms cannot describe multivariable interdependencies. Complexity is usually the price of generality.

4.2.1 Elastic Stress Concentration Analysis

For rotating structures designed to operate in the elastic range, a relatively simple design procedure can be outlined. Basic analyses are conducted to assure disk burst capability and acceptable creep deformation prior to cyclic life evaluation. These are stress-based analyses.

A shell or ring finite element stress analysis is usually conducted, including thermal boundary conditions to determine elastic stress distributions. Stress concentration regions can then be examined in more detail using either standard reference works to establish a stress concentration factor (K_t) or, if geometric complexity warrants, a further finite element analysis. The approach is then to use the product ($K_t \times \sigma_{\text{nominal}}$) as a local pseudo-stress to enter an appropriate LCF curve. For relatively high stress concentrations, this approach adequately estimates notched LCF capability; however, for $K_t < 2.5$, and $N < 10^4$ cycles, nonconservative estimates may result. This stems, in part, from the inaccuracy of the $k_t \sigma_{\text{nom}}$ product in describing notch root strain when some plastic strain is present. The preferred approach is to obtain the best analysis of notch root strain either by finite element methods or possibly, from a photoelastic model. If significant notch-root plasticity is anticipated, then a plastic finite-element analysis is recommended. The basis for life prediction is that the local strain range ($\epsilon_e + \epsilon_p$) is uniquely related to crack initiation lifetime. The stress concentration approach may introduce inaccuracies since the curves of k_t and k_e as functions of stress diverge for local stresses beyond yield. The elastic approach requires constraint at notch strain concentrations (i.e., high k_t) for successful application.

4.2.2 Summation of Time and Cycle Fractions

This approach is the oldest of the high-temperature creep-fatigue life prediction methods. The ASME Code (Boiler and Pressure Vessel Code, 1977) has sanctioned a variation of this approach and design curves are provided for a very few materials. In this method for predicting life, it is assumed that at high temperature, there are two independent types of damage that develop. The first is conventional

fatigue damage ϕ_f , analogous to the damage that occurs in the fatigue process at low temperature where creep is absent. The second is a high-temperature form of damage ϕ_c , that would eventually cause failure independent of cyclic loading, analogous to that obtained in elevated temperature creep-rupture tests. The total damage, therefore, is $\phi = \phi_f + \phi_c$ * and failure occurs when ϕ is a constant, usually unity or less.

The way in which the fatigue and creep damages are determined is as follows: The fatigue damage issuing from one applied cycle is defined as the reciprocal of the number of cycles to failure (N_f) that would be obtained (in the absence of creep) if the applied cycle were allowed to be continually repeated. This is represented by the ratio $1/N_f$. That portion of the total damage assigned to fatigue and due to all the applied cycles ϕ_f , is then computed as the summation of the calculated $1/N_f$ values for each applied cycle. If all the applied cycles are identical, then the summation is simply N/N_f where N is the total number of applied cycles.

The creep damage during each time increment t , of applied stress is defined as the ratio of the time for which this applied stress is held to the time to rupture t_r . This would result if this stress were maintained until failure. An ambiguity arises in defining creep damage under compressive stress, since monotonic creep rupture does not occur under compressive stress. The Code attempts to take this into account in a conservative manner by assuming compressive stresses to be as damaging as tensile stresses. The creep damage for each applied cycle is then defined as the summation over the cycle of the ratios t/t_r . The portion of the total damage attributable to creep ϕ_c , is obtained by summing the damage for all the applied cycles. If all the applied cycles are the same, the summation reduces to N times the summation obtained for one cycle.

This approach has an advantage over other approaches in that it requires only high frequency fatigue data and conventional monotonic creep rupture data. Also, it is very easy to take different types of cycles into account in the summations required, and it is compatible with current stress

* A nonlinear form of damage summation is sometimes used, i.e.,

$$(\phi_f)^m + (\phi_c)^n = \text{constant.}$$

analysis techniques. A disadvantage of the approach is the inability to realistically account for compressive creep effects.

The code-formulating bodies have recognized this and have attempted to deal with the fact that ϕ may indeed not be equal to 1 at failure. They have suggested ways of choosing values other than 1 as required, depending upon the specific material and test types involved.

Such modifications to the time and cycle fraction method, e.g., the nonlinear form $(\phi_f)^m + (\phi_c)^n = k$, can be made to work, but in doing so, they obviate the model suggested as the basis for this approach. A second disadvantage of this approach results from the fact that since rupture life is very sensitive to stress, it is necessary to know the stresses very accurately.

If a material undergoes cyclic hardening or softening, the rupture lives calculated on the basis of monotonic behavior could be very inaccurate. There is also a built-in computational problem for cases that have long times at low tensile stresses, where it could be necessary to calculate the t/t_r ratios associated with very large t_r values. These are usually calculated from time-temperature r parameters which are themselves in question when used for extrapolating well beyond the available data.

The time and cycle fraction method has been applied to LCF data from several steels (Ellis and Esztergar, 1971; Esztergar, 1972; Campbell, 1975). The results show considerable scatter resulting from at least two causes: an apparent creep-fatigue synergism, and inordinately high stresses in the cyclic tests and the subsequent calculation of creep damage from these stresses. For the latter, damage summations of 10 to 100 were not uncommon. The method has also been applied to a nickel-base superalloy U-700, tested under thermomechanical cycling with interspersed 10-hour creep (Vogel and Carden, 1967). A linear damage summation was always overly optimistic. Damage summations of 0.05 were typical where both extensive creep and fatigue were present.

4.2.3 Strain-Range Partitioning (SRP)

Historically, the dependence of LCF lifetime on inelastic strain range has been amply demonstrated by Manson and Coffin over a period of 25 years. Their studies have led the field of high-temperature design life prediction.

They have produced an impressive body of data, analytical concepts, and design rules, beginning with the familiar power-law relationship between plastic strain range and cycles to failure. An exhaustive presentation and critical evaluation of these methods is contained in Carden et al. (1977).

With the appreciation of the effects of time, temperature and cycle shape on fatigue lifetime came a departure in the development of design procedures, with Manson pursuing inelastic strain and its type as the controlling variables and Coffin pursuing total strain-range frequency.

Manson early associated time-dependent fatigue lifetime with intergranular cracking and reasoned that this damage mechanism was intimately associated with time-dependent inelastic strain (i.e., creep or relaxation) whereas time-independent plasticity was accompanied by transgranular cracking. Manson also conducted cyclic creep tests between fixed strain limits and found that the lifetime did not correlate with monotonic time to rupture in a creep test (i.e., t_r in the creep-rupture test). This led to tests of four simple uniaxial cycle types involving creep and plasticity in the increasing and decreasing halves of the strain cycle. Manson found different characteristic cyclic life dependencies on inelastic strain range for the four combinations, referred to as pp, pc, cc, and cp. For example, pc denotes plastic tensile strain reversed by creep strain in compression.

The conceptual basis of this separation of strain cycles is appealing. Manson suggests that intergranular cracking is caused by tensile creep strain, which is accommodated partly by grain boundary sliding and triple-point cracking. This sliding is not reversed by high strain rate or low temperature strain reversal in compression, since slip mechanisms and grain distortion are favored. Thus on successive tensile half cycles of straining, progressive intergranular cracking occurs, leading eventually to failure. Reversing the cycle results in a much slower rate of damage accumulation. Cycles of reversed creep could result in some of the cavitation damage in tension being "healed" in compression.

Manson generalized this concept into a procedure for evaluating any strain-time-temperature cycle, which he names strain-range partitioning (SRP) (Manson, 1972). There are various procedures for "partitioning" the cycle, depending upon the accuracy required. Also the four "basic" material

lifetime correlations must be developed and the net damage resulting from the relative contributions of these components computed. The influence of time enters through the partitioning process, which may be done experimentally or analytically (from constitutive behavior models for the material). Until recently there was no attempt to include other effects of time at elevated temperature such as embrittlement, oxidation of grain boundaries or stress-corrosion mechanisms in an aggressive environment. These effects are now accommodated (Manson and Zab, 1977) in the method through time-dependent ductility intercepts for the basic lifetime correlations.

Thus the creep and tensile ductilities for the four correlations would be measured for the specific time, temperature, and environment anticipated for the structure. This procedure would appear to account for bulk effects of service exposure, but not necessarily surface-related effects such as stress-corrosion cracking, which depends upon strain rate in addition to time, temperature, and environment. The distinction is important, and is an issue in the extrapolative ability of SRP.

The applicability of SRP to superalloys has recently been the subject of a major international cooperative investigation under NATO auspices (AGARD, 1978). While the success of the method in establishing bounds of a factor of two on life for a wide variety of materials and applications was generally acknowledged and several examples were given of excellent quantitative agreement between test results and predictions, certain difficulties were experienced by some of the participants. It was concluded, for example, that SRP could not, in its current formulation, be suitable to predict the effect of mean stress on lifetime, or be used to correlate crack propagation. In several cases, such as René-95 and IN-738 LC, the imposition of tensile strain hold periods resulted in significantly longer lives than found in tests with no hold time, an effect attributed to mean stress. The applicability of SRP apparently varied markedly with the material and test conditions. One source of this variability was thought to be the diversity of micromechanical mechanisms. In particular, this would appear to involve the importance of crack propagation relative to microcrack formation within the definition of lifetime. Some investigators found SRP inferior to stress-time in predicting the results of long hold-time tests. Normalized tensile

ductility was found to be a useful concept when predicting the debit associated with multiaxial stress fields and environmental effects, although the physical basis of the correlation of LCF results with ductility was questioned.

Several investigators found it difficult to partition inelastic strain. This is especially so at long lives connected with anelasticity, primary creep on load reversal, and very small time-dependent strain components. The interpolation or extrapolation of test results requires either a time-dependent, cyclic constitutive model for the material or an experimental simulation of the structural response with a test specimen. At the strain ranges encountered in turbine disks, the inelastic strain components are very small -- on the order of 10^{-4} -- so accurate computation or measurement is very difficult. For this reason, the accurate partitioning of strain range was concluded by some investigators to be impractical. Although it was pointed out that the problem of constitutive behavior is common to all other lifetime prediction methods, the sensitivity of SRP to the time-dependent strain computation was regarded as an impediment to its application to turbine disks.

In the summary of the AGARD report (1978), Manson stresses the usefulness of SRP in establishing bounds of lifetime for a specific material and design application. In the case of turbine disk design, the manufacturers already possess empirical data and lifetime correlations for bounding purposes; they need a method that minimizes the uncertainty of the prediction of high-temperature lifetime. It would appear, therefore, that SRP requires further specialization to the nickel-base superalloys. The method should include mean stress effects and, if possible, be based upon total strain. Inclusion of mean stress should allow the prediction of crack extension lifetime, i.e., from the scale of the grain diameter to 0.8 mm (0.031 in.). SRP should be applicable to crack propagation, where the analytical partitioning of inelastic strain at the crack tip may provide greater accuracy of crack growth prediction.

4.2.4 Frequency Modification and Separation

If all of the variables are held constant over the life of a structure except strain range, the cyclic life can be simply related to the cycles to failure by two material constants, e.g., b and C in the equation

$$\Delta \epsilon_p N_f^b = C$$

where $\Delta \epsilon_p$ is the plastic strain range (the internal width of the stress-strain hysteresis loop); N_f is the number of cycles to failure, b and C are constants dependent upon the material, temperature, frequency, etc. The simple relationship will not describe fatigue crack initiation under many different cycle frequencies, wave shapes, or temperatures. The methods devised by Coffin, described in detail by Carden et al. (1977, Chapter 3), are formulations devised to include these other effects observed at elevated temperature. Coffin has presented several methods; he recommends using the simplest form that will perform the prediction.

All of the methods presented have the following qualities:

- They are phenomenologically based and require testing to derive the constants of the equations relating dependent to independent variables.
- The algorithms are structured so that strain range is the governing independent variable.
- The method requires an analysis of the stress-strain hysteresis loop to determine the inelastic strain, but can also be applied on the basis of total strain.
- N_f represents the cyclic life for a defect to originate and grow to about 2.5 mm (0.1 in.) in a smooth 6.4 mm (0.25 in.) diameter specimen.
- If temperature is a variable, experiments are required over the range at discrete (isothermal) levels.
- As now formulated, the damage is a scalar quantity and linearly accumulates during the test period.

The more complex methods that deal with time-dependent fatigue and wave shape effects have a power function on cycle-period and do not predict a lower bound on life. In other words, if the cycle period continues to increase, the cyclic life continues to decrease. This finding is also predicted from parameter studies of fatigue crack growth (Carden, 1973).

Coffin's methods have been tested for their validity under isothermal, uniaxial, time-dependent and time-independent constant cycle boundary conditions. Nearly all of the results can be correlated to be within a scatter band of ± 200 percent about the mean cyclic life.

Certain hypotheses of the latter methods remain to be tested experimentally:

- Testing at intermediate and high homologous temperature. The homologous temperature is the temperature in question divided by the melting point temperature, both in absolute degrees.
- Cyclic temperature.
- Extreme conditions of tension-going to compression-going time (specific effects of hold conditions).
- Evaluation of disk materials for sensitivity to wave shape and mean stress.

Certain aspects of Coffin's methods are contrasted to the formulations proposed by Manson:

- Time is used directly in Coffin's later formulations.
- N_{tr} is the fatigue life for the condition where the elastic and plastic strain components are equal. The transition life N_{tr} , has significance in establishing regimes for dealing with the problem.
- The mean stress of the cycle is important, especially at lives greater than N_{tr} and there are procedures for dealing with mean stress.
- Elastic strains are used directly, and if the plastic strains are less than 10^{-4} they need not be calculated.
- The method does not infer that the time-dependency of fatigue crack initiation is uniquely related to the strain rate (or creep strain) effect. Environmental, metallurgical, and other time dependencies are accounted for through the terms:

$$\left(\frac{1}{\nu}\right)^{k-1} \quad \text{cycle period, or}$$

$$\left(\frac{\nu_t}{\nu_c}\right)^{-\beta C} \quad \frac{\text{tension-going (t) time}}{\text{compression-going (c) half-cycle}}^*$$

where: ν = frequency (Hz)

k, β, C - material constants to be determined.

One criticism of Coffin's later methods is the requirement of a large data base for evaluation of the parameters. A large data base does provide a statistical sample for evaluation of the distribution function of cyclic life. Such knowledge is required for quantitative calculation of reliability.

In the frequency modified fatigue equations, the damage is assumed to depend upon frequency as well as strain range. There are separate frequency effects for elastic and plastic strains. This method uses direct material data, recognizes time-dependency, is easily applicable to design, makes a simple transition from high frequency to low frequency conditions, and is applicable to the low plastic strain regime. The method does not distinguish between wave shapes (tension hold versus compression hold). The data base is relatively large, and it is not easily applied to cyclic thermal problems.

The frequency separation method was designed to determine the sensitivity of the material to wave shape and hold time. In one version of the method involving the use of the tension-going time, damage is assumed to occur predominantly while the strain rate is positive. A second version tacitly recognizes that damage may accumulate in both directions of the cycle; although at different rates. The frequency of the frequency-modified equations is replaced by the reciprocal of twice the tension-going time. The tension-going time is that period for which the strain rate is positive, that is, unloading from a maximum compression and loading to a maximum tension.

* The ratio of the period of the cycle during which strain is increasing to that in which it is decreasing.

This method accounts for time-dependent damage during this period and does not explicitly recognize wave shape effects. It assumes the damage to be a power function of the tension-going time. This method is useful only for materials where tensile hold is much more damaging than compression hold. The method is better for life prediction for alloys more sensitive to tension hold periods. It is not helpful for alloys more sensitive to compression holds.

4.2.5 Ostergren's Method

Ostergren (1976) proposed a time-dependent LCF life prediction model using hysteretic energy (the integrated damage under the trapezoidal waveform equation). The net tensile hysteretic energy of the fatigue cycle can be approximated by the damage function $(\sigma_t)(\Delta\epsilon_p)$. Thus peak tensile stress and plastic strain range are explicitly incorporated into the failure criterion. The time dependency is accounted for using Coffin's frequency separation and frequency modified equation. Ostergren applied his method to isothermal laboratory data from several materials with good correlation. Much work would be required to integrate this life prediction method into a confident design procedure, mainly because it has been applied to a small data base.

4.2.6 Majumdar's Method

The damage-rate approach (Majumdar 1977; Maiya and Majumdar 1977; Majumdar and Maiya, 1976a, b) is a phenomenological approach. It is based upon frequency modification that has to date provided reasonably accurate estimates for the high temperature fatigue life behavior of smooth (axially loaded) test specimens manufactured from types 304 and 316 stainless steels, Incoloy-800, 2½Cr-1Mo steel, and Inconel-718. The damage mechanism assumed in the development of the approach is microcrack propagation; both the transgranular and intergranular modes have been described.

The basic equation below describes the assumed relationship between the "damage" rate and the controlling mechanical factors, the plastic strain (ϵ_p) and the plastic-strain rate ($\dot{\epsilon}_p$). The damage rate is the \dot{D} time rate of increase of the characteristic length, a , of a microcrack.

$$\frac{d(\ln a)}{dt} = \begin{cases} T |\epsilon_p|^m |\dot{\epsilon}_p|^k & \text{tensile loading} \\ \text{or} \\ C |\epsilon_p|^m |\dot{\epsilon}_p|^k & \text{compressive loading} \end{cases}$$

The equation contains four explicit constants, T , C , m , and k , which are evaluated subsequent to its integration via a comparison with strain-life test data. Integrating this equation for the simplest case, where the cyclic loading is both symmetrical and applied at high frequency (no hold-times), results in the following estimate of the failure cycles:

$$N_f = \frac{m+1}{4A} \left(\frac{\Delta\epsilon_p}{2} \right)^{-(m+1)} \dot{\epsilon}_p^{1-k}$$

where

$$A = \frac{\frac{C+T}{2}}{\ln\left(\frac{a_c}{a_o}\right)}$$

The N_f equation above provides the basis for determining the constants A , m , and k from failure data by least squares fitting procedures.

Integration of the basic equation for other cyclic loading conditions allows one to establish a life equation similar to the equation for N_f . Note from the two equations above that the constants C and T from the basic equation and the constants of integration (a_c the final microcrack size, and a_o the initial microcrack size) are collected by the constant A . For the materials considered to date, m has been found to be independent of temperature and strain rate conditions. However, both A and k must sometimes vary in a precipitous and discontinuous fashion as a function of strain rate to account for transitions in microcracking from a transgranular (high loading frequency) to an intergranular (low loading frequency) propagation mode. While the constants have been shown to be independent of temperature for the experimental studies conducted to date, the

developers suggest that this assumption should be verified for each new set of data.

The most recent reports describing the application and development of the Majumdar method will be found in the Materials Science Division Reports of Argonne National Laboratory or in the Mechanical Properties Quarterly of Oak Ridge National Laboratory (Metals and Ceramics Division). Minimum basic advances required to bring this model closer to predicting the fatigue life of high temperature turbine disks are:

- Further experimental justification for the basic equation 1 hypothesis in the region of interest. Specifically, data are needed to extend the model to predict life behavior where lower cyclic (plastic) strain amplitude loadings are controlling failure behavior.
- Data and analysis documenting tensile and compressive hold-time effects need to be developed.
- A modeling procedure needs to be made available for relating the hypothesized microcrack life behavior observed in smooth test samples to that observed in notched members.

4.2.7 Applicability of Linear Elastic Fracture Mechanics Methods to Description of LCF Behavior

Efforts to predict fatigue lifetimes have ranged from empirical to physically based models, with varying degrees of success; but no totally acceptable method accounts for the many known factors that influence cyclic life. One difficulty is the identification of a single parameter to describe the fatigue damage process in a mathematically convenient manner. Observations that physical damage occurs at an early stage in nickel-base superalloys have been made by several researchers. For example, Coffin and Henry (1972) using a replication technique, showed nucleation and early crack growth in the first few percent of cyclic life of Inconel-718 and René-95 specimens. Also, it has been widely recognized that with increasing tensile strength, "defect sensitivity" increases and fatigue nucleation sites are

frequently associated with structural imperfections. Consequently, some proponents of fracture mechanics have attempted to describe the fatigue mechanism entirely in terms of crack propagation, using, for convenience, an initial flaw concept. The flaw size is either back calculated from fatigue data or identified as a structural parameter, e.g., characteristic grain size.

Some early work by Salt (1973) considered the former approach, identifying such a flaw as a statistically inherent parameter, to derive other fatigue life curves. Flaws that were smaller than the statistical flaw were ignored, while larger flaws were analyzed by conventional fracture mechanics macrocrack growth methods. Two U.S. patents (Salt, June 1975, September 1975) were awarded recognizing such a life prediction procedure. More recently, other researchers have attempted to further develop the approach, notably Mowbray (1976) who extended the analysis to nonlinear fracture mechanics, with some success in describing A355 steel cyclic capability.

The total integration of statistical or mathematical flaws to estimate lifetimes of components is not widely used in industry. Several pitfalls limit its universal use and require additional study and verification before the needed confidence in its utility is established. The potential problems are discussed briefly below.

Determination of Initial Flaw Size. Initial flaw size is usually back calculated from a low endurance (2000-5000 cycle) LCF life point, generally from constant strain amplitude tests. The laboratory specimen is tested under typically uniaxial loading and the strain life fatigue curves are generally reproducible in most low-cycle fatigue laboratories. However, frequently the stress intensity solution is not precise, merely proportional to the accurate value. This is so because $K = \sigma \sqrt{\pi a}$ for strain cycle data has the stress term represented by $(E\epsilon)$, whereas the integration to obtain cyclic life utilizes macrocrack growth rate data with a load-base stress in the stress intensity formulation. The defect size identified in this manner would not be readily translatable to other loading cases. Additional work has been performed and a more accurate approach can be postulated, but a detailed stress analysis is yet required.

Physical Basis. While the initial flaw concept is merely a convenient parameter and not a physically meaningful value, problems can arise for lower strength, ductile materials in that the calculated initial flaw size is often in the vicinity of 0.4 mm (0.015 in.) in depth. Natural flaws in the material, however, may not be detectable at the crack origin on one-tenth this scale. The predicted initial flaw depth for high-strength alloys may be in the range of 0.025-0.125 mm (0.001-0.005 in.). This is more in line with naturally occurring flaws in these materials but still an overestimate. The problem with these calculations is that at the earliest stage of crack initiation, the stress intensity of the flaw is below the threshold for crack propagation.

Fatigue Nucleation Sites. Current turbine disk alloys, in their wrought form, generally exhibit surface nucleation sites when subjected to fatigue loading. However, the advent of powder metallurgy as a manufacturing process has introduced the possibility of subsurface nucleation competing with the free surface. In fact, a trend is emerging that for low cyclic lifetimes (i.e., high strain ranges), surface nucleation sites dominate whereas for longer lives, subsurface crack initiation frequently occurs. The latter would present special problems in NDT inspection and reliability prediction. An initial flaw calibration based on a single observation (life) is unlikely to accurately describe the entire fatigue curve, and translation to components, notch details and complex loading is equally problematic.

Fatigue-Crack Propagation Compatibility. Application of the inherent-flaw concept to life prediction of components subjected to multiaxial fatigue presents possibly a further problem. Fatigue lifetimes to crack-initiation under biaxial stress have been shown to be relatable to the effective stress; crack growth data are generally described in terms of the crack normal stress. A situation can occur in which a part can be judged to have a longer crack propagation life than total fatigue life, using the two criteria. To enter such a situation from the inherent flaw (calculated from uniaxial data) standpoint may result in nonconservative life prediction.

Summary. While some technical problems are apparent in the total integration approach to fatigue life prediction, particularly the translation from laboratory specimen to component geometry and stress states, certain features remain attractive. It is convenient to think of the damage process as crack progression, or plastic zone size, rather than an empirical expression with no physical basis. The inherent flaw method has been used with some success in describing cumulative damage effects (e.g., Hurchalla et al., 1975) and its development should be continued to confidently define areas of application.

4.2.8 Application of the J-Integral Concepts to Microcracking

The J-integral, described in Appendix B, has shown considerable promise as a parameter for correlating fatigue crack growth rates under elastic-plastic conditions. For A533B steel, Dowling has been able to show that a J-integral concept can be used:

- To relate the crack growth rate behavior of macrocracks in compact (CT) specimens subjected to elastic-plastic cyclic conditions to that generated under typically linear elastic fracture mechanics conditions (Dowling and Begley, 1976),
- To correlate elasto-plastically generated fatigue crack growth rate behavior in CT specimens with that generated in center-cracked specimens (Dowling, 1976), and
- To compare the crack growth rate response of small surface cracks ($0.8 \text{ mm} < \text{length} < 3.8 \text{ mm}$ [$0.031 \text{ in.} < \text{length} < 0.15 \text{ in.}$]) in smooth fatigue test specimens subjected to quasi-plastic strain control conditions with that exhibited by macrocracks (Dowling, 1977).

Dowling's successful experimental correlations provide credibility to the LCF-crack growth model correlation studies suggested by Mowbray (1976, 1977). Both Mowbray and Dowling use an operational definition of ΔJ , the range in the J-integral parameters, which in a sense is equivalent

to stating that the fatigue cracking process is controlled by the work done in opening the crack surfaces and extending the crack an increment da . As Mowbray states, "The implication is that J defines the stress and strain fields near the crack tip during the loading half of the cycle despite intermittent unloading."

Mowbray's principle contribution has been to develop a framework for making a life calculation based on J -integral concepts. It applies specifically for uniformly loaded, smooth (initially uncracked) elements (A533B steel) subjected to plastic strain cycling. In developing the framework, he showed that similarities exist between the Coffin-Manson strain life equation and the life equation based on J -integral concepts. In demonstrating these similarities, Mowbray assumed that the initial crack size present in the smooth elements was on the size of a microstructural feature (grain size, pore diameter, etc.). Specifically, the assumed depth of microcrack was 0.025 mm (0.001 in.). The final microcrack depth was assumed to be 0.64 mm (0.025 in.). While the initial and final microcrack depths are not critical to the comparison between the Coffin-Manson and J -integral based life equations, the choice of initial and final microcrack depths can result in substantial differences in the calculated life.

Neither Mowbray nor Dowling have, to date, published estimates of crack growth life using the J -integral concept. Probably there are two reasons for this: all data thus far generated have been used to evolve the methodology, and the relationships between remote mechanical parameters (stress, strain, plastic strain, strain energy, etc.), crack length, and the ΔJ parameters presently are first order assumptions.

The J -integral does provide a mechanical transfer function that can be used to relate fatigue crack growth rate data between various geometries containing macrocracks. However, the calculational procedure for determining the crack growth behavior exhibited by small cracks in the vicinity of notches must await both additional experimental data and analysis. Insofar as the applicability of the previously described J -integral concepts to advanced turbine disk alloys can be demonstrated, two additional steps will be required to improve their readiness for design. The methodology must: correlate high temperature fatigue microcrack growth behavior in these alloys for various geometrical features, and account for time dependencies in crack initiation and crack growth phenomena.

4.2.9 Methods Combining LCF Relations and Crack Propagation

Carden et al. (1977, pp. 148, 414) concluded that:

Although the computational capability and certain mathematical representations of stress-strain behavior of materials are available, the more fundamental approach of analytically treating failure as a physical process of initiation and propagation of a crack has not been undertaken. . . . Our goal should be to consider fatigue damage as a progressive process of crack growth utilizing phenomenological relationships to determine the growth rate. The fatigue life is then determined as some predetermined end point of crack growth. We know that the physical process of damage is fatigue crack propagation. Sooner or later this physical process will be translated into the language of a design basis.

Gamble (1977) and Cruse and Meyer (1979) have shown one method for combining a crack initiation and propagation methodology into a design procedure. Strain-based LCF crack initiation correlations are employed to define the lifetime required to produce a crack that will then propagate according to a length- and strain-dependent crack growth law. Carden et al. (1977) showed that the use of N_f from smooth bar specimens 6.35 mm (0.25 in.) in diameter could mislead the reviewer as to the effect of hold time conditions. If, however, the crack growth from about two grain dimensions to about 3.2 mm (0.125 in.) were used to delineate the crack initiation period from the growth period, the effect of hold time could be more positively identified, understood, and utilized in a design method. More specifically, the failure lives under compression hold and no-hold conditions were practically identical. The tension-hold life was much less. The fatigue crack growth rate (da/dN) for all of the hold condition tests was more than double the no-hold condition tests. The period of fatigue crack growth, as evidenced by striations studies of the fracture surface, was about one-half the total life. The total life, N_f , in all LCF tests is therefore the sum of cycles to initiate a crack to 2-5 grain dimension plus the cycles to grow the crack to

a critical size and fracture the specimen (or terminate the test).

4.2.10 Summary

The five major life prediction methods discussed in this chapter are: strain-range partitioning, frequency modification, frequency separation, fracture mechanics, and time and cycle fraction summation. Each of these methods contains certain strengths and weaknesses. Strain-range partitioning is difficult to apply for plastic strains less than 10^{-4} . Constitutive behavior is required for design analysis, and a method for handling mean stress is not clear at this time. The frequency separation and frequency modification methods seem to be better designed to deal with environmentally dominated damage processes. All of the above should involve crack initiation for flaws out to about 2-grain dimensions. Beyond that, some form of growth law should be employed.

There are many unanswered questions regarding the application of fracture mechanics to the general problem of time-dependent, elevated temperature crack growth. Conventional fracture mechanics methods do not appear applicable to the growth of flaws less than 0.1 mm (0.004 in.) in length. The time and cycle fraction summation method has been used by ASME for many years and is relatively easy to use. It is one way to factor the effects of time into LCF behavior.

No published reports of applications of the ASME time and cycle fraction method to turbine disk design were discovered by this committee. A review of the utilization method for certain hold-time low-cycle fatigue data shows large differences in predicted and observed lives (Campbell, 1971). We do not recommend its further development.

Chapter Five

CONCLUSIONS

5.1 MICROMECHANISMS

The committee concluded from its review of microstructural mechanisms that the most significant effects of temperature and time on fatigue crack initiation were oxidation of grain boundaries, viscoplasticity and cavitation, in descending order. Grain boundary oxidation and cavitation only occur above a critical combination of temperature and hold time; thus their importance depends upon the assumed duty cycle of advanced turbine disks and the relative grain boundary strength of the alloys. The committee was unable to resolve fully the extent and significance of intergranular cracking in current advanced disks of IN-100 and René-95, partly as a consequence of the proprietary nature of this information. Based upon the experience with conventionally-wrought and recrystallized superalloys and limited data on the newer alloys, it was concluded that disks would not normally be designed to operate in the regime of intergranular cracking since premature crack initiation could result. On the other hand, it was apparent that some instances of intergranular cracking have occurred as a result of overheating and that in any event disks would be operated at as high a temperature as possible consistent with acceptable lifetime.

Therefore, the committee concluded that some degree of intergranular cracking would be expected and that an understanding of the limits of the material and the sensitivity of lifetime to this mechanism was necessary. It was further concluded that improvement in powder metallurgy processing would result in resistance to grain boundary sliding and cavitation, but that oxygen diffusion and oxidation kinetics would be accelerated by the increased service temperature.

In either case the significant effects of viscoplasticity would be time-dependent shakedown of the mean stress around strain concentrations, increase in the range of inelastic strain in the cycle (or in the range of crack tip displacement once a microcrack formed), and homogenization of slip, again in descending order of importance.

The committee concluded that the evidence to date supports the applicability of crack growth mechanisms from a crack length of about 0.1 mm (0.004 in.) to the termination of initiation, defined to be a length of 0.8 mm (0.031 in.), in the current advanced alloys, but not in earlier coarse-grained disk alloys.

The statistics of microcrack formation and link-up appear to present a formidable analytical problem; yet the crack initiation lifetime of carefully prepared LCF specimens does not reflect an inordinate amount of scatter (assuming the absence of significant flaws). Thus it is likely that the initiation of surface slip-band or grain boundary cracks is highly reproducible if a statistical ensemble of grains is subject to identical loading histories, and our current inability to predict lifetime from microstructural models may result from the lack of an adequate conceptual model rather than an inherent variability of the mechanism.

5.2 STRESS AND STRAIN ANALYSIS

The committee determined that the level of mean or peak stress in the cycle was of great importance and that methods to predict or measure its shake-down level were essential to reduce the uncertainty in life-time prediction. The macroscopic residual stress could be calculated adequately with existing finite element computational methods, but some concern was voiced over the adequacy of time- and cycle-dependent constitutive models. The microscopic residual stress resulting from manufacturing (machining and surface finishing), present to a depth on the order of 25-100 microns, is known to govern initiation lifetime substantially but its initial value and variation with service exposure defy current capabilities. Residual manufacturing stresses were concluded to be significant contributors to uncertainty in lifetime prediction; i.e., variability in residual stress would be a source of scatter in crack initiation lifetime.

5.3 TIME-DEPENDENT LCF MODELS

No existing model was found to meet all of the requirements dictated by design considerations. The committee concluded that models requiring explicit measurement or analysis of inelastic strain range would be inappropriate for turbine disks because the inelastic component is very small relative to the total strain range. This conclusion was based upon the reported behavior of IN-100 and René-95, but was believed applicable to advanced alloys, although the maximum allowable strain ranges and temperatures are not known for these materials. In addition, the analytical model would be required to include the mean or peak value of stress in the cycle and the gradient of manufacturing-related residual surface stress to allow exact application of specimen data to structural model and component tests. The committee further concluded that models which identified incremental damage with microcrack growth and link-up were to be preferred from the standpoint of physical relevance.

Crack propagation methods were considered to offer some potential advantages over current initiation models, especially in the case of very fine-grained, powder-metallurgy superalloys. The advantages are, first, that the damage mechanism is based to a large extent on metallurgical observations; second, that the crack growth law can be made explicitly mean-stress dependent and can be integrated over the stress gradients at a strain concentration and (potentially) the residual manufacturing stress; third, that the crack growth law can be modified either empirically or analytically to account for cycle shape. It was recognized, however, that some fraction of the lifetime, consisting of the formation and link-up of individual grain-diameter-size microcracks, could not be predicted currently by fracture mechanics. In the case of coarse-grained superalloys (relative to the 0.8 mm criterion), the applicability of crack growth laws would be marginal, but for fine-grained material, especially in the presence of processing flaws such as microporosity or inclusions, perhaps the major portion of the lifetime could be represented.

In addition to the problem of microcrack formation and link-up, certain obstacles to the application of fracture mechanics may exist, chiefly involving the mechanics of small flaws at high-strain amplitudes. Analytical techniques

for such situations are available but have not been verified for advanced disk materials and service environments. Scatter in growth rates may be especially severe where the crack front extends across only a few grains, and the fracture mechanics approach obviously breaks down if the crack length (or depth) is less than the threshold value for fatigue crack growth for the appropriate deformation mode.

The significant effects of viscoplasticity on slip band microcrack growth appear to be tractable as a first approach by a strain-rate-dependent flow stress. However, the effects of time and temperature on grain boundary crack growth are probably more analogous to stress-corrosion cracking or corrosion fatigue. Both the concept of a time-dependent surface ductility, used in conjunction with the strain-range partitioning method, and the incremental damage form of the frequency-modified fatigue life approach appear capable of representing intergranular cracking when applied to cases where total strain and mean stress are the independent mechanical variables.

Thus, even within the restricted family of advanced disk alloys it may be that no one single method suffices for all circumstances. The presence of processing flaws might strongly favor a fracture mechanics approach. If operation is restricted to the transgranular cracking regime, a statistical microcrack link-up model might be best, with either the strain-range partitioning or frequency-modified fatigue life required in the event of intergranular crack initiation. At the present time the subject of high-temperature fatigue of superalloys has not received detailed experimental attention. There are insufficient data to adequately compare predictive models with actual test programs or service life experience.

Chapter Six

RECOMMENDATIONS

The committee recommends first that the lifetime prediction method for fatigue crack initiation be made as consistent as possible with the microstructural mechanisms of fracture. Therefore, it is considered important to determine the mode of cracking for the alloy -- transgranular or intergranular -- at the upper limit of design service temperature, including any statistically significant overtemperature, and the relative importance of microcrack formation, linkup and propagation in the formation of the 0.8-mm (0.031-in.) crack length selected for this study. The preferred method would be suggested by these observations and would fall into one of several categories as described below.

In any case it is reasonably certain that the lifetime will be governed by mean stress and total strain range, and that the method must, as a minimum, contain explicit dependence on these variables. In the short term, it is recommended that the feasibility of reformulating the principal lifetime predictions methods, strain-range partitioning and frequency-modified fatigue life, be determined. The committee also encourages the representation of incremental damage as the growth of microcracks and the recognition of the nonlinearity of damage accumulation with increasing number of duty cycles.

The committee further recommends the longer-term adaptation of advanced fracture mechanics procedures to superalloys to incorporate the effects of viscoplasticity (for transgranular crack growth) and intergranular oxidation-assisted crack growth. The latter mechanism requires more research, since the kinetics of intergranular oxidation and the simultaneous effects of stress and strain have not been determined; modeling along the lines of intergranular stress-corrosion cracking is recommended.

In addition to the fracture mechanics approach, which the committee considers important in connection with crack growth from pre-existing flaws regardless of the mode of crack initiation, two additional paths of development are recommended. First, the rate-controlling step in the link-up process, by which microcracks extend to neighboring grains during the formation of a macrocrack, should be identified and statistically modeled for transgranular or intergranular crack initiation, as appropriate. Second, the accuracy with which the mean surface stress can be predicted in the vicinity of a strain concentration by existing models of constitutive behavior should be evaluated and improved if necessary. Such models should not be directed toward the detailed analysis of cyclic inelastic strain but toward predicting the stationary mean stress distribution after the shakedown process has been essentially completed.

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Appendix A

CAVITATION EFFECTS

A.1 NUCLEATION OF CAVITIES

Surface energy causes a small void to shrink and disappear. The stress tending to close the void is inversely proportional to the radius of the void. This stress must be opposed by a tensile stress of magnitude greater than a critical value if the void is to be stable. Raj (1978) has estimated the time required to form a vacancy cluster which is almost stable under a given tensile stress, σ , normal to the grain boundary. He finds that the "incubation time" to form a cluster of critical size is strongly dependent on σ . Raj and Ashby (1975) have calculated the rate at which stable voids are formed from such clusters. Since the rate is proportional to

$$\exp \left(- \frac{C}{\sigma^2 kT} \right)$$

where C is a material constant and k and T have the usual meaning, it appears as if nucleation occurred only above a threshold stress. A smaller critical volume is required if the cavity is located next to a second phase particle on the grain boundary, so nucleation should be seen primarily at such particles and spacing between cavities may be identified with the particle spacing. Nucleation next to particles and an incubation for nucleation were observed by Fleck et al. (1975) in a copper-base alloy. Raj (1977) interpreted the data of Fleck et al. and of Sadananda and Shahinian (1977) on the stress threshold effect, and incubation times required for creep crack growth in Inconel 718 on the basis of cavity nucleation. It should be noted that

Sadananda (1978) has used an entirely different mechanism, based on crack growth by diffusion, to interpret the Inconel-718 data.

A.2 CYCLIC LOADING -- EFFECT OF CYCLE SHAPE

Large transient stresses acting normal to a grain boundary can be produced during grain boundary sliding. Min and Raj (1978) calculated these stresses as a function of the loading cycle. It was assumed that diffusional accommodation around grain boundary particles was the rate limiting process for sliding. Min and Raj find that a compression hold followed by quick reversal to a tensile hold was an optimum loading pattern to produce the large tensile normal stresses over an appreciable time period which was needed to permit the nucleation of cavities and their growth to a stable size before fatigue cycling is resumed. The maximum tensile stress was $2\Delta\sigma_s/f_b$, where $\Delta\sigma_s$ is the magnitude of the hold stress and f_b is the area fraction of grain boundary particles. This stress will persist over a characteristic time which, for a 316 stainless steel tested at 625°C (1160°F), was calculated to be about a minute.

Although the cycle just described may nucleate cavities throughout the specimen and allow them to grow to a stable size, it was observed that their continued growth under a prolonged hold was much slower than the theory based on vacancy diffusion growth predicted. Min and Raj attributed this fact to the necessity for the accommodation somewhere in the specimen of the material flowing from the growing cavities. Somehow a large strain must be produced in the region of the grain boundaries if the voids are to continue to grow rapidly. Thus while nucleation and early growth were stress controlled and time dependent, growth to coalescence was strain controlled.

A.3 INTERGRANULAR FATIGUE CRACK PROPAGATION RESULTING FROM THE GROWTH AND COALESCENCE OF BOUNDARY CAVITIES

If cavities produced during a tensile hold period are to grow until they coalesce, a large displacement across the grain boundary must be provided, i.e., the strain must be localized in the region of the boundary. Min and Raj

(1978) and Pavinich and Raj (1977) proposed an empirical equation for the displacement U_n required for cavity coalescence:

$$U_n = 0.23 \bar{\lambda} e^{4/(n-1)}$$

where $\bar{\lambda}$ is the average cavity spacing and n is the stress exponent in power law creep.

The tip of a growing fatigue crack can provide the necessary displacement. Since the crack tip opening displacement (CTOD) is of the order of da/dN (the extension per cycle of a crack of length a), and da/dN generally increases as the crack lengthens (provided the stress is not decreasing abruptly), eventually the CTOD will become large enough to enable cavities (produced during a preceding tensile hold) located in the tip region to grow and coalesce during the tensile portion of a fatigue cycle. Min and Raj (1978) cited the results of experiments on the crack growth rate in 316 stainless steel after tensile hold. At first the value of da/dN was the same as before the hold, but when a , and thus da/dN , reached a critical value, the crack began to grow much more rapidly. Min and Raj explained that when da/dN , and thus the CTOD, reached the magnitude of the displacement U_n (or when the displacement fell on a grain boundary near the crack tip, which can also attain the value U_n), the cavities started coalescing. It was observed that when da/dN started to increase, crack propagation switched from a transgranular to an intergranular mode. A long compressive hold, which presumably sintered the cavities, brought da/dN back to the pre-hold value and crack propagation returned to transgranular.

Thus the enhancement of crack propagation rates in stainless steel can be explained on the basis of cavity nucleation and growth to stable size during a tensile hold. This is followed by coalescence of cavities in the vicinity of the crack tip viscoplastic zone when cycling resumes. The cause for the degradation of life by compressive holds observed in some other alloys was not so clear in terms of this model. Raj (1977) attributed the adverse effects of compressive holds to the development of a mean tensile stress as a result of the hold. Such a stress later would promote cavity growth.

A.4 STRAIN RATE AND FREQUENCY EFFECTS IN HIGH TEMPERATURE FATIGUE

If material is fatigued near the yield point, cavities formed at triple junctions can lead to cracking. Since the cracking involves grain boundary sliding and there is less constraint to sliding at the surface, triple point cracking is likely to start on the surface. Localized yielding in the neighborhood of the fracture is needed to accommodate the large grain boundary displacements required for cavity linkage.

Raj (1977) derived an expression for the net grain boundary sliding during a single fatigue cycle. An unbalance in sliding, needed for crack growth, will occur in the plastic zone of the crack tip, particularly if only small-scale yielding occurs. Thus triple point fracture should lead to an enhanced crack-growth rate when the plastic strain is small compared with the total strain. Raj's expression for the net grain boundary sliding during a cycle indicates that this sliding is inversely proportional to the strain rate. Therefore, the crack growth rate should drop with increasing frequency of cycling. An increase in the number of cycles to failure with increasing cycle frequency (at least over a certain frequency range) or with strain rate was observed in a number of superalloys. The mode of cracking changed from intergranular to transgranular.

A.5 ADVANTAGES OF THE MODEL

Raj provided detailed models for the nucleation, growth, and coalescence of grain boundary cavities under a variety of experimental conditions. His equations permit the calculation of the rates of these processes as functions of loading pattern, temperature, and some microstructural parameters (particularly area fraction of grain boundary particles). The equations may assist in materials selection for certain operating conditions. They may be used to predict life degradation of certain conditions of fatigue. It is possible to carry out experiments on fatigue life at high temperatures, where effects can be obtained in a reasonable length of time, then, using his equations, extrapolate the results to lower temperatures and very long times.

A.6 DISADVANTAGES OF THE MODEL

All of this modeling is based on one mechanism: grain boundary cavitation. If the failure stems from some other process, the method is inappropriate. Even if cavitation is operative, the phenomenon is so complex that there is no assurance that some factors were not omitted from the model.

It appears that a great deal of experimental fatigue behavior has been explained on the basis of cavitation in the circumstances where no cavities were observed by the investigators. However, in some cases the cavities may have been too small to be seen by optical microscopy. It seems desirable that, in future studies of intergranular failure where cavitation is suspected to be an important factor, an effort be made to detect the presence of voids.

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Appendix B

NONLINEAR (INELASTIC) CONCEPTS IN FRACTURE MECHANICS

There exists a logical development of mechanical analysis from the Griffith theory to G_c and K_{Ic} (LEFM) concepts. Several models have been developed^c which seem compatible with accepted physical theory concerning fatigue crack growth under LEFM conditions. When the specimen or structure exhibits inelastic or viscoelastic behavior, the use of strain-energy methods seems, to some, irrelevant. There has been a significant effort during the past decade in this arena, and some significant results have been published. While these frameworks do not constitute a life prediction method or offer a design method, it seems appropriate to review the status of nonlinear fracture mechanics in this section.

Localized inelastic strains frequently occur in notches and regions of stress concentration. Moreover, the inelastic strain usually has a large gradient directed inward and a residual stress is established upon removal of load or temperature gradient. The question regarding the effect of wave shape (load or strain time function), including so-called hold time effects, environment, overload, and history; and the question of the method of treating cumulative damage; while not completely resolved for simple laboratory tests of smooth specimens, are even more important and less clear for notches. Nonlinear concepts in fracture mechanics seem, to some of the committee, the best approach for treating the fatigue behavior of notches.

B.1 REVIEW OF CONCEPTS AND APPLICATION OF J-INTEGRAL TO MICROCRACKING

Linear elastic fracture mechanics assumes that there exists a characteristic distribution of stresses ahead of

the crack tip. The principal parameter of this stress distribution is the stress intensity factor K_I which relates the magnitude of the elastic stress field in the vicinity of the crack of loaded specimens to the applied load and crack geometry. The assumption that the dominating characteristic of stress intensity associated with the crack is given by the elastic stress field is true only if yielding associated with the crack tip is sufficiently small, or if the crack tip is stationary. Under those conditions, the analysis accounts for all geometric factors so that a value of K_I will be identical in specimens and components.

If the assumption of small-scale yielding is not valid, the characteristic of stress intensity dominating the behavior of the crack becomes less applicable, and alternate criteria must be examined.

McClintock (1960) considered the problem of slow stable crack growth on the basis of a stress-strain analysis. For the case of a nonwork-hardening material, the stress in the plastic zone becomes limited by the yield stress. Thus the fracture process becomes dominated by a strain criterion, in which the crack propagates if the strain at some distance ahead of the crack exceeds a critical value. For materials with strain hardening behavior, the cracking process is dominated by some combination of stress and strain behavior whose magnitude is determined by the local state of stress. Thus, no single value of stress or strain would have general applicability.

Most nonlinear concepts use some form of energy as the propagation criterion, either strain energy released G or J , crack growth resistance R , or strain energy density S , or a displacement criterion such as crack tip opening displacement.

In the case of elastic crack propagation, all of the available elastic strain energy in the body is available to the crack for potential crack propagation. Thus, the criterion for propagation can be determined at the point where the available elastic strain energy equals the needed strain energy. The strain energy in the elastic case G^* can be calculated from the elastic stress field. If sufficient

* G , the crack extension force, or strain energy release rate, is

$$\frac{F^2}{2} \frac{\partial(C)}{\partial a}$$

where F is the Mode I force, C is the compliance (measured at the load-line), and a is the crack length.

plasticity occurs, one must determine how much of the available strain energy is supplied to the crack for propagation and which portion is supplied to the material surrounding the crack for plastic deformation. This requires an exact calculation of the effect of plasticity on G , i.e., a rigorous elastic plastic solution to the crack tip stress field for all crack geometries.

Since such a solution is not available, an alternative method of approaching the problem is the J -integral defined by:

$$J = \int_{\Gamma} (W dy - T \frac{\partial u}{\partial x} ds)$$

where

$$W = \int_0^e \sigma_{ij} d\epsilon_{ij}$$

and Γ is a closed contour, T is the tension vector perpendicular to Γ , u is the displacement in the x direction, ds an element of Γ . If the contour is closed $J = 0$.

The J integral measures the strain energy flow across the contour Γ and is associated with the material behavior across the interface of the contour Γ , but places no limits on the location of the contour except to require that it begins and ends on the crack edges. In the elastic case, $J = G$, and is equivalent to the elastic energy release rate; in the more general case it is the energy release due to crack propagation and is valid if there is appreciable crack tip plasticity. This is because one may select a contour along which the integration can be carried out entirely within the elastic region.

The J value can be determined directly from the load displacement curves as the area between two load displacement curves associated with crack lengths a and $a+da$. Thus, the J value is determined in a manner analogous to the compliance methods for linear elastic behavior, but the curves can exhibit nonlinearity as a result of plastic deformation at the crack tips.

B.2 STRAIN ENERGY DENSITY

The strain energy density concept initiated by Sih (1973) requires no calculation of the energy release rate. For an elastic material the strain energy density is given by dW/dV , as

$$\frac{dW}{dV} = \left[\frac{1}{2E} (\sigma_{ii})^2 - \frac{\nu}{E} (\sigma_{ij}) + \frac{1}{2\mu} (\tau_{ij})^2 \right]$$

where the stresses are those given by the elastic solution associated with a crack tip under mixed mode loading.

$$\sigma_{ii} = \frac{K_I}{\sqrt{2\pi r}} f(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} g(\theta) + \frac{K_{III}}{\sqrt{2\pi r}} H(\theta) + \dots$$

Substitution of these equations gives a quadratic form of the strain energy density function as

$$\frac{dW}{dV} = \frac{1}{r} \left(a_{11} K_I^2 + 2a_{12} K_I K_{II} + a_{22} K_{II}^2 + a_{33} K_{III}^2 \right) = \frac{S}{r}$$

this includes the general form of the crack border stress fields and the three stress intensities K_I , K_{II} , K_{III} , for the three modes of deformation. The use of strain energy density requires two hypotheses: that the crack will grow when the intensity of the strain energy density reaches a critical given value, S_{IC} ; and that the direction of the growth of the crack will be in the direction of maximum potential energy density, i.e., in a direction in which the strain energy density is minimal.

The assumption of the first hypothesis is a generalized linear elastic concept extended to mixed-mode problems and assumes that the elastic stresses and hence the elastic strain energy can be superimposed with no interactive terms. This permits calculations to be made of the crack growth associated with mixed mode loading problems.

The direction of crack growth is predicted by the minimum in strain energy density. Why the onset of growth should be determined by maximal strain energy density, while the growth direction is determined by the minimal, is unclear. One would think that the direction of growth would be toward

a maximum in strain energy density so as to enable the growth to exceed the critical strain energy needed for propagation and hence accelerate, rather than decelerate.

If the crack sizes are small compared with the section size, the crack may not begin to propagate until general yield has occurred in the body. If the cracks are large compared with the structure, the local stresses associated with the crack will cause the uncracked section of the body to yield. In both cases the local stresses at the crack tip cannot increase very much after the general yield condition has occurred, and the fracture condition for propagation is due to the localization of a sufficiently high strain-stress combination. A measure of the plastic strain at the crack tip is given by the crack-tip opening displacements (CTOD). The occurrence of fracture due to the presence of a critical crack-tip displacement was first proposed by Wells (1963).

If the behavior of the material is elastic, the CTOD can be calculated using the elastic stress equations. One obtains the relationship (Broek, 1974)

$$\text{CTOD} = \frac{4}{\pi} \frac{K_1^2}{E\sigma_{ys}}$$

and the existence of a critical stress intensity factor K_{Ic} implies the presence of a critical CTOD. To extend the CTOD concept to general yielding one must assume that the local crack-tip stresses remain essentially within the magnitude of the yield stress. Crack-tip opening at crack propagation (CTOD_C) is measured directly using three-point bend specimens and assuming the presence of a plastic hinge effect that allows the CTOD to be estimated from crack mouth displacements on the surface of the specimen (Wells, 1963).

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