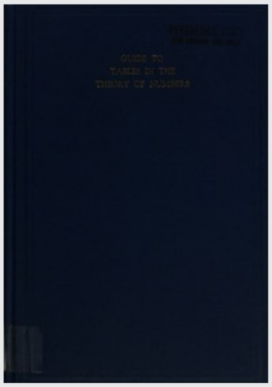

Guide to Tables in the Theory of Numbers

DETAILS

195 pages | 6 x 9 | PAPERBACK
ISBN 978-0-309-30025-4 | DOI 10.17226/18678

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TABLES IN THE THEORY OF NUMBERS

BULLETIN
OF THE
NATIONAL RESEARCH COUNCIL
NUMBER 105 FEBRUARY, 1941

DIVISION OF PHYSICAL SCIENCES
COMMITTEE ON MATHEMATICAL TABLES AND AIDS TO COMPUTATION
RAYMOND CLARE ARCHIBALD, *Chairman*

REPORT 1

Report of the Subcommittee on Section F: Theory of Numbers

GUIDE TO TABLES IN THE THEORY OF NUMBERS

BY
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PUBLISHED BY THE NATIONAL RESEARCH COUNCIL
NATIONAL ACADEMY OF SCIENCES
WASHINGTON, D. C.
1941

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December 1940

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FOREWORD

This Report of the Subcommittee on the Theory of Numbers is the first one to be published by the Committee. In broad outline it exhibits the general plan for all Reports in the series. In adopting this plan the Committee desires to make clear that the Reports are being prepared primarily for scholars and others active in scientific work throughout the world.

It is recognized however that, even in the United States, those using this and later Reports may often be greatly hampered through lack of library facilities. Because of this fact the bibliographic section of our present Report is more extended than it might otherwise have been. Information is there given concerning the holdings, in libraries of the United States and Canada, of the books and pamphlets to which reference has been made. It may thus frequently be found that a desired publication is near at hand. The *Union List of Serials* furnishes similar information concerning serials containing tables and errata in the tables discussed. But these errata are often in periodicals and books somewhat difficult of access. Hence it was finally decided, as a matter of policy, to list all known errata in tables surveyed. It seemed desirable in this Report to group all errata together in a special section; in later Reports, however, they may be included in the bibliographic section.

Authorities for all errata are indicated, and in the case of errata previously printed the sources are given. Professor Lehmer's personal contributions in this connection are very notable; where no authority is mentioned it is to be assumed that the discovery of the errata was due to him. The reader who makes checks will find that the reprinting in this Report of all known published errata has two other great advantages over giving mere references to sources, namely, that they are combined with other known unpublished errata, and that source notations (often difficult of comprehension, except by the expert) have been made to conform with those of this Report.

It is a pleasure to acknowledge notable courtesies extended to us. Doctor Arthur Beer, of the University of London Observatory, placed at our disposal for this Report the late Doctor Jiří Kaván's manuscript lists of errata in the tables of Chernac, Goldberg, and Inghirami, discovered while preparing his remarkable *Factor Tables*. Hence it may well be assumed that our lists of errata in the cases of the two latter are complete. The same may be said of the Gifford tables errata supplied by Doctor L. J. Comrie of London, the great authority on all that pertains to table making.

The directions for the use of this Report in the contents and index ought to render all of its material readily available.

The undersigned will be happy to hear from anyone who may notice in this Report any omission, inaccuracy, or misstatement. It is not expected that another Report will be ready for publication before 1942.

R. C. Archibald
Chairman of the Committee

December 1940

STYLE, NOTATIONS, AND ABBREVIATIONS

In the series of Reports of this Committee there will be references to Serials, Books and Pamphlets, and Manuscripts. It seemed desirable to be able readily to determine where such material might be consulted. The serial holdings of libraries of the United States and Canada are indicated in the *Union List of Serials* and its Supplements, of which a new and enlarged edition, in a single alphabet, is now in an advanced stage of preparation. The present custodian of all manuscripts is stated. From the hundreds of Libraries listed in the *Union List of Serials* the following 37 were selected, representing Canada and 22 states. These Libraries are as follows:

CPT	California Institute of Technology, Pasadena
CU	University of California, Berkeley
CaM	McGill University, Montreal
CaTU	University of Toronto
CoU	University of Colorado, Boulder
CtY	Yale University, New Haven, Conn.
DLC	Library of Congress, Washington
ICJ	John Crerar Library, Chicago, Ill.
ICU	University of Chicago
IEN	Northwestern University, Evanston, Ill.
IU	University of Illinois, Urbana
InU	University of Indiana, Bloomington
IaAS	Iowa State College, Ames
IaU	University of Iowa, Iowa City
KyU	University of Kentucky, Lexington
MdBJ	The Johns Hopkins University, Baltimore, Md.
MB	Boston Public Library
MCM	Massachusetts Institute of Technology, Cambridge, Mass.
MH	Harvard University, Cambridge, Mass.
MiU	University of Michigan, Ann Arbor
MnU	University of Minnesota, Minneapolis
MoU	University of Missouri, Columbia, Mo.
NhD	Dartmouth College, Hanover, N. H.
NjP	Princeton University, Princeton, N. J.
NIC	Cornell University, Ithaca, N. Y.
NN	New York Public Library
NNC	Columbia University, New York, N. Y.
NRU	University of Rochester, Rochester, N. Y.
NcD	Duke University, Durham, N. C.
OCU	University of Cincinnati

OU Ohio State University, Columbus
PBL Lehigh University, Bethlehem, Pa.
PU University of Pennsylvania, Philadelphia, Pa.
RPB Brown University, Providence, R. I.
TxU University of Texas, Austin
WvU West Virginia University, Morgantown
WU University of Wisconsin, Madison

In the case of all Books and Pamphlets mentioned in our Reports, the holdings of each of these Libraries are indicated in the Bibliographies. It may be noted that the forms of titles of Serials in our Bibliographies follow the forms in the newest *Union List*. Transliterations of Russian and Ukrainian names, and titles of articles and periodicals, are in accordance with *Manual of Foreign Languages*, third edition, Washington, 1936.

A few of the Abbreviations used in the Reports are as follows:

Abt. = Abteilung
Acad. = Academy, Académie, etc.
Akad. = Akademii͡a, Akademija, Akademie, etc.
Am. = America, American
App. = Appendix
Ass. = Association
Ast. = Astronomy, Astronomische, etc.
Biog. = Biography
Br. = British
Bull. = Bulletin
Cambridge = Cambridge, England
col. = column
d. = der, die, di, etc.
Dept. = Department
ed. = edited, edition
f. = för, für
Fis. = Fisiche
Gesell. = Gesellschaft
heraus. = herausgegeben
Inst. = Institute (English or French)
Int. = International
Ist. = Istituto (Italian)
Jahresb. = Jahresbericht
Jn. = Journal
Kl. = Klasse
Mat. = Matematica, Matematică, Matemática, etc.
Math. = Mathematics, Mathematical, Mathematische, etc.
Mo. = Monthly
n.s. = new series

Nach. = Nachrichten
Nat. = National
Natw. = Naturwissenschaften
no. = number
nos. = numbers
opp. = opposite
p. = page, pages
Phil. = Philosophical
Phys. = Physical, Physics, Physik, Physikalische
Proc. = Proceedings
Rev. = Review
s. = series
Sci. = Science, scientifique
Sitzungsb. = Sitzungsberichte
So. = Society
Sup. = Superiore, Supérieure, etc.
Trans. = Transactions
transl. = translated, translation
u. = und
Univ. = University, Universidade, Université, Università, etc.
v. = volume, volumes, voor
Wiss. = Wissenschaften
z. = zur
Z. = Zeitschrift

CONTENTS

INTRODUCTION

The theory of numbers is a peculiar subject, being at once a purely deductive and a largely experimental science. Nearly every classical theorem of importance (proved or unproved) has been discovered by experiment, and it is safe to say that man will never cease to experiment with numbers. The results of a great many experiments have been recorded in the form of tables, a large number of which have been published. The theory suggested by these experiments, when once established, has often made desirable the production of further tables of a more fundamental sort, either to facilitate the application of the theory or to make possible further experiments. It is not surprising that there exists today a great variety of tables concerned with the theory of numbers. Most of these are scattered widely through the extensive literature on the subject, comparatively few being "tables" in the usual sense of the word, i.e., appearing as separately published volumes. This report is intended to present a useful account of such tables. It is written from the point of view of the research worker rather than that of the historian, biographer, or bibliophile.

Another peculiarity of the theory of numbers is the fact that many of its devotees are not professional mathematicians but amateurs with widely varying familiarity with the terminology and the symbolism of the subject. In describing tables dealing with those subjects most apt to attract the amateur, some care has been taken to minimize technical nomenclature and notation, and to explain the terminology actually used, while for subjects of the more advanced type no attempt has been made to explain anything except the contents of the table, since no one unfamiliar with the rudiments of the subject would have any use for such a table.

There are three main parts of the report:

I. A descriptive account of existing tables, arranged according to the topical classification of tables in the theory of numbers indicated in the Contents.

II. A bibliography arranged alphabetically by authors giving exact references to the source of the tables referred to in Part I.

III. Lists of errata in the tables.

Brief comment on each Part may be given here.

Part I is not so much a description of tables as a description of what each table contains. It is assumed that the research worker is not interested in the size of page or type, or the exact title of column headings, or even the notation or arrangement of the table in so far as these features do not affect the practical use of the table. Since there is very little duplication of tables the user is seldom in a position to choose this or that table on such grounds as one does with tables of logarithms, for example. However, it is a well known fact that many tables in the theory of numbers have uses not contemplated by the author of the table. A particular table is mentioned as many times and in as many places as there are, to the writer's best knowledge, practical uses to which it may be put.

The practical viewpoint was taken in deciding what constitutes a table in the theory of numbers, and what tables are worthy of inclusion. Tables vary a great deal in the difficulty of their construction, from completely trivial tables of the natural numbers to such tables as those of the factors of $2^{2^n}+1$, one additional entry in which may require months of heavy computing. In general, old obscure tables, which have been superseded by more extensive and more easily available modern tables, have been omitted. Short tables, every entry in which is easily computed, merely illustrating some universal theorem and with no other conceivable use, have also been omitted. The present century with its improved mechanical computing devices has seen the development of many practical methods for finding isolated entries in number theory tables. In spite of this, many old tables, any single entry of which is now almost easier to compute than to consult, have been included in the report since they serve as sources of statistical information about the function or the problem considered.

Most of those tables prior to 1918 which have not been included here are mentioned in Dickson's exhaustive three-volume *History of the Theory of Numbers*. Under DICKSON 14 of the Bibliography in the present report will be found supplementary references to the exact places in this history where these tables are cited, arranged according to our classification of tables in the theory of numbers. For example the entry

d$_4$ v. 1, ch. I, no. 54: ch. III, no. 235.

means that two tables of class **d**$_4$ (solutions of special binomial congruences) are cited in vol. 1, chapter I, paper 54, and chapter III, paper 235.

For a fuller description of many of the older tables cited in this report the reader is referred to Cayley's valuable and interesting report on tables in the theory of numbers, CAYLEY 7.

The writer has tried to include practically all tables appearing since 1918, and on the whole has probably erred on the side of inclusion rather than exclusion.

A few remarks about nomenclature in Part I may be made here. The unqualified word "number" in this report means a positive integer and is denoted generally by n. The majority of tables have numbers for arguments. In saying that a table gives values of $f(n)$ for $n\leqq 1000$ it is meant that $f(1), f(2), \cdots, f(1000)$ are tabulated. If the table extends from 500 to 10 000 at intervals of 100 we write $n=500(100)10\ 000$. A great many tables have prime numbers as arguments, however. Throughout the report the letter p designates a prime which may be $\geqq 1$, >1, or >2 according to the context.

Introduction

To say that the function f is tabulated for each prime of the first million as argument, we write "$f(p)$ is given for $p<10^6$." Occasionally it is convenient to use the words decade, century, chiliad, or myriad to indicate an interval of 10, 100, 1000, or 10 000 numbers. Frequently the arguments of a table are numbers (or primes) of some special form, such as a multiple of 6 plus 1. In cases of this sort we use such notations as $n=6k+1<1000$, or $1013\leq p=6x-1\leq 10\,007$.

In Part I, tables are described as though entirely free from errors, with the exception of an occasional remark on the reliability of certain general utility tables where the user has some choice in his selection.

The uninterrupted description of tables in Part I is made possible by Part II, where one may find complete bibliographic references, arranged by authors, to the one or more places in which each of the tables mentioned in Part I appears. The various reprints, editions, or reproductions of a table are distinguished by subscripts on the number following the author's name. Thus, for example, CAYLEY 6_1 refers to the original table, while CAYLEY 6_2 refers to the same table as reprinted in his *Collected Mathematical Papers*. In Part I these distinctions are rarely used, but in Part III they are convenient.

Following each reference in Part II (except CAYLEY 7, CUNNINGHAM 40–42, DICKSON 14, and D. H. LEHMER 11) there appears in square brackets, [ ], an indication of the kind (or kinds) of tables contained in the work referred to, together with their location. The small boldface letters, with or without subscripts, refer to the classification of tables given in the Contents. The page numbers following any particular classification letter not only locate the table for the reader in possession of the publication, but give an idea of the extent of the table to the reader who may not have it, and will be of help in ordering photostats or a microfilm of the table from a distant library. In further explanation of the notation used, it should be noted that the absence of page numbers after a particular letter indicates that practically the whole work is devoted to a table, or tables, of this particular class. An asterisk placed on a classification letter indicates that errors in the corresponding table are cited in Part III. When a publication has tables capable of several classifications and errors are cited in all tables, an asterisk is placed after the closing bracket. The following examples with explanations should make these notations clear.

[$\mathbf{f}_1$] A list of consecutive primes occupying practically the whole work referred to.

[$\mathbf{d}_1$, 14–29: $\mathbf{d}_4^*$, 30–35: $\mathbf{f}_1$] Tables of primitive roots on pages 14–29. Solutions of special binomial congruences on pages 30–35, with errors cited in Part III. Lists of consecutive primes on practically every page.

As already mentioned, Part III gives errata in certain tables mentioned in Part I and is arranged alphabetically according to authors. The list of errors given for any particular table is not necessarily complete. Tables mentioned in Part I but not in Part III, so that no asterisk appears after the reference in Part II, may contain errors, either unknown to the writer or too trivial to be of any practical interest. In cases where errors have been found by others, the authority for the corrections, together with a reference to their source in case they have been published, is generally given in parentheses after the errors in question. In no case has an error been listed which was printed in connection with the table itself.

The writer has seen nearly all the tables mentioned in this report in at least one of the following libraries:

Brown University Mathematical Library, Providence, R. I.
Princeton University Mathematical Library, Princeton, N. J.
University of California Library, Berkeley, California
Cambridge University Library, Cambridge, England
The Science Library, London, England.

The writer's best thanks are due to the chairman of this Committee, Professor Archibald, whose unceasing efforts and expert knowledge have added greatly to the accuracy and reliability of Part II, and to Mr. S. A. Joffe, who has read all the manuscript and proof with great care, and has given many valuable suggestions.

The writer also wishes to acknowledge the frequent assistance of Miss M. C. Shields of the Princeton University Mathematical Library. Dr. N. G. W. H. Beeger has kindly supplied information about lists of primes, Mr. H. J. Woodall, information about works of Cunningham, and Dr. S. Perlis, information concerning the tables in the University of Chicago dissertations.

The part of the work on this report which was done abroad was made possible by a fellowship of the John Simon Guggenheim Memorial Foundation.

I

DESCRIPTIVE SURVEY

F. THEORY OF NUMBERS

a. PERFECT AND AMICABLE NUMBERS AND THEIR GENERALIZATIONS

The number n is called *perfect* if it is equal to the sum of its proper divisors (i.e., divisors $<n$). Only 12 perfect numbers are known. These numbers are given by $2^{n-1}(2^n-1)$ for $n=2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107$ and 127. A list of these 12 numbers, written in the decimal system, has been given recently by TRAVERS 1.

Chapter 1 of DICKSON 4 gives a very complete historical account of perfect numbers up to the year 1916 with many references to old lists of these numbers. ARCHIBALD 1 has given a complete up to date historico-bibliographic summary in tabular form.

If we use $\sigma(n)$ to denote the sum of all the divisors of n (including 1 and n), a perfect number is one for which $\sigma(n)=2n$. In case $\sigma(n)>2n$ the number n is called *abundant*. A list of all even abundant numbers <6232 is given in DICKSON 2 (Table III, p. 274–277). A rather special list of all primitive abundant numbers (i.e., numbers containing no abundant or perfect factors) with exactly four distinct prime factors of which the second in order of magnitude is 5, appears in DICKSON 3.

If n is such that $\sigma(n)=kn$, then n is called *multiply perfect* and k is the *index of perfection*. Thus a perfect number has an index of 2. The first real table of multiply perfect numbers is due to CARMICHAEL 1, who gave a list of 47 such numbers including all $<10^9$. Later CARMICHAEL AND MASON 1 extended this list to 251 numbers. Further lists of such numbers of index $k=5, 6$, and 7 appear in POULET 1. The most complete list to date is POULET 2, which gives 334 multiply perfect numbers with $3\leqq k\leqq 8$.

Two numbers n_1 and n_2 such that each is the sum of the proper divisors of the other, or in other words, such that $\sigma(n_1)=\sigma(n_2)=n_1+n_2$, are called *amicable*. Euler discovered 64 such pairs, which are tabulated in DICKSON 4. More recent lists are found in MASON 1 and in POULET 2 (p. 46–50), the latter containing 156 amicable pairs. A list of 21 new pairs, due to E. B. Escott, appears in POULET 5 together with a table of the distribution of amicable numbers $<10^{23}$.

A set of k numbers $n_1, n_2, \cdots, n_k$, not necessarily distinct, and such that

$$\sigma(n_1) = \sigma(n_2) = \cdots = \sigma(n_k) = n_1 + n_2 + \cdots + n_k$$

is called a set of *multiply amicable numbers of index k*. Lists of such sets of numbers with $2 \leqq k \leqq 6$ are found in MASON 1, while many more for the same range of k are given in POULET 2.

A series of numbers $n_1, n_2, \cdots$ each term of which is the sum of the proper divisors of the preceding term is called an *aliquot series* with *leader* n_1. The question of whether there exists an unbounded aliquot series is at present unanswered. DICKSON 2 has considered all aliquot series with leaders <1000. Table I (p. 267–272) gives most of these series complete. 13 incomplete series are given in Table II (p. 272–274). These are corrected and extended or completed in POULET 2 (p. 68–72) and POULET 3 (p. 188). The longest completed series has 138 as a leader and contains 178 terms.

In Table IV of DICKSON 2 (p. 278–290) the first few terms of aliquot series with leaders <6232 are given; in each case enough of the series is given to be sure it is not periodic with a period $\leqq 6$. If an aliquot series is purely periodic with proper period k then the k distinct members of the series are called *sociable numbers of index k*. Perfect and amicable numbers correspond to $k=1$ and 2. POULET 2 (p. 68) has discovered two sets of sociable numbers with indices 5 and 28 and with leaders 12496 and 14316 respectively.

Tables for facilitating the investigation of perfect, abundant, and amicable numbers, and their generalizations are described under b_2 (sum and number of divisors, and allied functions).

b. NUMERICAL FUNCTIONS

b_1. *Euler's totient function and its inverse, sum, and generalizations*

There are but two tables of Euler's totient function $\phi(n)$, defined as the number of numbers not exceeding n and prime to n. These are SYLVESTER 2 in which $\phi(n)$ is given for $n<1000$ and J. W. L. GLAISHER 27, where (in Table I) the function is tabulated to $n=10\,000$. The fact that there are only two tables of this fundamental function may be accounted for by the simple formula for $\phi(n)$, by means of which isolated values of ϕ may be quickly found, once the factorization of n into its prime factors is known, namely:

$$\phi(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_t^{\alpha_t}) = p_1^{\alpha_1-1}(p_1-1)p_2^{\alpha_2-1}(p_2-1) \cdots p_t^{\alpha_t-1}(p_t-1).$$

Both tables were in fact constructed with a view to obtaining numerical data for the less simple functions, the sum and inverse of ϕ.

There are several small tables of the inverse of ϕ giving all n's for which $\phi(n)$ has a given value. For $\phi(n) \leqq 100$ we may cite LUCAS 5, and KRAITCHIK 4. These also give the number of n's in each case. Two much larger tables exist: CARMICHAEL 2, which extends to $\phi(n)=1000$, and J. W. L. GLAISHER 27, where Table II gives all n's up to $\phi(n)=2500$.

A manuscript table of MILLER 1 gives odd solutions n of $\phi(n)=N$ for all possible $N \leqq 10\,000$ and was used to verify Glaisher's Table II.

The sum function

$$\Phi(n) = \sum_{\nu=1}^{n} \phi(\nu) = \frac{3n^2}{\pi^2} + 0(n \log n)$$

has been the subject of numerous papers. A table of $\Phi(n)$ for $n \leqq 100$ together with (for comparison purposes) the nearest integer to $3n^2/\pi^2$ is given in PEROTT 1. A more extensive table is SYLVESTER 2, which tabulates $\Phi(n)$ up to $n=1000$ together with $3n^2/\pi^2$, correct to the second decimal place. SARMA 1 has tabulated $\Phi(n)$ for $n=300(50)800$ and for 820, and gives for the same values of n the error function

$$E(n) = \Phi(n) - \frac{3n^2}{\pi^2},$$

which he states is positive for $n \leqq 1000$ except for $n=820$. Values of $\Phi(n)$ and $3n^2/\pi^2$ for $n=1000(1000)10\,000$ are given in GLAISHER 27. Isolated values of $\Phi(n)$, at least for $n \leqq 500\,000$, are most easily calculated by means of the formula

$$2\Phi(n) - 1 = \sum_{\nu=1}^{[\sqrt{n}]} \left\{ \left[\frac{n}{\nu}\right]^2 \mu(\nu) + M\left[\frac{n}{\nu}\right](2\nu - 1) \right\} - M(\sqrt{n})[\sqrt{n}]^2,$$

where the values of the Möbius function $\mu(n)$ may be taken from MERTENS 1, and its sum function

$$M(x) = \sum_{\nu \leqq x} \mu(\nu)$$

may be taken from the tables of STERNECK 1, 2.

A function $\psi(n)$, similar to Euler's $\phi(n)$, which may be defined as the least common multiple of the factors occurring in the above product for $\phi(n)$ or as the least positive exponent k for which the congruence

$$x^k \equiv 1(\bmod n)$$

holds for all x prime to n, has important applications in the theory of the binomial congruence. CAUCHY 1, 2 contain tables of $\psi(n)$ for $n \leqq 100$ and for $n \leqq 1000$ respectively, while MOREAU 1 has a table of the inverse of $\psi(n)$ giving all values of n below 1000 (and in most cases many larger values also) for which $\psi(n)$ has a given value $\leqq 100$.

Another special table dealing with the numbers less than and prime to n is due to BACKLUND 1, and gives the frequency of a fixed difference between consecutive members of the set of integers prime to $n=2\cdot 3\cdot 5 \cdots p_r$ for $1 \leqq r \leqq 8$. Thus for $r=6$, we find that among the $\phi(2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13)=\phi(30030) =5760$ numbers less than and prime to 30030 there are precisely 1690 consecutive ones differing by 6.

Lists of the actual numbers $\leqq n$ and prime to n are given for every $n \leqq 120$ in CRELLE 3.

If all irreducible fractions between 0 and 1 whose denominators do not exceed N be arranged in increasing order the resulting sequence of $\Phi(N)$ fractions is called the *Farey series of order N*. GOODWYN 1, 2 give the Farey series of orders 100 and 1000 respectively. The Farey series of order N less than 100 or 1000 may be read directly from the corresponding table simply by omitting those fractions whose denominators exceed N.

b₂. *Sum and number of divisors, and allied functions*

There exists only one large table of the sum $\sigma(n)$ and the number $\nu(n)$ of divisors of n (including 1 and n). These functions are given in Table I of GLAISHER 27 for all n up to 10 000. A table of $\sigma(n)$ for $n \leqq 100$ is in GLAISHER 17, where the function is denoted by $\psi(n)$. Table III of GLAISHER 27 gives all values of $n \leqq 10\,000$ for which $\nu(n)$ has a given value, while Table IV gives for each possible value of $\sigma(n) \leqq 10\,000$ all those n's for which $\sigma(n)$ has this value. DICKSON 2 has published a somewhat similar inverse table of σ extending only as far as $n = 1600$. These inverse tables are useful in finding multiply perfect numbers, amicable numbers, etc. Another kind of table useful in this connection gives the decomposition into prime factors of the values of $\sigma(p^\alpha) = (p^{\alpha+1}-1)/(p-1)$, two examples of which are EULER 1 and KRAITCHIK 7. The former table extends for each prime p as far as $\alpha = r_p$ as follows:

p	2	3	5	7	11	13	17	19	23	$29 \leqq p < 1000$
r_p	36	15	9	10	9	7	5	5	4	3

The latter table extends for each $p < 1000$ over $\alpha = 2, 3, 4, 5, 6, 8, 10, 12$ and for $p < 100$ (and for several larger primes) over $\alpha = 7, 9, 14, 15, 16, 18, 20, 24,$ and 30.

In connection with the function ν there is the concept due to Ramanujan of a *highly composite number*, that is, a number which has more divisors than any smaller number. A list of the first 103 highly composite numbers extending as far as 6 746 328 388 800, which is the first number to have as many as 10080 divisors, is given in RAMANUJAN 1.

Glaisher has given several tables of numerical functions which depend upon the difference between the number of divisors of n of one specified form, and the number of divisors of n of another specified form. These functions occur naturally in the series expansion of certain elliptic functions and are also connected with the number of representations of integers by certain binary quadratic forms. The function of this kind most frequently met with is $E(n)$, the difference between the number of divisors of n of the form $4k+1$, and the number of those of the form $4k+3$. Tables of $E(n)$ are given in GLAISHER 17, p. 164–165 to $n = 100$, in GLAISHER 15, to $n = 1000$. In GLAISHER 18, and in GLAISHER 19, the function $E(12n+1)$ is given for $n \leqq 100$. The function $H(n)$ denoting the excess of the number of $3k+1$ divisors of n over the number of $3k+2$ divisors is tabulated in GLAISHER 19 to $n = 100$, and in GLAISHER 24, to $n=1000$. The function $J(n)$ denoting the excess of the number of $8k+1$ and $8k+3$ divisors of n over the number of $8k+5$ and $8k+7$ divisors is tabulated for $n \leqq 1000$ in GLAISHER 25. The function $E_2(n)$ denoting the excess of the sum of the squares of the $4k+1$ divisors of n over the sum of the squares of the $4k+3$ divisors is given for $n \leqq 100$ in GLAISHER 17.

The sum of the first n values of the above functions has been given by Glaisher as follows:

function	range of n	asymptotic formula	reference
$\sum_{k=1}^{n} \sigma(k)$	$n=1000(1000)10\ 000$	$n^2\pi^2/12$	GLAISHER 27, p. viii
$\sum_{k=1}^{n} \nu(k)$	$n=1000(1000)10\ 000$	$n \log n+(2C-1)n$	GLAISHER 26, p. 42
$\sum_{k=1}^{n} E(k)$	$n=100(100)1000(1000)10\ 000$	$n\pi/4$	GLAISHER 26, p. 193
$\sum_{k=1}^{n} H(k)$	$n=100(100)1000(1000)10\ 000$	$n\pi/3\sqrt{3}$	GLAISHER 26, p. 204
$\sum_{k=1}^{n} J(k)$	$n=100(100)1000$	$n\pi/2\sqrt{2}$	GLAISHER 26, p. 213

In each case the values are compared with the corresponding asymptotic formula.

For a table of all the divisors of each number up to 10 000 see ANJEMA 1.

b_3. *Möbius' inversion function and its sum*

The function $\mu(n)$ defined for positive integers n by

$$\mu(1) = 1, \qquad \mu(p) = -1, \qquad \mu(p^\alpha) = 0 \quad \text{for} \quad \alpha > 1$$
$$\mu(mn) = \mu(m)\mu(n) \qquad\qquad (m \text{ and } n \text{ coprime}),$$

plays a very fundamental role in the theory of numerical functions, and has the value $+1$ or -1 if n is a product of an even or odd number of distinct primes, but vanishes for all other numbers $n>1$. This function is so easy to evaluate for isolated numbers whose factors are known that tables of $\mu(n)$ are rare and were constructed to study the behavior of a more complicated allied function. GRAM 1 gives $\mu(n)$ together with the sum $S_n=\sum_{k=1}^{n}\mu(k)k^{-1}$ for $n \leqq 300$. This was published before Euler's conjecture that $S_n \to 0$ as $n \to \infty$ had been rigorously proved. MERTENS 1 contains a table of $\mu(n)$ and of the sum $M(n) =\sum_{k=1}^{n}\mu(k)$ for $n<10\ 000$. STERNECK 1 tabulates $M(n)$ for all $n<150\ 000$, while in STERNECK 2, $M(n)$ is given for $n=150\ 000\ (50)\ 500\ 000$. Finally in STERNECK 4, 5 a table of $M(n)$ is given for 16 values ranging from 600 000 to 5 000 000. These tables were computed with the hope of shedding some light on the still unsolved problem of the order of magnitude of $M(x)$, a problem intimately connected with the Riemann hypothesis. These tables, however, may also be used to advantage in computing other sum functions, as indicated above in connection with $\Phi(n)$. A list of numbers $<10^4$ which are primes or products of distinct primes is given in boldface type in Table III of GLAISHER 27. These are arranged, in increasing order, into sets according as n is a product of 1, 2, 3, 4 or 5 distinct primes. This list is useful in evaluating and inverting series involving $\mu(n)$.

b_4. *The quotients of Fermat and Wilson*

The integer $q_a=(a^{p-1}-1)/p$, where p is a prime, is known as *Fermat's quotient* and occurs in several branches of the theory of numbers. Its connection with the so-called first case of Fermat's last theorem, which dates from 1909, accounts for most of the tables of q_a. MEISSNER 1 tabulated q_2 modulo p for $p<2000$. This table was extended from 2000 to 3697 by BEEGER 3, who discovered a second example $p=3511$ of $q_2\equiv 0 \pmod p$, the first being $p=1093$. The table of HAUSSNER 2 gives $q_2 \pmod p$ for $p\leqq 10\,009$. BEEGER 4 extended his table from 3697 to 13999, and recently this high limit has been raised to $p<16\,000$ in BEEGER 8. Extensive tables for q_a exist only for $a=2$. HAUSSNER 3 gives a table of all known cases of $q_a\equiv 0 \pmod p$ in which $a<p$. Tables such as MEISSNER 2, BEEGER 1, and CUNNINGHAM 5 which give all solutions x of $x^{p-1}\equiv 1 \pmod{p^2}$ are described under d_4 (solutions of special binomial congruences).

The integer $w_p=[(p-1)!+1]/p$, where p is a prime, is known as *Wilson's quotient*. Only two small tables of $w_p \pmod p$ exist, namely BEEGER 2, for $p<300$, and E. LEHMER 1, for $p\leqq 211$. The congruence $w_p\equiv 0 \pmod p$ has only two known solutions $p=5, 13$.

b_5. *Sums of products of consecutive integers*

Two tables may be cited in this connection: GLAISHER 23 which gives the sums of products, k at a time, of the integers $1, 2, 3, \cdots, n$ for all $k<n$ and for $n<22$, and MORITZ 1 which gives the sums of products k at a time of the integers $m+1, m+2, \cdots, m+n$ for $0\leqq m\leqq 10$, $1\leqq n\leqq 12$ and $1\leqq k\leqq 12$. Tables of the sums of like powers of $1, 2, \cdots, n$, as well as tables of Bernoulli numbers and polynomials, will be cited and described in another report of this Committee, Section I.

b_6. *Numerical recurring series*

There are a number of recurring series which have been computed to a great many terms, in particular LAISANT 1, in which the *Fibonacci series* $u_n(0, 1, 1, 2, 3, 5, \cdots)$ and its associated series $v_n=u_{2n}/u_n$ are both tabulated up to $n=120$. In most cases these series are rather special and were computed for factorization purposes. These will be described under e_2, but may be cited as follows: HALL 1, LAISANT 1, D. H. LEHMER 2, KRAITCHIK 4, LUCAS 1, POULET 3.

b_7. *Triangular numbers*

There are three tables of *triangular numbers* $n(n+1)/2$. The earliest and most extensive is JONCOURT 1, which gives the first 20 000 triangular numbers. KAUSLER 1 has a table of the first 1000 triangular numbers, with their doubles and their halves whenever these latter numbers are integers. More recently BARBETTE 1 has given the first 5000 triangular numbers. Figurate numbers of higher order, namely

$$n(n+1)(n+2)\cdots(n+k-1)/k! = \binom{n+k-1}{k},$$

are essentially binomial coefficients, tables of which numbers will be cited and described in another report, Section I, of this Committee.

c. PERIODIC DECIMALS

Although tables for the conversion of ordinary fractions into decimals belong properly to another report of this Committee, Section A, there are a few such tables which are of number-theoretic interest inasmuch as they give in each case the complete period of the repeating decimal.

Perhaps the best known table of this sort is due to GAUSS 5, and was intended for use (according to the title) in finding the complete period of the repeating decimal for P/Q, where $Q<1000$. Strictly speaking this is true only for $Q<467$ but the table is also available for an unlimited number of other fractions. We can, of course, suppose that $P<Q$ and prime to Q and by partial fractions we may express P/Q by

$$P/Q = \frac{P_1}{p_1^{\alpha_1}} + \frac{P_2}{p_2^{\alpha_2}} + \cdots + \frac{P_k}{p_k^{\alpha_k}}$$

where $Q=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$, the p's being distinct primes, and the P's being integers. Hence we need consider only fractions of the form P/Q, where Q is a prime or a power of a prime $\neq 2$ or 5. Therefore we set $Q=p^\alpha$, $\phi=\phi(Q)=p^{\alpha-1}(p-1)=e\cdot f$, where e is the exponent of 10 and f its residue-index (mod p^α). Let g be any primitive root of p^α so that $g^f\equiv 10 \pmod{p^\alpha}$. Then if $P\equiv g^i \pmod{p^\alpha}$ we can write

$$i = kf + \nu \qquad (0 \leqq k < e, 0 \leqq \nu < f).$$

Then

$$P \equiv g^i = (g^f)^k g^\nu \equiv 10^k g^\nu \pmod{p^\alpha},$$

which shows that P and g^ν have essentially the same decimal expression, or in other words it suffices to tabulate the f really distinct periodic decimals corresponding to the f fractions

$$\frac{1}{p^\alpha}, \frac{g}{p^\alpha}, \frac{g^2}{p^\alpha}, \ldots, \frac{g^{f-1}}{p^\alpha}.$$

In particular if 10 happens to be a primitive root of p^a there is only one period to give.

Such a table is given in GAUSS 5 for $p^a < 467$. For $467 \leq p^a < 1000$ only the periods for $1/p^a$ are given. The primitive roots used for each p^a are given on page 420. In actual practice it is seldom necessary to anticipate which of the f fundamental decimal periods corresponds to a given P/p^a, since after the first few digits are determined this can be recognized from the table.

There are three other tables similar to GAUSS 5. In fact this table is an extension of an earlier one for $p < 100$ given in GAUSS 1. Another table for $p \leq 347$ due to Hoüel is given in LEBESGUE 2 and is reproduced in HOÜEL 1.

Another and more complete set of tables which serve the same purpose more expeditiously is due to Goodwyn. In GOODWYN 3 are given the possible periods of every fraction P/Q with $Q \leq 1024$, while the possible non-periodic part of the decimal (if any) may be read from GOODWYN 2. GOODWYN 1 contains the same material as GOODWYN 2, 3 but is limited to fractions with denominators < 100. These rare tables are described in greatest detail in GLAISHER 4.

Tables giving a complete period when a rational fraction is converted into a "decimal" in a scale of notation different from 10 are as follows: BELLAVITIS 1 has given a table[1] similar to GAUSS 5 for $p \leq 383$ but with the base 2 instead of 10. CUNNINGHAM 12 gives the complete period of $1/n$ for base 2, for $n < 100$, while CUNNINGHAM 18 contains a table of the same extent for the bases 3 and 5.

d. THE BINOMIAL CONGRUENCE

The congruence $x^n - a \equiv 0 \pmod{m}$ is the subject of a great many tables many of which can be classified in several ways. The case $n = 2$ is not considered here but is discussed under **i**. There is, of course, an intimate connection between the binomial congruence and the binomial equation $x^n - a = 0$, especially when $a = 1$. Tables having to do with this equation are treated under **o**. Every solution x of the binomial congruence gives a factor m of the number $x^n - a$. Hence tables of factors of $x^n - a$ or even $x^n - ay^n$, which are described under $\mathbf{e}_2$, give, indirectly, solutions of the binomial congruence.

It is difficult to give an orderly description of the tables relating to the binomial congruence without making some conventions as regards nomenclature and notation. Thus the real integer x (if it exists) will be called the *base*, n will be called the *index* of a for the base x modulo m, and a will be called an nth *power residue* of m, and we shall write $(a/m)_n = 1$ to indicate that x exists. The term *solution* of a binomial congruence will be reserved to denote the re-

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[1] To save space such a decimal as

$$1/25 = .00001010001111010111^* \, 00001010001111 \cdots$$

is written simply 41113, thus indicating that the first half of the period (to the left of the star) begins with 4 zeros, followed by 1 one, 1 zero, etc. The second half of the period is complementary to the first.

sult of solving the congruence for the unknown base. The modulus m is almost always a prime or a power of a prime, and when a table extends to all such moduli not exceeding L we shall write $p^\alpha \leqq L$, where it will be understood that $\alpha \geqq 1$.

When $\alpha = 1$, the following nomenclature will be used. If $n = e$ is the least positive number for which $x^n \equiv 1 \pmod{p}$, then for brevity e is called the *exponent* of $x \pmod{p}$.[1] The integer $f = (p-1)/e$ is called the *residue-index* of $x \pmod{p}$, (after Cunningham), and is found more frequently than e in tables on account of its small average size. Moreover, if $f = 5$ for instance, then, by Euler's criterion, x is a fifth (but not higher) power-residue (mod p). Hence tables of f give indirectly, by setting $f = k, 2k, 3k, \cdots$, a list of those primes which have x as a kth power-residue, or a list of those x's which are kth power residues of a given prime. Those numbers x (positive or negative) for which $f = 1$ are called *primitive roots* of p.

d_1. *Primitive roots*

There are $\phi(p-1)$ incongruent primitive roots of p. The fact that there are so many primitive roots causes no difficulty in the theory of the binomial congruence but has caused considerable confusion in the tabulation of primitive roots. There are only four tables giving the full set of primitive roots of p. These are OSTROGRADSKY 1, for $p < 200$, reproduced in CHEBYSHEV 2, CAHEN 1, and GRAVE 3, and extended in CHEBYSHEV 2_5, to $p \leqq 353$; CRELLE 1 for $p \leqq 101$, except for $p = 71$, and KULIK 2, where $103 \leqq p \leqq 349$.

In most applications it is sufficient to know only one primitive root of p. All the others, if need be, may be generated from a single one by finding the residues of

$$g^{\tau_1}, g^{\tau_2}, \cdots, g^{\tau_\phi} \pmod{p}$$

where $\tau_1, \tau_2, \cdots, \tau_\phi$ are the $\phi(p-1)$ numbers less than and prime to $p-1$. For $p < 1000$ the *Canon Arithmeticus*, JACOBI 2, gives these various powers. For this reason authors of extensive tables of primitive roots have been content to give only one or sometimes two primitive roots for each p. A confusion exists, however, as to which root should be given, some authors giving always the least positive root, some the absolutely least root, some both, but frequently any convenient root, especially ± 10 when possible. It is often pointed out that primitive roots with small absolute values, especially ± 10, are easier to raise to high powers (an operation which is most frequently met with) than large roots. When p is quite large however this argument now-a-days has less weight, for in this case it is not a question of computing g^k by successive multiplications by g, but of calculating g^k for isolated values of k. This is best done by a computing machine writing k to the base 2 and using the method of

[1] Reuschle (1856) and, more recently, Cunningham use the terminology "e is the hauptexponent of x." This, and the above nomenclature is somewhat opposed to the older and more lengthy "e is the exponent to which x belongs," in which e is thought of as possessing x.

successive squarings modulo p in which case one soon loses sight of the original root g. Perhaps the best reasons for insisting on least positive primitive roots are 1) that this permits the collating of tables of primitive roots, and 2) that there is considerable theoretical interest in the question of the distribution of primes with large least primitive roots. The following is a tabular description of the 20 extensive tables of primitive roots arranged according to the highest value of p tabulated.

Tables of Primitive Roots

reference	range of p	factorization of $p-1$	type of root	indication that 10 is a root
JACOBI 2	1–1000	yes	3	yes
WERTHEIM 1	1–1000	no	1	yes
KULIK 2	1–1009	no	1	no
WERTHEIM 2	1–3000	yes	1	yes
CAHEN 3	200–3000	no	1	yes
WERTHEIM 3	3000–3500	yes	1	no
KORKIN 1	1–4000	yes	3	no
REUSCHLE 1	1–5000	yes	1, 3	no
WERTHEIM 4	3000–5000	yes	1	yes
POSSE 1	4000–5000	yes	3	no
POSSE 3	1–5000	yes	3	no
WERTHEIM 5	1–6200	no	1	yes
DESMAREST 1	1–10 000	no	3	no[1]
POSSE 2	5000–10 000	yes	2	no
POSSE 4	5000–10 000	yes	2	no
GOLDBERG 2	1–10 160		1	
KRAITCHIK 1	1–25 000	no	3	no
{CUNNINGHAM, WOODALL and CREAK 1}	1–25 409	no	1, 1′	no
{CUNNINGHAM, WOODALL and CREAK 2}	1–25 409	yes	1, 1′	no
{KRAITCHIK 4 (p. 131–145)}	1–27 457	no	3	no

[1] Yes on page 308.

The majority of tables give the factorization of $p-1$ into powers of primes, information essential to the application of primitive roots to the binomial congruence. Whether or not this is given in a particular table is indicated in the center column above. The types of roots tabulated are as follows:

1. least positive primitive root
1′. greatest negative primitive root
2. absolutely least root
3. some primitive root usually not exceeding 10 in absolute value modulo p.

REUSCHLE 1 gives the type 1 root for $p<1000$ and one or two roots of type 3 beyond 1000. CUNNINGHAM, WOODALL and CREAK 1, 2 give both 1 and 1′ for each p. These tables are perhaps the most reliable of all. The authors also give interesting data on the frequencies of least positive and greatest negative roots. Some tables give an indication whether or not 10 is a primitive root of p. Thus JACOBI 2 bases each table of his *Canon Arithmeticus* (described under d_3)

on the primitive root 10 whenever it exists. Other tables, as indicated in the last column of the above tabular description, mark with an asterisk the primes having 10 as a primitive root. Although this is not done in KRAITCHIK 4 (p. 131–145), he gives a separate list (p. 61) of the 467 primes $<10\,000$ of which 10 is a primitive root. On p. 55–58 are given lists of those primes $<10\,000$ whose least positive primitive root has a given value, and also the number of such primes. A more extensive table of the number of primes whose least positive and greatest negative primitive root have a given value is given in CUNNINGHAM, WOODALL and CREAK 1, for $p \leqq 25\,409$. It is remarkable that primes have such small primitive roots,[1] and this fact has been of great assistance in the preparation of tables described above.

d_2. *Exponents and residue-indices*

Interest in the exponent of x modulo p first arose in the special case of $x=10$. It was observed that for $p \neq 2$ or 5 the length of the period of the circulating decimal representing $1/p$ was a certain unpredictable factor e of $p-1$, and that the number 10^k-1 was divisible by p if and only if k was divisible by e, long before it was realized that these phenomena form only a part of a general theory of the binomial congruence (in which the base 10 is in no way peculiar), and in terms of which they are best described and investigated. As this bit of history has repeated itself in the case of countless individuals who have approached the theory of numbers from an interest in circulating decimals, we shall consider first the tables devoted to exponents of 10.

The earliest table is due to BURCKHARDT 1 who completed the last page of his factor table with a table of exponents of 10 for $p<2550$, and 22 larger primes. This table was reproduced with certain corrections by JACOBI 2 who used it in constructing his *Canon Arithmeticus*. Tables of exponents and residue-indices of 10 for various ranges of p may be given the following tabular description.

Tables of Exponents and Residue-indices of 10

reference	range of modulus	exponent	residue-index
BURCKHARDT 1	$p<2550$	yes	no
DESMAREST 1	$p<10\,000$	no	yes
REUSCHLE 2	$p<15\,000$	yes	yes
SHANKS 1	$p<20\,000$	yes	no
KRAITCHIK 1	$p<25\,000$	yes	no
CUNNINGHAM, WOODALL AND CREAK 1	$p^a<10\,000$	yes	yes
	$10000<p\leqq 25\,409$	no	yes
SHANKS 3	$20000<p<30\,000$	yes	no
KRAITCHIK 4 (p. 131–145)	$p\leqq 27\,457$	no	yes
[2]BORK 1	$p<100\,000$	no	yes if >2
HERTZER 1	$100000<p<112\,400$	no	yes if >2
SHANKS 4	$30000<p<120\,000$	yes	no

[1] All but 163 out of the 2800 primes under 25 410 have $2\leqq g\leqq 12$. The smallest prime known to have its least positive primitive root $\geqq 71$ is $p=48\,473\,881$.

[2] This table is due to F. Kessler.

A short table for composite as well as prime moduli (based on GOODWYN 3) has been given by GLAISHER 4. This has for argument every number $q \leqq 1024$ and prime to 10, and gives the exponent e of 10 modulo q as well as $\phi(q)$ and $\phi(q)/e$, where ϕ is Euler's totient function.

Tables of exponents and residue-indices of 2 may be tabulated in like manner as follows:

Tables of Exponents and Residue-indices of 2

reference	range of modulus	exponent	residue-index
CUNNINGHAM 4	$p^\alpha < 1000$	yes	yes
[1] MEISSNER 1	$p < 2000$	no	yes
[1] BEEGER 3	$2000 < p < 3700$	no	yes
REUSCHLE 1	$p < 5000$	yes	yes
[1] HAUSSNER 2	$p \leqq 10\,009$	no	yes
[1] BEEGER 4	$3700 < p < 14\,000$	no	yes
[1] BEEGER 8	$14000 < p < 16\,000$	no	yes
KRAITCHIK 1	$p < 25\,000$	yes	no
{CUNNINGHAM	{$p^\alpha < 10\,000$	yes	yes
{WOODALL and CREAK 1	{$10000 < p \leqq 25\,409$	no	yes
CUNNINGHAM and WOODALL 7	$p < 100\,000$	no	yes if >2
KRAITCHIK 4 (p. 131–191)	$p < 300\,000$	no	yes

There are four tables of Kraitchik which give residue-indices of 2 for primes of special forms up to high limits as follows:

KRAITCHIK 4, p. 53 $\begin{cases} p = 2^x 3^y 5^z + 1 < 10^7 \\ p = k2^n + 1,\ 3 \leqq k \leqq 99 \text{ (odd)},\ 22 \leqq n \leqq 36, \text{ and } 2\cdot 10^8 < p < 10^{12} \end{cases}$

KRAITCHIK 4, p. 192–204 $p = 512k + 1 < 10^7$

KRAITCHIK 6, p. 233–235 $p = k2^n + 1,\ 10^8 < p < 10^{12},\ k < 1000.$

Tables of exponents and residue-indices of other bases are less numerous and less extensive and give this information for several bases at once. They may be described as follows:

reference	bases	range of modulus	exponents	residue-indices
REUSCHLE 1	3, 5, 6, 7	$p < 1000$	yes	yes
KRAITCHIK 4 (p. 65)	2, 3, 5, 10	$p < 1000$	no	yes
{CUNNINGHAM	{2, 3, 5, 6, 7	{$p^\alpha < 10000$	yes	yes
{WOODALL and CREAK 1	{10, 11, 12	{$10000 < p \leqq 25409$	no	yes

Another special table of CUNNINGHAM and WOODALL 1 gives for $p \leqq 3001$ the least positive α for which $10^\alpha 2^x \mp 1 \equiv 0 \pmod p$ has a root x, and also the least such x.

Two, more elaborate tables, of the same type, are given in CUNNINGHAM, WOODALL and CREAK 1. These give for each $p^\alpha < 10\,000$, and for each of the four values $y = 3, 5, 7$, and 11, a set of three numbers (x_0, α_0, x_0') satisfying (for a certain choice of signs $\pm$) the two congruences

$$t^{x_0} \equiv \pm y^{\alpha_0},\quad t^{x_0'} y^{\alpha_0} \pm 1 \equiv 0 \pmod{p^\alpha},$$

[1] These tables give residue-indices as incidental data. The residue-indices were obtained from the other tables in this list.

where α_0 is the least possible such number for which x_0 and x_0' exist, and where x_0 and x_0' are also as small as possible. In the first table (p. 33–64), $t=2$, while in the second (p. 65–96), $t=10$. These tables were used by the authors for finding exponents of y (mod p) from the known cases $y=2$ and $y=10$.

There exist also two small tables of KRAITCHIK 4 (p. 63–65), which give the least positive number x for which $2^x \equiv h$ (mod p) for all $p<1000$ for which such an x exists, together with a list of all $p<1000$ for which no such x exists. The first table deals with $h=3$, and the second with $h=5$.

An analogue of the series of numbers a^n-1 ($n=0, 1, 2, \cdots$) is the Fibonacci series

$$0, 1, 1, 2, 3, 5, 8, 13, \cdots$$

defined by

$$u_n = u_{n-1} + u_{n-2}, \qquad u_0 = 0, \qquad u_1 = 1.$$

Corresponding to the exponent of a (mod p) we may define, after Lucas, the *rank of apparition* of p as the least positive value e' of n for which $u_n \equiv 0$ (mod p). Except for $p=5$, e' is a certain divisor of $p\pm1$ (more precisely $p-(5/p)$), and the quotient $f'=(p\pm1)/e'$ is the counterpart of the residue-index. KRAITCHIK 4 (p. 55) gives, for each $p<1000$, the corresponding value of f'.

An inverse table giving those p's for which the exponent of a (mod p) has a given value e, would be a table of so-called *primitive factors* of a^e-1. Such tables are discussed under $\mathbf{e}_2$. A similar table in which the residue-index f is given would be a table of those p's of which a is an exact fth power residue. Such tables are described under $\mathbf{d}_5$.

d₃. *Powers and indices*

If g is a primitive root of p then the $p-1$ successive powers

$$g^0, g^1, g^2, \cdots, g^{p-2}$$

taken modulo p are congruent, in some order, to the numbers

$$1, 2, 3, \cdots, p-1.$$

A table for a fixed prime p of powers of a primitive root g giving for each number i, $0 \leqq i \leqq p-2$, the least positive number n for which

$$g^i \equiv n \pmod{p}$$

may be thought of as similar to a table of the exponential function e^x. An inverse table, giving for each number $n \not\equiv 0$ (mod p) that index $i=\text{Ind}_g n$ (mod $p-1$) for which the above congruence holds, would correspond to a table of natural logarithms, and can be used, as suggested by GAUSS 1 who published such a table of indices for each prime $p<100$, in precisely the same way as a logarithm table for finding products, quotients, powers and roots modulo p. There is one practical difference, however, between a table of logarithms and a table of indices; the logarithm table can be used inversely to find anti-logarithms with perfect ease because log x is a strictly increasing function of x, whereas a table of Ind n (mod $p-1$) for $n=1, 2, 3, \cdots, p-2$ has its values scattered in such confusion that, except when p is small, there may be some difficulty in finding the value of n (mod p) corresponding to a given value of Ind n (mod $p-1$). It therefore adds considerably to the effectiveness of a table of indices to print a companion inverse table of powers of g (mod p). This appears to have been done first by OSTROGRADSKY 1 for all primes <200 in 1837–8. This table has been reproduced in CHEBYSHEV 2 and GRAVE 3, and extended to $p=353$ in CHEBYSHEV 2_5. CAHEN 3 has reproduced the table for $p<200$ from Chebyshev but has introduced many new errors.

An entirely independent calculation of a set of tables of similar extent (including also powers of primes <200) was made by HOÜEL 1, who based his table on absolutely least primitive roots. This table was first printed in LEBESGUE 2 in 1864.

Two years after the appearance of Ostrogradsky's work, JACOBI 2 published his monumental *Canon Arithmeticus*, which extends to $p^\alpha<1000$. The part for $p<200$ was reproduced from OSTROGRADSKY 1. Following Ostrogradsky, Jacobi uses the primitive root ± 10 whenever possible, otherwise usually a primitive root whose square, cube, or other low power is congruent to ± 10 (mod p). This exceedingly useful table is still in print after 100 years.

A small table for moduli p^α and $2p^\alpha<100$ appears in WERTHEIM 5. Another table for $p<100$ appears in USPENSKY and HEASLET 1.

A somewhat similar table entitled *A Binary Canon* has been given by CUNNINGHAM 4. This gives for each $p^\alpha<1000$ a pair of tables, one giving values of 2^i (mod p^α), and the other giving, inversely, whenever it exists, that value of $i<p^{\alpha-1}(p-1)$ for which 2^i has a given value (mod p^α). For such moduli p^α as have 2 for a primitive root this pair of tables is equivalent for most purposes to the corresponding pair in *Canon Arithmeticus*. For the other moduli the tables are smaller or have blank entries since not all the powers of 2 will be distinct, and certain indices are necessarily non-existent. This table is intended chiefly for studying the binomial congruence with base 2.

The *Canon Arithmeticus* may be said to have reduced any problem to which it is applicable to at most a simple pencil and paper calculation. The question of extending the *Canon* to, say, $p<10000$ is one which presents certain practical difficulties. If the original form were preserved it would occupy several thousand pages. With modern computing machines in use, however, such an extensive table is really unnecessary. In fact, as remarked above, the problem of finding g^k for an isolated value of k (mod p) is one that presents very little difficulty. This means that we may dispense with that half of the *Canon* comprising the tables giving powers of g. The remaining tables of indices of all numbers $<p$ may now be condensed by listing only indices of primes since we have the multiplicative relation

$$\text{Ind}\,(mn) \equiv \text{Ind}\,(n) + \text{Ind}\,(m) \pmod{p-1}.$$

Finally if q is a rather large prime then by use of one of the relations

$$\text{Ind}\,(q) \equiv \text{Ind}\,(q \pm p) \equiv \text{Ind}\,(q \pm 2p) \cdots \pmod{p-1}$$

one soon finds a number $q \pm kp$ all of whose prime factors are rather small. Hence we may tabulate only the indices of rather small primes. A similar condensation is possible for the modulus p^α $(\alpha > 1)$. A table based on such a scheme has been published in KRAITCHIK 4 (p. 216–267), which gives for each modulus $p^\alpha < 10000$ the indices of all primes <100. This is an extension of a previous table KRAITCHIK 3 giving for $p^\alpha < 1000$ the indices of every prime <50. KRAITCHIK 4 (p. 69–70) has also a table of the indices of odd primes $\leqq 37$ for moduli 2^n, $n \leqq 20$, and 5^n, $n \leqq 16$.

Tables giving powers but not indices are either of a small extent or else are of rather special types. There is the table of KULIK 2 which gives for each $p \leqq 349$ all powers (modulo p) of the least primitive root. This table is described in the introduction as extending to $p = 1009$ but its publication was abruptly discontinued in the middle of the table for $p = 353$. A small table giving all powers of all numbers (modulo p) is due to BUTTEL 1. It extends as far as $p = 29$.

LEVÄNEN 1 constructed a table giving for each $m < 200$ and prime to 10 the absolutely least value (mod m) of 10^n for $n = 0, 1, \cdots, e/2$, where e is the exponent of 10 (mod m).

CUNNINGHAM 11 has given for each $p < 100$ and for some much higher primes the values (modulo p) of the functions $E_n = 2^{2^n}$, 2^{E_n}, 3^{3^n}, 5^{5^n} for all values of n.

The primitive root tables of KORKIN 1 and POSSE 1, 2, 3, 4 described under d_1 give for each prime in the range considered certain powers of a primitive root modulo p. The notation for the various powers tabulated is as follows

$$f = g^{(p-1)/2^2}, \quad f' = g^{(p-1)/2^3}, \quad f'' = g^{(p-1)/2^4}, \ldots$$
$$z = g^{(p-1)/3}, \quad z' = g^{(p-1)/3^2}, \quad z'' = g^{(p-1)/3^3}, \ldots$$
$$u = g^{(p-1)/q}, \quad u' = g^{(p-1)/q^2}, \quad u'' = g^{(p-1)/q^3}, \ldots$$

where q is a prime factor >3 of $p-1$.

d_4. *Solutions of special binomial congruences*

Tables of powers and indices of a primitive root (described under d_3) such as JACOBI 2 serve to solve the general binomial congruence

$$x^n \equiv r \pmod{p}. \qquad (1)$$

In fact, armed with such a table, this congruence may be replaced by the equivalent linear congruence

$$n \text{ Ind}_g x \equiv \text{Ind}_g r \pmod{p-1}.$$

In spite of the availability of this general method, there exist many tables giving explicit solutions of binomial congruences of more or less special type, partly for the same reason that, in spite of the existence of tables of logarithms, there are numerous tables of square and cube roots, and partly because such tables have in most cases some important connection with the problem of factorization, a fact which accounts for the many extremely special tables described in what follows.

There is in fact only one table giving explicit solutions of the general congruence (1), and this table is very limited. It appears in CRELLE 1, and is reproduced in CRELLE 2, and gives for each n all solutions x (mod p), if any, of

$$x^n \equiv r \pmod{p} \qquad 1 \leqq r \leqq p-1, \qquad p \leqq 101.$$

The degree n ranges over all integers $<p$ in case $p<31$, but for $31 \leqq p \leqq 101$, n assumes only those values which are prime factors of $p-1$.

Tables of solutions of the more specialized congruence

$$x^n \equiv 1 \pmod{p^\alpha}$$

are more numerous and extensive. REUSCHLE 3 contains tables of solutions of this congruence with $\alpha=1$, $p<1000$, and $n \leqq 100$, besides $n=105$, 120, and 128. There is a table for each value of n giving only the $\phi(n)$ "primitive" solutions of the congruence for each $p=kn+1<1000$. The $n-\phi(n)$ imprimitive solutions can be taken, if need be, from the tables corresponding to the several divisors of n. Similar tables with $\alpha=1$ and 2 (and for small p's, many higher values of α) have been given in CUNNINGHAM 5. However, these extend only to $p \leqq 101$. A more extensive table is due to CUNNINGHAM and CREAK 1. This is arranged according to p^α and extends to $p^\alpha<10\,000$. For each such modulus p^α there is given the least positive solution x of $x^n \equiv 1 \pmod{p^\alpha}$, where n runs through the divisors of $\phi=p^{\alpha-1}(p-1)$ with the exception of the trivial cases $n=1$, and $n=\phi$. The other solutions can be found if necessary by taking successive powers (mod p^α) of the tabulated solutions.

Cunningham's *Binomial Factorisations* (CUNNINGHAM 28–34, 38, 39), 9 volumes of which have appeared, contain extensive tables of the $\phi(n)$ primitive solutions of $x^n \equiv 1 \pmod{p^\alpha}$ for $p^\alpha<100\,000$ and for numbers n from 3 to 17 and their doubles. Various smaller tables are given in which $p^\alpha<10\,000$ (in some cases 50 000) for the odd numbers $n<50$ and their doubles, and also for a few higher composite values of n. The more extensive tables for a fixed n are not confined to one volume but are distributed over three or four volumes as indicated below. The sequence of prime arguments in the tables is often interrupted to insert a sequence of prime power arguments. On the whole the arrangement leaves something to be desired. The following scheme gives some account of what values of n are considered in the various volumes.

values of n	volume numbers
4, 8, 16, 32: 3, 6, 12, 24: · · ·	1, 4, 8.
5, 10, 15, 20, · · ·	2, 6, 8, 9.
7, 14, 21, · · · : 9, 18, 27, · · ·	3, 7, 8, 9.
11, 22, 33, · · · : 13, 26, 39, · · ·	5, 7, 8, 9.
17, 34, · · · : 19, 38, · · · : 23, 46, · · ·	8, 9.
p, $2p$, $29 \leqq p \leqq 47$	9.

For $n=8$, Cunningham's table of the solutions of $x^4+1\equiv 0 \pmod p$ (CUNNINGHAM 28, 29) has been extended from $p=100\,000$ to $p=200\,000$ by HOPPENOT 2.

So far we have discussed the special congruence

$$x^n \equiv 1 \pmod{p^\alpha}$$

in which n is fixed throughout the table. There are several tables in which $n=p-1$. Tables of solutions x of the congruence

$$x^{p-1} \equiv 1 \pmod{p^2} \qquad (2)$$

which occurs, for instance, in the discussion of Fermat's last theorem date from JACOBI 1, who gave all solutions of (2) for $3\leqq p\leqq 37$. BEEGER 1 gives a more extensive table, in fact for $p<200$. MEISSNER 2 gives only one root x of (2) for $p<300$, and a root of

$$x^{p-1} \equiv 1 \pmod{p^3}$$

for $p<200$. A very short table of all solutions of

$$x^{p-1} \equiv 1 \pmod{p^\alpha} \qquad 1 \leqq \alpha \leqq 12, \qquad p \leqq 13$$

is given in BERWICK 1.

Another set of tables in which n depends on p is that of CUNNINGHAM 22 in which roots of

$$x^{p^k} \equiv \pm 1 \pmod{p^\alpha}$$

are tabulated, and in some cases roots of

$$x^{qp^k} \equiv \pm 1 \pmod{p^\alpha}, \qquad q = 2, 3, 5, 6 \qquad p^\alpha < 10\,000, \qquad p \leqq 19.$$

Special tables of the general binomial congruences

$$x^n \equiv r \pmod p \quad \text{or} \quad ax^n \equiv 1 \pmod p$$

may be cited as follows: CUNNINGHAM 21, giving solutions of

$$x^4 \equiv \pm 2 \pmod p \quad \text{and} \quad 2x^4 \equiv \pm 1 \pmod p \quad \text{for} \quad p < 1000,$$

and GÉRARDIN 3, giving all 4 solutions of

$$2x^4 \equiv 1 \pmod{p} \quad \text{for} \quad 1000 < p < 1600,$$

extended by VALROFF 1 up to $p<5300$.

CUNNINGHAM and WOODALL 9 have given tables of roots of the congruences

$$2^w \equiv w \pmod{p^\alpha},\ 2^z \equiv -z \pmod{p^\alpha},\ y2^y \equiv 1 \pmod{p^\alpha},\ x2^x \equiv -1 \pmod{p^\alpha},$$

for $p<50$, and $p=73$, 89, 127, 257, and for $p^\alpha<1000$ when $\alpha>1$ and $p\leqq 17$; for $43\leqq p\leqq 199$, values of w, z, y if $\leqq 100$ and values of x if $\leqq 250$; for $199\leqq p<10\,000$, values of w and z if $\leqq 100$; for $199\leqq p<1000$, and for certain selected primes $<10\,000$, values of y if $\leqq 100$, and values of x if $\leqq 250$.

Finally we may cite here a rather special table of LAWTHER 1 which gives for each integer $N<140$ the least positive solution x of

$$x^d \equiv \pm 1 \pmod{N},$$

which is "primitive" in the sense that

$$x^b \not\equiv \pm 1 \pmod{N}$$

for any positive $b<d$. Here d is the largest possible exponent for which such an x exists. For example if N is a prime, then $d=(p-1)/2$ and x is the least quadratic non-residue of p. This table is for use in the splicing of telephone cables.

d_5. *Higher residues*

By a higher residue modulo p we shall mean the residue of an nth power, where $n\geqq 3$. The case $n=2$ will be dealt with separately under i_2. A list of all the nth power residues modulo p may be found by taking every nth entry in a table of powers of a primitive root as described under d_2. If the greatest common divisor of n and $p-1$ is δ, this process will give in fact all the δth power residues, or in other words one can confine oneself to the case in which n divides $p-1$ so that p is of the form $nx+1$.

KRAITCHIK 3 has given a table of all nth power residues for

$$n = 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18$$

with respect to the first 20 or 25 primes $p=nx+1$ in each case.

A table of all three-digit cube-endings, i.e., cubic residues, modulo 1000, is given in MĂTIES 1.

For the case $n=4$, GAUSS 2 has a short table giving for each $p=4x+1<100$ not only its biquadratic residues but also those numbers $<p$ which have each of the other three biquadratic characters. The corresponding table for cubic characters by STIELTJES 1 extends to $p=6x+1\leqq 61$.

A rather special table of NIEWIADOMSKI 1 gives all the pth power residues (mod p^2) for each $p<200$, and is used in connection with criteria for Fermat's last theorem.

Some tables give lists of those primes p having a given number a as an

nth power residue. It has been pointed out before that such primes may be picked out of the tables, described under d_3, which give the residue-index of the base a modulo p. DESMAREST 1 and GÉRARDIN 1 each gave lists of primes $<10\,000$ of which 10 is an nth power residue. Similarly, KRAITCHIK 4 gives lists of primes $<10\,000$ of which 2 and 10 are nth power residues for all possible $n \geqq 2$. REUSCHLE 1 has listed all primes $p<50\,000$ having 10 for a cubic residue, and all primes $p<25\,000$ having 10 as biquadratic and octic residues. CUNNINGHAM 2 lists all primes $<25\,000$ having 2 as an octic residue, indicating those which have 2 as a 16th power residue.

GOSSET 1 has a table for finding the biquadratic character of q with respect to $p=a^2+b^2$ in case the value of b/a (mod q) does not exceed 8 in absolute value. Tables in CUNNINGHAM and GOSSET 1 serve to determine the biquadratic character $(q/p)_4$ when q contains no prime factor exceeding 41, and the cubic character $(q/p)_3$ when q contains no prime factor exceeding 47. These tables are reproduced in CUNNINGHAM 36 (p. 130–133). These restrictions on q are less drastic than would appear at first sight, since it is frequently easy to replace a given q by another congruent to it modulo p, and having only small prime factors. The "quadratic partitions"

$$p = a^2 + b^2 \quad \text{and} \quad 4p = L^2 - 27M^2$$

are supposed to be known. Tables of these partitions are cited and described under j_2.

Finally there are tables giving merely the frequency of primes having a given number a as an nth power residue. These have been obtained from tables of residue-indices by counting the number of p's having a given entry. CUNNINGHAM and WOODALL 7 give the number of primes p in each 10 000 up to 100 000 for which $(2/p)_n=1$ for all $n \leqq 40$. These are based upon corresponding enumerations of primes having given residue-indices.[1] CUNNINGHAM 23 has given similar tables for each of the bases 2, 3, 5, 6, 7, 10, 11 and 12. For the bases 2 and 10 the number of primes in each 10 000 up to 100 000 for which $(2/p)_n=1$ and $(10/p)_n=1$ respectively is given for $n \leqq 40$. A smaller table gives, for each of the bases mentioned above, the number of primes less than 10 000 having this base as an nth power residue ($n \leqq 40$). These are based on a set of tables giving for each base the number of primes having a specified residue-index.

d_6. *Converse of Fermat's theorem*

It is a well known fact that the fundamental theorem of Fermat

$$a^n \equiv a \pmod{n}, \text{ if } n \text{ is a prime} \tag{1}$$

has a false converse. Four tables giving examples of composite numbers n for which the congruence (1) holds may be cited here. Two of these are sufficiently

[1] The number of primes $\leqq x$ for which $(a/p)_n=1$ is clearly the sum of the numbers of primes $\leqq x$ for which the residue-index of a has the value kn ($k=1, 2, 3, \cdots$).

complete to be used in connection with the problem of identifying primes, and will be described from this point of view under **g**.

Isolated examples of composite numbers n satisfying the congruence (1), usually with $a=2$, date from 1819. In 1907 ESCOTT 1 gave a list of 50 miscellaneous composite numbers n for which

$$(2) \qquad 2^n \equiv 2 \pmod{n}.$$

Another miscellaneous list is given by MITRA 1 for $a=2$, 3, 5, 6, 7, and 10. D. H. LEHMER 6 gives a list of all 8-digit composite n satisfying (2), and having their least prime factor $p>313$. The factor p is given with each n. This list has been augmented by POULET 4, who has listed all composite $n<10^8$ for which (2) holds. Each n is given with its least factor provided this factor exceeds 30, otherwise the largest prime factor is given. The list comprises 2037 numbers. Many of these numbers n are such that (1) holds for every a prime to n and are accordingly marked with an asterisk.

e. FACTOR TABLES

No other kind of table in the theory of numbers is as universally useful as a factor table. The problem of factoring has long been recognized as a very fundamental one, and factor tables, as a partial solution of this problem, have a long and interesting history. This is especially true of the first of the two kinds of factor tables described below which we have called "ordinary." These factor tables were constructed for general use, the entries being found either by a sieve or by a multiple process. Tables of this sort, in which the entries are obtained readily, but not in their natural order, and in which an isolated entry cannot be easily found by direct calculations, exemplify the ideal table in the theory of numbers. The history of ordinary factor tables may be found in Chapter 13 of DICKSON 4, and in the sources there referred to, where an account will also be found of the numerous very old tables that are of historical interest. More recently, a bibliographic list of 16 ordinary factor tables, both old and new, beyond 100 000 has been given by Henderson in PETERS, LODGE and TERNOUTH, GIFFORD 1 (p. xiii–xv).

Tables of factors of numbers of special form are as a rule not published separately, but are scattered through periodical literature. An effort has been made to give a reasonably complete account of such tables.

e_1. *Ordinary factor tables*

By an ordinary factor table we mean a table which gives at least one divisor >1, or indicates the primality, either of every number within its range, or else of all the numbers not divisible by the first k primes. We can classify[1] such tables into types, according to values of k. A factor table of type 0 would

[1] To be sure, there are a number of small factor tables which omit only multiples of 2 and 5, and these escape our classification.

be a table dealing with all integers in its range, a type 1 table would consider all odd numbers in its range, a table of type 3 would deal with numbers prime to 30, etc. All large tables are of types 3 and 4. Theoretically the higher the type the more condensed the table becomes since a higher proportion of the natural numbers is thereby excluded. A table of type 25, for example, dealing with only those numbers whose least prime factor exceeds 100, would thus eliminate from consideration 88% of all numbers as compared with about 77% for a table of type 4. It is not difficult to see that the advantages of condensation gained by raising the type number are soon more than offset by the difficulties of arranging and ordering the table, if indeed one is to maintain the usual condensed form in which the number, whose least factor is given, is indicated merely by the position which that factor occupies in the body of the table. A factor table of high type and of very considerable extent could, however, be arranged in a form similar to a list of primes in which the last few digits of each number considered are given together with some symbol for its factor. The almost universal use of computing machines makes the omission of small factors from a table of high type a less serious objection than formerly.

Factor tables may also be classified according to the range of numbers about which information is given and also according to the amount of information given. The only table which gives the fullest information possible is that of ANJEMA 1. This rare table lists for each number $\leq 10\,000$ the complete set of all its divisors. A dash (or two dashes in case n is a square) separates those divisors which are $<\sqrt{n}$ from the others. This table is quite useful for experimental work on certain numerical functions and Dirichlet series. All other tables give either the canonical factorization into products of powers of primes, e.g., $360=2^3\cdot 3^2\cdot 5$, or else the least prime factor of each number considered.

Of the many small factor tables to 10 000 or thereabouts GLAISHER 27 (which was taken from BARLOW 1) and STAGER 2 are typical in that they give the canonical factorization of every number less than 10 000 and 12 000 respectively, and are at the same time quite reliable. An unusual table to 10 000 is due to CAHEN 2. It is a table of type 5, which omits primes as well. Only the least factor is given of each composite number, prime to $2\cdot 3\cdot 5\cdot 7\cdot 11$, and less than 10^4. Some idea of the condensation this achieves may be gained from the fact that it occupies only three and one half small pages.

Turning now to medium-sized factor tables we find 10 tables with upper limits ranging from 50 000 to about 250 000. The most useful and reliable of these are KAVÁN 1 and PETERS, LODGE and TERNOUTH, GIFFORD 1. Both are of type 0 and give canonical factorizations of all numbers up to 256 000 and 100 000 respectively. The arrangement in the latter table in which consecutive numbers lie in the same column rather than in the same line, is more convenient for many of the purposes for which such a table would ordinarily be used.

Other tables in this group giving canonical factorizations are of type 3. These are

reference	upper limit
POLETTI 2	50 000
CARR 1	99 000
GIFFORD 1	100 390
VEGA 1	102 000
LIDONNE 1	102 003
GOLDBERG 1	251 647

Lidonne's table is actually of type 0 as far as 10 000. The tables of Gifford and Goldberg should be used carefully, since each contains numerous errors. Poletti's table is of a handy pocket size but has quite a number of misprints.

The other tables in this group give only the least prime factor. LEBESGUE 1, which is a table of type 4, extends to 115 500. INGHIRAMI 1 deals with all numbers prime to 10 and less than 100 000, but is quite unreliable. GRAVE 3 is a type 3 table extending to 108 000.

The largest table giving canonical factorizations is the monumental *Cribrum Arithmeticum* of CHERNAC 1. This is a type 3 table and it extends to 1 020 000. It is remarkably accurate considering the number of entries and the era in which it was produced (1811), although a complete examination of this table has never been undertaken.

All other large tables list only the least prime factor of the number n considered, blank entries indicating that n is a prime. If the entry is a prime $p > \sqrt[3]{n}$, then the quotient n/p is also a prime. If $p \leq \sqrt[3]{n}$, it might be necessary to consult the table again (or perhaps some smaller more convenient table) for the least factor of n/p in case the complete decomposition of n is desired. A single examination of the table yields the often sufficient information that the number n is composite.

The nineteenth century saw the production and publication of such factor tables for the first 9 millions. BURCKHARDT 1, 2, 3 set the style with his table of the first three millions. These tables, almost always bound together, are, because they deal with the first three millions, more frequently useful than those of J. GLAISHER 1, 2, 3 for the fourth, fifth, and sixth millions and those of DASE 1, 2, 3, for the seventh, eighth, and ninth millions. All nine tables are of type 3 and are quite uniform in their arrangement. The page is split into 3 parts by two horizontal partitions, and entries in the same line, but in adjacent columns, refer to numbers differing by 300. This arrangement makes for ease in entering the table. This advantage to the user was paid for at the price of numerous errors (many of which occur in the eighth million) due to the fact that the practically mechanical and self-checking stencil or sieve process could not be employed to advantage for primes p much beyond 300 on account of the lengthy stencils required. Instead, recourse was had to the "multiple method," and numerous entries were put in the wrong place in the tables.

In referring to nineteenth century tables mention should be made of the huge manuscript table of KULIK 3 which extends from 4 000 000 to 100 330 201.

This is a type 3 table arranged exactly like Burckhardt's tables, except that the two horizontal partitions which divide the page into three parts are missing. Kulik used a system of one- and two-letter symbols to represent the primes, so that no entry requires more space than two letters. In this arrangement the use of stencils was feasible up to $p=997$, the "multiple method" being used for 4-digit primes.

Early in this century D. N. Lehmer began the construction of his monumental *Factor Table for the First Ten Millions* (D. N. LEHMER 1), which appeared in 1909. This is a type 4 table with a simpler arrangement than that used by Burckhardt, Glaisher, and Dase; that is to say, the arrangement is simpler for construction, but less simple for use. Entries in the same column, but in adjacent rows, refer to numbers differing by $210=2\cdot3\cdot5\cdot7$. There are naturally $\phi(210)=48$ columns. This arrangement enabled the use of stencils throughout the construction of the table. The user will find it a little more troublesome to enter this table than, for example, Burckhardt's. An auxiliary sheet enables one to find the exact row and column in which the least factor of one's number is given. This loose sheet, entitled "Auxiliary Table," which is reproduced on the reverse side of page 0, is apparently missing by now in many copies, since many writers contend that in order to enter the table it is necessary to divide the given number by 210, the quotient giving the page and line numbers, and the remainder giving the column number. Although this is not necessary, it is certainly sufficient and those users, to whom an electric calculating machine with automatic division is available, will find this method very effective where frequent use of table is required. The user can be turning to the proper page while the machine is operating. No error has as yet been found in the 2 372 598 entries of Lehmer's table.

A manuscript table for the sixteenth million was computed by DURFEE 1. The table, which is on 500 sheets of heavy paper, appears to have been copied from a type 5 table. Those numbers, whose least factor is 11, were later interpolated in red ink. The result is a type 4 table.

GOLUBEV 1 computed manuscript tables of the eleventh and twelfth millions.

Cunningham and Woodall have published many short tables beyond 10 million incidental to their determination of successive high primes. These tables will be cited under f_1 where these lists of primes are described. Similarly, KRAITCHIK and HOPPENOT 1 have two factor tables for the ranges from 10^{12} to $10^{12}\pm10^4$. These are of type 1, and give only the least divisor. The first of these from $10^{12}-10^4$ to 10^{12} was reproduced in KRAITCHIK 12.

e_2. *Tables of factors of numbers of special form*

The factorization of numbers defined in some special way has been the subject of countless investigations. In many cases short tables giving the results of a particular investigation have been published, mostly in periodical literature. Sometimes these results are used to obtain further factorizations. Often, however, each entry represents a great deal of hard work, in no way lessened by the existence of the other entries of the table. Occasionally, the complete factorization of a certain number is not known, but only one or two small prime factors are given. Again, there are often gaps in the table where even the prime or composite characters of the corresponding numbers are unknown, and may well remain so for centuries to come. It would therefore be difficult, and perhaps valueless, to give a precise account of just what factorizations are given in each of the many ancient and modern tables of factors of numbers of special form. Fortunately, writers have a tendency to reproduce the old tables along with their new entries. Thus it has been possible to neglect quite a number of historically interesting tables and to cite in each case the two or three modern ones by which a particular class of tables has been superseded.

By far the majority of tables of factors of numbers of special form deal with what are, in the last analysis, the factorization of certain cyclotomic functions. The Fermat numbers $2^{2^n}+1$, the Mersenne numbers 2^p-1, and more generally the numbers $2^n\pm1$, $10^n\pm1$, $a^n\pm1$, $a^n\pm b^n$, the Fibonacci numbers, the functions of Lucas and their generalizations comprise the class of numbers referred to.

If we denote by

$$Q_n(x) = x^{\phi(n)} + \cdots = \prod_{\delta/n} (x^{n/\delta} - 1)^{\mu(\delta)}$$

the irreducible cyclotomic polynomial whose roots are the $\phi(n)$ primitive nth roots of unity, so that we have the factorization

$$x^n - 1 = \prod_{\delta/n} Q_\delta(x),$$

then the tables referred to may be said to give the factors of x^n-1, when x is integral or rational, or (when x is algebraic) of the norm of x^n-1 taken with respect to the field defined by x or a subfield of that field. In all cases $Q_n(x)$ or its norm is the essential factor, the other factors $Q_\delta(x)$ $(\delta<n)$ having appeared before in the table. These other factors are quite often given separately and are called the *algebraic* or *imprimitive* factors; occasionally they are omitted entirely and only the factors of $Q_n(x)$, styled as the *irreducible* or *primitive* factors are given.

To begin with, up-to-date tables of the factors of the Fermat numbers $2^{2^n}+1$ are given in CUNNINGHAM and WOODALL 10 (p. xvi) and in KRAITCHIK 5, 6 (p. 221). These give one or more factors of $2^{2^n}+1$ for $n=5$, 6, 9, 11, 12, 15, 18, 23, 36, 38, and 73. For $n=0$, 1, 2, 3 and 4, $2^{2^n}+1$ is a prime as noted by Fermat. The numbers $2^{2^7}+1$ and $2^{2^8}+1$ are composite, but no factor of either number is known. Another table, lacking the entry for $n=15$, is given in KRAITCHIK 3 (p. 22).

A table of the latest results on Mersenne numbers 2^p-1, where p is prime, is given in ARCHIBALD 1. Here the reader will find a history of the problem with complete references to the original sources. A short table giving merely the number of prime factors of 2^p-1 for $p\leqq 257$ known in 1932 appears in D. H. LEHMER 4. Older tables of the factors of Mersenne numbers are in CUNNINGHAM and WOODALL 10 (p. xv), CUNNINGHAM 19 and WOODALL 1. This last table includes the forms of the factors of the numbers not then completely factored. KRAITCHIK 4 (p. 20) gives a list of small factors of 2^p-1 for 59 primes p, $79\leqq p<1000$, together with a list of the 85 primes p between 100 and 1000 for which no factor of 2^p-1 is known.

Tables of the factors of the numbers $2^n\pm 1$ really begin[1] with LANDRY 1, reproduced in LUCAS 1 (p. 236), who gave in 1869 the complete factorization of $2^n\pm 1$ for all values of $n\leqq 64$, except $2^{61}\pm 1$ and $2^{64}+1$. Recent tables are due to CUNNINGHAM and WOODALL 10 (p. 1–9), and KRAITCHIK 7 (p. 84–88). The first of these gives all information known in 1925 as to the factors of $2^n\pm 1$ for n odd and <500, and of 2^n+1 for n even and $\leqq 500$. Naturally, for such large ranges of n many entries are incomplete or even blank. However no factor $<300\,000$ has been omitted. The table really gives the factors of $Q_n(2)$ for n odd and <500, and for n even and $\leqq 1000$. The KRAITCHIK 7 (1929) table is an extension of one given in KRAITCHIK 3 (1922) and gives complete factorization of $2^n\pm 1$ as follows:

	2^n-1	n odd,	$n=1$–77, 81, 87, 89, 91, 93, 99, 105, 107, 117, 127.
	2^n+1	n odd,	$n=1$–65, 69, 75, 77, 81, 83, 87, 91, 97, 99, 105, 111, 135.
$2^{4k+2}+1$	$2^{2k+1}-2^{k+1}+1$,		$4k+2=2$–138, 150, 154, 162, 170, 174, 182, 198, 210, 270, 330.
	$2^{2k+1}+2^{k+1}+1$,		$4k+2=2$–130, 138, 146, 150, 154, 162, 170, 174, 182, 186, 190, 198, 210, 234, 258, 270.
	$2^{4k}+1$,		$4k=4$–84, 96.

Primitive and algebraic factors are given separately. The facts that

$$2^{101}-1,\ 2^{103}-1,\ 2^{109}-1,\ 2^{137}-1,\ 2^{139}-1,\ 2^{257}-1,\ 2^{128}+1,\ 2^{256}+1$$

are composite are also entered in the table. No factors of these numbers are known. Three factors of $2^{113}-1$ are also given.

[1] The comparatively insignificant table of REUSCHLE 1 (p. 22) antedates this by 13 years.

This table has been brought up to date in 1938 in KRAITCHIK 13. Factorizations are given here of

$$2^n - 1 \quad \text{for} \quad n = 79, 85, 95^*, 111.$$
$$2^n + 1 \quad \text{for} \quad n = 73, 93, 95.$$
$$2^{4k+2}+1\begin{cases} 2^{2k+1} - 2^{k+1} + 1 & \text{for} \quad 4k+2 = 146^*, 186^*, 190^*, 234^*, 250^*. \\ 2^{2k+1} + 2^{k+1} + 1 & \text{for} \quad 4k+2 = 142, 158^*, 222^*. \end{cases}$$
$$2^{4k} + 1 \quad \text{for} \quad 4k = 88, 100^*, 108, 120.$$

where * indicates that there is some doubt that certain large factors of these numbers are actually primes. The number $2^{241}-1$ is given as composite but without known factors, but there is no mention of the fact that the number $2^{149}-1$ belongs in the same category. A table giving the factors of $2^{4k}+1$ for $4k=4-88$, 96 appeared in KRAITCHIK 8. A table (p. 24–26) of KRAITCHIK 4 gives all prime factors $<300\,000$ of $2^n \pm 1$ for n odd and <257 and of $2^{4k+2}+1$ for $4k+2<500$ in those cases where the complete factorization of these numbers had not then been found.

Next to the numbers 2^n-1, the numbers 10^n-1 have been most frequently under consideration. These correspondingly larger numbers are especially interesting from the point of view of repeating decimals. The rational fraction k/p has a decimal expansion of period n if and only if p divides 10^n-1. This period is "proper" only in case p is a primitive factor of 10^n-1.

An early table of factors of 10^n-1 is due to REUSCHLE 1. It is limited to $n \leqq 42$, and is naturally incomplete in many of its entries. Another old but readily accessible table is due to SHANKS 2, which gives all factors $<30\,000$ of 10^n-1 for $n<100$. Actually only the factors of $Q_n(10)$ are given. In twenty-five cases n is marked with an asterisk to indicate that the factorization is complete. As a matter of fact it is also complete for $n=19$, 23, 25, 26, 27, 34, 36, 38, 46, 48, 50 and 62. Similar tables are found in BICKMORE 1, 2, and GÉRARDIN 1. In 1924 KRAITCHIK 4 (p. 92) gave the complete factorization of 10^n-1 for n odd, $n=1$–21, 25, 29 and of 10^n+1 for $n=1$–17, 21, 23 and 25. CUNNINGHAM and WOODALL 10 give all factors $<120\,000$ (if any) of $10^n \pm 1$ for n odd and $\leqq 109$ and of 10^n+1 for n even, $\leqq 100$. There are of course many incomplete entries. Many new complete factorizations have been discovered since the publication of this table.

The most up-to-date tables of the complete factorizations of $10^n \pm 1$ are in KRAITCHIK 7 (p. 95). These give all prime factors of 10^n-1 for all odd $n \leqq 29$, and of 10^n+1 for $n=1$–21, 23–25, 27, 30, 31, 36 and 50. (The case of $10^{35}-1$ is in doubt.)

Besides the numbers 2^n-1 and 10^n-1 other numbers of the form a^n-1 have been the subject of factor tables. Thus REUSCHLE 1 gives the factors of a^n-1 for $a=3, 5, 6, 7$, for $n \leqq 42$, and similar tables by BICKMORE 1 give corresponding results for $a=3, 5, 6, 7, 11, 12$ for $n \leqq 50$. (Both these tables deal also with $a=2$ and 10 as mentioned above.) These tables are far from complete.

The most extensive tables of this kind are CUNNINGHAM and WOODALL 10, reproduced in KRAITCHIK 7. For the bases $a=3, 5, 6, 7, 11$ and 12 Cunningham and Woodall give, together with many complete factorizations, all prime factors $p<100\,000$ dividing either $a^n \pm 1$ for n odd and $\leqq 109$, or a^n+1 for n even and $\leqq 100$. Only the factors of $Q_n(a)$ are given. For these bases little attempt to factor individual numbers was made by the authors, the results being obtained indirectly from tables of exponents. As a result a goodly number of blank entries have now been filled in by various computers since the volume appeared. Most of these for the bases 3, 5, 6, 7, have been included in KRAITCHIK 7 (p. 89–94). This gives the complete factorization of $a^n \pm 1$ as follows:

$3^n - 1$	n odd,	= 1–41, 45, 47, 49, 51, 75, 105.
$3^n + 1$	$n \neq 6k + 3$,	= 1–31, 35, 37, 40, 41, 42, 47, 48, 60, 84.
	$n = 6k + 3$,	= 3–117, 135, 165.
$5^n - 1$	n odd,	= 1–29, 33, 35, 45, 75.
$5^n + 1$		n = 1–22, 24, 25, 27, 30, 34.
$6^n - 1$	n odd,	= 1–23.
$6^n + 1$		n = 1–22, 24, 26, 28, 30, 33, 35, 42.
$7^n - 1$	n odd,	= 1–17, 27.
$7^n + 1$		n = 1–16, 18, 21, 22, 35.

A small separate table giving the latest information on the factors of 6^n+1 appears in KRAITCHIK 11. This gives the complete factorization of 6^n+1 for $n=1$–32, 42.

For the bases a from 13 to 30 (exclusive of 16, 25 and 27) CUNNINGHAM 37 has given as far as known the factors of $a^n \pm 1$ for all $n \leqq 21$.

Thus far we have spoken of tables of factors of numbers of the form $a^n \pm 1$ in which a may be thought of as small and fixed while n ran to high limits. There is also another set of tables in which n is small and fixed, while a varies. Obviously the numbers in these tables do not increase as rapidly as those in the tables in which a is fixed and n varies. On the other hand less information about the possible factors of these numbers is available.

The first table of this sort is due to EULER 2 (1762). This is a factor table for numbers of the form a^2+1 extending to $a \leqq 1500$. Only factors <1000 are given as the table was constructed by a sieve process.

Surprisingly enough, this is the only factor table of its sort ever published, although other such tables have existed in manuscript from which have been extracted lists of primes of the form x^2+1 to be mentioned under f₂. There are two tables giving factors of a^2+1 for very large but scattered values of a. The first of these is GAUSS 8, which gives the complete factorization of a^2+1 or of $(a^2+1)/2$ for 712 values of $a \leqq 14\,033\,378\,718$, in those cases in which no prime factor exceeds 200. This table is only one of a set of 9 tables giving the factors of a^2+b^2 to be described presently.

The other special table of factors of a^2+1 is CUNNINGHAM 8. If (x, y) is a solution of the Pell equation $x^2-Dy^2=-1$, then the factors of $x^2+1=Dy^2$ are obtained from those of D and y. A list of the 97 values of x between 10^4 and 10^{12} for which the factorization of x^2+1 is thus possible (for $D<1500$) is given, together with the factorizations of the corresponding D's and y's. This table is extended and greatly ramified in CUNNINGHAM 28 (p. 106–112).

Tables of factors of $a^n \pm 1$, with $n>2$ and fixed, occur in REUSCHLE 1 for $a<100$ and $n \leqq 12$, with many gaps.

The largest collection of such tables occurs in the first, second, third and fifth volumes of Cunningham's *Binomial Factorisations:* CUNNINGHAM 28, 30, 32, 33. Many of these tables are extremely special and short. The essential factor $Q_n(a)$ of a^n-1, though an irreducible polynomial in a, may become reducible as a polynomial in x when a is replaced by any one of a large number of appropriate functions of x. Thus we get cases of relatively easy factorizations of numbers of the form $Q_n(a)$ where a is of special type. The 185 factorization tables in these four volumes are largely of this special type. Nineteen refer to $Q_n(a)$ and are in no way special. Their extent and location are given as follows:

n	limit of a	volume	pages
5	1000	2	106, 108, 110, · · · 118
7	250	3	154–158
8	1000	1	113–119
9	250	3	178–181
10	1000	2	107, 109, · · · 119
11	100	5	104–105
12	1000	1	157–163
13	100	5	113–114
14	250	3	154–158
15	200	2	185–188
16	200	1	140–141
18	250	3	178–181
20	200	2	177–178
21	40	3	172
22	100	5	104–105
24	200	1	215–216
26	100	5	113–114
30	200	2	185–188
36	54	3	191

There are also tables where n is a multiple of the n's listed above, but these are more than half blank.

Most of those entries in the above tables which are complete factorizations have been reproduced, with a few additions, in a more compact form by KRAITCHIK 7. Here one finds tables of the factors of $Q_n(a)$ for $a<100$ and for

$n=1$–12, 14, 15, 16, 18, 20, 24, 30 (p. 96–107), with some gaps, together with many supplementary results for other values of n up to 60. Numerous tables are given of the factorization of $x^n \pm 1$ (really of $Q_n(x)$ and $Q_{2n}(x)$) for values of $n<50$. These are without gaps and extend to various limits of x as indicated in the following scheme. This description includes special tables in which the factorization of $Q_n(x)$ is rendered easier for the special values of x indicated, on account of an algebraic decomposition as mentioned above. A simple example of this phenomenon is

$$Q_{12}(2a^2)=(2a^2)^4-(2a^2)^2+1=(4a^4-4a^3+2a^2-2a+1)(4a^4+4a^3+2a^2+2a+1).$$

Factorization of

	x^n-1		x^n+1	
n	general x	special x	general x	special x
4	—	—	$x\leqq 409$ (p. 116–117)	—
5	$x<400$ (p. 118–119)	$x=5a^2$, $a\leqq 100$ (p. 122–123)	$x<400$ (p. 120–121)	—
6	—	—	$x<400$ (p. 126–127)	$x=2a^2$, $a<130$; $x=6a^2$, $a<70$ (p. 128–9)
7	$x\leqq 50$ (p. 130–131)	—	$x\leqq 50$ (p. 130–131)	$x=7a^2$, $a<20$ (p. 130)
8	—	—	$x\leqq 32, 34, 36$ (p. 132)	—
9	$x\leqq 50$ (p. 132–133)	—	$x\leqq 50$ (p. 132–133)	$x=3a^2$, $a\leqq 21$ (p. 134)
10	—	—	$x\leqq 60$ (p. 135)	$x=2a^2$, $a<25$; $x=10a^2$, $a<8$ (p. 135)

For larger values of n, in fact for $n=11$–16, 18, 21, 22, 24, 26, 27, 30, 33, 35, 39, 42, and 49 there are small tables with many gaps. For further addenda in Cunningham's tables see BEEGER 5 and HOPPENOT 1.

There are several tables giving factors of numbers of the form $p^a \pm 1$. We have already pointed out that several tables of primitive roots give in addition the factorization of $p-1$. These are described in **d₁**. Similarly, Cunningham's tables of quadratic partitions (described under **j₂**) also give this information. These tables are found in CUNNINGHAM 7, p. 1–240, and CUNNINGHAM 36, p. 1–55. These lists have been useful in discussing primes of the form $kn+1$.

CUNNINGHAM and CREAK 1 (p. 1–91) give all divisors of $p-1$ (except 1 and $p-1$) for $p<10^4$. EULER 1 gave factorizations of $\sigma(p^\alpha)=(p^{\alpha+1}-1)/(p-1)$ as noted under **b₂**. A more extensive table is in KRAITCHIK 7 (p. 152–159). This gives factors of $p^\alpha \pm 1$ for $\alpha \leqq 15$, as also noted under **b₂**. Two small special tables may be noted. GÉRARDIN 2 gives a table of the factorization of those numbers of the form $(p+1)(p^2+1)$, $p<1000$, all of whose factors are less than 1000. GLAISHER 21 gives the factors of $p^6-(-1)^{(p-1)/2}$ for all $p<100$, except $p=79$ and 83.

Finally, there are two small tables of factors of n^n-1. LUCAS 1 (p. 294) gives the complete factorizations of $(2m)^{2m}-1$ for $m=7$, 10, 12, 14 and 15, while CUNNINGHAM 24 gives factors of $y^y \pm 1$ for $y \leqq 50$, many incomplete.

Turning now to more general numbers $a^n \pm b^n$ with $b>1$, we find a few tables of their factors. The earliest is a special table of GAUSS 8 for $n=2$, already referred to in connection with a^2+1. The complete table gives for 2452 numbers of the form a^2+b^2, $b \leqq 9$ their complete factorization. The numbers a are so chosen that all the prime factors of a^2+b^2 are less than 200. For each value of b there is an inverse table showing for each possible p all those a's for which the greatest prime factor of a^2+b^2 is p. The fact that the number of these a's appears to be finite in each case must have led Gauss to conjecture for the first time that the largest prime factor of x^2+A tends rapidly to infinity with x, a fact established by Ivanov in 1895. However, this table was really intended to be used in discovering arccotangent identities.

Numerous small tables of factors of $a^n \pm b^n$ occur in Cunningham's *Binomial Factorisations* as follows:

form	v.	pages	form	v.	pages
x^2+y^2	1	99	$x^{11} \pm y^{11}$	5	106, 107, 109, 111
x^3+y^3	1	149–150, 152, 221	$x^{12}+y^{12}$	1	217
x^4+y^4	1	120–129, 220	$x^{13} \pm y^{13}$	5	115–116
x^5-y^5	2	120–123, 130, 133–146, 148, 154, 158	$x^{14}+y^{14}$	3	169–171
			$x^{15}-y^{15}$	2	189, 193
x^5+y^5	2	124–129, 131–133, 147–149, 155, 159	$x^{15}+y^{15}$	2	192–193
			$x^{16}+y^{16}$	1	143
x^6+y^6	1	164–171, 174–179, 181–189, 220	$x^{18}+y^{18}$	3	191–192
			$x^{21} \pm y^{21}$	3	173–174
$x^7 \pm y^7$	3	160–168	$x^{22}+y^{22}$	5	112
x^8+y^8	1	142–143	$x^{27}-y^{27}$	3	193
$x^9 \pm y^9$	3	185–187, 189	$x^{30}+y^{30}$	2	195
$x^{10}+y^{10}$	2	179, 183			

KRAITCHIK 7 (p. 107–109) contains the complete factorization of $3^n \pm 2^n$ as follows:

$3^n - 2^n$, n odd, $=$ 1–27, 33, 35, 105

$3^n + 2^n$, $n =$ 1–27, 29, 30, 31, 33, 35, 36, 42, 45, 54, 63, 70 and 75.

CUNNINGHAM 26, which is an extension of CUNNINGHAM 17, contains tables of the factors of $x^n \pm (x-1)^n$ for $n=3$, 5, 7, 9, 11, and 15, with $x \leqq 100$, 100, 50, 50, 40, and 40, respectively. CUNNINGHAM 27 gives factors of $x^n \pm (x-n)^n$ for $n=3$, 5, 7, 9, 11, and 15 and for $x \leqq 74$, 187, 60, 74, 43, and 49 respectively, with some gaps. Both of these tables reappear in *Binomial Factorisations* as noted above.

A short table of the factors of $x^{xy} \pm y^{xy}$ for 15 pairs (x, y) is given in CUNNINGHAM 24 (p. 74).

The essential factor $a^{\phi(n)}Q_n(a/b)$ of $a^n - b^n$ can, by a formula of Aurifeuille, be expressed in the form $X^2 - nabY^2$, where X and Y are certain homogeneous polynomials in a and b, tables of whose coefficients are described under o. In case n, a and b are so chosen that nab is a perfect square this essential factor, generally irreducible, breaks up into two factors. Tables of factors of $a^n \pm b^n$

in this case have been given by KRAITCHIK 2. Here b, $a \leqq 100$, and n is usually less than 50. Quite a large number of factorizations are within the range of factor tables. There are comparatively few blank entries.

The technique of factorization developed for $a^n \pm b^n$ is also applicable, with slight modifications, to the function $U_n = (\alpha^n - \beta^n)/(\alpha - \beta)$ of Lucas (and its generalizations) in which α, β are algebraic integers. For example the Fibonacci series

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \cdots, U_n, \cdots; U_{n+1} = U_n + U_{n-1},$$

where $\alpha = (1+\sqrt{5})/2$, $\beta = (1-\sqrt{5})/2$, has been the subject of factor tables.

The first such, due to LUCAS 1 (p. 299), gives the complete factorization of U_n for $n \leqq 60$. KRAITCHIK 4 (p. 77–80) gives the factors of both U_n and $V_n = U_{2n}/U_n$ as follows:

U_n, n odd, = 1–71, 75, 81, 85, 87, 95, 99, 105, 129

V_n, $n \not\equiv 5 \pmod{10}$, = 1–72, 77, 78, 80, 81, 84, 87, 90, 93, 99, 102, 111, 120

V_n, $n \equiv 5 \pmod{10}$, = 5–175, 195, 205, 215, 225.

Another factor table of what is essentially a Lucas function is due to D. H. LEHMER 2, and gives for $n \leqq 30$ the factors of y_n, where $x_n^2 - 2y_n^2 = 1$, (x_n, y_n) being successive multiple solutions of this Pell equation.

The Fibonacci series increases more slowly than the series of numbers $2^n - 1$, and hence more terms can be factored before the numbers become too large. A more slowly increasing series than the Fibonacci series has been factored by HALL 1. Here the complete factorization of the norm $N(\alpha^n - 1)$, a function introduced by T. A. Pierce, in the field defined by the root α of $x^3 - x - 1 = 0$, is given for $n \leqq 100$.

Still slower series are factored by POULET 3. In case $f(x)$ is an irreducible reciprocal equation, the norm $N(\alpha^n - 1)$ taken with respect to the field defined by the root α of $f(x) = 0$ will be a perfect square. The sequence $U_n = \sqrt{N(\alpha^n - 1)}$ is a recurring series of order at most 2^r, where $2r$ is the degree of $f(x)$, and the possible factors of U_n are restricted to certain linear forms $nx + b$, permitting the factorization of quite large numbers U_n, especially when U_n increases slowly. POULET 3 has published a number of series U_n and $V_n = U_{2n}/U_n$, the terms of which are completely factored. He gives

7 series of order 2 (Lucas' functions)
7 series of order 4
1 series of order 8 to 138 terms
1 series of order 16 to 250 terms
1 series of order 32 to 382 terms
1 series of order 64 to 230 terms.

The least rapidly increasing of these is the series of order 32 defined by the reciprocal equation

$$x^{10}+x^9-x^7-x^6-x^5-x^4-x^3+x+1=0.$$

In fact $U_{383}=360\ 429\ 381\ 874\ 489=16199093\cdot 22249973$. The average value of U_{n+1}/U_n is only about 1.0845. The author mentions the construction of about 40 other series of this sort and gives many algebraic formulas of use in constructing such series. The conjectured parts of this memoir have been proved by the present writer.

We turn now to the consideration of tables which give factors of binomials which are not cyclotomic, such as for example ka^n+1 or a^4+4b^4. Some of the methods and tables employed in the cyclotomic case are applicable here also. KRAITCHIK 6 (p. 222–232) has given a complete factor table of all numbers of the form $k2^n+1$ lying between 10^8 and 10^{12} with $k<1000$. Only the least factor is given. This is an extension of the previous table, KRAITCHIK 4 (p. 12–13), for numbers of this form between $2\cdot 10^8$ and 10^{12}, with $k<100$, and $21\leqq n\leqq 38$ with some gaps.

D. H. LEHMER 10 gives factors of numbers of the form $k2^n-1$ for $k=3, 5, 7$, and 9, and $n\leqq 50$ with some gaps. Factors of $6^k s\pm 1$ have been given by BEEGER 6.

CUNNINGHAM and WOODALL 1 gave a table of factors of $10^a 2^x\pm 1$ for $a\leqq 10$, $x\leqq 30$ (with gaps) and for several higher values of a and x. CUNNINGHAM 25 gives the factors of $x^y\pm y^x$ for 128 pairs of integers (x, y).

CUNNINGHAM AND WOODALL 9 has considered the factors of $2^q\pm q$ and of $q2^q\pm 1$. All factors are given for $q\leqq 66$. For $67\leqq q\leqq 260$ only small factors are given of $q2^q\mp 1$. These numbers are remarkable for being nearly all composite.

CUNNINGHAM 21 has tables of factors of $y^4\pm 2$ and of $2y^4\pm 1$. These extend to $y\leqq 100$, and to several higher values of y.

A short table in KRAITCHIK 4 (p. 14) gives the complete factorization of the numbers $p_1p_2\cdots p_n\pm 1$, where p_n is the nth prime, for all $n\leqq 8$.

LEBON 1 contains a table (part II) of all factors of numbers of the form $N_k=2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13k+1$ for such values of $k\leqq 4680$ as make N_k composite.

Tables of factors of numbers not associated with binomials are as follows:

CUNNINGHAM and WOODALL 8 contains the factorization of numbers of the form $2^a\pm 2^x\pm 1$ for $x<a<27$, and several higher numbers of this form.

VANDIVER 1 gives a list of small factors of Bernoulli numbers B_n. All values of $n\leqq(p-3)/2$ are given for which the numerator of B_n is divisible by p for $317\leqq p\leqq 617$.

ALLIAUME 1 has published decompositions of $n!$ for all $n<1200$. In Table I he gives the factorization of $n!$ into products of powers of primes, while in Table II, $n!$ is expressed as a product of powers of "prime factorials" $p_1p_2\cdots p_r$, where p_r is the rth prime. This table is useful in computing values of $\log n!$. PETERS and STEIN 1 have a table of the canonical factorization of the binomial coefficients up to those of the 60th power.

Finally, we may cite the table of CUNNINGHAM 36 (p. 162–170) which gives canonical factorizations of all numbers $\leq 10^6$, all of whose prime factors are <13. This table has a number of interesting uses, especially in connection with the calculation of logarithms and the binomial congruence. Tables of the same sort, but very much more extensive, are given in WESTERN 4, together with tables showing the mere number of numbers N having small prime factors only, for many very large values of N. MILLER and LODGE 1 gives the number of numbers $\leq 10^6$ having a given prime p as least (and also greatest) prime factor for all possible primes p.

f. LISTS OF PRIMES AND TABLES OF THEIR DISTRIBUTION

Tables of this sort naturally fall into two groups according as the primes considered are consecutive or not. Tables giving information on distribution phenomena are mostly concerned with consecutive primes. Lists of primes themselves have mainly two uses: 1) they may enable one to decide whether a given number is a prime or not, and 2) they serve as a source of statistical information about properties of primes. In spite of the existence of ordinary factor tables, the first use, whose importance is often not fully appreciated by those interested in distribution phenomena, is perhaps the best reason for the publication of lists of primes. Here, again, lists of consecutive primes are more useful than lists of primes of special form.

f_1. *Consecutive primes*

Lists of consecutive primes are of two sorts, those giving all primes less than a given limit, and those giving all primes between two high limits. Most lists of primes of the first sort occur as *arguments* in numerous tables, such as those of the binomial congruence (d_1, d_2), and certain "quadratic partition" tables cited under j_2. Among the more extensive of these lists we may cite for example KRAITCHIK 4 (p. 131–191), giving a list of primes to 300 000. The tables of SIMONY 1 and SUCHANEK 1 contain a list of primes to $2^{14}=16\ 384$, and from 2^{14} to 100 000 respectively. These tables give the primes also in the binary scale, or rather in a condensed form of binary scale in which, for example, the prime 2243 instead of being written 100011000011 is abbreviated to .3242, the dot being the symbol for 1.

Among those lists of primes which are not the incidental arguments of other tables, many small ones are to be found in textbooks on the theory of numbers, and even in certain handbooks for engineers. J. GLAISHER 1 has a convenient list of primes to 30 341, giving also a column of differences which occasionally is useful. LEBESGUE 1 has given a list of primes to 5500 at the same time showing how each prime may be represented by a pair of symbols, a device similar to that employed by Kulik in his factor table.

By far the most extensive list of primes is Lehmer's list of primes from 1 to 10 006 721 (D. N. LEHMER 2), which appeared in 1914. This list, containing 665 000 primes, 5000 on each page, is based on his *Factor Table* (mentioned under e_1), and on previous factor tables. The arrangement makes possible the rapid determination of n when the prime p_n is given; the user should be careful to note that here 1 is counted as a prime.

Other fairly extensive lists of primes are VEGA 1, for primes from 102 001 to 400 313, and POLETTI 2 (p. 3–67) for primes under 200 000. The latter, being in a handy pocket size, is quite convenient for occasional use.

We turn now to lists of consecutive primes between high limits (beyond 10 million). There are, surprisingly enough, as many as 69 such lists. Most of these cover only a short range of natural numbers and all but 8 of them have their lower limit ≦ 100 000 000.

The following list has been kindly prepared by Dr. N. G. W. H. Beeger, who is the best authority on large primes. In each case are given the upper and lower limits of the range in which all the primes are determined. If, in addition, the author includes a factor table for the range considered, this fact is indicated by an asterisk.

range		reference
10 000 000–	10 001 020*	CUNNINGHAM and WOODALL 5
10 000 000–	10 100 000	POLETTI 1
10 000 000–	10 100 009	POLETTI 2
10 076 676–	10 078 712*	CUNNINGHAM 35
10 088 152–	10 088 651	CUNNINGHAM 35
10 324 364–	10 324 517	CUNNINGHAM 16
10 761 411–	10 761 949*	CUNNINGHAM 16
11 000 000–	11 000 250	CUNNINGHAM 20
11 110 889–	11 111 333	CUNNINGHAM and WOODALL 5
11 184 451–	11 185 169	CUNNINGHAM and WOODALL 4
11 184 451–	11 185 169	CUNNINGHAM and WOODALL 2
12 093 036–	12 093 435	CUNNINGHAM 35
12 201 521–	12 201 702	CUNNINGHAM 16
12 206 762–	12 207 301*	CUNNINGHAM 16
12 499 750–	12 500 250	CUNNINGHAM and WOODALL 5
13 421 558–	13 421 988	CUNNINGHAM and WOODALL 6
13 450 870–	13 451 536	CUNNINGHAM 35
14 285 429–	14 286 000	CUNNINGHAM and WOODALL 5
14 285 715–	14 300 000	POLETTI and STURANI 1
14 347 889–	14 349 923*	CUNNINGHAM and WOODALL 4
14 912 970–	14 913 191	CUNNINGHAM 16
15 116 295–	15 116 794	CUNNINGHAM 35
16 275 683–	16 276 399*	CUNNINGHAM 16
16 666 334–	16 667 000	CUNNINGHAM and WOODALL 5
16 776 197–	16 778 233*	CUNNINGHAM and WOODALL 4
16 776 197–	16 778 233	CUNNINGHAM and WOODALL 3
19 173 819–	19 174 103	CUNNINGHAM 16

range	reference
19 486 153– 19 488 187*	CUNNINGHAM 35
19 999 600– 20 000 400	CUNNINGHAM and WOODALL 5
20 155 059– 20 155 725	CUNNINGHAM 35
20 176 304– 20 177 303	CUNNINGHAM 35
21 522 822– 21 523 899*	CUNNINGHAM 16
22 369 263– 22 369 980*	CUNNINGHAM and WOODALL 6
24 413 524– 24 414 600	CUNNINGHAM 16
24 999 500– 25 000 500	CUNNINGHAM and WOODALL 5
26 843 346– 26 843 745	CUNNINGHAM 16
30 232 589– 30 233 587	CUNNINGHAM 35
32 258 065– 32 261 290	POLETTI 2
33 332 667– 33 334 000	CUNNINGHAM and WOODALL 5
33 553 417– 33 555 451*	CUNNINGHAM and WOODALL 4
33 553 417– 33 555 451	CUNNINGHAM and WOODALL 3
34 482 759– 34 486 206	POLETTI 2
40 352 608– 40 354 606*	CUNNINGHAM 35
43 045 643– 43 047 800*	CUNNINGHAM 16
43 478 261– 43 482 608	POLETTI 2
44 738 910– 44 739 575	CUNNINGHAM 16
48 827 047– 48 829 201*	CUNNINGHAM 16
49 999 000– 50 001 000	CUNNINGHAM and WOODALL 5
52 631 579– 52 636 842	POLETTI 2
58 823 530– 58 829 411	POLETTI 2
60 465 177– 60 467 175*	CUNNINGHAM 35
61 621 560– 61 711 650*	BEEGER 9
67 107 787– 67 109 941*	CUNNINGHAM and WOODALL 6
76 923 077– 76 930 769	POLETTI 2
99 998 000– 100 002 000*	CUNNINGHAM and WOODALL 5
100 000 000– 100 001 000	KRAITCHIK 4 (p. 10)
100 000 000– 100 001 000	CUNNINGHAM 36 (p. 76)
100 000 000– 100 001 699	W. DAVIS 1
100 000 000– 100 005 000	PAGLIERO 1
100 000 000– 100 010 011	POLETTI 2
100 000 000– 100 100 000	POLETTI and STURANI 1
134 216 729– 134 218 727*	CUNNINGHAM 16
999 999 001– 1 000 119 119*	BEEGER 9
1 000 000 000– 1 000 001 000	KRAITCHIK 4 (p. 10)
1 000 000 000– 1 000 010 000	POLETTI 1
1 000 000 000– 1 000 100 009	POLETTI 2
999 999 990 000–1000 000 000 000*	KRAITCHIK and HOPPENOT 1
999 999 990 000–1000 000 000 000*	KRAITCHIK 12
1000 000 000 000–1000 000 010 000*	KRAITCHIK and HOPPENOT 1

Tables having to do with the distribution of consecutive primes p_n are of 5 types.

(A) Tables of $\pi(x)$, the number of primes $\leq x$, with or without the corresponding values of some approximating function.

(B) Tables of $\pi(nh)-\pi\{(n-1)h\}$, i.e., tables of the number of primes in each successive interval of length h of the natural numbers.

(C) Frequency tables, giving the number of centuries having a prescribed number of primes in each of a series of intervals.

(D) Tables of anomalies in the distribution of primes.

(E) Tables of $\sum_p p^{-n}$ and of $\Pi_{p\leq x}(1-p^{-1})$.

In tables of type A values of $\pi(x)$ usually have been extracted from lists of primes and factor tables. Meissel and Bertelsen have however evaluated $\pi(x)$ independently for use in checking factor tables. As already mentioned, isolated values of $\pi(x)$ for $x<10^7$ can be determined at a glance from D. N. LEHMER 2. A graph of $\pi(x)$ for $x<12000$ is given in STAGER 1, 2. A small table of $\pi(x)$ for consecutive integers x is included in GRAM 1, where $\pi(x)$ is tabulated along with the function

$$\psi(x) = \sum_{p^\alpha \leq x} \log p$$

for all $x<300$, and for $x=p^\alpha$, $300<p^\alpha<2000$.

All other tables give $\pi(x)$ for wide intervals of x. The best such table is D. N. LEHMER 2, where $\pi(x)$ is tabulated for $x=50\,000(50\,000)10^7$ and for $x=k\cdot 10^7$, $k=2$, 9, 10, 100. These last four entries, due to Bertelsen and Meissel, are compared with the corresponding values by Riemann's formula

$$P(x) = \sum_{n=1}^{\infty} \frac{\mu(n)\ Li\ (x^{1/n})}{n}.$$

All other entries of this table are compared not only with Riemann's formula, but also with those of Chebyshev and Legendre, which are

$$\int_2^x \frac{dx}{\log x} \quad \text{and} \quad \frac{x}{\log x - 1.08366} \text{ respectively.}$$

Other tables of $\pi(x)$ may be cited and described briefly as follows:

GLAISHER 16. This gives $\pi(x)$ for $x=100\,000(100\,000)9\,000\,000$ compared with formulas of Riemann, Chebyshev and Legendre. This table is reproduced in J. GLAISHER 3. GLAISHER 5 gives $\pi(k\cdot 10^6)$ for $k=.25(.25)4$ compared with various modifications of Legendre's formula.

GRAM 2 gives $\pi(x)$ for $x=k\cdot 10^6$, $k=.1(.1)1(.025)3(.1)7(.05)9(.1)10$, as well as $k=20$, 90, 100, 1000. These values due to Bertelsen as already mentioned were computed directly by Meissel's method. All but the last four were verified by direct count in Lehmer's *Factor Table*. POLETTI 2 (p. 243) gives $\pi(x)$ for

$x=k\cdot 10^6$, $k=.2(.2)1(1)10$ and $k=20$, 90, 100, and 1000, with comparisons with the formulas of Riemann, Legendre, Chebyshev, and Cesàro.

Tables of type B are more numerous than those of type A and are more indicative of the average density of primes in the region under consideration. The scope of each type B table which gives the number of primes in each successive interval of h natural numbers between the limits a and b may be given the following tabular description:

reference	a	h	b
GAUSS 6	1	1 000	1 000 000
	1 000 000	10 000	3 000 000
	1 000 000	1 000 000	3 000 000
GLAISHER 1	1	50 000	1 000 000
	8 000 000	50 000	9 000 000
GLAISHER 3	1	10 000	100 000
	100 000	50 000	400 000
	400 000	100 000	3 000 000
GLAISHER 2	6 000 000	100 000	9 000 000
GLAISHER 6	1	250 000	3 000 000
	6 000 000	250 000	9 000 000
GLAISHER 11	3 000 000	10 000	4 000 000
	1	250 000	4 000 000
GLAISHER 13	4 000 000	10 000	5 000 000
	1	100 000	5 000 000
	1	250 000	5 000 000
GLAISHER 14	5 000 000	10 000	6 000 000
	1	100 000	9 000 000
	1	250 000	9 000 000
	1	1 000 000	9 000 000
HUSQUIN 1	1	1 000 000	10 000 000
DURFEE 1	15 000 000	100	16 000 000

Tables of type C date from GAUSS 6 and give for all possible n the number of centuries containing n primes in each successive interval of h natural numbers between the limits a and b, and the total number of such centuries for the whole range a to b. The distribution always has a single mode about which there is a vague symmetry. The frequency tables in GAUSS 6 are due to Goldschmidt and, though inaccurate, are more detailed than any published later. There are 20 tables, each covering a range (a, b) of 100 000 between 1 000 000 and 3 000 000 for which $h=10$ 000, and two summarizing tables for the second and third million in which h is now 100 000. Other tables of type C may be given the following description:

reference	a	k	b
GLAISHER 7	1	10^6	$3\cdot10^6$
	10^6		$9\cdot10^6$
GLAISHER 3	1	10^6	$3\cdot10^6$
GLAISHER 2	$6\cdot10^6$	10^6	$9\cdot10^6$
GLAISHER 11	$3\cdot10^6$	10^5	$4\cdot10^6$
GLAISHER 13	$4\cdot10^6$	10^6	$5\cdot10^6$
	1	10^6	$5\cdot10^6$
GLAISHER 14	1	10^6	$9\cdot10^6$
KRAITCHIK 4 (p. 16)	1	10^6	10^6
HUSQUIN 1	1	10^6	10^7
KRAITCHIK and HOPPENOT 1	$10^{12}-10^4$	10^3	10^{12}
KRAITCHIK and HOPPENOT 1	10^{12}	10^5	$10^{12}+10^4$
KRAITCHIK 12	10^{12}	10^3	$10^{12}+10^4$

The table of HUSQUIN 1 and that of GLAISHER 13 show several discrepancies. Presumably this is due to errors in old factor tables.

Tables of type D mainly relate to large gaps in primes, that is, long series of consecutive composite numbers. A few tables relate to the distribution of "twin primes" differing by 2, "triplets," etc.

GLAISHER 7 gives for the ranges 1–3 000 000 and 6 000 000–9 000 000 all those gaps of 99 or more (79 or more for the first million) in the list of primes. Gaps of 111 or more for the same millions are given in GLAISHER 6. Gaps of 99 or more for the fourth million are listed in GLAISHER 11, for the fifth million in GLAISHER 13, for the sixth million in GLAISHER 14, where one also finds the largest gap in each of the first nine millions. Finally in GLAISHER 16 all gaps greater than 130 in the nine millions are given, arranged in order of length of gap. The largest gap in the first 10 millions is 153, following the prime 4 652 353. DURFEE 1 discovered an equally large gap following the prime 15 203 977.

All the tables of types A–D cited so far that are due to Glaisher have been reproduced by J. GLAISHER 1, 2, 3 in the introductions to his factor tables of the fourth, fifth and sixth millions. Tables relating to the full 9 millions appear in the introduction to the last of these volumes.

WESTERN 3, using in part the data of Glaisher, constructed a table of those primes $p_n < 10^7$ whose difference $p_n - p_{n-1}$ exceeds that of all smaller primes. This useful table has been reproduced by CHOWLA 1. The first 13 such primes had been listed by KRAITCHIK 4 (p. 15).

There are a few tables giving facts about the distribution of twin primes. GLAISHER 8 gives the number of twin primes in each successive chiliad (1000) in each of the first hundred chiliads of the first, second, third, seventh, eighth, and ninth millions. There is also a companion table giving the number of these chiliads containing a prescribed number of twin primes. A summary of these results in GLAISHER 9 gives the number of twin primes in each of the ten successive myriads of the first 100 000 numbers of the above mentioned millions.

POLETTI 2 (p. 244–245) gives the number of twin primes in each of the first 10 myriads beyond 10^k for $k=0, 5, 7$, and 9.

STÄCKEL 1 gives the number of twin primes not exceeding x for $x=1000(1000)100\ 000$, while SUTTON 1 tabulates the same function for the same values of x and for the more extensive range $x=10\ 000(10\ 000)800\ 000$.

Short tables relating to twin primes and triplets appear in HARDY and LITTLEWOOD 1.

Five tables of type E, which date from Euler, may be cited. MERRIFIELD 1 gives 15-place values of

$$\Sigma_n = \sum_p p^{-n} \quad \text{for} \quad 1 < n \leqq 35.$$

This table is reproduced in GRAM 1 (p. 269). GLAISHER 20 gives 24-place values of Σ_{2h} and of $(1/h)\Sigma_{2h}$ for $2 \leqq 2h \leqq 80$. The corresponding entries Σ_n for n odd have been supplied by H. T. DAVIS 1, where Σ_n is given to 24 decimals for all integers n from 2 to 80.

There are only 2 tables of the function

$$P(x) = \prod_{p \leqq x} (1 - p^{-1}).$$

In LEGENDRE 1, $2P(x)$ is given to 6 decimal places for $x \leqq 1229$, except in LEGENDRE 1_1, where $x \leqq 353$. In GLAISHER 22, $P(x)$ and its common logarithm are given to 7 decimals for $x < 10\ 000$.

f_2. *Primes of special form*

As in the case of consecutive primes, lists of primes of special form often occur as arguments of tables giving further information about these numbers. Thus in giving primitive solutions of the binomial congruence

$$x^k \equiv 1 \pmod{p}$$

one needs to consider only those primes that are of the form $kn+1$. Lists of these primes therefore occur in tables cited under d_4, especially in CUNNINGHAM 28–34, 38, 39, where lists of primes of the form $kn+1$, for $n \leqq 17$, are given up to $p < 100\ 000$, and for many larger values of n up to $p < 10\ 000$. These lists are sometimes useful in searching for small factors of numbers of the form $a^k - b^k$.

Other lists of primes of the form $kn+1$ are in GLAISHER 17 for $p=4n+1 < 13\ 000$, and KRAITCHIK 4 (p. 192–204) for $p=512n+1 < 10\ 024\ 961$.

Other important special forms of primes are those linear forms associated with a given quadratic residue. Those primes p for which the Legendre symbol (D/p) has a given value ($+1$ or -1) belong to certain linear forms depending on D, tables of which are described under i_3. Tables of "quadratic partitions" of primes $p=x^2-Dy^2$ naturally extend over those primes p for which $(D/p)=+1$. Hence such tables (described under j_2) give incidentally lists of these primes. In particular the tables of CUNNINGHAM 7, 36 give at a glance those primes $p<100\ 000$ and $100\ 000<p\leqq 125\ 683$ respectively, for which the symbols $(-1/p)$, $(-2/p)$, $(-3/p)$ have given values, taken separately or together. The factor stencils of D. N. LEHMER 3, 4 (described under g) give, in effect, all those primes $\leqq 48593$ and $\leqq 55073$ respectively for which (D/p) has a given value for $|D|<239$, and $|D|<250$ respectively.

A rather special table relating to primes belonging to linear forms is due to DICKSON 1. This gives all sets of 3 primes which for a fixed value of $n\leqq 10$, are values of a_1n+b_1, a_2n+b_2, a_3n+b_3 for 64 selected sets of three such linear forms.

Tables giving the number of primes belonging to given linear forms and less than certain limits date from SCHERK 1 (1833). This table gives the number of primes belonging to each of the forms $4n\pm 1$ in each chiliad up to 50 000. GLAISHER 12 gives this information for each myriad between $k\cdot 10^6$ and $k\cdot 10^6+10^5$ for $k=0, 1, 2, 3, 6, 7, 8$. These results are summarized in GLAISHER 10. GLAISHER 9 gives this information for $k=0$, 1, and 2 only.

CUNNINGHAM 14 gives for $n=4, 6, 8, 10, 12$ the number of primes $<10^6$ of each of the forms $nx+\alpha_i$ $(x=0, 1, \cdots)$ for each $\alpha_i<n$ and prime to n. For the special form $nx+1$ and for $n=2k<60$, and 15 higher values $\leqq 210$, the number of primes $\leqq x$ of this form is given for $x=10^4$ and 10^5. For $n=8p$, $100<p<250$, the same information is given for $x=10^5$ and $5\cdot 10^5$. These results extend those given in CUNNINGHAM 7. These numbers are compared with $\pi(x)/\phi(k)$. The number of primes in each successive myriad up to 10^6 and belonging to the form $kn+1$ for all n from 1 to 30, for all even n from 30 to 60, and for 19 other values >60 is given in CUNNINGHAM 23.

POLETTI 2 (p. 244–245) gives the number of primes >5 of each of the 8 possible forms $30n+r$ $(r=1, 7, 11, 13, 17, 19, 23, 29)$ in each of the 10 successive myriads beyond 10^k for $k=0, 5, 7, 9$.

Tables of the number of primes of each of the forms $6n\pm 1$, $10n\pm 1$, $10n\pm 3$ in the first and second 100 000 numbers, and of the form $4n+1$ and $8n+1$ in the first myriad appear in KRAITCHIK 4 (p. 15–16), where also is given the number of primes of the form $512n+1$ in each 100 000 numbers and in each million from 1 to 10^7. KRAITCHIK and HOPPENOT 1 give the number of primes of each possible form (modulo m) in each chiliad between $10^{12}-10^4$ and 10^{12} for $m=4, 6, 8, 10$ and 12. The same information for the range 10^{12} to $10^{12}+10^4$ is given in KRAITCHIK and HOPPENOT 1 and also in KRAITCHIK 12.

Twin primes $(p, p+2)$, $p>5$, are of three sorts, according as $p=10n+1$, $10n+7$ or $10n+9$. The number of twin primes of each sort in the 100 000 numbers beyond 10^k for $k=0$, 7 and 9 is given in POLETTI 2 (p. 246).

We turn now to lists of primes of the form $a^n\pm b^n$ or prime factors of such numbers. These are in large part by-products of factor tables of numbers of this form (described under e₂).

Under this heading come lists of primes of the form $p=a^2+b^2$ already mentioned under the "quadratic partitions" tables of j₂, to which section of the report the reader is again referred. The special case in which $b=1$ is, however, particularly interesting, and more extensive lists of these primes have been prepared. These date from EULER 2, who gave a list of all primes $p=a^2+1$ less than 2 250 000 as well as all values $a<1500$ for which $(a^2+1)/k$ is a prime for $k=2$, 5, and 10.

CUNNINGHAM 6 gives lists of all primes beyond $9\cdot 10^6$ of the forms a^2+1 and $(a^2+1)/2$ with $a\leq 5000$. KRAITCHIK 4 (p. 11) lists the 312 numbers a for which a^2+1 is a prime $<10^7$.

WESTERN 1 gives the number of primes of the form a^2+1 less than x for various values of x up to $x=225\ 000\ 000$ as compared with Hardy's conjectured formula: .68641 $Li(x^{1/2})$

Many lists of high primes dividing $a^n\pm b^n$ appear in Cunningham's *Binomial Factorisations*. These may be given the following tabular description:

form	limit	no. of primes	v.	pages
x^2+1	$225\cdot 10^6$	4430	1	238–244
x^2-1	$225\cdot 10^6$	4884	1	245–252
x^3-y^3	10^6	472	1	259–260
x^4+y^4	10^{10}	778	1	253–255, 281–284
$x^5\pm 1$	10^{10}	1565	2	200–210
x^6+y^6	10^{10}	1065	1	256–257, 261–264, 285–288
$x^7\pm y^7$	10^{10}	183	3	196–198, 203
x^8+y^8	$4\cdot 10^{12}$	9	1	258
$x^9\pm y^9$	10^{10}	182	3	200–202
$x^{10}+y^{10}$	10^{10}	87	2	211–212
$x^{11}\pm y^{11}$	10^{10}	42	5	119
$x^{12}+y^{12}$	10^{10}	20	1	258
$x^{15}\pm y^{15}$	10^{10}	172	2	211–214

Some of these lists have been published separately as follows:

form	limit	reference
$(a^3-1)/(a-1)$	16 000 000	CUNNINGHAM 6
$a^3-(a-1)^3$	1 000 000	CUNNINGHAM 17
a^4+b^4, a^8+b^8	10^{10}	CUNNINGHAM 10
$a^6\pm b^6$	10^{10}	CUNNINGHAM 13
$a^5\pm 1$	25 000 000	CUNNINGHAM 3

We now turn to lists of primes which are binomials other than of the cyclotomic form $a^n\pm b^n$ just considered.

Lists of primes represented by the binary quadratic form $x^2 \pm Dy^2$, in which x and y are also given, are listed under j2. However, we take this occasion to cite the list of 188 primes of the form $x^2+1848y^2$ lying between 10 000 000 and 10 100 000 of CUNNINGHAM and CULLEN 1, reproduced in CUNNINGHAM 36 (p. 74–76).

Three tables have been published of large primes of the form $k\cdot 2^n+1$. KRAITCHIK 4 (p. 53) gives 43 primes of the form $k\cdot 2^n+1$ between $2\cdot 10^8$ and 10^{12} with $3 \leqq k < 100$. This was later extended in KRAITCHIK 6 (p. 233–235) to include all such primes between 10^8 and 10^{12} with $k<1000$. CUNNINGHAM 36 (p. 56–73) gives a list of primes of the form $k\cdot 2^n+1$, $9 \leqq n \leqq 21$ up to various high limits $<10^8$.

DINES 1 has lists of k and s, $6 \leqq k \leqq 10$, for which $6^k s \pm 1$ are primes for all s less than 100, and in some cases 400. Certain cases left in doubt have been disposed of by BEEGER 6.

All primes of the form $2^x 3^y 5^z+1<10^7$ have been given by KRAITCHIK 4 (p. 53). The 184 sets (x, y, z) corresponding to these primes appear on p. 9–10.

Poletti, Sturani and Gérardin have constructed by a sieve process factor tables up to high limits of numbers of the form An^2+Bn+C $(n=1, 2, \cdots)$ for different choices of A, B, C, from which they have extracted long lists of high primes. The actual primes are not always given, but instead the values of n for which the function An^2+Bn+C is a prime are tabulated. The chief interest in such tables lies in the empirical information which they give concerning the distribution of primes of this form. Whether their number is finite or not is an unsolved problem.

The first such table is due to POLETTI 2 (p. 249–255), and gives all primes of the form n^2-n-1 up to 10 400 000. POLETTI 3 gives all primes between 10 018 201 and 24 123 061 of the form $5n^2+5n+1$. POLETTI and STURANI 2 list those n's for which either of the two numbers $2n^2+2n\pm 1$ is a prime $<250\ 000\ 000$. A table is given of the number of primes in each 1000 terms of the series

$$2n+1, \qquad n^2+n\pm 1, \qquad 2n^2+2n\pm 1$$

up to $n=11\ 000$.

POLETTI 4 gives all primes <121 millions of the form $n^2+n\pm 1$ with a table of their distribution. POLETTI 5 contains a table of nearly 17 200 primes, arranged in increasing order, and extracted from the series $n^2+n\pm 1$, $2n^2+2n\pm 1$, n^2+n+41, $41n^2+n+1$, $6n^2+6n+31$.

In attempting to construct quadratic functions containing more primes than Euler's $E(n)=n^2+n+41$, Lehmer and Beeger have suggested

$$L(n) = n^2+n+19\,421, \quad B'(n) = n^2+n+27\,941, \quad B''(n) = n^2+n+72\,491.$$

Poletti has investigated the frequency of primes in all four functions and his results are given in BEEGER 7 (p. 50), where is found the number of primes represented by each function for $n<x$, $x=1000(1000)10\ 000$. These facts

would suggest that $B'(n)$ and $B''(n)$ are more, and $L(n)$ less, fruitful sources of primes than $E(n)$.

POLETTI 7 gives all primes represented by $L(n)$, $B'(n)$, and $B''(n)$ between 10^7 and $2\cdot 10^7$.

POLETTI 6 contains primes represented by about 200 different quadratic functions An^2+Bn+C, $10^7<p<3\cdot 10^9$.

GÉRARDIN 6 contains over 2000 values of n for which An^2+Bn+C is a prime $>10^7$ for the following polynomials and ranges of n

$$\begin{array}{lll} 2n^2+2n+1 & \text{for} & 15800 \leqq n \leqq 23239 \\ 101n^2+20n+1 & \text{for} & 315 \leqq n \leqq 1542 \\ 122n^2+22n+1 & \text{for} & 286 \leqq n \leqq 1369 \\ 10n^2 \pm 6n+1 & \text{for} & 3161 \leqq n \leqq 4620 \\ 26n^2 \pm 10n+1 & \text{for} & 1216 \leqq n \leqq 1774. \end{array}$$

High primes represented by the trinomial $2^\alpha \pm 2^x \pm 1$ for $x<\alpha<27$ and many more pairs (x, α) are given in CUNNINGHAM and WOODALL 8.

KRAITCHIK 9 gives a list of the 94 largest primes known, and in KRAITCHIK 10 is a list of 161 primes exceeding 10^{12}.

g. TABLES FOR FACILITATING FACTORING AND IDENTIFYING PRIMES

Besides factor tables and lists of primes there are tables available for the easy application of known methods of factoring and tests for primality.

For instance, the method of factoring depending on quadratic residues, as described by Legendre, makes use of certain lists of linear forms or "linear divisors" of quadratic forms x^2-Dy^2. These are described in detail under i_3. To render this method still more effective, D. N. LEHMER 3, 4 devised the *factor stencils*. These give in place of the linear forms an actual list of the primes belonging to these forms. More particularly, in D. N. LEHMER 3, all the primes $\leqq 48593$ belonging to linear forms dividing x^2-Dy^2, or what is the same thing, all the primes having D for a quadratic residue are given for $|D|<239$. Actually the primes for each D are not printed but are represented by holes punched in a sheet of paper. Since the primes for $D=k^2D_1$ are the same as those for D_1, only the D's without square factors, of which there are 195, need be considered. Each of the 195 sheets is ruled in 5000 square cells, 25 to the square inch, 50 columns by 100 rows. A cell, by virtue of its row and column number, represents one of the first 5000 primes p, and if $(D/p)=+1$, it is punched out. All factors $\leqq 48593$ of a given number N having D as a quadratic residue are among those primes whose corresponding cells are punched out of the stencil for D. Having found a suitable number of quadratic residues D of N, the mere superposition of the corresponding stencils reveals only a few open holes, among which the factors $\leqq 48593$ of N must then lie. In this way, the discovery of all factors of N (if any) below this limit is reduced to the discovery of a certain number, not exceeding ten or a dozen, of quadratic residues less than 239 in absolute value. Thus the device will handle completely all numbers less than the square of 48611, i.e., 2 363 029 321, and, of course, can be used in factoring much larger numbers.

In D. N. LEHMER 4, the same method is used in a different form. Here use is made of Hollerith cards of 80 columns and 10 rows. For each D there are 7 cards of different color, each color dealing with 800 primes. By superposing cards of the same color for different D's, all prime factors of N less than 55 079 may be found. Besides extending the number of primes from 5000 to 5600 Elder has extended the range to $|D|<250$. The new edition has been entirely recomputed by Elder and, in addition to being more reliable, is more convenient to use than the old, especially when N is comparatively small, so that only two or three colors are needed.

The tables of D. H. LEHMER 6 and POULET 4 serve to test for primality any number n below 10^8 in 20 to 25 minutes at most. These tables give lists of composite numbers n for which $2^n \equiv 2 \pmod{n}$ together with a factor of n. The table of Lehmer is restricted to contain only such entries $n>10^7$ as have their least factor >313, while the more extensive table of Poulet contains all possible n's up to 10^8. In using the Poulet table one notes first if the given number n is in the table. If so, a factor of n is given. If not, then $2^n \equiv 2 \pmod{n}$ is a necessary and sufficient condition for primality of n. Whether this congruence holds or not can be decided quickly by a method of successive squarings described in D. H. LEHMER 6. In using Lehmer's table, there is the additional possibility that n contains a small factor $\leqq 313$. If so, this factor can be quickly discovered by a greatest common divisor process therein described.

Factorization methods, depending upon the representation of the given number by quadratic forms such as x^2-y^2, x^2+y^2, x^2-Dy^2, are greatly facilitated by the use of certain tables cited and described under i_1.

The list of 65 *Idoneal numbers D* (such that the unique representation of n by x^2+Dy^2 insures the primality of n) is given for example in MATHEWS 1 and CUNNINGHAM 36 (p. ii).

SEELHOFF 1 contains lists of binary quadratic forms especially devised for factorization purposes.

Tables giving the final digits of squares are sometimes used in factoring small numbers. These are cited under i_1 and i_2. A table of this sort, especially designed for representing n by $x^2 \pm y^2$ with $x<y<2500$, is given in KULIK 1 (p. 408–418).

A table of reciprocals of primes $<10\ 000$ to 8 significant figures with differences is given in PETERS, LODGE and TERNOUTH, GIFFORD 1. This is intended to replace trial divisions by a series of multiplications by the given number, and is useful when the available computing machine has no automatic division or no keyboard.

The detailed account of the application of commercial and specially made computing devices to the problem of factoring numbers and identifying primes will appear in another report of the Committee: Z.

h. TABLES OF SOLUTIONS OF LINEAR DIOPHANTINE EQUATIONS AND CONGRUENCES

The solution of the general linear Diophantine equation

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = k$$

depends ultimately upon the solution of

$$(1) \qquad ax + by = c$$

and tables of solutions of such equations with more than 2 unknowns are non-existent. A solution (x, y) of (1) gives at once a solution x of the linear congruence

$$(2) \qquad ax \equiv c \pmod{b}$$

and conversely, a solution of (2) gives a solution of (1) with $y=(c-ax)/b$. The equation (1), if it has a solution, can be reduced by cancelling common factors of a and b to the case in which a and b are coprime. All solutions (x, y) of (1) are then given by the formulas

$$x = kb + \xi c$$
$$y = -ka + \eta c,$$

where k is any integer, and (ξ, η) is any solution of

$$(3) \qquad a\xi + b\eta = 1.$$

Hence it is sufficient to tabulate a solution of (3) or of the congruence

$$(4) \qquad a\xi \equiv 1 \pmod{b}.$$

CRELLE 3 gives a solution of (3) for each coprime pair (a, b) with $b<a\leqq 120$. A similar table by CUNNINGHAM 36 extends to $a<100$, $b<100$.

Tables of solutions of the linear congruence (4) may be found in WERTHEIM 5, KRAITCHIK 4 (p. 27), and CUNNINGHAM 36. In these tables b is taken as a prime p, the composite case being readily reducible to this case. In Wertheim's table $a<p<100$. It is even possible to restrict a to be a prime, and this is done in Kraitchik's table where $a, b<100$, and are both primes. Cunningham's table (p. 158–161) gives for each prime $p<100$ solutions of both congruences $a\xi\equiv\pm 1 \pmod{p}$ for every $a<p$.

KRAITCHIK 4 (p. 69) has a table for the combination of two linear congruences, whose moduli are 2^n and 5^n. In fact the two congruences

$$N \equiv r_2 \pmod{2^n}$$
$$N \equiv r_5 \pmod{5^n}$$

give when combined

$$N \equiv A_2 r_2 + A_5 r_5 \pmod{10^n}.$$

The coefficients A_2 and A_5 are tabulated for $n \leqq 16$.

For the combination of many linear forms, a problem which arises in many different ways, graphical and mechanical methods are available. These are discussed in another report of this Committee (Z).

A special table due to J. L. BELL 1, useful in checking Bernoulli numbers $B_{2k} = N_{2k}/D_{2k}$, gives for each $n \leqq 62$ a solution (a_n, b_n) of the congruence

$$a_n N_{q+2n-1} \equiv b_n D_{q+2n-1} \pmod{q},$$

where q is any odd prime not in the set of excluded primes there listed.

i. CONGRUENCES OF THE SECOND DEGREE

i₁. *Solutions of quadratic congruences*

The general quadratic congruence in one unknown may be reduced by a linear substitution to one of the form

(1) $$x^2 \equiv D \pmod{m}.$$

When m is not too large, this congruence, when possible, is easily solved[1]. Nevertheless, it is very convenient in many applications to have these solutions tabulated. Existing tables are of two sorts: according as m is a power of 10, or a prime (or prime power). Tables of the first sort occur in tables of the endings of squares. KULIK 1 gives for each possible D, the two solutions of $x^2 \equiv D \pmod{10^4}$, which are < 2500 from which all solutions may be discovered. Similar tables for the moduli 10^3 and 10^4 occur in CUNNINGHAM 36 (p. 90–92).

Tables of the second sort date from EULER 2, who gave solutions $\pm x$ of

(2) $$x^2 + 1 \equiv 0 \pmod{p^\alpha} \qquad (\alpha \geqq 1)$$

for all primes $p = 4k+1$ up to $p^\alpha < 2000$. A table of solutions of (2) with $\alpha = 1$, and $p = 4k+1 < 1000$ is given in KRAITCHIK 4 (p. 46) and for $p < 100\ 000$ in CUNNINGHAM 28, 29.

The quadratic residue tables of GÉRARDIN 4 and CUNNINGHAM 36 give solutions $\pm x$ of (1) with $m = p$, for all possible $D \pmod{p}$ and for $p < 100$. The latter table contains in addition solutions of (1) for $m = p^\alpha \leqq 125$, $(\alpha > 1)$.

There are several very useful tables of solutions of quadratic congruences in two unknowns. The first of these is due to GAUSS 10 and gives all solutions $(x, y) \pmod{p}$ of the congruence $fx^2 + gy^2 \equiv A \pmod{p}$

[1] Especially if one uses the new stencil device of ROBINSON 1.

for all possible congruences of this sort with $p \leqq 23$. KULIK 1 gives solutions x of the congruence

$$x^2 \pm y^2 \equiv N \pmod{10^4}.$$

That is to say, the last four digits of possible numbers x in the equation $N = x^2 \pm y^2$ are given for all possible four-figure endings of N. A similar table for 3-digit endings is given in BIDDLE 1.

KRAITCHIK 3 (p. 187–199) gives tables of all solutions a, b, x, y, z, t of the congruences

$$x^2 - y^2 \equiv N \pmod{p^\alpha},\quad a^2 + b^2 \equiv N \pmod{p^\alpha},\quad z^2 + rw^2 \equiv N \pmod{p^\alpha}$$

and $t^2 + nv^2 \equiv N \pmod{p^\alpha}$ for all possible $N \pmod{p^\alpha}$, where r is any quadratic residue, and n any quadratic non-residue $\pmod{p^\alpha}$ for all primes $p < 50$, and all $p^\alpha \leqq 128$, except 121. An abridged table (p. 200–204) gives solutions x of the congruence

$$(3) \qquad x^2 + Dy^2 \equiv N \pmod{p^\alpha}$$

for all possible D and for $N \equiv 1$ and $n \pmod{p^\alpha}$, where n is the least non-residue $\pmod{p^\alpha}$, for $p < 100$, and for all powers of 2, 3, 5, 7, 11 up to 2^{12}, 3^6, 5^4, 7^2 and 11^2.

A short table showing all solutions x of (3) with $D = -1$ in case certain numbers R are known to be quadratic residues of N occurs in KRAITCHIK 4 (p. 87). The moduli considered are $p^\alpha = 8, 16, 32, 3, 5, 7, 11$ and 13, while the values of R are -1 and ± 2, when p is even, and $(-1/p)p$, when p is odd. A more complete table of the solutions of (3) with $D = -1$ occurs in KRAITCHIK 6. Here are found the solutions x for all possible N and for all primes $p \leqq 67$. The table is in two parts, thus separating the two cases $(N/p) = \pm 1$. In case $(N/p) = +1$, half the solutions x are impossible if it is known that $(-1/p)p$ is a quadratic residue of N.

CUNNINGHAM 36 (p. 103–123) gives, for all possible N, solutions x of (3) for $D = -1, 1, 2, 3$, for all $p \leqq 41$, and $p^\alpha \leqq 64$ $(\alpha > 1)$, as well as for the modulus 100.

D. H. LEHMER 8 gives, in effect, all solutions x of

$$ax^2 + bx + c \equiv y^2 \pmod{p^\alpha}$$

for all possible $a, b, c, \pmod{p^\alpha}$ and for all $p^\alpha \leqq 128$ with the exception of 125 and 127.

All these tables are, of course, designed for the application of Gauss' method of exclusion and serve to reduce such problems as the representation of a number by a given binary quadratic form to the mere combination of linear forms, and thus to make applicable a certain graphical and mechanical technique fully described under another report of this Committee: **Z.**

i₂. *Quadratic residues and characters and their distribution*

There are many small tables of quadratic residues giving for the first few primes p the positive quadratic residues of p arranged in order of their size. The more extensive of these may be described as follows:

BUTTEL 1 gives for each $p \leqq 101$, the list of its quadratic residues and non-residues. FROLOV 1 gives quadratic residues for all $p \leqq 97$, omitting, for $p \geqq 23$, those residues which are actual squares. CUNNINGHAM 36 (p. 100–102) gives quadratic residues and non-residues for all primes $p \leqq 131$. KRAITCHIK 3 (p. 205–207) gives quadratic residues for all $p < 200$. CUNNINGHAM 36 (p. 93–95) gives lists of residues (mod p^α) for $p < 100$, and $p^\alpha \leqq 169$. These are arranged in the order of their least positive square roots.

Since the even powers of a primitive root of p^α are the quadratic residues of p^α, while the odd powers are the quadratic non-residues, a table of powers of a primitive root gives in particular a table of residues and non-residues. Such tables were cited and described under d₃. Quadratic residues and non-residues for $p^\alpha < 1000$ are thus obtainable from JACOBI 2.

There are several tables of quadratic residues modulo 10^k. These are usually described as tables of "square endings," since they give the possible last k digits of squares. These are of two kinds: those which list all the actual endings in order of magnitude, and those tables which enable the user to decide at a glance whether a given ending is a square ending or not.

A list of all 159 three-digit square endings appears in SCHADY 1. KULIK 1, SCHADY 1, and THÉBAULT 1 list all the 1044 four-digit square endings. In Schady's table with each four-digit ending are given all possible fifth digits. Thus the entries ·2489 and g4676, for example, indicate respectively that any digit may precede 2489, while only even digits may precede 4676.

CUNNINGHAM 1 has a one page table showing at a glance whether a proposed 1, 2, 3, or 4-digit ending is a square ending or not. This table is reproduced in CUNNINGHAM 36 (p. 89). A similar table of three-digit square endings to the base twelve, due to Terry, appears in E. T. LEHMER 2.

A few tables give the values of the Legendre symbol (a/b) of quadratic character. GAUSS 1 gives the value of (a/q) for every odd prime $q < 100$, and for every prime $a < 100$, as well as $a = -1$. This table is extended in GAUSS 4 to $q \leqq 503$ and $a < 1000$. In both these tables a dash indicates either that $(a/q) = +1$ or that $a = q$, while the absence of any entry indicates that $(a/q) = -1$.

A small table in WERTHEIM 5 gives (p/q) for all $p < 100$ and $q < 100$, and is intended to give a graphic representation of the Law of Reciprocity.

A. A. BENNETT 2 gives for all odd primes $p \leqq 317$ and for all positive numbers $n < p$, the value of x in $(n/p) = (-1)^x$. That is, the table gives $x = 0$ or $x = 1$ according as n is or is not a quadratic residue of p.

D. N. LEHMER 3, 4 give the values of (a/q) for all $q \leqq 48\,593$ and $q \leqq 55\,073$

and for $|a| < 239$, and $|a| < 250$ respectively. In fact in the stencil (or stencils) for a the nth cell is punched out or not according as a is or is not a quadratic residue of the nth prime.

Finally four tables relating to the distribution of quadratic residues may be cited.

GAUSS 3 gives the number of quadratic residues in each of the r intervals of length p/r of the numbers from 1 to p for the following values of r and the corresponding ranges of p:

$$r = 4, \quad p = 4n + 1 < 400$$
$$r = 8, \quad p < 400$$
$$r = 12, \quad p = 4n + 1 < 275.$$

For $r=4$ the actual number of quadratic residues in each quadrant is not given, but follows at once from the given values of m by the formula $(p-1-(-1)^k 4m)/8$, where $k=1, 2, 3, 4$, is the number of quadrant.

A. A. BENNETT 3 gives the number of consecutive quadratic residues and non-residues for all primes $p \leqq 317$.

KRAITCHIK 4 (p. 46) gives for each $p \leqq 47$ the least non-square N_p of the form $8n+1$ which is a residue of all odd primes $\leqq p$. This table is extended to $p \leqq 61$ in D. H. LEHMER 1.

D. H. LEHMER 7 gives for each $r < 28$, the positive integer $N_r = 8n+3$ such that $-N_r$ is a quadratic non-residue of all odd primes not exceeding the rth prime p_r. Also $N_{29} > 5 \cdot 10^9$.

i_3. *Linear forms dividing $x^2 - Dy^2$*

The term *linear divisor* of $x^2 - Dy^2$ is due to Legendre, who published the first real tables of such forms. These linear forms are nothing more nor less than the arithmetic progressions in which lie all primes p for which $(D/p) = +1$, these being the only primes which will divide numbers of the form $x^2 - Dy^2$, in which x is prime to Dy. The linear forms dividing $x^2 - k^2D_1y^2$ are clearly those dividing $x^2 - D_1y^2$, so that tables of these forms deal only with those D's which have no square factor >1. In general we may speak of the linear divisors of any binary quadratic form f as the set of arithmetic progressions to which any prime factor of a number properly represented by f must belong. The tables of LEGENDRE 1 list the linear forms for each of the reduced (classical) quadratic forms of determinant D for $-79 \leqq D \leqq 106$. If all such forms for a fixed D are taken together we obtain the set of linear divisors of $x^2 - Dy^2$. Legendre's tables are the only ones in which this separation of the linear divisors of $x^2 - Dy^2$ is attempted. The tables of CHEBYSHEV 1 really give a part of this information, however; in fact for each $D < 33$ are given the possible forms (mod $4D$) of those numbers which are properly represented by the forms $x^2 - Dy^2$ or $Dy^2 - x^2$.

The tables of Legendre were reproduced and extended somewhat by CHEBYSHEV 2, who gave all linear forms dividing x^2-Dy^2 for $|D|\leq 101$, carrying over most of the many errors in Legendre's table. Chebyshev's table is reproduced in CAHEN 1 with more errors.

KRAITCHIK 3 published a table of linear forms for $|D|<200$. When $D>0$, the form $4D+x$ is always accompanied by the form $4D-x$ so that only those x's which are $<2D$ are given with the understanding that both $\pm x$ are to be taken. This table is extended in KRAITCHIK 4 where $200<|D|<250$.

These are the only large tables of linear divisors published. Unfortunately all contain numerous errors. D. N. LEHMER 5 extends to $|D|=300$ and was used by him and Elder in the preparation of the factor stencils (D. N. LEHMER 3, 4).

Three small tables of linear forms may be cited. LUCAS 4 gives the linear forms dividing x^2-Dy^2 for $|D|<30$. WERTHEIM 5 has a similar table for $|D|<23$. The table of CAHEN 3 gives linear forms $mx+r$ ($m=2D$ or $4D$) dividing x^2-Dy^2 for $|D|<50$. This table is peculiar in that the r's are chosen to be absolutely least (mod m) and are arranged according to increasing absolute values.

CUNNINGHAM 36 gives a small table of linear divisors and non-divisors of x^2-Dy^2. That is to say, the forms of primes for which $(D/p)=+1$ and $(D/p)=-1$ are listed for $|D|<12$. The tables of LEVÄNEN 2 give for 62 selected binary quadratic forms of negative determinant >-385 the corresponding linear divisors.

j. DIOPHANTINE EQUATIONS OF THE SECOND DEGREE

The solution of a large number of interesting problems in the theory of numbers, algebra and geometry may be made to depend on Diophantine or "indeterminate" equations. Problems resulting in equations which are of the second degree are particularly interesting, and solutions of such equations have been subjects of a great many tables. These tables fall naturally into three classes: those giving information about the equations

$$(1) \qquad x^2 - Dy^2 = \sigma, \qquad \sigma = \pm 1, \pm 4,$$

where D is a positive non-square integer, those dealing with the more general equation

$$(2) \qquad x^2 - Dy^2 = N,$$

and those dealing with quadratic equations involving more than two unknowns such as $x^2+y^2=z^2$. The equations (1) have long been recognized as fundamental and are known as *Pell equations*, although the term "Pell equation" is sometimes restricted to the case of $\sigma=1$, and sometimes generalized to cover (2). The equations (1) and those equations (2) for which $N<\sqrt{D}$ are intimately connected with the continued fraction expansion of $\sqrt{D}$, tables of which are cited and described under **m**.

j_1. *The Pell equations* $x^2-Dy^2=\sigma$, $\sigma=\pm 1, \pm 4$

Although these equations, especially with $\sigma=1$, have a very long history, the first tables of their solutions (x, y) were given by Euler. Since Euler's time the importance of the Pell equation to the theory of binary quadratic forms and of quadratic fields $K(\sqrt{D})$ has been fully realized and Euler's original tables have been greatly extended.

The first sizable table of the solutions of

$$x^2 - Dy^2 = \pm 1$$

appeared in LEGENDRE 1 in 1798. The table extends to non-square D's $\leqq 1003$, except in the second edition of LEGENDRE 1, where $D \leqq 135$. The fundamental solution of

$$(3) \qquad x^2 - Dy^2 = -1$$

is given whenever possible, otherwise of

$$(4) \qquad x^2 - Dy^2 = +1.$$

A glance at the final digits of x, y and D tells which of these two equations is satisfied by the given x, y. This table for $D \leqq 1003$ is reproduced in LEGENDRE 1_5, 1_6.

Table 1 of DEGEN 1 gives solutions of (4) for $D \leqq 1000$. Table 2 gives solutions of (3) for all possible D not of the form n^2+1, in which case the fundamental solution is obviously the trivial one $(x, y)=(n, 1)$. Unlike Legendre's table, Degen's Table 1 contains also the elements of the continued fraction for $\sqrt{D}$.

CAYLEY 6 may be considered as a continuation of Degen's Table 1 for $1000 < D \leqq 1500$, except that when (3) has a solution, that solution is given in place of the solution of (4) as indicated by an asterisk. This table was computed by Bickmore.

WHITFORD 1 gives for $1500 < D \leqq 1700$ solutions of (4) and also of (3) if possible, the latter being easily distinguished by their relative smallness. The corresponding continued fraction developments are given separately for $1500 < D \leqq 2012$.

These are the main published tables of the solutions of (3) and (4). D. H. LEHMER 9 gives these solutions for $1700 < D \leqq 2000$. An announced table, GÉRARDIN 7, up to $D=3000$, is probably incomplete.

There are 6 small tables giving the solutions of (3) and (4) for non-square D's <100. These are CAYLEY 6 (p. 75–80), WERTHEIM 5, CUNNINGHAM 7, PERRON 1, CAHEN 3 and KRAITCHIK 4 (p. 48–50). The tables of Cayley and Perron give in addition the continued fractions for $\sqrt{D}$. The tables of Cayley and Cunningham give solutions of (4) and also of (3) when possible. The others give solutions of (3) when possible, and otherwise of (4).

A special table of NIELSEN 1 gives solutions of (4) for those D's of the form a^2+b^2 for which the expansion of the continued fraction for $\sqrt{D}$ has an odd period with $D<1500$.

INCE 1 gives solutions of (3) or (4) with $D \neq k^2 D_1$ whenever

$$x^2 - Dy^2 = +4 \tag{5}$$

has no coprime solutions (x, y) for $D<2025$.

The omission of the solutions of (4) when (3) has a solution (x, y) is not important since the fundamental solution of (4) is in that case $(2x^2+1, 2xy)$.

The problem of telling "in advance" whether or not (3) is solvable has never been satisfactorily solved. Three small tables give information on this question. SEELING 3 gives the list of all those D's <7000 for which (3) is solvable. A similar list only for $D \leqq 1021$ appears in KRAITCHIK 4 (p. 46). NAGELL 1 gives the number $B(n)$ of D's not exceeding n for which (3) has a solution together with the number $A(n)$ of non-squares $\leqq n$ which are sums of two coprime squares, and also the difference $A(n)-B(n)$ for $n=100, 500, 1000(1000)10\ 000$.

Thus far we have been speaking of fundamental (or least positive) solutions of (3) and (4). If (x_1, y_1) is such a solution of (4), the successive multiple solutions of (4) are given by (x_n, y_n), where

$$x_n + \sqrt{D}\, y_n = (x_1 + \sqrt{D}\, y_1)^n \qquad (n = 1, 2, \cdots)$$

and are connected by the second order recursion formulas

$$x_{n+1} = 2x_1 x_n - x_{n-1}$$
$$y_{n+1} = 2x_1 y_n - y_{n-1}.$$

If, on the other hand, (x_1, y_1) is the fundamental solution of (3), then (x_{2n}, y_{2n}) and (x_{2n+1}, y_{2n+1}) are all the solutions of (4) and (3) respectively.

Only two tables give multiple solutions of (3) and (4). CUNNINGHAM 7 gives the first multiple solutions of both equations for $D \leqq 20$. A table of y_n in $x_n^2 - 2y_n^2 = +1$ is given for $n \leqq 30$ in D. H. LEHMER 2.

Four tables give solutions of (5) and of

$$x^2 - Dy^2 = -4. \tag{6}$$

These are used in constructing automorphs of indefinite binary quadratic forms, or the units of the real quadratic field $K(\sqrt{D})$.

Equation (5) is always possible since a solution is $(2x, 2y)$ where (x, y) is a solution of (4) and by the same device (6) may be solved when (3) is possible. These solutions are uninteresting, however. For some D's (5) and (6) have no coprime solutions. In fact it is necessary that $D \equiv 0$, 4, or 5 (mod 8).

The first two cases are uninteresting since, if $D=4D_1$, a solution (x, y) of (5) or (6) implies and is implied by the solution $(x/2, y)$ of (4) or (3) respectively (with $D=D_1$). Therefore tables of the solutions of (5) and (6) are concerned solely with $D\equiv 5 \pmod 8$. The first such table is ARNDT 2 which gives solutions of (6) when possible, otherwise of (5) if possible for $D\leqq 1005$. A similar table for $D\leqq 997$ is due to CAYLEY 1. Solutions of (5) and (6) are given whenever possible with $1005<D\leqq 1997$ in WHITFORD 2. INCE 1 gives solutions of (5) or (6) whenever possible for $D<2025$ as units of the field $K(\sqrt{D})$.

If (6) has a fundamental solution (x, y) (for $D\equiv 5 \pmod 8$) the solutions of (5), (3) and (4) are respectively

$$(x^2+2,\ xy),\qquad ((x^3+3x)/2,\ y(x^2+1)/2)$$

and

$$((x^6+6x^4+9x^2+2)/2,\ y(x^2+1)(x^3+3x)/2).$$

If (5) has a fundamental solution (x, y) with $D\equiv 5 \pmod 8$, then that of (4) is $((x^3-3x)/2,\ y(x^2-1)/2)$. Hence these tables may be used to find solutions of (3) and (4) if necessary.

Many writers have suggested methods for solving Pell equations, which avoid the explicit use of continued fractions. It is safe to say however that those methods which are practical for solving isolated equations like, for example, $x^2-1141y^2=1$ in which x and y have 28 and 26 digits, are equivalent to the continued fraction method. The application of the continued fraction algorithm by modern mechanical methods will be treated in another report of the Committee: **Z.**

j_2. *Other equations of the form $x^2\pm Dy^2=\pm N$*

Besides the tables of the Pell equations, there are tables of solutions of the equation

$$(1)\qquad x^2-Dy^2=\pm N \qquad (N\neq 1, 4).$$

These are of two kinds, according as D is positive or negative. In tables of the former kind, N is comparatively small. Those of the latter kind extend over prime values of N up to high limits for a very few negative values of D.

An important special case of (1) for $D>0$ is that in which $N<\sqrt{D}$. In this case the continued fraction development of $\sqrt{D}$ will disclose whether or not (1) is possible. In fact, by a theorem of Lagrange, $\pm N$ will appear in the denominator of a complete quotient (when these are taken with alternating signs) if and only if (1) is possible, and the corresponding convergent x/y will be the solution of (1). Hence tables of the continued fraction developments of $\sqrt{D}$ (cited and described under **m**) give information for solving (1) in this case.

The table of KRAITCHIK 4 gives the least positive solution (x, y) of (1) for $N<\sqrt{D}$, and for $D<100$. CAYLEY 6 gives for each non-square $D<100$ the least positive solution of (1), where $\pm N$ are the denominators of the complete quotients in the continued fraction of $\sqrt{D}$ taken with alternating signs, so that $N<2\sqrt{D}$. For larger values of N the continued fraction method is no longer applicable. There remains however the multiplicative property implied by the formula

$$(x_1^2 - Dy_1^2)(x_2^2 - Dy_2^2) = (x_1x_2 \pm Dy_1y_2)^2 - D(x_1y_2 \pm x_2y_1)^2$$

known to Brahmagupta. This product to which Cunningham has given the descriptive name "conformal multiplication" enables one to derive from solutions of

$$x_1^2 - Dy_1^2 = N_1 \quad \text{and} \quad x_2^2 - Dy_2^2 = N_2$$

a pair of solutions of

$$x_3^2 - Dy_3^2 = N_1N_2.$$

In particular from the infinity of multiple solutions of

$$x^2 - Dy^2 = 1,$$

we can derive an unlimited number of solutions of (1) from a single initial solution. Conformal multiplication is the basis of the extensive tables of solutions of (1) prepared by Nielsen. His largest table is NIELSEN 4 (p. 1–195) which gives small solutions of (1) for $N<1000$ and for $2\leqq D\leqq 102$, and for several larger D's up to 401. NIELSEN 2 contains a smaller table for $N<1000$ and for $D=34$, 79, 82 and 101, and certain products of these numbers by squares. NIELSEN 3 gives similar results for $D=30$, 41, 51 and 130.

Information about the solvability of (1) is given in NIELSEN 5, which lists for each $N\leqq 10$, all those D's $<10\,000$ for which (1) has a solution. With each D is given a solution (t, u) of $t^2-Nu^2=D$.

A small table of solutions of the equation (1) appears in OETTINGER 1, where fundamental and five multiple solutions are given for $D\leqq 20$, and $N=1, 2, \cdots, 10, 3^k, 5^k, 7^k$ with $1\leqq k\leqq 4$.

Conformal multiplication is also applicable to equations of the type

$$(2) \qquad Ax^2 - By^2 = N \qquad (AB = D),$$

which become of type (1) on multiplication by A.

Three tables of solutions of such equations have been given. ARNDT 1 gives solutions (x, y) of

$$Ax^2 - By^2 = 2 \qquad (AB = D)$$

when possible, otherwise of

$$Ax^2 - By^2 = 1$$

for all $D \leqq 1003$ which have no square factor, nor are primes or doubles of primes of the form $4n+1$. That pair of factors (A, B) of D is chosen which gives the smallest solution (x, y).

NIELSEN 4 (p. 199–234) gives small solutions (x, y) of (2) with $N<1000$ and $AB=D$ ranging over composite numbers from 10 to 346 with some gaps. This is an extension of the smaller table in NIELSEN 3, where $AB=30$, 41, 51 and 130, mentioned above.

Turning now to tables of the second kind in which D is negative, we find that in almost all cases N is a prime. This is permissible in view of conformal multiplication. These tables give the solutions (x, y) of

$$x^2 + y^2 = p \qquad p = 4m + 1 \tag{3}$$

$$x^2 + 2y^2 = p \qquad p = 8m + 1, 3 \tag{4}$$

$$x^2 + 3y^2 = p \qquad p = 6m + 1 \tag{5}$$

$$x^2 + 27y^2 = 4p \qquad p = 6m + 1. \tag{6}$$

These representations or "quadratic partitions" (to use Cunningham's terminology) of p are possible if and only if p is of the linear form (or forms) indicated, and when possible, are essentially unique (in (3) it is customary to insist that y be even). These quadratic partitions are chiefly used in determining the character $(a/p)_n$ for $n=3$, 4, 8, 16 for small bases a, especially $a=2$, and have been a great aid as a preliminary to finding the exponent of a (mod p). The distribution of quadratic residues (mod p), and certain class number and cyclotomic problems also depend upon these partitions.

The first extensive tables of quadratic partitions were published by JACOBI 3 in 1846, and were computed by Zornow and Struve. These give the partitions (3) for $p \leqq 11981$, (4) for $p \leqq 5953$ and (5) for $p \leqq 12\,007$. A table of the partitions of (3) for $p \leqq 10\,529$ occurs in KULIK 1.

REUSCHLE 1 gave the partitions (3) and (4) for $p \leqq 12\,377$ and for all those primes p from 12 401 to 25 000 of which 10 is a biquadratic residue. The partition (5) is given for $p \leqq 13\,669$ and for all those primes from 13 669 to 50 000 of which 10 is a cubic residue. The partition (6) is given for $p \leqq 5743$.

CUNNINGHAM 7 gives all four partitions for $p < 100\,000$. This table is extended from 100 000 to 125 683 in CUNNINGHAM 36, where also are found several other tables giving quadratic partitions of primes of special form as follows. On p. 56–69 are given the partitions (3), (4) and if possible (5) of all primes of the form $2^k\omega+1$, $k \geqq 9$ up to high limits L depending on k as follows:

k	9	10	11	12	13	14
L	10^9	$1.25 \cdot 10^9$	$2.5 \cdot 10^9$	$5 \cdot 10^9$	$8.5 \cdot 10^9$	$9 \cdot 10^9$

On p. 70–73 are given the partitions (3) and if possible (5) of all primes p of the form $2^k\omega+1$, $k \geqq 9$, $10^7 < p < 10^8$. These tables were used in factoring Fermat's numbers $2^{2^n}+1$.

A similar table occurs in KRAITCHIK 4 (p. 192–204) where the partition (3) and if possible (4) and (5) are given for all primes $p=2^9k+1\leqq 10\ 024\ 961$. As a matter of fact a, b, c, are given in the equations

$$x^2 + (4a)^2 = p, \qquad x^2 + 2(4b)^2 = p, \qquad x^2 + 3(4c)^2 = p.$$

CUNNINGHAM 36 also gives partitions (3) and (4) whenever possible of all primes between 10^8 and 10^8+10^3. The representation of all possible primes p by the idoneal form

$$p = x^2 + 1848y^2 \quad \text{for} \quad 10^7 < p < 10^7 + 10^6$$

appears on p. 74–76. Actually $(x, 2y)$ is tabulated. All solutions (x, y) are given of

$$x^2 + y^2 = n^2 \quad \text{and} \quad x^2 + 3y^2 = n^2$$

for all possible $n<3000$ together with the corresponding partition of n when n is composite (p. 77–87).

Other tables of quadratic partitions different from (3), (4), (5) and (6)may be given the following tabular description. These give the least solutions (t, u) of

$$t^2 - Du^2 = kp$$

for the values of k and D indicated, and for all possible primes p not exceeding the limit L:

Reference	D	k	L
CUNNINGHAM 7	2	1	25 000
	3	1	10 000
TANNER 2	5	4	10 000
CUNNINGHAM 7	5	4	10 000
	$-5, \pm 6, \pm 7, \pm 10, 11$	1	10 000
	11	4	10 000
	$\pm 13, \pm 14$	1, 2	1 000
	$\pm 15, \pm 17$	1, 9	1 000
	± 19	1, 4	1 000
BICKMORE and WESTERN 1	2	1	25 000.

In the last mentioned table, p is restricted to the form $8n+1$, and t to the form $4x+1$. This paper also contains a small table giving all the representations of each possible number less than 1000 as the sum of two squares.

j₃. *Equations in more than 2 unknowns, rational triangles*

All but a few tables of this sort have to do with rational triangles, and most of these are lists of rational right triangles. Many such lists have been given in obscure places, and have been superseded by larger lists in more readily available sources.

It is well known that the sides of all integral right triangles are given by the formulas

$$a = 2mn, \qquad b = m^2 - n^2, \qquad h = m^2 + n^2,$$

where h is the hypotenuse, and where m and n are integer parameters. If one wishes to exclude the less interesting non-primitive right triangles in which a, b, and h have a common factor one restricts m and n to be coprime, and to be of different parity. There remains only the question of arranging the list of triangles thus generated.

Two extensive tables arranged according to values of m and n may be cited. The first, BRETSCHNEIDER 1 gives all primitive triangles generated with $n < m \leqq 25$. With each triangle is given also its area and its acute angles to the nearest 10th of a second. A more extensive list is given in MARTIN 1 (p. 301–308). This contains 864 triangles arranged according to m and n with $n < m \leqq 65$, and is the largest list of rational triangles ever published. With each triangle is given its area.

An old list of 200 right triangles was published by SCHULZE 1 in 1778. These are arranged according to the size of the smallest angle of the triangle. The tangent of half this angle is made to assume every rational value between 0 and 1 whose denominator does not exceed 25.

The arrangement most frequently used is according to increasing values of the hypotenuse. Such tables for $h \leqq 1109$ are given in SAORGIO 1 and SANG 1. The latter gives also the angles to within 1/100 of a second. The most extensive tables with this arrangement are found in MARTIN 2 and CUNNINGHAM 36. Both these tables give all 477 primitive triangles whose hypotenuses h do not exceed 3000. The Cunningham table is in two parts in which h is respectively prime and composite. This same arrangement is used in KRAITCHIK 6 which extends only to $h < 1000$ however. CUNNINGHAM 28 (p. 190–194) has another table complete to $h = 2441$ with 28 other h's < 3000.

A table of TEBAY 1 (p. 111–112) gives a list of right triangles arranged according to their area A up to $A = 934\ 800$. This table is reproduced in HALSTED 1 (p. 147–149) with nine additions.

BAHIER 1 (p. 255–258) gives a list of all primitive triangles one of whose legs has a given value < 300.

A table of KRISHNASWAMI 1 is arranged according to semi-perimeters and lists all primitive right triangles whose semi-perimeters do not exceed 5000.

References to other tables of right triangles, mostly small and obscure, are given in MARTIN 2.

Several small tables giving special right triangles may be mentioned. MARTIN 1 (p. 322–323) gives 40 right triangles whose legs are consecutive integers and 313 triangles whose hypotenuses exceed one leg by 1 or by 2. BAHIER 1 (p. 260–261) gives the values of certain recurring series for use in solving such problems. There is also given (p. 259) the list of 67 triangles one of whose sides is 840. WOEPCKE 1 has given for each of the 33 primitive right triangles with $h<205$, 12 associated congruent numbers. MARTIN 2 contains many sets of right triangles with special properties too numerous to mention.

There are a few tables of rational triangles which are not right triangles. TEBAY 1 (p. 113–115) lists 237 rational triangles arranged according to area, the greatest area being 46 410. This table is reproduced in HALSTED 1 (p. 167–170) and amplified in MARTIN 1 by 168 additions. ŠIMERKA 1 lists all 173 rational triangles with sides <100. There is also given the area, the tangents of the half angles and the coordinates of the vertices of each of these triangles. SANG 1 gives the list of 137 triangles, one of whose angles is 120°, and whose largest side is less than 1000.

CORPUT 1 has listed all primitive rational isosceles triangles (a, a, c) of altitude h, and base angles A, arranged according to a from $a=25$ to $a<160\ 000$. The table gives for each triangle the values of $\sqrt{a}$, $c/2$, $h/24$, $\tan(A/4)$, $\sqrt{a}\cos(A/2)$ and $\sqrt{a}\sin(A/2)$. PARADINE 1 gives 1120 triangles, each having integral sides and one integral median.

Finally we cite tables of solutions of diophantine equations of the second degree in more than 2 unknowns which do not refer to triangles.

CUNNINGHAM 28 (p. 185–189, 194) gave solutions of $x^2=y^2-3z^2$ arranged according to y complete to $y\leqq 1591$ with 99 more y's <2774. EELLS 1 has tabulated 125 solutions of $x^2+y^2+z^2=a^2$ for various a's from 13 to 88 621. JOFFE 1 has given a complete list of 347 solutions of this equation for $1<a\leqq 100$. BISCONCINI 1 has given 50 solutions of

$$x_1^2 + x_2^2 + x_3^2 = x_4^2.$$

k. NON-BINOMIAL CONGRUENCES OF DEGREE $\geqq 3$

Very few tables exist in this category. The term non-binomial is used here in its technical rather than its strict sense. That is to say, tables of solutions of such congruences as

$$(x^{12}-1)/(x^4-1) = x^8 + x^4 + 1 \equiv 0 \pmod{p}$$

have been classified under the binomial congruences (d_4) in spite of the fact that it is a trinomial congruence of the eighth degree.

We may cite here however the table of REUSCHLE 3 which gives not only the primitive solutions of the congruence

$$x^n - 1 \equiv 0 \pmod{p},$$

but also the solutions of

$$F(x) \equiv 0 \pmod{p}$$

where $F(x)$ are the polynomials whose roots are the several sets of "periods" of the nth roots of unity (described more fully under o) for all $n=2$–100, 105, 120 and 128, and for all $p<1000$.

Another table having to do with cyclotomy may be cited here also. JACOBI 3 gives for each $m<p-1$ and different from $(p-1)/2$ a number m' such that

$$1+g^m \equiv g^{m'} \pmod{p},$$

where g is a given primitive root of p for $7 \leqq p \leqq 103$. This table has been extended by DICKSON 10 to $p<500$, and for those primes between 500 and 700 which are not of the form $kq+1$, where q is a prime and $k=2, 4, 6, 12$.

The table of FLECHSENHAAR 1 gives for each prime $p=6m+1$ from 7 to 307 a pair of numbers (b, c) such that

$$bc \equiv 1 \pmod{p}$$
$$b^p+1 \equiv (b+1)^p \pmod{p^2}$$
$$c^p+1 \equiv (c+1)^p \pmod{p^2}.$$

BANG 1 gives a list of primes $p=mx+1<1000$ for which the congruence

$$a^m+b^m-c^m \equiv 0 \pmod{p}$$

has solutions for $m \leqq 25$.

A rather special table of KRAITCHIK 4 gives for each $n \leqq 1019$, except 4, 5, and 7 a number a, and a prime p such that

$$n!+1 \equiv a \pmod{p}, \qquad \left(\frac{a}{p}\right) = -1,$$

thus showing that except for $n=4$, 5, and 7 the diophantine equation

$$n!+1 = m^2$$

has no solutions (n, m) with $n \leqq 1019$.

1. DIOPHANTINE EQUATIONS OF DEGREE >2

Actual tables of solutions of Diophantine equations of degree $d>2$ exist only for $d=3$ and 4, although short notes giving occasional solutions of such equations with $d>4$ are scattered throughout the literature on the subject.

A list of about 6000 solutions of equations of the form

$$x^3 \pm y^2 = D,$$

arranged according to $|D|$, with $|D| \leqq 2000$, is found in GÉRARDIN 3.

A table of all integral solutions (x, y), when possible, of

$$x^3 - y^2 = D$$

with $1 \leqq x \leqq 101$ and $D \leqq 1024$ is given in BRUNNER 1 together with the class number $h(\sqrt{-D})$.

Rational solutions of such equations are given in BILLING 1. "Base points" are given here from which all rational solutions of

$$y^2 = x^3 - Ax - B \qquad 1 \leqq |A| \leqq 3, \quad 1 \leqq |B| \leqq 3$$
$$y^2 = x^3 - B \qquad |B| \leqq 25$$
$$y^2 = x^3 - Ax \qquad |A| \leqq 50$$

may be generated.

KULIK 1 gives solutions (x, y) of

$$n = x^3 - y^3 \quad \text{and} \quad n = x^3 + y^3$$

for all possible odd n not exceeding 12097 and 18907 respectively.

The rare table of LENHART 1 gives, for more than 2500 integers $A < 100\,000$, solutions of

$$x^3 + y^3 = Az^3$$

in positive integers. A small table of solutions of this equation for each of the 22 possible A's $\leqq 50$ is given in FADDEEV 1.

Two tables of Delone relate to the integral solutions of the binary cubic

$$ax^3 + bx^2y + cxy^2 + dy^3 = 1 \tag{1}$$

with a negative discriminant D. DELONE 1 gives all solutions (x, y) of (1) for all non-equivalent equations with $-300 < D < 0$. This table is reproduced in DELONE 2, where also are given all sets of integers (n, p, q) for which the discriminant D of the cubic $x^3 - nx^2 - px - q$ has a given value with $-172 \leqq D < 0$.

The ternary cubic

$$x^3 + Dy^3 + D^2z^3 - 3Dxyz = 1,$$

like the Pell equations, has an infinity of solutions. A table of solutions (x, y, z) for each positive non-cube $D < 100$ is given in WOLFE 1.

A list of 16 solutions of

$$x^2 - y^2 = z^3$$

in OETTINGER 1 may be cited.

CUNNINGHAM 28 (p. 229) gives 44 solutions of

$$x^3 - y^3 = z^2$$

and (p. 234–235) solutions of

$$x^3 \pm cy^3 = z^2$$

for $c \leqq 100$.

A. A. BENNETT 1 gives a table of solutions of what is in effect a ternary cubic equation

$$\text{arccot } x_1 + \text{arccot } x_2 = \text{arccot } y_1 + \text{arccot } y_2.$$

All solutions are given in which $0 < x_1 + x_2 < 25$.

Finally the quaternary cubic equations

$$t^3 \pm x^3 \pm y^3 \pm z^3 = 0$$

are considered in RICHMOND 1. All solutions (t, x, y, z) in which the variables do not exceed 100 are given.

Turning to quartic equations we find only a few tables. CUNNINGHAM 15 gives all solutions (x, y, z) of

$$x^4 + y^4 = mz^2$$

in which the right member does not exceed 10^7, and all solutions in which $x=1$, and $y<1000$.

CUNNINGHAM 28 gives two or more solutions of

$$x^4 \pm ky^4 = \pm z^2 \qquad k \leqq 100 \text{ (p. 230, 236)},$$

and of

$$x^4 - kx^2y^2 + y^4 = z^2 \qquad k < 200 \text{ (p. 232–233)}.$$

OETTINGER 1 gives 16 solutions of

$$x^2 - y^2 = z^4.$$

VEREBRIŪSOV 1 tabulates all non-trivial solutions of

$$x^4 + y^4 + z^4 = x_1^4 + y_1^4 + z_1^4$$

in which the variables do not exceed 50. This table is reproduced in VEREBRIŪSOV 2.

m. DIOPHANTINE CONTINUED FRACTIONS

A number of useful tables of the continued fraction developments of algebraic irrationalities have been published. Most of them refer to the regular binary continued fraction

$$\theta = q_0 + \frac{1}{q_1 +}\frac{1}{q_2 +}\frac{1}{q_3 +}\cdots$$

and, of these, nearly all refer to the case in which θ is a pure quadratic surd $\sqrt{D}$.

If we write

$$x_0 = \theta = q_0 + 1/x_1 \qquad q_0 = [\theta]$$
$$x_1 = q_1 + 1/x_2 \qquad q_1 = [x_1]$$
$$\cdots\cdots\cdots$$
$$x_k = q_k + 1/x_{k+1} \qquad q_k = [x_k]$$

then the x_k are called complete quotients, and the q_k incomplete (or partial) quotients of θ, and

$$\theta = q_0 + \frac{1}{q_1 +} \frac{1}{q_2 +} \cdots + \frac{1}{x_k}$$

for every $k>0$.

In case $\theta=\sqrt{D}$ the complete quotient x_k takes the form

$$x_k = (\sqrt{D} + P_k)/Q_k$$

where P_k and Q_k are integers such that

$$0 \leqq P_k < \sqrt{D}$$

$$0 < Q_k < 2\sqrt{D}.$$

Several tables give Q_k as well as q_k. The numbers Q_k are important in many applications, especially in connection with the question of solving the equation

$$x^2 - Dy^2 = N.$$

The numbers P_k are less useful, and have (with one exception) never been tabulated. They may be obtained from the Q's by the formula

$$P_k^2 = D - Q_k Q_{k-1}$$

and have been used for solving quadratic congruences (mod D). All three sequences P_k, Q_k, q_k, are periodic for $k>0$.

The main tables of the continued fraction development of $\sqrt{D}$ are DEGEN 1, CAYLEY 6, and WHITFORD 1. Each table gives both q_k and Q_k up to the middle of the period, about which point the period is symmetric.

The table of DEGEN 1 extends from $D=2$ to $D=1000$, that in CAYLEY 6, which was computed by Bickmore, from $D=1001$ to $D=1500$, while that of WHITFORD 1 extends from 1501 to 2012.

The table of SEELING 2 gives for $D\leqq 602$ the first half of the period of the partial quotients q_k, but not Q_k. In addition it gives in each case the number of terms in the period of the continued fraction, a function about which little is known. Lists of D's are given which correspond to periods of given length and type.

Those D's <7000, which have an odd number of terms in the expansion of $\sqrt{D}$, are listed in SEELING 3.

A tabular analysis of the continued fraction for $\sqrt{D}$ arranged according to the length of the period is given for $D<1000$ in KRAITCHIK 6, where also is given a similar analysis of $(-1+\sqrt{4A+1})/2$ for $A<100$. Only the partial quotients are given.

The table of ROBERTS 1 gives partial quotients only for the expansion of $\sqrt{D}$ for D a prime of the form $4n+1$ not exceeding 10 501.

Another special table is that of VON THIELMANN 1, which gives partial quotients for $\sqrt{pq}$ where both p and q are primes of the form $4k+1$, and $pq<10\,000$. The trivial cases $pq=x^2+1$, $x^2\pm4$ are excluded. The table is in two parts, the first of which contains expansions with an odd number of terms in the period.

NIELSEN 1 gives for $D<1500$ and the sum of two squares both q_k and Q_k for the expansion of $\sqrt{D}$ in case the period has an odd number of terms.

A small table of the partial quotients in the first half of the period for $\sqrt{D}$ is given in PERRON 1 and extends to $D<100$.

INCE 1 gives in effect P_k and Q_k, but not q_k in the expansion of $\sqrt{D}$ for all $D<2025$ of the form $D=4k+2$, $4k+3$, and without square factors. These occur in the first cycle of reduced ideals. Thus for $D=194$, the first cycle given is

$$1,\ 13 \sim 25,\ 12 \sim 2,\ 12$$

This may be taken to indicate that P_k and Q_k have the values

k	0	1	2	3	4	5	6	7	8	...
P_k	0	13	12	12	13	13	12	12	13	...
Q_k	1	25	2	25	1	25	2	25	1	...

The other cycles, if they exist, correspond to certain irregular continued fractions for $\sqrt{D}$. For $D=4n+1$ the corresponding information is given for $(1+\sqrt{D})/2$.

Those convergents A_n/B_n to continued fractions which satisfy the equation $A^2-DB^2=\pm1$, ±4 occur in tables of the Pell Equation as described under j_1. Other convergents are given only rarely. A small specimen table in CAYLEY 6 gives all convergents in the first period of $\sqrt{D}$ for $D<100$.

SEELING 1 gives expansions of many higher irrationalities such as $\sqrt[3]{D}$ for $D=2, 3, 4, 6, 7, 9, 10, 15$ and several other numbers of the form $D^{1/k}$ up to $k=13$. Since by a theorem of Lagrange none of these expansions can be periodic the entire expansions cannot be given, so that only the beginnings of the expansions are found. Complete as well as partial quotients are given.

Daus has published three tables of the expansion of cubic irrationalities in a ternary continued fraction (Jacobi's algorithm). Such expansions are ultimately periodic. In place of partial quotients q_k we have partial quotient pairs (p_k, q_k) which determine the expansion. DAUS 1 gives a table of partial quotient pairs in the expansion of $\sqrt[3]{D}$ for $D\leqq30$. Similar expansions of the largest root of the cubic equation

$$x^3+qx-r=0 \qquad |q|\leqq 9,\ 1\leqq r\leqq 9$$

occur in DAUS 2. DAUS 3 gives expansions of cubic irrationalities in certain cubic fields with a minimal basis. The fields are defined by a root θ of the cubic equation

$$x^3 - px + q = 0 \qquad |p| \leq 9, |q| \leq 9$$

in which $(1, \theta, \theta^2)$ is not a basis.

n. NON-LINEAR FORMS, THEIR CLASSES AND CLASS NUMBERS

The theory of forms, especially of binary quadratic forms, has a number of applications in other parts of the theory of numbers. Tables having to do with the application of forms have been cited under other sections of this report, in particular under b_2, e_2, f_2, g, i_3, j, l, o and p.

There remains however a large number of tables without view to immediate exterior application, giving information about the theory of forms itself. To the amateur number-theorist, not an expert in the arithmetical theory of forms, most of the tables about to be described will doubtless appear to be sterile, if not useless. If so, the writer has been successful in his classification of these tables, as the tables here described are of interest mainly to experts.

Existing tables refer to four sorts of forms: binary quadratic, ternary quadratic, quaternary quadratic, and binary cubic forms.

The theory of binary quadratic forms arose from the problem of solving Diophantine equations of the second degree, and early tables reflect this origin. We have on the one hand the tables of the Pell equations

$$x^2 - Dy^2 = \pm 1,$$

fully described under j_1, and on the other hand tables for the representation of a large number N by the form

$$x^2 - Dy^2 = N,$$

described under g, i_1, i_3, and j_2.

This latter problem was at once seen to be a key to the question of factoring large numbers N and it was with this application in mind that Gauss began his epoch-making investigation into the theory of binary quadratic forms. Among the many by-products of this research three may be mentioned as being the source of tables described elsewhere. These are the theory of the number of representations of a number by a binary quadratic form, the representation of cyclotomic functions as binary quadratic forms, and the theory of quadratic fields.

Tables of the functions $E(n)$, $H(n)$ and $J(n)$ have been cited under b_2 and are contained in GLAISHER 15, 17, 18, 19, 24, 25 and 26. These functions are related to the number $N(n=f(x, y))$ of representations of n by the binary quadratic form $f(x, y)$ as follows:

$$N(n = x^2 + \ \ y^2) = 4E(n)$$
$$N(n = x^2 + 2y^2) = 2J(n) \ (n \text{ odd})$$
$$N(n = x^2 + 3y^2) = 2H(n) \ (n \text{ odd}).$$

GLAISHER 19 also contains a table of the function

$$G(n) = N(n = (6x)^2 + (6y + 1)^2) = N(n = (6x + 2)^2 + (6y + 3)^2)$$

for $n = 12k + 1 \leqq 1201$.

Tables of the coefficients of the polynomials $Y(x)$ and $Z(x)$ in the representation of the cyclotomic function

$$4(x^p - 1)/(x - 1) = Y^2(x) - (-1)^{(p-1)/2} pZ^2(x)$$

and related tables are cited under **o** and begin with GAUSS 1.

The theory of quadratic fields is of course very closely related to that of binary quadratic forms, their difference being largely one of nomenclature. Hence many of the tables cited under **p** are instances of tables related to binary quadratic forms.

Tables of reduced binary quadratic forms begin with LEGENDRE 1. Table I gives all reduced forms

$$ay^2 + 2byz - cz^2$$

of determinant $A = b^2 + ac$ for all possible $A \leqq 136$. Table II gives similarly reduced forms

$$Ly^2 + Myz + Nz^2 \ (M \text{ odd})$$

with $0 < M^2 - 4LN \leqq 305$. Tables III, IV, VI, and VII list the reduced forms

$$ay^2 + 2byz + cz^2$$

of determinant $A = b^2 - ac$ with $-106 \leqq A \leqq 79$ together with the corresponding linear forms of the odd divisors of $t^2 - au^2$ (as described under $\mathbf{i}_3$). Similarly Table V gives for the reduced forms

$$Ly^2 + Myz + Nz^2$$

with $a = 4LN - M^2$ or $LN - M^2/4$, according as M is odd or even, for $0 < a = 4k - 1 \leqq 103$.

Gauss must have constructed extensive tables of reduced forms but never published any. He in fact considered the publication of such tables as unnecessary since any isolated entry can be so easily obtained directly. His table of the classes of binary quadratic forms to be cited presently was published posthumously.

CAYLEY 2 tabulated the representatives of each class of forms of non-square determinant D with their characters and class group generators for

$|D|<100$ together with 13 irregular determinants D between -100 and -1000 noted by Gauss. For $D>0$, the periods of the reduced forms are given. This table was continued from $D=-100$ to $D=-200$ by COOPER 1.

CAHEN 3 gives a table of primitive classes of positive definite forms of discriminant $D<200$ omitting those cases in which there is but a single class. There is a similar table for indefinite forms of discriminant >-200.

WRIGHT 1 has given an interesting table of reduced forms $ax^2+2bxy+cy^2$ of determinant $-\Delta$ with $\Delta\leqq 150$, and $800\leqq\Delta\leqq 848$ arranged so that b and c can be read on entering the tables at a, Δ. The values of b are periodic functions of Δ for each fixed a. This table has been extended to $\Delta\leqq 1200$ in ROSS 1.

Two tables of indefinite binary quadratic forms are included in ROSS 1. The "basic" table gives reduced forms $(a, b, -c)$ with $0<a\leqq c$ and $2b\geqq c$, for determinants up to 1500. A second table lists the periods of reduced forms, as in CAYLEY 2, for determinants from 100 to 1000.

GAUSS 7 gave extensive tables of the number of classes for mostly negative determinants. More definitely the determinants considered are $-D$ for all D's of the nth century for $n=1$–30, 43, 51, 61–63, 91–100, 117–120 and, in another arrangement, for D's of the 1st, 3rd and 10th chiliad and for D of the form $-(15n+7)$ and $-(15n+13)$, $n<800$. The positive determinants considered are those of the nth century for $n=1$, 2, 3, 9, 10.

For each group of determinants above mentioned are listed those determinants which have a prescribed number of genera (I, II, IV, VIII, $\cdots$), and a prescribed number of classes in each genus. Under each specified number of genera are given the number of determinants having that number of genera, and the total number of classes. At the end of each group these numbers are combined to give the total number of genera and classes in that group together with the number of improperly primitive classes and the number of irregular determinants, the latter being indicated in the tables by asterisks, and in most cases the index of irregularity is also given.

E. T. BELL 1 contains a table of the number of odd classes of binary quadratic forms of determinant $-D$ for $D<100$.

SURYANARAYANA 1 gives a list of primes D of the form $4n+3$ for which the class number of D is 2 and $0<D<5000$.

For the purpose of factoring large numbers N or proving their primality, forms which have only a few classes in each genus are advantageous to use in representing the given number N. The 65 "idoneal" forms

$$x^2+\Delta y^2, \qquad \Delta>0$$

of Euler are such that each genus contains but a single class. The idoneal Δ's have been given in numerous places such as MATHEWS 1 and KRAITCHIK 6 (p. 119). Besides these idoneal forms, SEELHOFF 1 has given 105 others for which each reduced form in the principal genus is of binomial type ax^2+cy^2 to be used for factoring as mentioned under g. Forms of practical use in factoring are not confined to definite ones. CHEBYSHEV 1 has given for each indefinite form x^2-Dy^2 $(0<D\leqq 33)$ limits on x and y depending on N between which it is sufficient to look for representations of N.

The applications of the theory of binary quadratic forms to elliptic modular functions have produced tables of class invariants and other tables relating to the complex multiplication theory. These tables will be described in another report of this Committee under G: *Higher Algebra*.

We turn now to the consideration of tables related to ternary quadratic forms.

Interest in such forms originated from the problem of representing binary quadratic forms by ternary forms, and the earliest table involving ternary forms is concerned with this problem, and is found in LEGENDRE 1. Table VIII (in the first edition Tables VIII and IX) lists for all possible $c\leqq 251$, the reduced forms

$$py^2 + 2qyz + rz^2, \qquad c = pr - q^2$$

and expresses each of these as a sum of three squares of linear forms.

SEEBER 1 gave the first table of reduced ternary forms. This gives the classes of positive ternary forms of odd Gaussian determinant $-D$ for $D<25$. This table was revised by EISENSTEIN 1 who gave the characters and classes in each genus. EISENSTEIN 2 gives a table of primitive reduced positive ternary forms of determinant $-D$ for all $D<100$ as well as $D=385$. EISENSTEIN 3 lists all automorphs of positive ternary forms. These are given also in DICKSON 6 (p. 179–180).

BORISOV 1 gave a table of properly and improperly primitive reduced (in the sense of Selling) positive ternary forms for all determinants from 1 to 200, assigning to each representative form a type and the number of automorphs.

Tables, due to Ross, of reduced (in the sense of Eisenstein) positive ternary forms, both properly and improperly primitive of determinant $d\leqq 50$, giving also the number of automorphs, occur in DICKSON 6 (p. 181–185). Forms without "cross product" terms are listed separately. With each form is given the number of automorphs. This table has been extended to $d<200$ by JONES 1.

JONES and PALL 1 list all 102 so-called regular forms $f=ax^2+by^2+cz^2$. These are reproduced in DICKSON 9 (p. 112–113) where also are given in each case the numbers not represented by f.

A special table giving certain arithmetic progressions and generic characters of reduced positive ternary forms whose Hessian does not exceed 25 appears in HADLOCK 1.

Only three tables of indefinite ternary forms have been published. The first is due to EISENSTEIN 3 and lists non-equivalent indefinite forms whose determinants have no square factors and are less than 20.

MARKOV 1 tabulated reduced indefinite ternary forms, not representing zero, of determinant $\leqq 50$. This table was recomputed and extended to determinants $\leqq 83$ by Ross and appears in DICKSON 6 (p. 150–151). A similar table for determinants $4n \leqq 124$ occurs in ROSS 1.

CHARVE 1 lists all positive quaternary quadratic forms reduced in the sense of Selling of determinants $\leqq 20$. A similar table of such forms reduced in the sense of Eisenstein for determinants $\leqq 25$ is given in TOWNES 1.

There are a few tables of binary cubic forms, all with negative discriminants. Two of these by DELONE 1, 2 have been described under l.

ARNDT 3 gave all reduced binary cubics of negative discriminant $-D$, their classes and characteristic binary quadratic forms for all possible $D<2000$.

CAYLEY 3 reproduced part of this table in revised form. His table gives the reduced forms with their order, characteristic and composition for the following values of the discriminant D:

$$0 > D = 4k > -400 \quad \text{and} \quad 0 > D = 4k+1 \geqq -99$$

and $D=-4k$, $k=243, 307, 339, 459$, and 675.

MATHEWS 2 contains a table due to Berwick of all non-composite reduced binary cubics with discriminant $-D$, $D<1000$.

o. TABLES RELATED TO CYCLOTOMY

The problem of dividing the circle into an equal number of parts, or what is the same thing, the study of the roots of the binomial equation $x^n=1$ would seem at first sight to have little connection with tables in the theory of numbers. Gauss was the first to recognize, however, the intimate connection between cyclotomy and various branches of number theory, when he showed that the construction of regular polygons by Euclidean methods depends ultimately on the factorization of Fermat's numbers $2^{2^n}+1$. A list of the 32 regular polygons with an odd number of sides known to be constructible with ruler and compasses is given in KRAITCHIK 4 (p. 270). The theory of cyclotomy is of much wider application to number theory, however, and tables described under **b**$_1$, **b**$_3$, **d**, **e**$_2$, **f**$_2$, **j**$_1$, **j**$_2$, **n**, **p** and **q**$_1$ either depend upon cyclotomy or are of use in its applications.

We have in fact already introduced in various connections the cyclotomic polynomial

$$Q_n(x) = \prod_{\delta | n} (x^\delta - 1)^{\mu(n/\delta)},$$

where μ is Möbius' function and δ ranges over the divisors of n, which has for roots all the primitive nth roots of unity. This polynomial is often loosely spoken of as "the" irreducible factor of x^n-1, and is often written as X_n and $F_n(x)$. Tables of coefficients of $Q_n(x)$ are scarce. REUSCHLE 3 gives $Q_n(x)$ for $n=3$–100, 105, 120 and 128 with the exception of $n=4k+2$ for which $Q_{4k+2}(x)=Q_{2k+1}(-x)$. SYLVESTER 1 gives $Q_n(x)$ for all $n \leqq 36$. KRAITCHIK 7 gives, for all products n of two or more primes not exceeding 105 (except 77), the coefficients of $Q_n(x)$ or of $Q_n(-x)=Q_{2n}(x)$ according as $n=4k+1$ or $4k+3$, and for $n=2pq\leqq 102$, those of $Q_{2n}(x)$.

The need for tables of $Q_n(x)$ is not acute since for any particular n, $Q_n(x)$ may be readily found from the application of one or more of the following formulas:

$$(1)\qquad \begin{aligned} Q_p(x) &= x^{p-1}+x^{p-2}+\cdots+x+1 \\ Q_n(x) &= Q_{n_0}(x^m) \\ Q_{2n}(x) &= Q_n(-x),\ (n \text{ odd}) \\ Q_{np}(x) &= Q_n(x^p)/Q_n(x) \end{aligned}$$

where $n=n_0m$ and n_0 is the product of the distinct prime factors of n, and where n is not divisible by the prime p.

Several tables give data on the "f-nomial periods" of the primitive nth roots of unity where $\phi(n)=e\cdot f$. The most elaborate such table is REUSCHLE 3, which gives for every divisor f of $\phi(n)$ the set of fundamental relations between the f-nomial periods which express the product of any two of them as a linear combination of the periods for $n=1$–100, 105, 120, 128, except $n=4k+2$. In most cases the irreducible equation of degree e satisfied by the periods is given also, though when n is composite and e is large this equation is not given.

SYLVESTER 1 gives the polynomials whose roots are the binomial periods $\eta=\alpha+\alpha^{-1}$, where α are the primitive nth roots of unity, for all $n\leqq 36$, and 12 other polynomials whose roots are the f-nomial periods, $f>2$, for $n=15$, 21, 25, 26, 28 and 33.

D. H. LEHMER 3 contains a table of all irreducible polynomials of degree $\leqq 10$, whose roots are of the form $\alpha+\alpha^{-1}+2$, where α are the primitive nth roots of unity, $n\neq 4k+2$.

CAREY 1 contains tables of the coefficients in the linear expressions for the squares and products of two f-nomial periods of imaginary pth roots of unity for all primes $p<500$ and for $e=(p-1)/f=3$, 4, and 5.

TANNER 1 gives for each $p=10n+1<1000$ the quintic equation for the five $(p-1)/5$-nomial periods.

Many tables give the representation of $Q_n(x)$ as a quadratic form. The first of these is due to Gauss, who discovered the polynomials $Y_p(x)$ and $Z_p(x)$ of degrees $(p-1)/2$ and $(p-3)/2$ respectively such that

$$(2)\qquad 4(x^p-1)/(x-1) = Y_p^2(x) - (-1)^{(p-1)/2}pZ_p^2(x).$$

These are tabulated in GAUSS 1 for $p\leqq 23$. Dirichlet and Cauchy later pointed out that (2) can be generalized to the case of p, replaced by a composite number n, as follows:

$$(3)\qquad 4Q_n(x) = Y_n^2(x) - (-1)^{(n-1)/2}nZ_n^2(x),$$

where n is a product of distinct odd primes. (A quadratic form exists in the case of a perfectly general n, as may be seen at once from (1) by replacing x in (3) by $\pm x^m$).

Tables giving $Y_n(x)$ and $Z_n(x)$ may be given the following tabular description, where by "general" we mean prime or the product of distinct odd primes (the trivial case of $p=3$ is usually not given).

reference	character of n	range of n
GAUSS 1	prime	$p \leqq 23$
MATHEWS 1	prime	$p \leqq 31$
KRAITCHIK 2 (p. 3)	prime	$p \leqq 37$
KRAITCHIK 4 (p. 126)	prime	$p \leqq 37$
HOLDEN 1, 2	general	$n \leqq 57$ (with gaps)
POCKLINGTON 1	prime	$41 \leqq p \leqq 61$
LUCAS 2	general	$n = 5$–41, 61
GOUWENS 1	prime	$67 \leqq p \leqq 97$
TEEGE 1	general	$n \leqq 101$
KRAITCHIK 7 (p. 2–4)	general	$n \leqq 101$
GRAVE 1	prime	$23 \leqq p = 4m+3 \leqq 131$
GRAVE 2	prime	$29 \leqq p = 4n+1 \leqq 197$
GOUWENS 2	prime	$101 \leqq p \leqq 223$.

For some reason Gauss and his followers failed to discover another quadratic form representing $Q_n(x)$ which is, for some applications, more important than (2) or (3). The existence of polynomials $T_n(x)$ and $U_n(x)$ such that

$$Q_n(x) = T_n^2(x) - (-1)^{(n-1)/2} nxU_n^2(x)$$

was discovered 70 years after Gauss' discovery of (2) by Aurifeuille. Tables of the coefficients of T_n and U_n were first published by LUCAS 2 for odd $n \leqq 41$ not divisible by a square, as well as for $n=57$, 69 and 105. LUCAS 3 gives in effect the coefficients of the polynomials $V_n(x)$ and $W_n(x)$ such that

$$V_n^2(x) - nxW_n^2(x) = \begin{cases} Q_n(x) & \text{if} \quad n = 4k+1 \\ Q_{2n}(x) & \text{if} \quad n = 4k+2, \text{ or } 3 \end{cases}$$

for all $n \leqq 34$, having no square factor. This table was reproduced by CUNNINGHAM 23 with the additional entries for $34 < n \leqq 42$, and $n=46$, and also by KRAITCHIK 2 (p. 6), and KRAITCHIK 4 (p. 88), where in both tables the additional entries $n=35$, 39, 42 and 51 are given.

LUCAS 2 gives in reality the coefficients of the polynomials $R_n(x)$ and $S_n(x)$ in the identity

$$Q_{4n}(x) = R_n^2(x) - 2nxS_n^2(x)$$

for odd $n \leqq 35$, as well as $n=39$, 51 and 57. The importance of Aurifeuille's formula lies in the fact that for suitably chosen x, $Q_n(x)$ becomes the difference of two squares, and hence decomposable into rational factors.

p. TABLES RELATING TO ALGEBRAIC NUMBER THEORY

Algebraic number theory, like the theory of forms, is a rather technical subject. The more extended parts of the theory are so ramified that tables are apt to be little more than mere illustrations of theorems. In fact, many articles on the subject contain numerical illustrations too numerous, too special and too diverse to permit description here. Although these numerical illustrations serve to make more real the abstract subject matter being considered, they cannot fairly claim to be described as useful tables.

Tables described under other sections of this report are of use in parts of algebraic number theory. In fact, the theory of binary quadratic forms is practically identical with quadratic field theory, and many tables relating to the former subject (described under **n**) are applicable in the latter, and conversely. Other sections containing tables useful in various parts of algebraic number theory are $\mathbf{b}_2$, $\mathbf{b}_4$, **d**, $\mathbf{e}_3$, $\mathbf{f}_2$, $\mathbf{i}_2$, **j**, **l**, **m** and **o**. Other useful tables, more algebraic than number theoretic, such as tables of irreducible polynomials (mod p), modular systems, Galois field tables, class invariants, singular moduli, etc. will be described in another report of this committee under **G.** *Higher Algebra.*

Tables relating to algebraic numbers may be classified according to the degree of the numbers considered. Many tables pertain to quadratic number fields.

The tables of SOMMER 1 contain tables of both real and imaginary quadratic fields $K(\sqrt{D})$ for $|D| < 100$, and not a square, giving in fact for each such D a basis, discriminant, principal ideal, the classes of ideals, genera and characters. The fundamental unit is given when $D>0$.

A more comprehensive account of real quadratic fields is given by the table of INCE 1. This table gives data on the fields $K(\sqrt{m})$ for all $m<2025$ having no square factor. Ideals $(a, b+\omega)$, where $\omega=\sqrt{m}$ for $m\equiv 2, 3 \pmod 4$ and $\omega=(1+\sqrt{m})/2$, when $m\equiv 1 \pmod 4$, are written simply a, b. Reduced ideals fall into classes of equivalent ideals, and the ideals in any one class form a periodic cycle which is palindromic. The table lists the first half of these cycles. In addition the table gives the number of genera in the field, and the number of classes in each genus, their generic characters and finally the fundamental unit $\epsilon=x+y\sqrt{m}$ or $(x+y\sqrt{m})/2$, also written in the form $(u+v\omega)^2/n$, whenever possible.

The table of SCHAFFSTEIN 1 gives the class number of real quadratic fields whose discriminant is a prime $p(=4k+1)$ for $p<12\,000$, $10^5<p<10^5+10^3$, and $10^6<p<10^6+10^3$.

A number of tables refer to the Gaussian numbers $a+b\sqrt{-1}$, and their powers.

The first such table occurs in GAUSS 2 and gives for each of 19 complex primes $p=a+ib$ with norm $a^2+b^2\leqq 157$ those complex numbers (mod p) which have each of the 4 different biquadratic characters (mod p).

GAUSS 9 has a table of indices for 45 complex primes $p=a+ib$. This table was extended to all prime and composite moduli in $K(\sqrt{-1})$, whose norms do not exceed 100, by G. T. BENNETT 1.

BELLAVITIS 1 contains a table of powers

$$(a + ib)^k \pmod{p, x^2 + 1}$$

of a primitive root $a+ib$ for $p=4m+3\leq 67$, for $k=r(p+1)$, $s(p-1)$ and $s(p-1)+1$, where $r=1, 2, \cdots (p-1)/2$, $s=1, 2, \cdots (p+1)/4$.

The table of VORONOĬ 1 gives for each prime $p<200$, a pair of companion tables, one of which gives the powers (mod p) of a primitive root $E=a+ib$, where $i^2\equiv N \pmod p$, N being the least positive quadratic non-residue of p. The other table gives the index of that power of E whose real part is specified and whose imaginary part is positive.

GLAISHER 17 has tabulated three functions which depend on "primary" Gaussian numbers, that is, numbers of the form

$$(-1)^{(a+b-1)/2}(a \pm ib)$$

where $a>0$ is odd and b is even.

Let $S_k(n)$ denote the sum of the kth powers of the primary Gaussian numbers whose norm is n. Glaisher denotes the functions $S_1(n)$ and $S_2(n)$ by $\chi(n)$ and $\lambda(n)$ respectively. In fact $\chi(n)$ is tabulated for odd $n<1000$, and for all primes and powers of primes $<13\,000$, while $\lambda(n)$ is tabulated for $n<100$. The function $S_0(n)$ is designated by $E(n)$, several tables of which are described under $\mathbf{b_2}$.

Tables relating to cubic fields are much less numerous than those for quadratic fields.

The tables of REID 1, 2 are in two parts. Part 1 gives for each reduced cubic equation

$$x^3 + px + q = 0, \qquad |p| \leq 9, 1 \leq q \leq 9$$

the discriminant of the field thus defined, the class number, a basis and a system of units as well as the factorization of certain small rational primes in the field. Part II gives the same information for 19 other cubics of the general form

$$ax^3 + bx^2 + cx + d \qquad (b \neq 0).$$

The tables of DAUS 2, 3 (described under m) give the units in the cubic fields under consideration.

DELONE 1, 2 give information about units of cubic fields of negative discriminant. In particular DELONE 2 lists all fields with discriminant $-D$ with $D<172$.

Extensive tables of relative cubic fields are given in ZAPOLSKAÎÂ 1.

Quartic field tables are all of special type. DELONE, SOMINSKIĬ and BILEVICH 1 give a list of all totally real quartic fields with discriminant not exceeding 8112. With each such field is given a basis.

The tables of TANNER 1, 2 refer to the quartic field defined by ω, a primitive 5th root of unity. These give the "coordinates" q_i of the "simplest" complex factor

$$f(\omega) = q_0 + q_1\omega + q_2\omega^2 + q_3\omega^3 + q_4\omega^4$$

of a prime $p=10n+1$ as well as the coordinates of the "simplest primary" factor and the "reciprocal" factor $\psi(\omega)$, the latter being such that $\psi(\omega)\psi(\omega^{-1})=p$. In TANNER 1, $p<1000$ while in TANNER 2 the information is given for $p<10\ 000$ except that the reciprocal factor is tabulated only for $1000<p<10\ 000$.

A similar table for the quartic field defined by a primitive 8th root of unity is given in BICKMORE and WESTERN 1. This gives the coordinates of a canonical complex prime factor of every prime $p=8n+1<25\ 000$.

These tables really belong under cyclotomic fields, concerning which extensive tables were published by Reuschle, and are in fact extensions of similar tables occurring in REUSCHLE 2, 3. REUSCHLE 2 gives the complex factors of rational primes p in the cyclotomic field $K(\exp 2\pi i/n)$ and the subfields generated by the periods for $p=kn+1<1000$ and for all primes n from 7 to 29 as well as for $n=5$ and $p=10k+1<2500$. These tables are superseded by REUSCHLE 3 where $n=3$–100, 105, 120, 128 ($n\neq 4k+2$). For n a prime <20 two factors of $p<1000$ are given, one "simple" and one "primary" after Kummer. For other values of n only "simple" factors of p are given. In many cases complex factors of p^α are given where $\alpha>1$ is the index of ideality. In all cases $p<1000$. For n large and composite many of the tables pertaining to the subfields are wanting.

q. TABLES RELATING TO ADDITIVE NUMBER THEORY

Of the many and varied problems of additive number theory, three have been the source of tables. These are the problem of partitions or the representation of numbers as sums of positive integers of no special type, the problem of Goldbach, or the representation of numbers as sums of primes, and the problem of Waring, or the representation of numbers as sums of powers.

q₁. *Theory of partitions*

Tables relating to partitions are of two types according as the parts contemplated are or are not restricted in some way as to size or number. We take up the unrestricted partitions first.

The actual partitions of a number n into the parts $1, 2, \cdots, n$, giving for $n=5$, for example, the 7 entries (11111), (1112), (113), (122), (14), (23), (5) occur as arguments of tables of symmetric functions and other algebraic tables to be considered in another report of the Committee under G. *Higher Algebra*. We may cite here, however, a table of all partitions of n for $n\leq 18$ due to

CAYLEY 4. The parts 1, 2, 3, $\cdots$ are represented by the letters $a, b, c, \cdots$ and the 7 entries under $n=5$ thus appear as a^5, a^3b, a^2c, ab^2, ad, bc, e.

The theory of partitions is concerned more with the mere number $p(n)$ of partitions rather than the actual partitions themselves. The function $p(n)$ increases so rapidly that Cayley's table could not be carried much farther. For $n=30$, for example, it would have 5604 entries.

The first real table of $p(n)$ occurs as a by-product of the table of EULER 3 and is there denoted by $n^{(\infty)}$ and tabulated for $n \leqq 59$. This table was not extended until 1917 when the analytic researches of Hardy and Ramanujan made it desirable to examine the magnitude of $p(n)$ for large n. MacMahon accordingly computed $p(n)$ for $n \leqq 200$, his table being published by HARDY and RAMANUJAN 1. GUPTA 1 has given $p(n)$ for $n \leqq 300$ and for $301 \leqq n \leqq 600$. The complete table for $n \leqq 600$ is reproduced in GUPTA 7.

Two tables give values of $p(n)$ (mod p). GUPTA 3 gives $p(n)$ (mod 13) and (mod 19) for $n \leqq 721$. MACMAHON 1 lists those values of $n \leqq 1000$ for which $p(n)$ is even.

Thanks to recent investigations the asymptotic series of Hardy and Ramanujan now offers an effective and reliable method of obtaining isolated values of $p(n)$. This series contains certain coefficients $A_k(n)$, tables of which, as functions of n, are given in HARDY and RAMANUJAN 1 for $k \leqq 18$. D. H. LEHMER 5 contains a table of actual values of $A_k(n)$ for $k \leqq 20$, and for all n, (since $A_k(n+k)=A_k(n)$), the number of decimal places being sufficient for computing $p(n)$ for n up to three or four thousand. This table is reproduced in GUPTA 7.

In investigating an approximate formula for $p(n)$, HARDY and RAMANUJAN 1 have given the value of

$$.9 \operatorname{Log}_{10} p(n) - \sqrt{10+n}$$

for $n=10^k$ and $3 \cdot 10^k$ $(k=0, 1, \cdots, 7)$.

The generating function for $p(n)$ is the modular function

$$f(x) = \prod_{n=1}^{\infty} (1-x^n)^{-1} = 1 + \sum_{n=1}^{\infty} p(n)x^n.$$

The coefficients of the related function

$$x\{f(x)\}^{-24} = \sum_{n=1}^{\infty} \tau(n)x^n$$

have been studied to some extent and are given for $n \leqq 30$ in RAMANUJAN 2.

Turning now to tables of the number of partitions in which the parts are restricted in some way we find two tables of the function $q(n)$ which may be regarded either as the number of partitions of n into distinct parts or as the number of partitions of n into odd parts, so that $q(5)=3$. The first table is due to Darling and is published in HARDY and RAMANUJAN 1 (Table V), and gives $q(n)$ for $n \leqq 100$. WATSON 1 extends this table to $n \leqq 400$, and gives for the same values of n the function $q_0(n)$, which denotes the number of partitions of n into distinct odd parts. UMEDA 1 gives for $n \leqq 100$ the values of the function

$$\frac{1}{p(n)} \sum_{m=1}^{n} m p_m(n)$$

where $p_m(n)$ denotes the number of partitions of n into exactly m parts.

A small table of CAYLEY 5 gives for $n \leqq 100$ the number of partitions of n into the parts 2, 3, 4, 5, and 6.

Other tables of restricted partitions are double entry tables. The first of these is EULER 3, which gives the number $n^{(m)}$ of partitions of n into parts $\leqq m$, or what is the same thing, the number of partitions of n into not more than m parts for $n \leqq 59$, $m \leqq 20$. The differences $n^{(m)} - n^{(m-1)}$ are also tabulated.

The table of GUPTA 7 gives the number (n, m) of partitions of n in which the smallest part is precisely m, so that $p(n) = (n+1, 1)$. Table II (p. 21–79) gives (n, m) for $n \leqq 300$ and $2 \leqq m \leqq [n/5]$. On p. 81 is a table giving the number of partitions of n into parts exceeding $[n/4]$ for $n \leqq 300$.

A small table of TAIT 1 gives the number of partitions of n into parts $\geqq 2$ and $\leqq r$ for $n \leqq 32$ and $r \leqq 17$.

GIGLI 1 gives the number $N_n(r)$ of partitions of n into precisely r distinct parts not exceeding 10 for $r \leqq 10$ and all possible values of n.

The subject of partitions is of course not to be confused with the so-called quadratic partitions discussed under j_2, giving the actual partitions of numbers into several squares, all but one being equal. In this connection we may cite a table of GAUSS 3 having to do with the number $R(n)$ of representations of n as a sum of two squares. Gauss tabulates the sum

$$\sum_{n=1}^{A} R(n) \quad \text{for} \quad A = k \cdot 10^m, \ k \leqq 10, \ m = 2, 3, \text{ and } 4.$$

This is also the number of lattice points inside a circle of radius $\sqrt{A}$.

q_2. *Goldbach's problem*

Goldbach's, as yet unproved, conjecture is that every even number >2 is the sum of two odd primes[1] >1. Tables have been constructed to test the validity of this conjecture as well as to obtain some information as to the order of magnitude of the number $G(x)$ of representations of $2x$ as a sum of two primes.

CANTOR 1 gives all decompositions of $2n$ into a sum of two primes by listing

[1] Some writers admit 1 as a prime, however.

the lesser of the two primes in each case for $2n \leqq 1000$. The number of such decompositions is also given.

HAUSSNER 1 gives the same information as CANTOR 1, but for $2n \leqq 3000$, and in addition gives the number of decompositions of $2n = p_1 + p_2$ $(p_1 < p_2)$ for $2n \leqq 5000$. As an auxiliary table the values of $P(n) - 2P(n-2) + P(n-3)$ and of $P(n)$, the number of odd primes $\leqq n$, are given for each odd $n \leqq 5000$.

PIPPING 1 lists for each even number $2n \leqq 5000$ the smallest and largest primes $< n$ which enter into the representation of $2n$ as a sum of two primes, together with the value of $G(2n)$, the number of pairs of primes (p_1, p_2) such that $p_1 + p_2 = 2n$, the pairs (p_1, p_2) and (p_2, p_1) being reckoned as distinct if $p_1 \neq p_2$.

PIPPING 2 gives $G(2n)$ for $2262 \leqq 2n \leqq 2360$, $4902 \leqq 2n \leqq 5000$ and $29\,982 \leqq 2n \leqq 30\,080$ together with the corresponding values of two approximating functions. PIPPING 3 gives $G'(2n)$, the number of decompositions of $2n$ as a sum of two primes in Haussner's sense in which $2n = p_1 + p_2 = p_2 + p_1$ are reckoned as one decomposition, for the same values of $2n$ as occur in PIPPING 2, and also the values of $G(2n)$ for $120\,072 \leqq 2n \leqq 120\,170$.

HAUSSNER 4 has a table of the number of representations of $2n$ as a sum of two numbers divisible by no prime $\leqq p_r$, where $p_r^2 < 2n < p_{r+1}^2$ for $2n \leqq 500$, and eleven other values of $2n$ between 4000 and 4166.

STÄCKEL 1 has a similar table due to Weinreich for $n = 6k \leqq 16\,800$.

PIPPING 4 has a table of those even numbers $2n$ which exceed the largest prime less than $2n-2$ by a composite number for $5000 \leqq 2n \leqq 60\,000$. With each such number $2n$ is given the least prime p such that $2n - p$ is also a prime.

GRAVE 3 gives $G'(2n)$ for $2n \leqq 1500$.

Two tables give verifications of Goldbach's conjecture at isolated points up to high limits.

CUNNINGHAM 10 has tested the conjecture for even numbers $2n$ of the form $k \cdot 2^m$, $k = 1, 3, 5, 7, 9, 11$,

$$12^m,\ 20^m,\ 2\cdot 10^m,\ 6^m,\ 10^m,\ 14^m,\ 18^m,\ 22^m,\ 2^m(2^m \pm 1)$$

and also $2 \cdot k^n$, $k = 3, 5, 7, 11$ and $2(2^n \pm r)$, $r \leqq 11$ and odd, up to, in some cases, $2n \leqq 200\,000\,000$.

SHAH and WILSON 1 give the number of decompositions of $2n$ into the sum of two primes, and also into the sum of two powers of primes for 35 values of $2n$ from 30 to 170 172.

A curious table of SCHERK 1 expresses the nth prime p_n in terms of all previous primes as a sum of the form

$$p_n = 1 + \sum_{k=1}^{n-1} \epsilon_k p_k$$

where $\epsilon_k^2 = 1$ for $k < n-1$, while $\epsilon_{n-1} = 1$ or 2.

q3. *Waring's problem*

The eighteenth century conjecture of Waring that every number is the sum of at most 9 positive cubes, at most 19 fourth powers and so on, has given rise to a large number of tables. The Waring problem has been generalized in many ways, but almost all tables refer to the problem of representing numbers as sums of positive kth powers.

These tables are of two sorts: basic tables dealing with the representation of numbers from 1 to N as sums of some limited number of kth powers, and special tables giving such information for miscellaneous ranges of numbers between certain high limits. Tables of this latter type are more recent and owe their existence to attempts to connect with results obtained analytically proving a "Waring theorem" for all large numbers, say $n>N$, and thus to prove the Waring theorem completely. The practical importance of many of these tables has been greatly reduced due to refinements in the analytical methods and a consequent lessening of the number N, a process which is likely to continue in the future.

Tables relating to Waring's problem for kth powers naturally classify themselves according to the value of k, and begin with $k=3$.

Tables of this sort for cubes date from 1835, when ZORNOW 1 gave the least number of cubes required to represent each $n \leqq 3000$, together with the number of numbers between r^3 and $(r+1)^3$ which are sums of no fewer than a specified number of cubes, for $1 \leqq r \leqq 13$.

This table was recomputed and extended by Dase to $n \leqq 12\,000$, and published in JACOBI 4. Besides the corresponding distribution tables there is also the list of those numbers $\leqq 12\,000$, which are sums of 2 cubes and sums of not less than 3 cubes.

The table of STERNECK 3 gives the minimum number of cubes required to represent every number $\leqq 40\,000$ as a sum of cubes. There also appears a table of the number of numbers in each chiliad which require a specified number of cubes from 1 to 9.

A. E. Western has made a special study of the numbers represented as a sum of 4 or 5 cubes. In particular, he has determined for each $n=9k+4 <810\,000$, whether the number of representations by 5 cubes is 0, 1 or >1. These results, and others for selected ranges between $4\cdot 10^6$ and $4\cdot 10^9$ are summarized in WESTERN 2, where the densities of the various numbers in various ranges are given and compared with empirical formulas.

DICKSON 12 is a manuscript table extending STERNECK 3 from 40 000 to 270 000. DICKSON 13 is a manuscript table of the sum of 4 cubes from 270 000 to 560 000. From 300 000 on the minimum number of summands required to represent such numbers is indicated.

A small table of KO 1 gives the representation of every $n \leqq 100$, except 76 and 99, in the form $x^3+y^3+2z^3$, where x, y, and z are integers, positive, negative or zero. The cases $n=6k$ are omitted from the table, since in this case we have $(x, y, z)=(k+1, k-1, -k)$.

Three tables on fourth powers may be mentioned. BRETSCHNEIDER 2 gives "minimum decompositions" for numbers $n \leqq 8^4=4096$. If s is the least number of biquadrateswhose sum is n then all decompositions involving s biquadrates are given. Those numbers n whose minimum decompositions are derived merely by adding 1^4 to those of the preceding number are omitted from the table. A second table lists all numbers representable by s, but no fewer than s biquadrates for $s=2, 3, 4, \cdots, 19$. A more elaborate table for the same range is D. H. and E. T. LEHMER 1. This gives all decompositions of each number $\leqq 4096$ into a sum of not more than 19 biquadrates. A table sufficient for finding one minimum decomposition into fourth powers for $4096 < n \leqq 28\ 561$ together with a summarizing table appears in CHANDLER 1.

A special table of SPARKS 1 is used to prove that every number $\leqq 4184$ is represented by the form $x_1^4+x_2^4+x_3^4+x_4^4+2x_5^4+2x_6^4+4x_7^4+7x_8^4$.

Three tables of fifth powers may be cited. WIEFERICH 1 shows the least number of 5th powers required to represent each number $n \leqq 3011$. DICKSON 7 gives a minimum decomposition into 5th powers for all $n \leqq 150\ 000$, and the minimum number of such decompositions for $n \leqq 300\ 000$.

DICKSON 11 gives a minimum decomposition into sums of fifth powers for the ranges 839 000 to 929 000, and 1 466 800 to 1 600 000. This information for the range 3 470 000 to 3 500 000 is given in DICKSON 8 (p. 84–154). On p. 154–257 are given the minimum numbers of fifth powers required to represent all numbers between 3 500 000 and 3 600 000.

Tables relating to Waring's problem for higher powers are all very special and may be cited as follows:

For sixth powers—SHOOK 1; seventh powers—YANG 1, MAUCH 1 and DICKSON 8 (p. 25–81); eighth powers—SUGAR 1; tenth powers—DICKSON 8 (p. 1–7); thirteenth powers—ZUCKERMAN 1; fifteenth and seventeenth powers —DICKSON 8 (p. 8–24).

HARDY and LITTLEWOOD 2 give values or lower bounds for the number $\Gamma(k)$ which is the least number s such that every arithmetic progression contains an infinity of numbers which are sums of at most s positive kth powers, for $k \leqq 200$.

Finally, there is the table of PILLAI 1, which gives for each $n \leqq 100$, the values of 2^n, l_n and r_n in the equation

$$3^n = l_n 2^n + r_n,$$

quantities which are important in Waring's problem for nth powers.

Gupta has published 4 tables dealing with the representation of numbers by sums of like powers of primes. In this case 1 is counted as a prime.

GUPTA 2 has a table showing that every number $\leqq 100\ 000$ is a sum of not more than 8 squares of primes. GUPTA 6 has a special table for this problem of all integers $\leqq 2000$ of the form $A=(p^2-1)/120$, $B=(p^2-49)/120$, $C=A+B$, where p is a prime. GUPTA 4 gives the least number of cubes of primes required to represent each number $\leqq 11^3=1331$, and a list of 150 numbers between 11^3 and 20 828 which require 6 or fewer cubes of primes. GUPTA 5 gives tables showing that every number $\leqq$ 20 875 (except 1301) is a sum of not more than 12 cubes of primes.

Waring's problem with polynomial summands is responsible for a number of special tables due to Dickson and his pupils. The summands in question are polygonal numbers and certain cubic functions. For polygonal numbers we may cite DICKSON 5, ANDERSON 1, GARBE 1, and for cubics, BAKER 1, and HABERZETLE 1.

II

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g v. 1, ch. XIV, no. 37.
h v. 2, ch. II, no. 41.
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DLC, IEN, IaAS, IaU, IU, KyU, MdBJ, MB, MH, MiU, NhD, NjP, NNC, NcD, OCU, OU, PU, RPB, TxU, WU

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Libraries: v. 3, 4, CPT, CU, CaM, CaTU, CoU, CtY, DLC, ICJ, ICU, IU, InU, IaAS (not v. 4), IaU, KyU, MdBJ, MB, MCM, MH, MiU, MnU, NhD, NjP, NN, NNC, NcD, OCU, OU, PU, RPB, TxU, WU

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Libraries: CPT, CU, CaM, CaTU, CtY, DLC, ICJ, ICU, IaU, MdBJ, MB, MH, MnU, MoU, NhD, NjP, NIC, NNC, NcD, OCU, OU, RPB, WvU, WU

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Libraries: CU, CtY, DLC, IU, MB, MnU, NjP, NN, NNC, OU, PU, RPB, WU

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Libraries: DLC, ICU, NN, NNC
1₂. *Zweiter Abdruck*, Leipzig, 1849, xxiii+840 p. [e_1^*, 360–423: f_1^*, 424–454.]
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Libraries: CU, ICU, MH, MoU, RPB

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W. WEINREICH, see STÄCKEL.

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Libraries: CU, DLC, ICU, IU, InU, MdBJ, MB, MCM, MH, MiU, NN, NNC, PBL, PU, RPB, WU
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   Libraries: ICU

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H. S. ZUCKERMAN.

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   Libraries: ICU

III

ERRATA

Arndt 2.

Insert 397 3447:173

Barlow 1.

n	read	n	read	n	read
465	$3 \cdot 5 \cdot 31$	4364	$2^2 \cdot 1091$	7668	$2^2 \cdot 3^3 \cdot 71$
1431	$3^3 \cdot 53$	5598	$2 \cdot 3^2 \cdot 311$	7795	$5 \cdot 1559$
1917	$3^3 \cdot 71$	5798	$2 \cdot 13 \cdot 223$	7894	$2 \cdot 3947$
2140	$2^2 \cdot 5 \cdot 107$	5912	$2^3 \cdot 739$	7936	$2^8 \cdot 31$
2799	$3^2 \cdot 311$	6517	$7^3 \cdot 19$	7964	$2^2 \cdot 11 \cdot 181$
2862	$2 \cdot 3^3 \cdot 53$	6660	$2^2 \cdot 3^2 \cdot 5 \cdot 37$	8560	$2^4 \cdot 5 \cdot 107$
2956	$2^2 \cdot 739$	6786	$2 \cdot 3^2 \cdot 13 \cdot 29$	8618	$2 \cdot 31 \cdot 139$
3580	$2^2 \cdot 5 \cdot 179$	6868	$2^2 \cdot 17 \cdot 101$	8728	$2^3 \cdot 1091$
3718	$2 \cdot 11 \cdot 13^2$	7160	$2^3 \cdot 5 \cdot 179$	9244	$2^2 \cdot 2311$
3834	$2 \cdot 3^3 \cdot 71$	7322	$2 \cdot 7 \cdot 523$	9275	$5^2 \cdot 7 \cdot 53$
4280	$2^3 \cdot 5 \cdot 107$	7436	$2^2 \cdot 11 \cdot 13^2$		

(Cunningham 41(a), p. 27)

Beeger 1.

p	for	read
109	5947	5934
109	3936	3717
179	16614	15427
197	2768	2668

(Meissner 2, p. 96)

Beeger 2.

p	for	read
127	$W = 51$	71
127	$w = 107$	117
167	$W = 115$	21
173	$W = 16$	106
211	$w = 90$	121
211	−	+

p	for	read
223	$w = 56$	167
223	−	+
227	+	−
241	$W = 34$	196
263	−	+
271	$w = 194$	77
271	−	+

(Beeger)

Bickmore 1.

n	column	for	read
47	$2^n - 1$	2251	2351
16	$5^n - 1$	11439	11489
2	$6^n - 1$	$5 \cdot 7$	7
49	$6^n - 1$	$883 \cdot x$	x
16	$12^n - 1$	26053	260753
20	$12^n - 1$	x	$5^2 \cdot x$
44	$12^n - 1$	$2697 \cdot x$	$2377 \cdot 3697 \cdot x$

(Cunningham, *Messenger Math.* v. 26, 1896, p. 38)

BICKMORE 2.

p	read
29	43037
33	1344 62821 03132 98373
64	504 00685 44932 21107 80706 61761

(HERTZER, *Archiv Math. Phys.*, s. 3, v. 13, 1908, p. 107)

BORISOV 1.

n	for	read
184	(3, 8, 9, −4, −1, 0)	(3, 8, 10, −4, −1, 0)
193	(7, 7, 5, 0, −1, −2)	(7, 7, 5, 0, −1, −3)

(JONES 1, p. 6; see also *Scripta Math.*, v. 4, 1936, p. 104)

BORK 1.

p	q	p	q	p	q	p	q	p	q
1753	3	46229	7	49831	110	78031	10	87881	40
41221	5	46489	4	50221	9	82307	14	87973	6
41651	7	48679	38	51341	17	84067	6	89041	28
42491	7	49069	9	51767	181	84653	2	90067	6
43051	7	49787	62	53327	13	85639	6	93151	10
45767	7	49801	8	57191	38	86923	22		

(CUNNINGHAM 40, p. 154)

BRETSCHNEIDER 2.

page	number	for	read
	977	0, 6, 0, 11	0, 6, 0, 1, 1
4	1134	0, 0, 1	0, 0, 14
5	1289	3, 0, 5, 5, 1	3, 0, 5, 1, 1
	1610	0, 0, 9, 11	0, 0, 9, 1, 1
6	2067	0, 1, 3, 2, 0	0, 1, 3, 2, 0, 1
	2323	not listed	3, 0, 0, 4, 0, 1
			0, 1, 3, 3, 0, 1
7	2384	0, 4, 0, 49, 1	0, 4, 0, 4, 0, 1
	2516	0, 1, 0, 0, 0, 4	0, 1, 0, 0, 4
	2532	0, 2, 0, 0, 0, 4	0, 2, 0, 0, 4
8	3025	0, 6, 1, 1, 0, 1	0, 6, 1, 1, 0, 2
10	3522	0, 2, 3, 0, 2	0, 0, 2, 3, 0, 2
	3541	0, 4, 0, 1, 2	0, 0, 4, 0, 1, 2
	3603	0, 0, 3, 3, 0, 1	0, 0, 3, 3, 0, 2
11	3723	0, 0, 10, 2, 0, 1, 1	0, 0, 10, 2, 0, 0, 1
12	4011	5, 0, 0, 1	5, 0, 0, 1, 6

page	table	for	read
16	VI	3424	3524
22	XVIII	379	479

(CHANDLER 1, p. 10)

BURCKHARDT 1, [d₂].

p	for	read	p	for	read
911	450	455	1979	1976	1978
1213	1212	202	1993	1992	664
1597	266	133	2311	462	231
1831	915	305	2437	2436	1218
1951	390	195	3467	3466	1733

(SHANKS 1, p. 202)

1, [e1].

n	for	read
9899	blank	19
307849	11	211
446021	573	577
446023	197	193
854651	prime	7
854647	7	prime
895339	7	17

(D. N. LEHMER 1, col. xi)

BURCKHARDT 2.

n	for	read
1019681	17	13
1037051	53	17
1130023	881	prime
1130323	prime	881
1138027	prime	11
1207517	blank	229
1233473	37	prime
1249843	23	7
1250111	57	53
1270471	223	prime
1307377	1013	1019
1330001	1123	prime
1359233	277	prime
1397647	589	587
1411679	11	prime
1412047	13	7
1420847	97	prime
1459699	499	449
1496693	prime	11
1504741	41	7
1556257	prime	37
1588633	23	17
1618087	1069	prime
1619173	prime	151
1623703	prime	151
1627081	169	167
1748209	101	19
1782899	1153	1151
1785169	147	149
1787471	prime	7
1793023	7	prime
1793029	prime	7
1857997	14	41
1916683	prime	193
1936159	1123	prime
1979687	73	47
1984891	797	prime
1996399	83	67

(D. N. LEHMER 1, col. xi)

BURCKHARDT 3.

n	for	read
2012603	prime	887
2071301	69	79
2077529	prime	131
2114693	103	7
2193923	1429	1433
2214413	31	37
2214931	31	37
2222417	1129	1123
2501261	prime	7
2511893	2	29
2518817	17	7
2542283	1197	1193
2619887	7	17
2755189	63	163
2763907	1213	1297
2768683	449	prime
2868407	683	prime
2882699	blank	19
2891813	2	23
2903591	1697	1699
2913833	29	13
2915899	prime	7
2954939	prime	13
2976227	549	547
2976881	311	prime
3026279	79	prime

(D. N. LEHMER 1, col. xi)

CAHEN 1, [d1].

page	p	for	read
377	59	57	56
384	137	8	3
384	137	62	67
387	173	96	76
389	191	*insert*	175

1, [d_3].

page	p	table	arg.	for	read
375	17	I	15	3	2
379	79	I	6	34	43
380	101	N	41	74	72
380	101	N	81	62	67
382	109	N	25	66	69
385	149	N	101	82	92
386	157	N	118	22	33
386	163	I	72	131	137
386	163	I	92	137	133
389	193	I	58	161	191
389	193	I	78	191	161
389	193	N	24	144	184

1, [i_3].

Contains all errors of CHEBYSHEV 2_4, and also

Δ	for	read	D	for	read
31	−	+	47	879	79
74	27	29	77	283	285
			101	04z	404z

CAHEN 3, [d_1].

page	p	for g	read
55	1021	7	10
56	2161	14	23

3, [d_3].

page	p	table	argument	for	read
40	17	I	15	3	2
40	19	I	19	5	−
40	23	N	9	90	20
41	37	I	31	37	27
41	41	I	27	2	5
42	59	I	30	32	33
43	79	I	6	34	43
45	101	N	41	74	72
45	101	N	81	62	67
45	103	I	26	11	10
45	103	I	89	37	27
46	109	I	14	y3	73
46	109	N	25	66	69
47	131	I	37	33	23
47	131	I	65	117	112
47	131	I	85	102	107
47	131	I	113	1	10
48	139	I	9	08	98
48	139	N	136	57	37
49	149	N	101	82	92
50	157	N	118	22	33
50	163	I	72	131	137
50	163	I	92	137	133

3, [d_2].—*continued*

page	p	table	argument	for	read
51	167	N	164	84	162
52	179	I	109	133	113
52	181	N	56	17	170
52	181	N	66	61	67
52	181	N	76	155	102
52	181	N	86	109	4
52	181	N	96	93	25
52	181	N	106	174	111
52	181	N	116	92	15
52	181	N	126	32	139
52	181	N	136	19	9
52	181	N	146	164	11
52	181	N	156	120	114
52	181	N	166	26	79
52	181	N	176	72	177
53	191	N	170	51	52
53	193	I	58	161	191
53	193	I	78	191	161
53	193	N	24	144	184

3, [i_3].

a	for	read	a	for	read
26	...	± 9	−38	12,35	13, −35
−29	−55	55	−39	−33	−23
−30	7	− 7	−42	−39	−19
−31	− 2	− 3	−43	35	31
−33	−47, −57	47, 65	−43	...	− 5
−34	...	−13	−46	...	41
35	±53	±17			

CARMICHAEL 2.

$\phi(m)$	for	read	insert	delete
768			1785, 3570	
792	2384	2388		
880				1043, 2086
888			1043, 2086	
960			1309, 2618	
972			1467, 2934	

(GLAISHER 27, p. vii)

CAYLEY 1_2.

D	for	read
253	1177: 74	1861:117
597	7949:399	9749:399
645	203: 8	127: 5
917	1181: 31	1181: 39

CAYLEY 2_3.

page	D	changes
144	−17	*Erase* the long bar under 1, 0, 17
144	−20	*For* 2, 0, 5 *read* 4, 0, 5
144	−34	*For* 7, −1, 7 *read* 5, −1, 7
145	−40	*Insert* a short bar under 0, 40
145	−40	*Insert* a short bar under 0, 8
145	−40	In cols. of δ, ε, *enter* ++ in l. 1, −− in l. 2, *enter* +− in l. 3. −+ in l. 4
145	−40	*Cancel* all entries in col. of δε
145	−56	*For* 2, −1, 19 *read* 3, −1, 19
147	−88	*Insert* a short bar under 0, 88
147	−88	*Insert* a short bar under 0, 11
147	−88	In col. of δ, *enter* +, −, −, + in lines 1, 2, 3, 4
149	29	L. 2, the period should be 2, 5, −2, 5, 2
149	37	L. 3, *reverse* the period, thus −3, 5, 4, 3, −7, etc.
149	41	L. 2, the period should be 2, 5, −8, 3, 4, 5, −4, 3, 8, 5, −2, 5, 8, 3, −4, 5, 4, 3, −8, 5, 2
150	50	In col. of ε, *enter* + in l. 1, + in l. 2
150	50	*Cancel* the entries in col. of δε
151	65	L. 1, the period should be I, 8, −I, 8, I
152	91	L. 2, *for* 3, 7, −14 *read* 3, 7, −I4

(CUNNINGHAM 42, p. 59–60)

CAYLEY 6_1.

page	a	for	read
76	29	1, −6, 5, −3, 2, −1	1, −4, 5, −5, 4, −1
76	1014		146246
76	1051	x, y	y, x
109	1361	a=1361	a=1361*
	1366		61 98787 71121 28467 93128 64853 64042

(CUNNINGHAM 42, p. 67 and D. H. LEHMER 11, p. 550)

CHEBYSHEV 1_1, [i_3].

p. 273, x^2-11y^2, *for* $N=44n+27$ *read* $44n+25$, 27; this is correct in 1_2.

CHEBYSHEV 2_4, [d_3].

p	table	argument	for	read
13	I	12	—	6 (also in 2_1)
17	I	15	3	2 (peculiar to 2_4)
109	N	25	66	69 (peculiar to 2_4)

2_4, [i_3].

form	insert	delete
x^2+42y^2	157	159
x^2+61y^2	215	
x^2+66y^2	71	77
x^2+70y^2	239	233
x^2+74y^2		89
x^2+77y^2	159, 237	119, 143, 297
x^2+86y^2	87	89
x^2+89y^2	345	354
x^2+91y^2	115, 297	7, 189
x^2+101y^2	281, 309, 317, 325, 333	287, 305, 313, 321, 329

2_4, [i_3].—*continued*

form	insert	delete
x^2-38y^2	21, 131	23, 129
x^2-62y^2	107, 141	103, 145
x^2-87y^2	25, 323	91, 257
x^2-91y^2	33, 55, 73, 89, 97, 267 275, 297, 309	17, 63, 115, 143, 175 189, 221, 245, 249, 347
x^2-95y^2	161, 219	29, 351
x^2-101y^2	71, 79, 87, 95, 309 317, 325, 333	75, 83, 91, 99, 305 313, 321, 329

All these errors (except the misprint in x^2+89y^2) occur also in 2_1 and 2_5 while none is in 2_3.

CHERNAC 1.

L. J. COMRIE found (PETERS, LODGE and TERNOUTH, GIFFORD 1, p. ix) misprints in the factors of 66 011=11·17·353 and (in some copies) of 44393=103·431. RPB has two editions of this table, one with the correct factors, and one with the factors 10·3431.

number	factors	authority
19697	prime	CUNNINGHAM
19699	prime	CUNNINGHAM
38963	47·829	CUNNINGHAM
39859	23·1733	BURCKHARDT
65113	19·23·149	BURCKHARDT
68303	167·409	BURCKHARDT
68303–68399	raise each line of factors one line up	BURCKHARDT
68987	149·463	CUNNINGHAM
76769	7·11·997	CUNNINGHAM
354029	13·113·241	BURCKHARDT
469273	7·7·61·157	CUNNINGHAM
494543	7·31·43·53	CUNNINGHAM
545483	prime	BURCKHARDT
580807	prime	BURCKHARDT
637447	prime	BURCKHARDT
769469	prime	BURCKHARDT
783661	prime	BURCKHARDT
795083	prime	BURCKHARDT
795089	67·11867	BURCKHARDT
795091	11·11·6571	BURCKHARDT
931219	29·163·197	BURCKHARDT

(CUNNINGHAM 41, v. 34, p. 26 and v. 35, p. 24; BURCKHARDT 1, p. 1)

CRELLE 1, [d_1].

page	col.	line	for	read
52	8	101	·	101
52	57	61	61	·
52	57	67	·	67
52	65	83	89	·
52	65	89	·	89

Same errors in CRELLE 2, Tafel III.

CUNNINGHAM 4, [d_3].

page	p	argument	for	read
10	139	$p-1$	$2\cdot 69$	$2\cdot 3\cdot 23$
60	547	$x=435$	102	192
104	773	$x=699$	873	73
120	839	$x=315$	541	548
122	853	R	{300 / 210	{300 / 310
127	859	$x=526$	776	770
147	947	R	{910 / 820	{910 / 920

(CUNNINGHAM 42, p. 68)

CUNNINGHAM 5.

page 174, table of $\rho^{35}\equiv+1$ (mod 71^2) *for* $\rho^{11}\equiv 60$ *read* 5030
page 177, table of $\rho^{13}\equiv+1$ (mod 53^2) *read*
752, 895, 1689, 460, 413, 1586, 1656, 925, 1777, 2029, 521, 1341.
table of $r^{13}\equiv-1$ (mod 53^2) *read*
2057, 1914, 1120, 2349, 2396, 1223, 1153, 1884, 1032, 780, 2288, 1468.
(CUNNINGHAM, *Messenger Math.*, v. 30, 1900, p. 60, v. 43, 1914, p. 155)

CUNNINGHAM 7, [e_2].

p	for	read
87481	$8\cdot 5\cdot 27\cdot 81$	$8\cdot 27\cdot 81\cdot 5$
96661	$4\cdot 5\cdot 27\cdot 179$	$4\cdot 27\cdot 5\cdot 179$

7, [j_2].

Interchange a and b for
$p=45289$, 55633, 70289, 77549, 79609, 80809, 95101.
$p=60169$, *read* $A, B=37$, 140; $L, M=383$, 59.

(CUNNINGHAM 42, p. 69)

CUNNINGHAM 10.

page 169 *for* $p=8124461$ *read* 8124161
(CUNNINGHAM, *Messenger Math.*, v. 40, 1910, p. 36)

CUNNINGHAM 24, [o].

$n=42$, coefficients in Q, *read* 1, 7, 15, 14, 1, −12, −12, 1, 14, 15, 7, 1.

CUNNINGHAM 28.

	page	x	y	for	read
	143		15	*insert*	257
B	152		$3^4\cdot 2^8$	20 155 393	$61\cdot 330413$
	163		984	14877921	114877921
	217		12		$7681\cdot 40609\cdot 592734049$
	281	1	71	12708841	12705841
B	284	line	6	12084217	12004217

(WOODALL and BEEGER 5; errata marked with "*B*," BEEGER)

CUNNINGHAM 29.

p. 86 in heading, *for* (y^7+1) *read* (y^6+1)

(CUNNINGHAM 39)

CUNNINGHAM 30.

page	x	y	for	read
145	7^4	5^5	4.193051	4193051
183	81	2	10730221	10730021
183	49	72	15543281	1143281
185		32	180801	100801
193	64	75	10545971	151·211·331
193	9	125	25437261	125437261
214			25613261	25813261

(BEEGER 5)

CUNNINGHAM 31.

p. 81, prime 9901, *for* 5004 *read* 5304

(CUNNINGHAM 39)

CUNNINGHAM 32.

page	for	read
166	12207171	15450197
174	98068509	127·211·3697
189	15801871	15801571

(BEEGER 5)

CUNNINGHAM 33.

page 112 bottom *for* 29105 · · · *read* 39105 · · ·
page 115 $\eta=2$, $y=12$, *for* 29105 · · · *read* 39105 · · ·

(BEEGER 5)

CUNNINGHAM 35, [f₁].

page 7 *for* 19487569 *read* 19487579
page 7 *for* 19487969 *read* 19487959

(BEEGER)

CUNNINGHAM 37.

page	y		for	read
80	22	y^8+1	4·121	5·97
80	28	$\frac{y^{18}+1}{y^6+1}$	28481	24481

(J. C. P. MILLER)

CUNNINGHAM 38.

p. 125, line 3 from bottom, *for* 38014 *read* 38012

(CUNNINGHAM 39)

CUNNINGHAM and WOODALL 7, [d₂].

for $p=2241$ *read* 2341
for $p=40152$ *read* 40153
for $p=44029$ (bis) *read* 44089
for $p=27551$ *for* $\nu=10$ *read* $\nu=50$.

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, 1924, p. 73)

CUNNINGHAM and WOODALL 10.

	page	n		for	read
	3	155	2^n-1	*insert*	31
	14	19		48713705353	48713705333
	16	25		*delete* 29251	
M	22	25	12^n-1	*delete* entry	
M	23	22	12^n+1	6836860537	68368660537

(Errata marked with "*M*," J. C. P. MILLER)

CUNNINGHAM, WOODALL and CREAK 1.

page 26, $p=8011$, *for* $g=13$ *read* $g=14$
page 108, $p=14009$, base 7, *for* $=8$, *read* $=824$
page 120, $p=19009$, *for* $g=+29, -29$ *read* $g=+23, -23$.

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, p. 180)

CUNNINGHAM, WOODALL and CREAK 2.

page 353, $p=8011$, *for* $g=13$, *read* $g=14$
page 356, $p=19009$, *for* $g=+29, -29$, *read* $g=+23, -23$.

(WOODALL)

DASE 1.

number	for	read
6027133	blank	7
6036637	blank	prime
6075451	21	421
6403117	9	7
6408679	33	83
6722999	217	127
6736409	7	71

(D. N. LEHMER 1, col. xi)

DASE 2.

number	for	read
7022623	prime	1913
7040029	prime	1627
7047113	1997	prime
7047413	prime	1997
7110881	prime	1861
7141793	prime	2617
7220819	prime	1877
7224053	1143	2143
7295077	prime	2683
7295081	2683	prime
7324523	prime	2467
7345979	1801	prime
7346279	prime	1801
7366739	13	23
7384631	prime	2179
7385993	prime	1933
7410421	173	179
7412899	23	13
7430573	prime	2089
7489961	prime	181
7548199	553	353
7556273	prime	1949
7556573	1949	prime
7576799	prime	149
7601003	prime	2437
7601303	2437	prime
7614461	prime	2539
7680451	prime	1811
7732871	prime	1783
7741093	41	prime
7790381	prime	2311
7802999	prime	2179
7810963	prime	1847
7820201	prime	1831
7845427	prime	1901
7855549	29	13
7856147	prime	13
7857343	prime	13
7860931	101	13
7861517	prime	2383
7861529	prime	13
7863323	107	13
7864519	prime	13
7865117	prime	13
7866911	prime	13
7868107	prime	13
7887931	67	367
7918819	31	131
7927501	prime	1879
7933649	prime	2341
7941047	prime	1831

(D. N. LEHMER 1, col. xii)

DASE 3.

The entries for 8236079 and 8245589 are given correctly in some copies and incorrectly in others. Two copies, one correct and one incorrect, are in RPB.

number	for	read
8057743	prime	2617
8068211	prime	2617
8083913	prime	2617
8136253	prime	2617
8162423	prime	2617
8167657	prime	2617
8167987	prime	181
8169797	prime	181
8170159	prime	181
8209529	prime	2617
8236079	23	73
8245589	41	11
8277571	prime	2617
8282197	prime	7
8288039	prime	2617
8293273	prime	2617
8318393	73	43
8324677	prime	2617
8340379	prime	2617
8350847	prime	2617
8382251	prime	2617
8397953	prime	2617
8409631	79	379
8409917	7	17
8418889	prime	2617
8427193	97	67
8429357	prime	2617
8431151	prime	1613
8431169	1613	prime
8450293	prime	2617
8456059	prime	239
8477669	prime	1361
8477671	1361	prime
8478889	233	prime
8486449	227	277
8491187	769	569
8496181	1123	1223
8499737	prime	829
8499763	829	prime
8500853	227	277
8507867	prime	2617
8513101	prime	2617
8523569	prime	2617
8525317	prime	877
8528803	prime	2617
8536319	13	11
8560057	31	11
8560207	prime	2617
8562461	23	43
8593507	43	13
8626981	41	11
8633483	prime	2617
8636011	11	31
8638717	prime	2617
8654419	prime	2617
8670121	prime	2617
8684609	prime	233
8684903	233	prime
8685823	prime	2617
8696291	prime	2617
8711993	prime	2617
8717227	prime	2617
8748631	prime	2617
8754887	2627	1627
8759099	prime	2617
8783693	5171	571
8783699	49	149
8788069	prime	2017
8790503	prime	2617
8795737	prime	2617
8821907	prime	2617
8827141	prime	2617
8869013	prime	2617
8874247	prime	2617
8916119	prime	2617
8930137	1949	1049
8931821	prime	2617
8964901	13	11
8965801	11	13
8984161	prime	2617
8995513	2767	prime
8995517	prime	2767

(D. N. LEHMER 1, col. xii)

W. DAVIS 1.

delete 10^8+0013, 0391, 0657, 0723, 1221, 1353, 1549, 1647.

(CUNNINGHAM and WOODALL 5, p. 78)

DEGEN 1.

A	read
853	*for* 10th entry of upper line 14, *not* 15.
929	30, 2, 11, 1, 2, 3, 2, 7, 5, (2, 2),
	1, 29, 5, 40, 19, 16, 25, 8, 11, (23, 23)
238	$y=756$
277	$x=15915007379898 0475849$
421	$y=189073995951839020880499780706260$
437	$x=4599$
613	$y=187415457848319978803087843 40$
641	$x=2609429220845977814049$
	$y=1030662575509627 37720$
653	$x=104999865686772 99849$
672	$x=337$
751	$x=72933184667948824244 18960$
823	$x=2351704749036440 06168$
919	$y=14783444239653675978149 9589$
945	$x=275561$
949	$y=19789181711517243032971740$
951	$x=224208076$

(D. H. LEHMER 11)

DESMAREST 1, [d₂].

p	for	read	p	for	read	p	for	read	p	for	read
3	·	2	3517	2	4	5519	1	2	8087	2	1
277	2	4	3541	59	177	5557	4	6	8093	1	2
317	2	4	3547	1	2	5827	1	2	8101	1	5
397	2	4	3637	1	4	6101	2	5	8219	2	1
409	1	2	3677	4	2	6277	2	4	8423	1 }	1
449	2	14	3769	4	2	6287	2	1	8423	2 }	
787	1	2	3821	2	1	6781	1	5	8521	24	12
1409	1	44	3911	1	2	6997	2	4	8609	18	8
1657	6	3	4049	4	2	7001	2	4	8681	2	10
1733	4	2	4397	28	14	7127	14	7	8893	2	4
1889	32	16	4621	1	5	7481	2	10	8999	1	2
1997	4	2	4651	2	1	7561	2	4	9067	1	2
2087	8 }	7	4943	2	1	7717	2	4	9187	1	2
2087	9 }		5081	2	4	7741	3	9	9397	2	116
3253	12	6	5107	1	2	7841	20	140	9521	10	16
3373	6	4	5407	6	3	7853	·	2	9629	2	1
3413	4	2	5479	1	2	8011	6	3	9649	8	16
									9941	1	5

Primes misprinted:

for	read
4167	4157
5871	4871
8421	6421

(CUNNINGHAM 40, p. 151)

DICKSON 2, [b_2].

Table III, *add* 2750, 2990, 3250, 3430.

Table V

$\sigma(n)$	for	read	$\sigma(n)$	for	read
224	233	223	1440	—	1195
240	158, 135	135, 158	(*add*) 1524	—	704, 1083, 1523
289	$\sigma(n)$	288	1536	—	1023
372	—	305	1620	—	1513
468	196	198	(*add*) 1776	—	1022, 1095, 1329
1170	1069	—	2400	1068	1064
1248	993	933	2448	1513	1515
1344	—	546	2736	1587	1582
1368	814, 735	735, 814	2880	—	1434
1404	—	1165			

TABLE VI

$\sigma(n)=280$, *for* 106 *read* 108
add $\sigma(n)=399$ 196, 242
add $\sigma(n)=1374$ 914, 1373
delete entries under 1124, 1134, 1304 and 1524.

Table VII

add $\sigma(n)=1134$ 544
$\sigma(n)=1862$ *for* 1571 *read* 1573
delete entries under 372, 399, 1151, 2860.

(GLAISHER 27, p. vii)

DICKSON 6.

page 184, $d=47$, *for* 1, 3, 6, −1, 0, 0 *read* 1, 3, 16, −1, 0, 0.

(JONES 1, p.6)

DINES 1.

page 114 range 10 *delete* 53

(BEEGER 6)

DURFEE 1, [e_1].

$n=15485303$ *for* prime *read* 109

EULER 1_4.

n	$\sigma(n)$	factors of $\sigma(n)$
7^{10}	329554457	$1123\cdot 293459$
37^3	52060	$2^2\cdot 5\cdot 19\cdot 137$
61^3	230764	$2^2\cdot 31\cdot 1861$
insert 79	80	$2^4\cdot 5$
insert 79^2	6321	$3\cdot 7^2\cdot 43$
insert 79^3	499360	$2^5\cdot 5\cdot 3121$

(POULET 2, p. 10)

GAUSS 6.

Contains many errors.

(GLAISHER)

GAUSS 7.

negative determinants

page	cent.	for				read			
451	5	II.	9	459*		II.	9	459	(*3*)
451	5	IV.	4	468		IV.	4	468	(*2*)
451	5	IV.	4	485		IV.	5	485	
451	6	IV.	4	544		IV.	4	544	(*2*)
451	6	I.	9	547		I.	9	547	(*3*)
451	6	II.	9	557		II.	13	557	
451	7	I.	25	647		I.	23	647	
452	9	IV.	6	894		IV.	7	894	
452	10	II.	9	931		II.	9	931	(*3*)
452	10	IV.	3	933		IV.	4	933	
452	10	IV.	4	993		IV.	3	993	
452	12	IV.	9	1116		IV.	6	1116	
453	13	II.	10	1261		IV.	5	1261	
453	14	I.	27	1367		I.	25	1367	
453	14	IV.	7	1396		II.	14	1396	
454	16	IV.	8	1508		IV.	8	1508	(*2*)
454	16	IV.	8	1598		IV.	8	1598	(*2*)
454	17	II.	9	1683		II.	6	1683	
454	18	IV.	9	1701		IV.	9	1701	(*3*)
454	18	VIII.	4	1725		VIII.	4	1725	(*2*)
454	18	IV.	10	1796		II.	20	1796	
455	19	IV.	9	1836		IV.	9	1836	(*3*)
455	19	VIII.	4	1872		VIII.	4	1872	(*2*)
455	20	IV.	8	1940		IV.	10	1940	
455	21	VIII.	5	2085		VIII.	4	2085	
456	22	centas 2 (at top)				centas 22			
456	22	II.	9	2188		II.	9	2188	(*3*)
456	22	IV.	12	2196		IV.	12	2196	(*2*)
456	22	IV.	16	2180		IV.	16	2180	(*2*)
456	23	IV.	11	2204		IV.	13	2204	
456	24	IV.	12	2331		IV.	12	2331	(*2*)
456	24	IV.	8	2304		IV.	8	2304	(*2*)
456	24	VIII.	4	2320		VIII.	4	2320	(*2*)
457	25	VIII.	4	2448	(*2*)	VIII.	4	2448	
457	27	II.	33	2636		II.	33	2636	(*2*)
458	29	IV.	12	2900		IV.	12	2900	(*2*)
459	61	VIII.	8	6032		VIII.	8	6032	(*2*)
459	61	IV.	24	6068		IV.	24	6068	(*2*)
459	61	II.	27	6075	(*3*)	II.	27	6075	(*9*)
459	61	IV.	12	6084		IV.	12	6084	(*2*)
460	62	IV.	8	6148		IV.	8	6148	(*2*)
460	62	IV.	20	6176		IV.	20	6176	(*2*)
461	92	IV.	32	9104	(*2*)	IV.	32	9104	
461	92	VIII.	4	9108		VIII.	4	9108	(*2*)
461	92	VIII.	8	9156		VIII.	8	9156	(*2*)
461	94	VIII.	12	9324		VIII.	12	9324	(*2*)
462	96	VIII.	8	9513	(*2*)	VIII.	8	9513	
462	96	IV.	40	9554		IV.	40	9554	(*2*)

GAUSS 7—*continued*

		positive determinants							
		for				read			
page	cent.								
475	1	*G* IV.	1	99		*G* IV.	2	99	
475	2	*G* IV.	1	136		*G* IV.	2	136	
475	2	*G* VIII.	1	150		*G* IV.	1	150	
475	2	*G* IV.	1	156		*G* IV.	2	156	
475	2	*G* II.	1	174		*G* IV.	1	174	
475	3	[at head of table]				excidunt 3			
475	3	omitted				*G* IV.	1	208	
475	3	omitted				*G* II.	1	209	
475	3	*G* II.	1	229		*G* II.	1	227	
476	9	*G* IV.	1	850		*G* IV.	2	850	
476	9	*G* IV.	1	885		*G* IV.	2	885	
476	10	*G* IV.	1	904		*G* IV.	2	904	

(CUNNINGHAM 42, p. 55–56)

E. GIFFORD 1.

N	for	read	*N*	for	read
121	11×11	11^2	54353	13×31×113	13×37×113
4193	7×559	7×599	553	too low	
8477	7×7×173	7^2×173	57553	67×889	67×859
20567	121×157	131×157	613	too low	
21329	7×11×227	7×11×277	64643	113×509	127×509
22331	127×163	137×163	65069	29×2099	31×2099
26413	61×233	61×433	660	too low	
26443	31×253	31×853	69781	31×3251	31×2251
26567	31×257	31×857	71801	19×3719	19×3779
28733	59×457	59×487	75293	17×43×101	17×43×103
28873	13×2201	13×2221	76879	11×19×241	11×29×241
289	too low		79237	17×51×79	17×59×79
29351	7^2×559	7^2×599	79439	19×31×113	19×37×113
30523	121×233	131×233	79583	7×10369	7×11369
30589	13^2×781	13^2×181	82081	73×1039	79×1039
32131	11×127×23	11×23×127	82477	65×1231	67×1231
32671	37×853	37×883	87203	29×3007	29×31×97
39931	—	73×547	90493	13×6161	13×6961
39937	73×547	—	90571	13×6167	13×6967
43589	7×6197	7×13×479	91681	17×5303	17×5393
46711	7^2×6673	7×6673	99433	17×5894	17×5849
47081	23×23×89	23^2×89	99731	19×29×281	19×29×181
50059	103×443	113×443	100051	17×14293	7×14293

(This previously unpublished list of errata was furnished by L. J. COMRIE after comparison with PETERS, LODGE and TERNOUTH, GIFFORD 1, and is believed to be complete. Dr. COMRIE notes also the following two errors in Mrs. GIFFORD's "Errata": *for* "9307" *read* 93; *after* 50519, *for* 7^2×1039, *read* 7^2×1031.)

J. GLAISHER 1, [e_1].

number	for	read
3039709	5	53
3043027	1	13
3063523	127	1277
3081121	1	31
3081733	46	467
3082109	5	53
3083273	1	17
3083561	1	13
3085219	57	577
3089489	1	13
3093503	blank	7
3230309	53	59
3230317	prime	1721
3230321	1721	prime
3234043	57	157
3347717	199	109
3464011	223	233
3539017	prime	1699
3539021	1699	prime
3543737	181	prime
3563659	1	11
3621197	prime	1097
3621199	1097	prime
3776569	1789	prime
3776579	prime	1789
3826601	373	prime
3826607	prime	373
3903341	19	13

(D. N. LEHMER 1, col. xi)

J. GLAISHER 2, [e_1].

number	for	read
4610243	1	11
4782811	1	11
4793477	1	13
4801751	prime	167
4905281	4	41
4986869	prime	29

(D. N. LEHMER 1, col. xi)

J. GLAISHER 3, [e_1].

number	for	read
5580421	23	7
5581823	3	13
5581829	1	11

(D. N. LEHMER 1, col. xi)

J. W. L. GLAISHER 9.

Second million, first myriad *for* 391,362 *read* 390,363;
third million, third myriad *for* 349,344 *read* 350,343.

(GLAISHER 12, p. 193)

J. W. L. GLAISHER 15.

page 106, *insert* $E(802) = 2$, $E(922) = 2$.
page 107, column "sum of values"
at 800–899 *for* 73 *read* 75
at 900–999 *for* 79 *read* 81

(GLAISHER 25, p. 66, and 27, p. 185)

GOLDBERG 1.

page	for	read
5	4367	4267
5	5387	4387
6	5939	5039
7	5369	5569
9	7973	7073
13	10667	10867
15	12237	13237
26	21687	22687
27	22669	23669
38	33347	33247
39	K 34389	34289
40	K 54571	34571
40	34093	35093
43	37517	37417
43	K 39547	37547
43	37899	37799
44	38139	38239
47	K 41193	41093
48	K 42953	42053
50	42507	43507
51	K 45641	45041
54	56939	46939
55	48627	48629
56	K 38793	48793

GOLDBERG 1—*continued*

page	for	read	page	for	read	page	for	read
60	53313	52313	159	K110111	140111	225	K188973	198973
60	K 32861	52861	162	K443269	143269	227	200834	200831
67	K 58159	59159	166	146437	146537	228	K211583	201583
70	91321	61321	167	K147979	147079	229	251893	201893
75	65599	65699	168	K147959	147859	231	253539	203539
75	K 63813	65813	169	148781	148789	233	K235933	205933
76	K 69529	66529	170	K449797	149797	234	206638	206639
76	K 69553	66553	172	131409	151409	234	207001	207007
76	69883	66883	176	K455357	155357	235	907463	207463
77	K 67751	67651	178	156691	156697	236	207943	207947
78	K 69401	68401	182	K166559	160559	236	298073	208073
80	70627	70727	186	K463763	163763	236	K298661	208661
81	71197	71297	187	164776	164779	237	209329	209323
83	12889	72889	189	166872	166873	237	K200341	209341
83	73919	73019	191	169703	168703	238	K210263	210269
86	K 65371	75371	192	166813	169813	238	K510503	210503
87	76051	76951	193	K176407	170407	239	K240733	210733
91	79729	79829	194	171084	171083	239	110767	210767
93	82481	82181	194	K151587	171587	240	411673	211673
94	92733	82733	195	472121	172121	240	211781	211771
96	K 34757	84757	196	172754	172751	240	211791	211781
97	K 35183	85183	196	172929	172927	241	312407	212407
98	K 68333	86333	197	K473581	173581	242	212914	213913
100	K 67653	87653	198	K177877	174877	244	215301	215303
108	95079	95077	201	K777527	177527	245	246733	216733
108	95329	95429	202	K478687	178687	247	517811	217811
109	93917	95917	204	K179141	179641	253	223278	223273
109	96123	96023	204	480377	180377	255	225470	225479
114	K199409	100409	206	191511	181511	256	325863	225863
115	K106967	100967	207	K162539	182539	256	225597	225997
118	204027	104027	207	K162711	182711	258	227668	227669
118	104287	104387	207	182848	182849	259	223329	228329
121	106181	106183	208	188407	183407	259	228403	228409
123	408127	108127	209	134673	184673	259	258673	228673
123	K103373	108373	210	195213	185213	260	K329531	229531
123	K408521	108521	210	175431	185431	261	230928	230923
124	409241	109241	210	485471	185471	262	231311	231317
126	119857	110857	211	155837	185837	262	K281361	231361
133	117438	117433	211	168373	186373	262	221467	231467
134	418367	118367	211	86697	186697	262	321793	231793
139	112419	122419	212	187138	187139	263	K531863	231863
145	138159	128159	212	157157	187157	263	232250	232259
145	K138161	128161	215	189364	189367	264	332739	232739
146	K138419	128419	218	792511	192511	264	332801	232801
153	K434819	134819	218	192760	192769	264	238877	232877
154	125409	135409	219	198681	193681	264	232801	232901
154	185673	135673	220	191339	194339	264	333077	233077
154	125809	135809	222	106213	196213	265	293741	233741
154	126241	136241	223	K197993	196993	265	K234043	234049
156	K127461	137461	223	196017	197017	265	294091	234091
157	198541	138541	223	191129	197129	266	233067	235067
157	188871	138871	224	147881	197881	267	236112	236113
158	439001	139001	225	198434	198439	267	K336221	236221
159	K138849	139849	225	188791	198791	268	336507	236507

page	for	read
268	K337031	237031
269	337671	237671
270	238061	238081
271	238971	238981
273	240820	240829
273	241009	241007
273	241274	241271
273	241468	241469
274	241840	241849
275	245143	243143
277	244241	244249
277	K241811	244811
278	K345381	245381
278	245497	245407
278	245407	245497
279	246917	246017
280	347043	247043
283	249917	249919
284	K280789	250789
284	250937	250931
285	K221387	251387

number	corrected factors
5951	11·541
8891	17·523
9571	17·563
9937	19·523
11429	11·1039
13559	7·13·149
17651	19·929
18361	7·43·61
19907	17·1171
20009	11·17·107
22919	13·41·43
23047	19·1213
23441	11·2131
24173	23·1051
27347	23·29·41
28259	7·11·367
29597	17·1741
29971	17·41·43
32477	47·691
37631	11^2·311
48719	11·43·103
50813	7^2·17·61
51209	41·1249
52693	23·29·79
53011	7·7573
53021	37·1433
53041	29·31·59
53071	73·727
53293	137·389
53731	prime
53761	37·1453
55033	11·5003
58729	11·19·281
K 58993	11·31·173
59807	11·5437
61807	19·3253
63631	17·19·197
64199	43·1493
64277	17·19·199
K 65639	7·9377
71623	67·1069
K 74191	13^2·439
74461	19·3919
76729	277^2
77371	7^2·1579
K 79679	17·43·109
80357	107·751
81617	17·4801
84797	19·4463
86483	197·439
89987	29^2·107
90419	7·12917
K 90721	257·353
K 91877	79·1163
93547	139·673
94001	23·61·67
94831	11·37·233
94987	43·47^2[43·47 in some copies]
95567	227·421
96301	23·53·79
96631	71·1361
96883	17·41·139
96937	31·53·59
K 97481	43·2267
K 98099	263·373
K 99769	19·59·89
99997	19^2·277
101857	7·14551
102283	29·3527
104303	37·2819
105473	29·3637
K105919	11·9629
K105961	17·23·271
108619	7·59·263
108809	53·2053
109939	17·29·223
111523	229·487
113627	37^2·83
K114733	17^2·397
114959	13·37·239
116093	17·6829
118559	7·16937
118859	13·41·223
119287	7·17041
119669	11^2·23·43
119843	37·41·79
121033	11·11003
122483	53·2311
123763	23·5381
124763	17·41·179
K127801	227·563
128527	7^2·43·61
128851	269·479
133429	29·43·107
134057	7·11·1741
138379	71·1949
K138761	7·43·461
K139621	17·43·191
K139829	67·2087
140519	83·1693
141187	59·2393
142769	11·12979
143311	7·59·347
144859	11·13·1013
K145597	19·79·97
146189	29·71^2
146591	17·8623
147017	13·43·263
147389	11·13399
147581	7·29·727
148613	353·421
149593	227·659
149603	prime
152573	271·563
K154447	41·3767
154813	23·53·127
155011	379·409
155489	61·2549
156263	307·509
157117	59·2663
157453	19·8287
157663	11^2·1303
158273	163·971
158503	31·5113
158899	13·17·719
160261	43·3727
160283	29·5527
160693	13·47·263
161299	23·7013
162667	47·3461
165997	13·113^2

GOLDBERG 1—*continued*

number	corrected factors
166249	$83 \cdot 2003$
166573	$11 \cdot 19 \cdot 797$
169907	$131 \cdot 1297$
170951	$11 \cdot 15541$
172231	$29 \cdot 5939$
172339	$23 \cdot 59 \cdot 127$
172891	$23 \cdot 7517$
174247	$163 \cdot 1069$
174643	$7 \cdot 61 \cdot 409$
176879	$73 \cdot 2423$
K177467	prime
K179183	$59 \cdot 3037$
179467	$197 \cdot 911$
179597	$11 \cdot 29 \cdot 563$
179711	$7 \cdot 25673$
181561	$47 \cdot 3863$
182117	$13 \cdot 14009$
182177	prime
182399	$7 \cdot 71 \cdot 367$
182527	$349 \cdot 523$
184423	$311 \cdot 593$
184937	$173 \cdot 1069$
186083	$53 \cdot 3511$
186313	$211 \cdot 883$
186517	$37 \cdot 71^2$
187537	$7 \cdot 73 \cdot 367$
187829	$31 \cdot 73 \cdot 83$
191423	$107 \cdot 1789$
191839	$41 \cdot 4679$
192203	$11 \cdot 101 \cdot 173$
192449	$223 \cdot 863$
193781	$7 \cdot 19 \cdot 31 \cdot 47$
195151	$11 \cdot 113 \cdot 157$
K195671	$7 \cdot 27953$
196301	$7 \cdot 29 \cdot 967$
196411	$59 \cdot 3329$
197041	$13 \cdot 23 \cdot 659$
197501	$23 \cdot 31 \cdot 277$
198053	$23 \cdot 79 \cdot 109$
198401	$7^2 \cdot 4049$
198547	$367 \cdot 541$
198617	$31 \cdot 43 \cdot 149$
198947	$7 \cdot 97 \cdot 293$
200167	$11 \cdot 31 \cdot 587$
203917	$7 \cdot 29131$
204853	$11^2 \cdot 1693$
204901	$17^2 \cdot 709$
207107	$71 \cdot 2917$
207167	$223 \cdot 929$
207413	$211 \cdot 983$
207557	$7 \cdot 149 \cdot 199$
K208349	$89 \cdot 2341$
210217	$7 \cdot 59 \cdot 509$
211459	$103 \cdot 2053$
K213251	$107 \cdot 1993$
213793	$439 \cdot 487$
213871	$7 \cdot 30553$
215101	$17 \cdot 12653$
215171	$11 \cdot 31 \cdot 631$
215441	$17 \cdot 19 \cdot 23 \cdot 29$
K215729	$31 \cdot 6959$
216581	$19 \cdot 11399$
216737	$73 \cdot 2969$
217039	$17^2 \cdot 751$
217897	$193 \cdot 1129$
219209	$223 \cdot 983$
219379	$431 \cdot 509$
219859	$43 \cdot 5113$
220087	$7 \cdot 23 \cdot 1367$
220439	$17 \cdot 12967$
220993	$223 \cdot 991$
221029	$83 \cdot 2663$
K223109	$47^2 \cdot 101$
223459	$19^2 \cdot 619$
K224647	$277 \cdot 811$
224719	$11 \cdot 31 \cdot 659$
224729	prime
K225121	$13 \cdot 17317$
225877	$107 \cdot 2111$
K225899	$223 \cdot 1013$
225901	$13 \cdot 17377$
226249	$61 \cdot 3709$
226279	$41 \cdot 5519$
227689	$7 \cdot 11 \cdot 2957$
228967	$101 \cdot 2267$
229471	$11 \cdot 23 \cdot 907$
229537	$7 \cdot 11^2 \cdot 271$
229579	$7 \cdot 32797$
229907	$149 \cdot 1543$
230261	$19 \cdot 12119$
231601	$31^2 \cdot 241$
232427	$13 \cdot 19 \cdot 941$
K233263	$19 \cdot 12277$
233927	$223 \cdot 1049$
235093	$17 \cdot 13829$
235801	$37 \cdot 6373$
236099	$229 \cdot 1031$
236281	$277 \cdot 853$
K237949	$17 \cdot 13997$
238271	$11 \cdot 21661$
239603	$7 \cdot 13 \cdot 2633$
K240329	$17 \cdot 67 \cdot 211$
241399	$283 \cdot 853$
242611	$19 \cdot 113^2$
242791	$97 \cdot 2503$
K245743	$397 \cdot 619$
246863	$43 \cdot 5741$
247019	$19 \cdot 13001$
247109	$29 \cdot 8521$
247751	$7 \cdot 35393$
247979	$17 \cdot 29 \cdot 503$
248029	$97 \cdot 2557$
249241	$47 \cdot 5303$
251587	$7 \cdot 127 \cdot 283$
K251593	$43 \cdot 5851$

(Practically all corrections in this list were given in Dr. JIŘÍ KAVÁN's MS. list, but those without a "K" were first given, 1904–05, in CUNNINGHAM 41, KAVÁN added 94 new corrections. Mr. H. J. WOODALL has pointed out that $54131 = 7 \cdot 11 \cdot 19 \cdot 37$; the broken type for the first factor makes it uncertain.)

GOUWENS 1.

$p = 97$, *in* Y *for* 446 *read* 466.

GRAVE 1.

p	for coefficient of	read
59	y^5	+35
59	z^{14}	− 1
67	z^6	+ 4
71	z^5	− 5
79	y^5	+69

GRAVE 2.

p	for coefficient of	read
113	y^{18}	353
157	y^{10}	1084
197	y^5	353

GRAVE 3, [d_1].

page 377, $p=131$, *insert* 57
page 380, $p=149$, *insert* 32, *delete* 35

3, [e_1].

page 330, $n=9899$ *for* — *read* 19

3, [q_2].

p. 21–22

A	for	read
1230	56	55
1232	27	29
1234	26	25
1236	41	42
1240	36	34
1242	42	44
1244	23	22

A	for	read
1252	24	23
1254	50	51
1272	41	40
1274	25	26
1396	25	24
1398	44	45

HALSTED 1.

page 149, *for* 330, 644, 725, 107226 *read* 333, 644, 725, 107226; also change order of entry.
page 149, *for* area 863550 *read* 934800

(MARTIN 2, p. 309,321)

page 167, *for* 21, 61, 65, 420 *read* 14, 61, 65, 420

HARDY and RAMANUJAN 1_1, 1_2.

Table I. log $\omega_{3,16}/\pi i$. *for* $-27/32$ *read* $5/32$
log $\omega_{13,16}/\pi i$. *for* $27/32$ *read* $-5/32$
Table II. In A_{15} *for* $-\pi/90$ *read* $89\pi/90$
In A_{16} *for* $+27\pi/32$ *read* $-5\pi/32$
for $A_{11}(n)=0$ ($n\equiv 1, 2, 3, 5, 7 \pmod{11}$) *read* $A_{11}(n)=0$ ($n\equiv 1, 2, 3, 5, 8 \pmod{11}$)
for $A_{16}(n)=0$ ($n\equiv 0 \pmod 2$) *read* $A_{16}(n)$ never vanishes
for $A_{15}(n)$ never vanishes *read* $A_{15}(n)=0$ ($n\equiv 1, 2 \pmod 5$).

(D. H. LEHMER 5, p. 118)

HAUSSNER 1.

n	for	read	omit	insert	for s =	read
670	...	...	103	281	...	...
1014	...	...	171	47	...	...
1026	...	...	...	433	41	42
1038	...	...	...	131, 337	38	40
1040	413	313	...	...	...	..
1060	506	503	...	...	...	...
1106	83	97	...	...	...	...
1108	103	131	...	47	24	25
1126	19	17	...	...	...	...
1136	...	...	393	...	24	23
1146	...	...	433	...	39	38
1164	587	577	...	...	...	...
1170	89	83	...	...	...	...
1184	...	...	...	193, 277	18	20
1186	...	...	...	593	19	20
1232	...	...	157	...	30	29
1244	...	...	...	613	22	23
1284	...	...	...	47	46	47
1380	...	...	...	499	60	61
1454	...	...	...	601	26	27
1568	...	...	...	97	25	26
1584	...	...	...	151	58	59
1606	...	...	...	5	29	30
1664	53	43	...	...	...	...
1690	...	...	...	137	36	37
1696	...	...	...	...	27	28
1722	691	631	...	...	...	...
1726	903	503	...	...	...	...
1790	...	...	...	181	36	37
1808	...	...	41	...	29	28
1818	...	...	...	41	52	53
1824	...	...	...	887	58	59
1840	...	...	...	227	36	37
1842	233	223	227	...	55	54
2020	829	929	...	...	...	...
2026	171	179	...	...	...	...
2050	147	149	...	...	...	...
2102	...	...	227	...	32	31
2104	...	...	...	227	34	35
2136	489	389	...	...	...	...
2142	...	...	...	433	81	82
2228	...	...	...	67	27	28
2238	...	...	67	...	60	59
2262	...	...	...	1061, 1069, 1091	72	75
2304	1097	1091	...	...	...	...
2402	233	223	...	...	...	...
2404	...	...	17	...	37	36
2406	...	...	...	17	71	72
2442	...	...	...	53	75	76
2444	233	223	...	...	...	...
2446	...	...	1193	...	41	40
2448	...	...	...	337	73	74
2470	1071	1061	...	...	...	...
2472	1192	1193	...	...	...	...

n	for	read	omit	insert	for ν =	read
2508	...	...	229	...	75	74
2510	233	223	...	229	44	45
2530	...	...	1213	...	56	55
2532	...	...	...	1061, 1093, 1213	68	71
2584	1123	1223	...	...	...	...
2598	...	...	...	5	70	71
2606	...	...	157	...	36	35
2616	...	...	...	157	71	72
2630	...	...	...	487	45	46
2636	...	...	313	...	35	34
2646	...	...	...	313	80	81
2654	...	...	733	...	36	35
2656	...	...	733	...	42	41
2664	...	...	...	733	72	73
2666	...	...	...	733	36	37
2674	...	...	571	...	49	48
2684	...	...	...	571	42	43
2688	...	...	571	...	90	89
2692	151	251	...	...	...	...
2698	...	...	571	...	43	42
2802	...	...	...	11	72	73
2804	...	...	1217	...	36	35
2808	...	...	139	...	91	90
2810	...	...	...	139	50	51
2814	...	...	...	1217	96	97
2856	2956	2856	...	...	...	...
2870	...	...	...	73	63	64
2900	...	...	1427	...	52	51

(Table II)

n	for	read	n	for	read	n	for	read
1026	41	42	1842	55	54	2654	36	35
1038	38	40	2102	32	31	2656	42	41
1108	24	25	2104	34	35	2664	72	73
1136	24	23	2142	81	82	2666	36	37
1146	39	38	2228	27	28	2674	49	48
1184	18	20	2238	60	59	2684	42	43
1186	19	20	2262	72	75	2688	90	89
1232	30	29	2404	37	36	2698	43	42
1244	22	23	2406	71	72	2802	72	73
1284	46	47	2442	75	76	2804	36	35
1380	60	61	2446	41	40	2808	91	90
1454	26	27	2448	73	74	2810	50	51
1568	25	26	2508	75	74	2814	96	97
1584	58	59	2510	44	45	2870	63	64
1606	29	30	2530	56	55	2900	52	51
1690	36	37	2532	68	71	3036	91	92
1696	27	28	2598	70	71	3038	45	44
1790	36	37	2606	36	35	3102	93	92
1808	29	28	2616	71	72	3108	101	100
1818	52	53	2630	45	46	3112	47	46
1824	58	59	2636	35	34	3210	110	111
1840	36	37	2646	80	81	3228	84	86

ERRATA

HAUSSNER 1—*continued*

n	for	read
3264	88	89
3268	47	46
3288	77	78
3332	56	55
3352	44	45
3392	45	46
3408	87	86
3418	49	50
3480	122	123
3492	84	85
3528	103	104
3584	57	56
3588	107	108
3594	90	89
3598	57	58
3610	67	66
3612	111	112
3614	54	53
3616	47	48
3646	51	52
3658	46	47
3688	45	46
3710	80	79
3712	48	49
3714	94	93
3724	60	62
3772	55	54
3774	102	103
3804	94	93
3808	65	64
3810	129	130
3814	48	49
3818	45	44
3828	108	109
3840	127	128
3844	51	50
3846	97	98
3852	93	94
3882	97	98
3954	101	100
3958	52	53
4008	106	105

n	for	read
4018	65	66
4056	108	109
4098	101	102
4100	68	67
4104	105	104
4110	138	139
4114	60	61
4144	63	64
4172	61	62
4178	54	53
4188	103	104
4190	65	64
4198	52	53
4200	164	165
4206	104	105
4216	55	54
4222	55	56
4242	122	123
4248	105	106
4258	54	53
4284	133	132
4308	101	100
4310	67	68
4350	143	144
4352	56	55
4374	102	103
4388	49	48
4398	106	107
4422	113	114
4428	110	108
4438	68	70
4444	61	60
4446	124	125
4470	148	147
4474	56	57
4522	76	77
4608	116	117
4618	58	57
4628	60	59
4638	107	108
4640	71	70
4642	64	65
4662	135	136

n	for	read
4664	71	69
4674	122	123
4690	96	95
4692	119	120
4708	66	65
4710	147	148
4718	61	60
4724	58	57
4734	119	120
4736	59	58
4738	56	57
4740	151	154
4746	138	139
4754	61	60
4764	113	114
4782	107	108
4792	61	60
4808	52	51
4814	62	61
4816	73	75
4818	128	129
4854	113	114
4884	125	126
4894	61	60
4900	95	94
4902	121	123
4904	58	59
4908	104	103
4914	149	150
4916	53	54
4918	56	57
4920	152	153
4930	83	84
4942	72	73
4944	121	122
4950	166	167
4956	143	145
4972	67	68
4986	116	117
4990	83	84
4996	62	63
5000	76	77

(PIPPING 1, p. 2–5)

HERTZER 1.

$p=101009$ *read* $q=16$
add $p=106321$, $q=4$, and $p=109873$, $q=7$.

(CUNNINGHAM 40, p. 155)

INGHIRAMI 1_3.

In these tables a prime number is denoted by a dot (.). The following 55 primes (p. 17 is not considered) do not have a dot clearly printed:

1867	3593	5879	10909	14243	12479	17393	21001	29819	35831
36191	41257	41983	46747	47629	42169	43063	43579	44159	47699
55127	56809	57131	56993	61223	63149	64433	61291	70001	69463
69959	72901	75619	77081	79337	82207	82241	82507	83003	83231
78191	84347	90947	91129	95813	91757	92387	92959	93559	94463
95279	97231	98101	99523	98867					

(MRS. JIRÍ KAVÁN)

page	number	for	read
1	4241	1	·
3	8249	−3	73
5	15707	13	113
7	18703	39	59
7	B23641	77	47
8	B20159	18	19
9	B29327	3	·
9	B29427	·	3
11	B30531	13	3
13	B36843	·	3
13	B36943	3	·
15	B42431	15	151
15	B47507	7	·
15	47539	37	137
16	B42583	17	97
16	44579	,	·
16	45383	19	13
19	48457	37	47
19	51377	33	83
19	B53693	3	·
19	B53793	·	3
20			N = 49
20	B55101	·	3
20	B55201	3	·
20	55609	3	·
21	55473	7	3
21	55573	3	7
21	B55793	3	·
21	B55893	·	3
21	55979	·	7
21	B59069	3	·
21	B59463	7	3
21	B59563	3	7
22	B61129	3	·
22	61329	·	3
22	62321	3	7
22	62421	7	3
22	B65703	·	3
22	65723	·	7
22	B65847	7	3
22	65939	223	233
22	B65947	3	7
23	B60471	7	3
23	B60571	3	7
23	60587	42	43

page	number	for	read
23	62087	43	47
24	B72001	39	89
25	66481	18	19
25	68353	19	29
25	68771	3	·
25	68871	·	3
25	B70467	7	3
25	B70567	3	7
26	72209	103	163
26	B74607	·	3
26	B74707	3	·
26	B74907	·	3
26	75009	7	3
26	76221	7	3
26	76321	3	7
26	76701	·	3
26	76801	3	·
26	77819	17	7
26	78041	3	·
27	74351	148	149
27	74367	·	3
27	74773	13	23
27	75471	·	3
27	75571	3	·
27	B77699	3	·
27	B77799	·	3
27	B77999	3	·
28	80837	129	229
28	82739	11	17
28	B84047	3	·
29	83099	11	23
29	83357	7	·
29	B83573	17	7
30	84641	83	53
30	B86849	·	7
31	84157	213	23
31	84653	1	·
31	B85377	7	3
31	B85477	3	7
31	89881	1	11
31	89979	13	3
32	B91839	·	3
33		N = 58	N = 59
33	B90287	117	17
33	91887	7	3

INGHIRAMI 1_3—*continued*

page	number	for	read
33	91987	3	7
33	96089	27	7
34	96109	·	13
34	B96749	3	·
34	B96849	·	3
34	98941	103	163
35	B96173	3	7
35	B96273	7	3
35	B96459	·	3
35	97183	137	157

(The errata marked with "B" were given by T. BARINAGA in *Revista Matem. Hispano-Americana*, v. 3, 1921, p. 27—these and the others (except four by Dr. COMRIE) were found by Dr. JIŘÍ KAVÁN. There were errors in BARINAGA's errata for 55609 and 55709, and the correction for 42583 was not given. Errors on p. 17, duplicating p. 16, not considered.)

JACOBI 2, [d_2].

contains the same errors as BURCKHARDT 1 [d_3]

2, [d_1, d_3].

Numbers

page	p	argument	for	read
61	449	219	374	364
62	457	453	325	320
64	463	134	2	29
82	557	503	427	437
225	243	12	206	208
228	361	131	169	165
234	841	192	233	223
193	929	Col. I	91	90
193	929	Col. I	92	91
193	929	Col. I	93	92
		Value ($p-1$)		
63	461	$p-1$	2^4	2^2
76	523	$p-1$	$3\cdot 87$	$3^2\cdot 29$
77	523	$p-1$	$3\cdot 87$	$3^2\cdot 29$
219	997	$p-1$	2^3	2^2

Indices

page	p	argument	for	read
116	677	35	368	308
139	757	565	168	468
222	25	14	8	6
224	169	33	41	71
228	361	122	43	93
228	361	216	87	78
228	361	353	144	174
232	729	196	204	304
234	841	353	394	694
237	961	Col. N	61	61
			61	62
Cancel this correction in Jacobi's corrigenda				
245	571	109	190	109

(CUNNINGHAM 42, p. 59, and VANDIVER)

JACOBI 3_1, 3_2, partly corrected in 3_3, [j_2].

Table I of (a, b)

read $p=$ 2357; 3253; 3469; 3529; 5693;
instead of 2457; 2253; 3459; 2529; 5093;

Table II of (A, B)

read 3631; 6427; *instead of* 2631; 6433;

Omissions, Table I

p	a	b
197	1	14
2713	3	52
6997	39	74
11173	97	42

Omissions, Table II

p	A	B
883	4	17
6427	80	3
11311	106	5

Corrigenda of a, b

p	a	b
5261	19	70
8609	47	80

Corrigenda of A, B

p	A	B
6481	41	40

(CUNNINGHAM 41, p. 132–133)

KAVÁN 1_1, 1_2.

page 32, $N=15280$ *for* $2^4\cdot 3\cdot 191$ *read* $2^4\cdot 5\cdot 191$
page 39, argument left hand column, *for* 1800 *read* 1870

(J. C. P. MILLER)

KRAITCHIK 2, [o].

page 6, $n=29$ *for* X *read* 1, 15, 33, 13, 15, · · · , 15, 13, 33, 15, 1

KRAITCHIK 3, [d_3].

page 214, $N=199$, *for* $\rho=197$ *read* 127
page 215, $N=293$, col. 37, *for* 23 *read* 230.

3, [i_1].

pages 188–189

N	for $x=$	read $x=$
73	167	157
89	191	91
233	21	11
265	233	253
385	243	193
489	21	11
601	49	69
641	193	191
745	21	11
841	167	157
1001	21	11

page	p	N	for	read
193	17	9	$t=3$	$t=6$
195	31	5	$a=8$	$a=11$
195	31	5	$t=11$	$t=8$
199	47	6	$t=10$	$t=11$
199	47	34	$t=19$	$t=22$

[i_3].

D	for	read
+38	59	53
−38	116	117
−42	55	53
−42	159	157
+69	55	53
−86	89	87
−102	147	145
−103	67	79
−103	177	179
−105	57	67
−106	73	71
−107	191	193
−109	333	103
−110	39	49
−110	207	217
−113	397	171
+122	195	199
−138	163	169
−141	413	415
+146	77	119
−146	77	303
−149	367	365
+151	183	189
+157	107	109
−157	471	529
+165	112	113
−166	473	477
−173	655	309
+174	203	61
−181	359	357
−181	491	461
−181	719	721
−185	661	253
+190	119	197
+191	173	175
+191	271	275
+193	155	129
−193	541	155
−193	617	231
+194	41	47
−194	453	455
−197	191	199
+199	309	257
−199	309	257
−199	insert	371

KRAITCHIK 3 [i_3]—*continued*

Correct Tables for $D = \pm 182$

$D = +182$

$728n \pm$	1	9	15	19	25	33
	37	41	43	51	55	59
	61	69	71	73	81	83
	85	87	89	93	97	101
	103	107	109	113	115	121
	135	141	145	149	151	155
	157	159	171	173	179	181
	187	197	199	201	211	225
	227	233	235	237	239	241
	253	265	269	285	289	297
	307	311	317	319	333	335
	337	341	347	353	359	361

$D = -182$

$728n +$	1	3	9	11	23	25
	27	31	33	37	41	47
	61	67	69	73	75	79
	81	85	89	93	95	97
	99	101	109	111	113	121
	123	127	131	139	141	145
	149	157	163	167	173	181
	183	191	197	201	207	215
	219	223	225	233	237	241
	243	251	253	255	263	265
	267	269	271	275	279	283
	285	289	291	295	297	303
	317	323	327	331	333	337
	339	341	353	355	361	363
	369	379	381	383	393	407
	409	415	417	419	421	423
	435	447	451	467	471	479
	489	493	499	501	515	517
	519	523	529	535	541	543
	549	551	557	563	569	573
	575	577	591	593	599	603
	613	621	625	641	645	657
	669	671	673	675	677	683
	685	699	709	711	713	723

(D. H. LEHMER, Am. Math. So., *Bull.* v. 35, p. 866–867)

KRAITCHIK 4, [d_1].

pages 55–58, 61

art.	*p*	delete	insert
129	3	2003	5347
129	3	2383	7867
129	3	5153	9043
129	3		9413
129	3		9967
130	5	1753	2083
130	5	5167	2383
130	5	5347	5153
130	5	6793	
130	5	7867	
130	5	9043	
130	5	9413	
130	5	9967	
131	6	6337	5167
132	7	8089	1753
132	7		2593
133	10	2593	6337
133	10		6793
133	11		1511
133	11		8231
133	12		7841
133	13	7841	
133	13	8231	
133	17	1559	8089
133	17		8191
133	19		1559
133	19		5711
133	23	1511	
133	29	5711	
133	29	8191	

4, [d_1]—*continued*

art.	primes misprinted for	read
128	3213	3203
129	6251	6151
129	6877	6977
135	2093	2099
135	8763	8663

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, 1924, p. 181)

4, [d_1].

pages 131–145

p	for *ρ*	read *ρ*	*p*	for *ρ*	read *ρ*	*p*	for *ρ*	read *ρ*
9257	2	3	16633	5	15	24181	6	17
10369	11	13	16921	13	17	25261	6	7
10487	2	−2	16927	3	6	25309	15	13
10631	2	−2	17209	7	14	25321	11	19
10639	2	−2	17293	6	7	25759	10	−10
11251	7	13	17401	7	11	26083	3	7
11491	−7	7	18049	7	13	26161	7	13
12007	3	13	18121	7	23	26317	35	6
12703	−3	3	18233	5	3	26431	−10	3
12973	6	14	18307	7	11	26641	2	7
13841	3	6	18397	5	6	26681	3	6
14281	13	19	19081	7	17	26701	6	22
14407	7	19	19477	5	6	27031	−5	6
14449	11	22	19843	−7	19	27109	30	7
15277	5	6	20011	3	12	27241	13	17
15601	7	23	21283	3	11	27281	3	6
15679	−7	11	21787	−7	23	27409	11	13
16061	7	12	22279	2	3	27427	10	−10
16111	−5	7	23609	3	6	27457	5	7
16249	11	17	24007	7	17			

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, 1924, p. 185)

4, [d_2].

pages 63–65

art.	*p*	for *x*	read *x*	for *p*	read *p*	for mod	read mod
138	577	107	105	...	...	...	...
138	797	569	563	...	...	...	...
139	...	...	...	457	449	...	...
139	...	...	...	449	457	...	...
140	...	blank	3	blank	3	blank	2
140	569	...	...	...	...	568	284
140	769	241	141	...	...	...	...
140	...	...	...	893	883	892	882

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, 1924, p. 183)

4, [d_2]—*continued*

pages 131–145

γ of base 2

p	for	read	p	for	read	p	for	read
947	2	1	34519	1	6	65543	1	2
1609	4	8	34543	1	2	68099	2	1
9257	1	2	34897	2	16	71503	1	2
10487	1	2	35671	1	2	74143	1	2
10631	1	2	36847	1	2	74729	4	8
10639	1	2	36929	1	2	77041	1	2
18451	15	25	37529	1	2	78259	2	1
18859	2	1	42187	1	3	80239	1	2
22279	1	2	49033	1	2	90019	6	3
24943	1	2	50951	1	2	93871	7	70
26641	1	2	53609	1	2	97849	1	2
27551	10	50	58679	1	2	98543	1	2
29671	1	2	61057	1	2	99839	1	2
31649	16	32	61631	1	2	99871	1	2
31849	1	2	63671	2	10	250867	2	1
						255071	1	2

γ' of base 10

p	for	read	p	for	read	p	for	read
797	2	4	21739	2	3	25667	1	2
15601	4	40	22343	2	1	25759	1	2

(CUNNINGHAM AND WOODALL, *Messenger Math.*, v. 54, 1924, p. 184)

4, [d_3].

page 219
$N=293$, col. 37, *for* 23 *read* 230
$N=509$, col. 47, *for* 270 *read* 207.

4, [d_5].

pages 59–64

art.	base	n	for	read
134	2	2	1999	1993
134	2	2	3773	3793
134	2	2	blank	4583
134	2	2	5279	omit
134	2	3	7669	7699
134	2	3	9723	9739
134	2	6	1993	1999
134	2	14	6957	6959
134	2	17	1427	1429
134	2	56	6557	6553
136	10	2	7273	7243
137	10	6	7551	7351
137	10	6	7573	omit
137	10	12	blank	7573
137	10	76	4673	4637

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, 1924, p. 182)

4, [e_2].

page	n	$2n+1$	for	read
20	163	...	160287	150287
24	163	...	160287	150287
24	177	...	174081	184081
24	253	...	85009	blank
25	...	177	12097	12037

4, [f_1].

page 10, art. 23, *for* 961 *read* 963

4, [f_1].

pages 131–191

for p	read p	for p	read p	for p	read p	for p	read p
17623	17923 C	116537	116437 K	179489	179989 K	234381	234383
27289	27299 C	118047	118043	183253	183259	234899	234893
65331	65831 C	126069	126079	192111	192611 K	240171	240173
68097	68099 C	136643	136649	192669	192667	258723	258733
69041	69941 C	138153	138157	194747	194749	262101	262103
74141	74143 C	147797	147793	201213	201233	262251	262253
78257	78259 C	150167	150169	204527	204557	263757	263759
80241	80251 C	153429	153929 K	204713	204719	274343	274349
92957	92857 C	169443	169343 K	205011	205111 K	280551	280561
100557	100559	171837	171937 K	209577	209579	281133	281153
103383	103387	172083	172093	210961	210967 K	283511	283501 K
104797	104707 K	174479	174469	211019	211039	284687	284689
106183	106181	174973	174673 K	211613	211619	286559	286589
106263	106273	176387	176389	215151	215153	290969	290999
106657	106957 K	176679	176699	221901	221909	292891	292841
113443	113453	179063	179083	224949	224947	295557	295553
				227847	227947		

(CUNNINGHAM and WOODALL, *Messenger Math.*, v. 54, 1924, p. 184, and KRAITCHIK 7, p. 182)

4, [f_2].

page 15, art. 30, *interchange* entries 2115 and 2414.
page 11, *insert* 1736, 2646, 2960.

Table II

for P	read
128441	125441
414259	414209
498629	498689
938353	932353
	insert $P = 3911681$

KRAITCHIK 7, p. 182)

4, [i_3].

D	for	read
+211	287	289
−217	319	317
−218	533	535
+222	99	95
−222	483	485
−226	375	373
−226	385	395
−226	387	397
+227	241	261
−229	197	199
+230	23	33
−233	915	925
−241	607	357
−241	697	693
−241	731	733
−246	387	389
−247	105	449
−249	197	695
−249	301	799

4, [j_1].

page 49, $A=61$, $D=4$, *for* −39, 4 *read* −39, 5.
page 50, $A=76$, $D=1$, *for* 57769 *read* 57799 (S. A. Joffe).

4, [j_2].

page	p	for a	read
192	15361	15	30
193	890881	234	179
193	918529	115	215
197	*insert* 3911681		385

(KRAITCHIK 7, p. 182)

4, [o].

page 88, $n=29$, *for* X *read* 1, 15, 33, 13, 15, · · · , 15, 13, 33, 15, 1.

KRAITCHIK 6, [d_2].

page 233, *add* entry $k=115$, $n=20$, $j=4$

6, [e_2].

page 224, $k=115$, $n=20$, *for* 379 *read* prime

6, [i_1].

page 159, $p=59$, $n=23$, *for* $x=14$ *read* $x=15$
page 159, $p=59$, $n=44$, *for* $x=14$ *read* $x=17$

6, [j_3].

page 242, line 1, column 4, *for* $z=541$ *read* $z=841$

6, [m].

A	read
19	2, 1, 3
45	1, 2, 2
296	4, 1, 7
498	3, 6, 22
514	1, 2, 22
590	3, 2, 4
649	2, 9, 1, 2, 3, 1, 1, 2, 1, 4, 1, 16, 6, 3, 4 (29 termes)
700	2, 5, 2, 1, 1, 1, 1, 12 (15 termes)
725	1, 12, 2

6, [m].—*continued*

A	read
813	1, 1, 18
994	1, 1, 8
539	4, 1, 1, 1
808	2, 2, 1, 5, 1, 1, 1, 1, 13 (17 termes)
814	1, 1, 7, 1, 1, 1, 5, 18, 1, 5, 2, 1, 1, 4 (27 termes)
927	2, 4, 5, 3
939	1, 1, 1, 4, 20 (9 termes)
116	1, 3, 2, 1, 4
369	4, 1, 3, 2, 7, 4 (11 termes)
415	2, 1, 2, 4, 6, 1, 1, 3
999	1, 1, 1, 1, 5, 6, 1, 5, 2 (17 termes)

(KRAITCHIK 7, p. 182–183)

KRAITCHIK 7, [b_2].

page 153, column $(p^4+1)/2$, $p=79$, *for* 233 *read* 433

7, [e_2].

page	line	column	for	read
84	$n=67$		19370721	193707721
86, 87	94, 114, 150		*interchange* primitive factors	
88	$n=56$		3153	5153
88	$n=120$		1851 · · · 521	394783681 · 46908728641
95	$n=41$		*delete* entry	
96	$a=26$	$\frac{a^5-1}{a-1}$	2641	8641
97	$a=18$	last	61	601
97	$a=24$	$\frac{a^{11}-1}{a-1}$	13467047	134367047
97	$a=42$	$\frac{a^{11}-1}{a-1}$	5942675703	5942675707
97	$a=44$	$\frac{a^9-1}{a^3-1}$	13	19
97	$a=61$	$\frac{a^9-1}{a^3-1}$	603870199	903870199
98	$a=52$		152987077	152787077
99	$a=75$	first	10922367593	109 · 22367593
99	$a=85$	first	193	163
99	$a=40$		338839937	7879999
100	$a=23$	$\frac{a^9+1}{a^3+1}$	2711117	271 · 1117
105	$a=58$	$\frac{a^{10}+1}{a^2+1}$	41 · 941	41941
105	$a=68$	$\frac{a^5+1}{a^2+1}$	106177	196117
106	$a=19$		537	5237
106	$a=19$		35533211573	35533 · 211573
127	60	8	106117	196117
137	$x=79$	N		$x=81$
140	$x=19$	M	537	5237
143	$a=10$	M	341 · 334661	541 · 534661
144	$y=11$		*insert* 51329	

7, [e_2].—*continued*

page	line	column	for	read
145	$a=6$		207544361	20754361
146	$x=40$	N	338839937	7879999
147	$x=12$	M	1377	2377
149	middle of page		$x=1, 2, 3$	$a=1, 2, 3$
149	$a=1$		99151	991651
153	$p=79$	first	233	433

(BEEGER, *Nieuw Archief v. Wiskunde*, s. 2, v. 16, no. 4, 1930, p. 42;)

7, [o].

page 2, $n=41$, *for* $Y=1, 1, 1, 4, \cdots$ *read* $1, 1, 2, 4, \cdots$
page 3, $n=97$, *for* $Y=1, 1, 5, 9, 17, 30, 40, 69, \cdots$ *read* $1, 1, 5, 9, 17, 30, 44, 69, \cdots$

KRAITCHIK 9.

no.	for	read
38	760765 · · ·	760965 · · ·
52	549767 · · ·	549797 · · ·
71	160242 · · ·	166242 · · ·

(BEEGER, *Mathematica*, Cluj, v. 8, 1934, p. 212)

LEGENDRE 1_1, [i_3].

form	for	read
t^2-29u^2	3	7
t^2-38u^2	23	21
	129	131
t^2-61u^2	see below	
t^2-62u^2	103	107
t^2-77u^2	53	137
	255	171

form	for	read
t^2+77u^2	89	61
	113	101
	149	153
	257	237
t^2+91u^2	7	115
t^2+101u^2	305	309
	313	317
	321	325
	329	333

t^2-61u^2 *read* $122n\pm 1, 3, 5, 9, 13, 15, 19, 25, 27, 39, 41, 45, 47, 49, 57.$

(D. N. LEHMER, Am. Math. So., *Bull.*, v. 8, 1902, p. 401–402)

1_1, [j_1].

N	read
133	$x=2588599$
214	$x=695359189925$
	$y=47533775646$
236	$x=561799$
301	$y=339113108232$
307	$x=88529282$
331	$x=2785589801443970$
343	$x=130576328$
	$y=7050459$
344	$y=561$
355	$y=50676$
365	$x=3458$
397	$x=20478302982$
	$y=1027776565$

N	read
718	$x=8933399183036079503$
722	$x=22619537$
	$y=841812$
753	$y=11243313484$
771	$x=2989136930$
	$y=107651137$
801	$x=500002000001$
	$y=17666702000$
806	$x=6166395$
809	$x=433852026040$
	$y=15253424933$
833	$x=9478657$
851	$x=8418574$
856	$x=695359189925$

1_1, [j_1].—*continued*

N	read	*N*	read
526	$x=8405609154695293 3775$		$y=23766887823$
	$y=3665019757324295532$	865	$x=348345108$
532	$x=2588599$	871	$x=19442812076$
613	$x=481673579088618$		$y=658794555$
619	$x=517213510553282930$	878	$x=9314703$
	$y=20788566180548739$		$y=314356$
629	$x=7850$	886	$y=2601487964640241948 50378$
655	$x=737709209$	944	$x=561799$
	$y=28824684$	965	$x=14942$
664	$y=66007821$		$y=481$
673	$x=48813455293932$	995	$x=8835999$
694	$x=38782105445014642382885$	1001	$x=1060905$
	$y=1472148590903997672114$		

(D. H. LEHMER 11, p. 548–549)

LEGENDRE 1_2, [i_3].

form	for	read	form	for	read
t^2-29u^2	3	7	t^2+77u^2	89	61
			t^2+77u^2	113	101
t^2-38u^2	23	21	t^2+77u^2	113	117
t^2-38u^2	129	131	t^2+77u^2	119	153
t^2-61u^2	see below		t^2+77u^2	149	159
			t^2+77u^2	257	237
			t^2+78u^2	102	103
t^2-62u^2	103	107	t^2+91u^2	7	115
t^2-73u^2	99	69	t^2+101u^2	305	309
			t^2+101u^2	313	317
t^2-77u^2	53	137	t^2+101u^2	321	325
	255	171	t^2+101u^2	329	333

t^2-61u^2 *read* $122n\pm 1, 3, 5, 9, 13, 15, 19, 25, 27, 39, 41, 45, 47, 49, 57$

(D. N. LEHMER, Am. Math. So., *Bull.*, v. 8, 1902, p. 401–402)

LEGENDRE 1_3, 1_4 [i_3].

form	for	read	form	for	read
t^2-14u^2	51x	56x			
t^2-34u^2	123	127			
t^2-38u^2	23	21	t^2+61u^2		215
t^2-38u^2	129	131			
t^2-51u^2	13	31	t^2+77u^2	119	159
t^2-61u^2	see LEGENDRE 1_2				
			t^2+77u^2	297	237
t^2-62u^2	103	107	t^2+91u^2	7	115
t^2-73u^2	99	69	t^2+101u^2	305	309
			t^2+101u^2	313	317
t^2-77u^2	53	137	t^2+101u^2	321	325
t^2-77u^2	255	171	t^2+101u^2	329	333

(D. N. LEHMER, Am. Math. So., *Bull.*, v. 8, 1902, p. 401–402)

1_3, 1_4 [j_1].

N	read	*N*	read
94	$x=2143295$	667	$y=4147668$
116	$x=9801$	749	$x=1084616384895$
149	$y=9305$	751	$x=7293318466794882424418960$
271	$x=115974983600$	809	$x=433852026040$
308	$x=351$	823	$x=2351704749036440006168$
479	$y=136591$	1001	$x=1060905$
629	$x=7850$		

(D. H. LEHMER 11, p. 550)

D. H. LEHMER 4.

move $n=233, 241$ to next higher classifications.

D. H. LEHMER 5.

for $A_{30}(n)$ *read* $A_{30}(n+5)$

D. N. LEHMER 2.

page	col.	line	for	read
11	13	1	8151	8051
14	30	55	51	47
99	20	heading	224	724

D. N. LEHMER 3_1.

In D. N. LEHMER 3_2 about 1200 errors of this edition have been corrected.

(J. D. ELDER)

LEVÄNEN 2.

$D=-77$, *for* 297 *read* 237

LUCAS 2.

table of	page	col.	line of *n*	for	read
Y, Z	165	*Y*	23	$\cdots -7-2]$	$\cdots -7-4]$
	165	*Z*	11	$[1+3]$	$[1+0]$
	165	*Z*	21	$[1+1+1]$	$[1-1+1]$
	165	*Z*	19	$[1+1-1-2]$	$[1+0-1+1]$
Y_1, Z_1	168	Z_1	7	$[1+1]$	$[1-1]$
	168	Z_1	23	$[\cdots 1+7]$	$[\cdots 1-7]$
	168	Y_1	33	$[\cdots -32-19]$	$[\cdots -32-59]$
	168	Y_1	29	$[1+15+33+15+\cdots$	$[1+15+33+13+15+\cdots$
	168	Y_1	41	$[1+21+57+\cdots$	$[1+21+67+\cdots$
	168	Z_1	41	*Interchange* the lines of $n=41$ and 69	
	168	Z_1	69		

(CUNNINGHAM 42, p. 65)

LUCAS 3.

table of	page	col.	line of *n*	for	read
Y, Z	6	*Y*	22	$+x^5y^5$	$+11x^5y^5$
	6	*Y*	33	$-19x^6y^6+$	$-59x^6y^6-$
	6	*Y*	29	$+15x^{11}y^3$	$+13x^{11}y^3$

(CUNNINGHAM 42, p. 65)

MERRIFIELD 1.

page 10, $n=3$, *for* 17096 · · · , *read* 17476 · · · .

OSTROGRADSKY 1.

numbers				indices			
p	argument	for	read	p	argument	for	read
127	105	107	108	71	16	15	22
	116	31	71		26	22	15
137	108	88	87	83	25	8	80
181	78	94	64	167	57	128	28
193	155	173	174	173	57	72	92
				181	16	165	172
					26	172	165

(JACOBI 2, p. 243)

PAGLIERO 1.

delete 100 004 539

(BEEGER)

POLETTI 2, [e₁].

number	for	read	number	for	read
667	23·39	23·29	26243	7·23163	7·23·163
1771	7·11·13	7·11·23	26527	41·467	41·647
2563	11·223	11·233	29729	7·13·137	7·31·137
5239	13^2·21	13^2·31	30667	7·13·137	7·13·337
5243	7^2·207	7^2·107	33943	7·13·173	7·13·373
8483	7·499	17·499	34561	11·19·107	17·19·107
9299	15·547	17·547	34621	83·389	89·389
9401	7·17·19	7·17·79	35329	7^2·103	7^3·103
12299	7^2·51	7^2·251	37939	13·3449	11·3449
13181	7^2·69	7^2·269	42511	7·6063	7·6073
17303	11^2·13	11^3·13	42601	12·29·113	13·29·113
18193	7·23·313	7·23·113	43423	171·251	173·251
18271	11^2·251	11^2·151	44671	11·31·141	11·31·131
19339	82·233	83·233	46699	41·67·17	17·41·67
20293	7·13·123	7·13·223	48739	47·17·61	17·47·61
25009	29·281	89·281	49067	139·343	139·353

2, [f₁].

page	for	read
7	9867	9967
19	44903	44909
31	82863	82963
63	186833	186883
97	100 000 961	100 000 963
97–98	*delete* 10^8+2271, 4291, 4909, 7129, 8709, 8793, 9891, 10011	
98	*insert* 100 010 017	
101	*delete* 10^8+46617, 50307, 55293, 70327, 86809, 94219	
101	*insert* 10^8+2149, 47989, 53053, 94881	

(BEEGER, *Boll. di Mat.* (CONTI), v. 21, 1925, p. lxv–lxvi and S. A. JOFFE)

POULET 2.

page	line	D	for	read
15		2	$2^7·3^4·5$ · · ·	$2^7·3^5·5$ · · ·
68	11 from bottom		(3·5·15299)	(2·5·15299)
70	6		3412776	3212776
70	last correct entry is 290504024 (2^3·17·41·53·983)			
72	5 from bottom, 50th term should be 1635524 result incorrect			

(POULET 3, p. 187–188)

POULET 4_1.

page	line	col.	for	read
77	31	9	831045	831405
78	35	5	976587	976487
	49	10	409	109
79	2	8	4178	4177
81	28	1	953683	953673
	44	3	887421	877421
82	3	3	39016841	39016741
83		1, 2	*insert*	*56052361 631
83	16	5	739073	729073
	16	6	578	577

(POULET 4_2)

POULET 4_2.

page 51, *insert* *56052361 631.

(BEEGER)

RAMANUJAN 1_1.

page 360, *insert* 293 318 625 600

(RAMANUJAN 1_2, p. 339)

REUSCHLE 1, [d_1].

p	read *w* =
3221	10
3251	6
3301	6
3361	22
3739	7
3881	13
4099	2
4231	3
4729	17
4969	11

misprinted primes pages 42–61 for *p*	read
5457	3457
3901	3907
7923	7927
11491 (bis)	11497
12511 (bis)	12541
12801	12809
blank	14731

(WERTHEIM 4, p. 153)

1, [d_2].

pages 42–46

p	base	*e*	*n*
179	7	178	...
193	6	...	2
311	2	155	...
311	3	155	...
311	5	155	...
311	10	155	...
313	2	156	...
367	7	61	6
409	6	17	24
457	7	114	4
463	7	154	3
503	5	502	...
523	2	...	1

p	base	*e*	*n*
523	3	58	9
739	6	369	2
757	7	189	4
821	5	410	...
821	6	410	...
821	7	410	...
919	7	...	1
939	3	369	...
947	3	...	2
997	2	332	...
997	5	332	...
997	6	332	...

1, [d_2]—*continued*

base 2

p	*e*	*n*	*p*	*e*	*n*	*p*	*e*	*n*
1487	743	2	3169	1584		4099	4098	1
1613	52	31	3191	55	58	4139	4138	
1747		1	3221	644	5	4271	305	
2053	2052		3251	650	5	4339	1446	
2161	1080		3259	1086		4391	2195	
2293	2292		3301	660	5	4597	1532	
2473	618		3739	534	7	4663	777	
2677	2676		3881	388	10	4751	475	10
2753	1376		3919	1959		4831	2415	
3079	1539		4051		81	4993	624	

base 10

p	*e*	*n*	*p*	*e*	*n*	*p*	*e*	*n*
1163	581		7129	594		12119	6059	2
2687	2686		7561	1890	4	12149	12148	
3301	3300		7823	7822	1	12289	384	32
3347	1673		7923	not prime		12301	2460	5
3671	367	10	7927	7926	1	12421	12420	
3697	1232		8387	599	14	12637	3159	
3797	949		8521	710	12	12721	2120	
3851	770	5	8681	868	10	12791	6359	
4139	4138		8689	2172	4	12853	459	28
4157	2078		8893	2223		13151	1315	10
4391	2195		8929	144	62	13487	13486	
4397	314	14	9151	1525		13553	1936	7
4637	61	76	9277	4638		13627	6813	
5647	1882		9613	267	36	13687	4562	
5779	5778	1	9661	1380		13697	13696	
6133	1533		10343	10342		13729	3432	
6299	94	67	10433	10432		13757	362	38
6359	3179		10597	5298		14081	1760	8
6373	1062		11047	11046		14221	2844	5
6379	2126		11113	3704		14533	519	28
6421	2140		11173	5586		14551	485	30
6491	1298	5	11423	11422	1	14731	14730	1
6529	1088	6	11491	766	15	14741	14740	1
6581	1316	5	11801	2950	4	14827	2471	6
6761	1690	4	11839	5919		14929	1866	8
6763	161	42	12043	2007	6	14983	4994	
6899	6898		12071	355	34			

1, [e₂].

pages 42–61
Errata occur in factors of $(p-1)$ for $p=$

101	2539	3989	7687	9049	10651	11827	12853
601	2617	4231	7723	9257	10831	11887	12923
937	2777	4397	7927	9277	10903	11933	12959
977	2969	4409	7937	9349	10939	11953	13553
1597	3259	5647	8039	9781	11071	12097	13687
1879	3547	5897	8447	9901	11383	12113	14149
1973	3697	6379	8461	10039	11549	12289	14593
2029	3719	6389	8563	10093	11597	12487	14713
2237	3739	6581	8747	10151	11677	12539	14731
2309	3793	6763	8893	10369	11681	12553	14779
2347	3797	6823	8969	10343	11719	12613	
2503	3877	7669	8971	10427	11813	12757	

(CUNNINGHAM 40, p. 151–153)

1, [j₂].

pages 23–32

corrigenda in p		insert omissions						corrigenda in L, M		
for	read	p	A	B	p	A	B	p	L	M
17136	17137	883	4	17	25453	95	74	139	23	1
25183	25189	11311	106	5	25747	160	7	397	34	4
5579	25579	12553	101	28	27631	166	5	1123	35	11
26459	26479	12739	8	65	32353	175	24	2377	79	11
30763	30703	12967	110	17	33037	65	98	2713	103	3
51051	31051	12973	65	54	34519	38	105	4003	107	13
32553	32353	12979	76	49	35437	65	102	4339	128	6
40659	40759	13477	107	26	37699	68	105	5437	146	4
49277	49279	13537	113	16	39181	191	30	5503	148	2
		19891	104	55	43201	1	120			
		20443	100	59	44563	200	39	omission		
		21499	68	75				883	47	7

corrigenda in A, B

p	A	B	p	A	B	p	A	B
313	11	8	18427	100	53	27691	104	75
5011	56	25	18481	127	28	29059	128	65
5653	19	42	18553	35	76	29179	152	45
8293	91	2	19423	130	29	30529	23	100
8707	92	9	19477	35	78	35257	53	104
9871	38	53	20071	86	65	37363	20	111
10957	47	54	21391	146	5	37507	160	63
12211	56	55	22651	76	75	38449	193	20
12823	106	23	23557	37	86	45307	212	11
16561	127	12	25147	140	43	45361	193	52
18301	7	78	26317	145	42			

pages 26, 31, *omit* the non-primes 6433 and 41197
pages 26, 27, *insert* asterisk after $p=8167$, 8317
pages 29–32, *omit* the primes 16561, 18301, 18481, 23557, 35257, 45307, 45361

1, [j_2]—*continued*

pages 32–41

Primes misprinted—Table IVa, page 34; *for* 3459 *read* 3469

Table IVb, page 40; *for* 29893 *read* 23893

Primes wrongly inserted—Table IVb, pages 39, 40; *omit* 12697, 16981, 19381, 21101 with their a, b as $(10/p)_2 = -1$

Table IVc, page 41; *omit* 16649 with its c, d, as $(10/p)_4 = -1$

Asterisks omitted or superfluous—

Table IVa, b, pages 33–41, *insert* one * after p=733, 2213, 2477, 2677, 2729, 3169, 3373, 6997, 11117, 14293, 14929, 17317, 20357, 21613, 21649, 22277, 23293, 24733

Table IVa, pages 34–38, *insert* two ** after p=2161, 12289

Tables IVa, b, pages 33–40, *omit* the * after p=1213, 2437, 16649, 22093

Table IVa, pages 34–38, *omit* one * after p=2129, 6761, 7561, 8521, 8689, 11801, 12329

Table IVc, page 41, *insert* one * after p=14081, 15601, 15641, 15761, 17489, 17729, 19489, 24809, 24889

Table IVc, page 41, *omit* the * after p=13729, 14321, 15361, 16249, 17209, 17449, 18329, 19289, 20681, 23561

Tables IVa, b, pages 32–41						Table IVa, pages 32–38			Table IVc, page 41		
omissions			corrigenda in a, b			corrigenda in c, d			corrigenda in c, d		
p	a	b	p	a	b	p	c	d	p	c	d
197	1	14	4421	65	14	17	3	2			
11173	97	42	14009	115	28	1777	25	24	14009	69	68
12269	13	110	15361	31	120	4177	55	24	14081	117	14
12301	99	50	16249	43	120	6553	55	42	14369	111	32
12373	103	42	17317	129	26	6653	—	—	14929	121	12
12973	83	78	18289	135	8	7481	57	46	17489	99	62
15493	97	78	19489	105	92	8969	63	50	17729	111	52
16253	37	122	21613	147	2	11057	105	4	19001	123	44
17077	119	54	23197	101	114	11113	49	66	19489	133	30
17117	91	94	23561	131	80	11329	31	72	23929	139	48
17929	125	48	24281	155	16	12049	41	72			
18517	119	66				12097	107	18		omission	
21493	87	118				12161	63	64	22129	77	90
22129	15	148				12281	27	76			

(CUNNINGHAM 41, p. 134–135)

REUSCHLE 3.

page	λ, n	table	p	for	read	auth.
2	5	I	691	$\alpha = +220$	+320	
8	11	I	199	$\alpha^3 = -69$	− 60	
8	11	I	199	$\alpha^{10} = -73$	− 78	
8	11	I	331	$\alpha^4 = +55$	+ 85	
8	11	I	661	$\alpha^3 = -214$	−204	
193	15	I	881	$p = 881$	811	
199	21	I	463	$\omega^{10} = -44$	− 14	
226	39	I	541	$p = 541$	547 (in 3 places)	
239	45	I	631	$\omega^{43} = +71$	+121	CREAK
239	45	I	631	$\omega^{44} = +223$	− 11	CREAK
273	57	I	457	$\omega^{48} = -7$	− 6	
273	57	I	457	$\omega^{52} = +230$	−227	
285	63	I	379	$\omega^{13} = -132$	−112	CREAK
285	63	I	757	$\omega^{13} = +202$	−202	CREAK
285	63	I	631	$\omega^{20} = -26$	− 24	CREAK
285	63	I	757	$\omega^{52} = -203$	−183	CREAK
285	63	I	883	$\omega^{24} = +18$	−355	CREAK

REUSCHLE 3—*continued*

page	λ, n	table	p	for	read	auth.
446	16	I	113	$\omega = -\ 43$	$-\ 48$	
450	32	I	257	$\omega = +\ 85$	$+\ 15$	
461	128	I	641	$\omega^{19} = -275$	-305	
461	128	I	769	$\omega^{20} = -138$	$-\ 38$	
476	24	I	601	$\omega^{11} = -306$	$+295$	
495	40	I	641	$\omega^{19} = +324$	-317	
513	48	I	97	$\omega^{17} = -\ 10$	$-\ 11$	
513	48	I	337	$\omega^{5} = -174$	$+163$	
513	48	I	337	$\omega^{11} = +\ 57$	$+\ 38$	
513	48	I	337	$\omega^{9} = -154$	-153	
533	56	I	673	$\omega^{27} = -\ 83$	$-\ 85$	CREAK
635	96	I	577	$\omega^{28} = -197$	-196	
643	100	I	701	$\omega^{19} = -353$	$+348$	
643	100	I	601	$\omega^{27} = -\ 46$	$-\ 26$	
643	100	I	601	$\omega^{49} = +341$	$+241$	

page	λ, n	table	p	for	read	auth.
3	5	I	601	$5-\alpha^2+2\alpha^4$	$5-\alpha^3+2\alpha^4$	TANNER
3	5	I	751	$12+2\alpha+5\alpha^2+9\alpha^3$	$12+2\alpha+8\alpha^2+9\alpha^3$	TANNER
3	5	I	821	$4+4\alpha-4\alpha^2+3\alpha^3$	$1+4\alpha+4\alpha^2+3\alpha^3$	TANNER
3	5	I	881	$4-5\alpha+5\alpha^4$	$4-5\alpha^2+5\alpha^4$	TANNER
5	7	I	491	$\alpha^3+3\alpha^3$	*Cancel* this entry	BICKMORE, WESTERN
5	7	I	547	$2-\alpha+2\alpha^5+2\alpha^5$	$\alpha^2+3\alpha^6$	BICKMORE, WESTERN
37	29	V	. . .	$6+\eta_1$	$8+\eta_1$	WESTERN
106	43	. . .	. . .	$\lambda=43$ [at top]	$\lambda=67$	
108	67	VI	. . .	Tab. VI.	Tab. VIII.	
108	67	VI	. . .	$p=2, 7, 11, 31$	$p=2, 7, 11, 13, 31$	
176	25	I	401	$f(\alpha)=1-\alpha^3-\alpha^4$	$f(\alpha)=1-\alpha^3+\alpha^4$	WESTERN
187	49	. . .	. . .	$\lambda=89$ [at top]	$\lambda=49$	
249	51	. . .	. . .	[line 4] $107\cdot 409$	$103\cdot 409$	WESTERN
282	57	VI, 1	. . .	$\omega^2+\omega-14=0$	$\omega^2-\omega-14=0$	
487	28	III	. . .	$\omega^4-2\omega^2+4=0$	$\omega^4-3\omega^2+4=0$	
511	44	VI	. . .	Tab. VI. [line 2]	Tab. IV.	
511	44	IV, 1	. . .	$p=40m-5, +7$	$p=44m-5, +7$	
546	56	IV, 4	. . .	$\omega^4-49=0$	$\omega^4+49=0$	
621	68	. . .	. . .	$n=68$ [at top]	$n=88$	
628	76	. . .	. . .	$n=76$ [at top]	$n=88$	

(CUNNINGHAM 42, p. 61–62)

ROBERTS 1.

page 107, $n=1553$, col. $2n+1$, *for* 17 *read* 15
in denominators, *after* 9 *read* (3, 3).
page 108, $n=1777$, col. $2n+1$, *for* 61 *read* 43
in denominators, *after* 27 *read* 2, (1, 1).

(CUNNINGHAM 42, p. 66)

SANG 1.

page 760, add entries

576	943	1105
744	817	1105

SARMA 1.

All entries contain last figure errors.

SCHULZE 1.

for			read			for			read		
6°	43′	58″	6°	43′	59″	38°	21′	28″	38°	21′	29″
12	1	4	12	1	5	50	41	33	50	41	32
12	40	50	12	40	49	55	6	2	55	6	20
13	25	10	13	25	11	61	10	29	61	9	30
15	11	24	15	11	21	61	55	57	61	55	39
16	16	24	16	15	37	64	56	32	64	56	33
17	28	35	17	29	32	68	45	39	68	45	38
17	56	44	17	56	43	70	39	21	70	37	21
21	13	38	21	14	22	72	56	16	72	56	18
21	33	55	21	34	7	74	48	38	74	48	39
23	45	22	23	46	38	75	45	54	75	45	0
24	32	14	24	31	46	78	12	44	78	11	16
25	35	25	25	36	31	79	8	50	79	7	10
26	59	25	26	59	29	79	35	56	79	36	40
33	23	54	33	23	55	81	12	2	81	12	9
35	2	44	35	3	4	81	49	43	81	49	44

ω			for perp.	for hyp.	for bas.	read perp.	read hyp.	read bas.
12°	1′	4″	78	—	—	76	—	—
13	41	8	—	308	317	—	317	308
30	30	37	23	—	—	33	—	—
42	44	28	203	—	—	207	—	—
60	30	46	—	183	—	—	193	—
70	30	28	—	929	—	—	949	—
71	40	31	476	—	—	468	—	—

for tan ½ω	read	for tan ½ω	read
0,2608691	0,2608696	0,6086965	0,6086957
0,2941179	0,2941176	0,6956526	0,6956522
0,3076938	0,3076923	0,7058823	0,7058824
0,3157363	0,3157895	0,7619047	0,7619048
0,5909001	0,5909091	0,9523809	0,9523810

(BRETSCHNEIDER 1, p. 100)

SHANKS 1, 3.

for p	read	insert p	insert p
2670	2671	6311	25999
9199	6199	6917	28949
11137	15137	19553	29243
11559	15559	22013	29387
12539	18539	22963	

p	ξ	p	ξ	p	ξ	p	ξ
4517	2258	19777	6592	22963	11481	28663	9554
5779	5778	19841	64	23041	1152	28687	28686
6311	3155	20071	3345	23599	437	28751	575
6917	3458	20143	20142	24443	12221	28843	759
10193	1456	20353	6784	25667	12833	28949	28948
10753	512	20359	10179	25759	12879	29243	14621
14437	7218	21277	1182	25999	12999	29387	2099
19423	6474	21821	21820	27427	13713	29443	14721
19553	19552	22013	5503	27739	27738	29527	1554

(CUNNINGHAM 40, p. 153)

SOMMER 1_1, 1_2.

in $K(\sqrt{31})$

$$\textit{for} \begin{Bmatrix} (1) & J^3 & J^3 \\ (3, 1-\sqrt{-31}) & J^3 & J^3 \\ (3, 1+\sqrt{-31}) & J & J \end{Bmatrix} \textit{read} \quad (1) \quad 1 \quad 1$$

(H. H. MITCHELL)

VON STERNECK 1.

page 969

n	for	read	n	for	read
106553	−28	−26	106561	−28	−30
106554	−29	−27	106562	−27	−29
106555	−30	−28	106563	−26	−28
106556	−30	−28	106564	−26	−28
106557	−31	−29	106565	−25	−27
106558	−30	−28	106566	−26	−28

(VON STERNECK 2, p. 1058)

VON STERNECK 3.

$n=32822$ *for* 4 *read* 3

(DICKSON 9, p. 125)

SYLVESTER 1_1.

n	for	read	n	for	read
9	-1	$+1$	23	$36u^3$	$36u^7$
10	$+1$	-1	25	$-5u$	$+5u$
14	$+2u$	$-2u$	29	$28u^3$	$28u^2$
18	$+1$	-1	30	$u^8-9u^6+\cdots+1$	$u^4+u^3-4u^2-4u+1$
19	$10u^3$	$-20u^3$	31	$-4u$	$-8u$
22	$+1$	-1	33	-1	$+1$
22	$-3u$	$+3u$	36	$9u^6$	$9u^2$

(D. H. LEHMER, *Annals of Math.*, s. 2, v. 31, 1930, p. 436)

SYLVESTER 2_1, 2_2.

$n=688$ *for* 536 *read* 336.

(GLAISHER 27, p. vii)

TANNER 2.

page 257, in reciprocal factor of $p=2161$, *for* $q_3=14$, *read* 24

page 258, the prime 3371 is missing. *Insert* line

p	X	Y	$3XY$	simplest	primary	reciprocal	Q
3371	127	$\dot{2}\dot{3}$	2	$3, 1, 1, 2, \dot{6}$	$0, \dot{5}, \dot{4}, 2, 8$	$6, 8, \dot{4}\dot{6}, 15, 16$	55

(CUNNINGHAM 42, p. 66)

TEBAY 1.

page 111, *for* 34, 143, 145, 1716 *read* 24, 143, 145, 1716

(HALSTED 1)

page 112, *for* 330, 644, 725, 107226 *read* 333, 644, 725, 107226
also change order of entry.

page 112, *for* area 863 550 *read* 934 800

(MARTIN 2, p. 309, 321)

page 113, *for* 21, 61, 65, 420 *read* 14, 61, 65, 420

TEEGE 1.

in $n=41$, coefficient of x^3 in z, *for* 1 *read* 2
in $n=97$, coefficient of x^7 in z, *for* 40 *read* 44

VEGA 1_1, 1_2, [e_1].

N	factors	N	factors
27293	7·7·557	82943	7·17·17·41
33293	13·13·197	90983	37·2459
41779	41·1019	93137	11·8467
55403	17·3259	95017	13·7309
55517	7·7·11·103	95623	11·8693
57103	17·3359		

(CUNNINGHAM 41, p. 27)

1_1, 1_2, [f_1].

delete 173279, *insert* 177347
(CHERNAC 1; correction of the corresponding table in Vega's *Logarithmisch-trigonometrische Tafeln*, v. 2, Leipzig, 1797, reprinted in VEGA 1_1, 1_2.)

VEREBRÍ͡USOV 1.

	for			read
32, 19, 1:	29, 26, 1		32, 19, 1:	29, 26, 11
49, 28, 3:	41, 37, 6		49, 28, 3:	41, 37, 36
		insert	44, 12, 2:	38, 36, 8

WERTHEIM 2.

delete asterisk on $p=1213$, 1993, 2437, 2729
insert asterisk on $p=2731$, 2887

(WERTHEIM 4, p. 157)

page	p	for $g=$	read
316	1013	2	3
	1021	7	10
318	2161	14	23
	2593	10	7
319	2999	7	17

WERTHEIM 4.

page	p	for $g=$	read
154	3181	11	7
	3191	17	11
	3631	21	15
155	3967	13	6
	4111	17	12
	4657	5	15
	4751	37	19

WHITFORD 1 [m].

A	
1733	The 6th partial quotient should be 3 and not 2.
1822	The 23d partial quotient and denominator of the 23d complete quotient are missing. They are 1 and 54 respectively.
1852	The 29th partial quotient should be 20 and not 16.
1963	The entry here should be:

WHITFORD 1—*continued*

44	3	3	1	2	3	2	29	9	1	4	(3)
	27	22	51	29	23	38	3	9	66	17	(26)

1549 y=12223 09542 82674 74959 34242 68334 63805–
08818 07626 31786 81966 09867 28279 63220
1566 y=308792110
1615 y=81732
1669 y= 572 84717 32803 87374 12405 68998 80229 34138 39259 82496 64340

(D. H. LEHMER 11, p. 548, 550)

WIEFERICH 1.

page 75

for		read	
232 . . . 239	17 . . . 24	232 . . . 238	17 . . . 23
240 . . . 250	8 . . . 18	239 . . . 250	6 . . . 17
264 . . . 271	18 . . . 25	264 . . . 270	18 . . . 24
272 . . . 282	9 . . . 19	271 . . . 282	7 . . . 18
291 . . . 303	3 . . . 15	291 . . . 302	3 . . . 14
304 . . . 314	10 . . . 20	303 . . . 314	8 . . . 19
323 . . . 335	4 . . . 16	323 . . . 334	4 . . . 15
336 . . . 346	11 . . . 21	335 . . . 346	8 . . . 20
355 . . . 367	5 . . . 17	355 . . . 366	5 . . . 16
368 . . . 378	12 . . . 22	367 . . . 378	10 . . . 21
387 . . . 399	6 . . . 18	387 . . . 398	6 . . . 17
400 . . . 410	13 . . . 23	399 . . . 410	11 . . . 22

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