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CK-12 Texas Instruments Algebra I

Teacher's Edition

Texas Instruments Algebra I Teacher's Edition

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CHAPTER 1 TE Introduction - TI

CHAPTER OUTLINE

1.1 Algebra I TI Resources Flexbook

Teacher Edition

Introduction

This flexbook contains Texas Instruments (TI) Resources for the TI-83, TI-83 Plus, TI-84, and TI-84 SE. All the activities in this flexbook supplement the lessons in the student edition. Teachers may need to download programs from www.timath.com that will implement or assist in the activities. Each activity included is designed to help the teacher with each activity in the TI Resources Flexbook, Student Edition. All activities are listed in the same order as the Student Edition.

There are also corresponding links in the 1st Edition of Algebra I, 2nd Edition, and Basic Algebra.

- Algebra I, first edition: http://www.ck12.org/flexr/flexbook/1374
- Algebra I, second edition: http://www.ck12.org/flexr/flexbook/3659
- Basic Algebra: http://www.ck12.org/flexr/flexbook/2876

Any activity that requires a calculator file or application, go to http://www.education.ti.com/calculators/downloads and type the name of the activity or program in the search box.

CHAPTER 2 TE Equations and Functions - TI

CHAPTER OUTLINE

2.1 Back in Time?

This activity is intended to supplement Algebra I, Chapter 1, Lesson 5.

ID: 11572

Time Required: 15 minutes

Activity Overview

Students will explore the definition of a function graphically, with a set of ordered pairs, and by using an input-output model with the graphing calculator. This model dynamically allows students to discover the function by experimenting with input values that produce the desired output. Function notation is also reinforced.

Topic: Functions #38; Relations

- *Function definition #38; notation*
- *Using ordered pairs, function machines and graphs.*

Teacher Preparation and Notes

- *This activity can serve as a good introductory exploration to functions.*
- *If time is limited, Problem 4 can be used as an extension or homework problems.*
- ***To download the calculator file, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=11572 and select "MACHINE.8xp" to download.***

Associated Materials

- Student Worksheet: Back In Time? http://www.ck12.org/flexr/chapter/9611
- **MACHINE.8xp** (program)

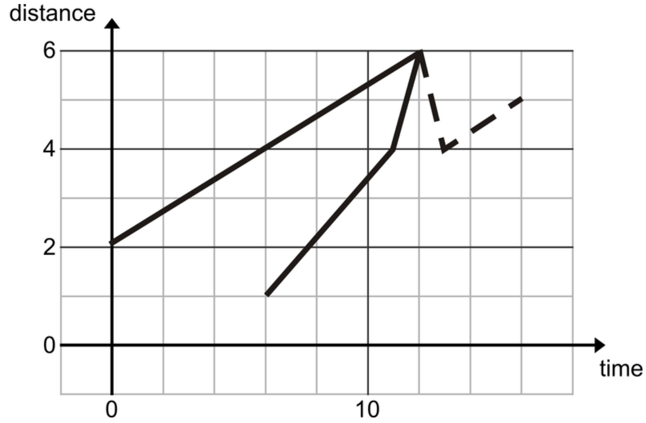
Problem 1 – Graphical

The students are asked to consider a graph of position vs. time. They are asked if this is a function. Since there is not one unique output of d for each value of t , it is not a function.

Students then discuss with their neighbor why the answer is 'no' based on the definition, and under what circumstance could the graph occur. It could only occur if Marty went back in time.

Students then redraw the graph so that it is a function. An example of a solution is to the right.

Use this opportunity to discuss the concept of the vertical line test.



Problem 2 – Set of ordered pairs

To review the notation of a set of ordered pairs this section begins with a multiple choice question with multiple solutions. Students should realize that as long as there is only one unique output for each input then it describes a function, otherwise it can only be referred to as a relation.

Another question that can be asked is “What are some examples of relations that are not functions?” Answers include inequalities, like $y > 3x, x$ as a function of y when $y = x^2$, and x as a function of y when $y = |x+2|$.

Students will calculate the output values for given input values using the formula $d = \frac{1}{2} \cdot a \cdot t^2$ for when $a = 12\ ft/s^2$. The formula can be simplified to $d = 6 \cdot t^2$.

To compute d , students can substitute the $t-$ values directly into the formula as shown in the first screenshot.

To compute t , students need to either solve for t $\left(t = \pm\sqrt{\frac{d}{6}}\right)$ and then substitute the $d-$ values into the formula as shown in the second screenshot.

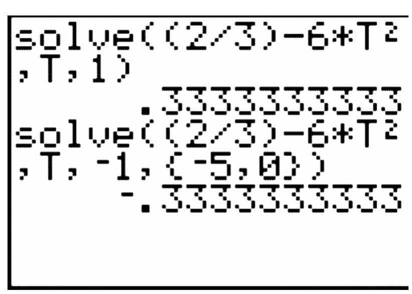
Or they can use the **solve** command.

The solve command has parameters

solve(expression, variable, guess, low bound, high bound).

Note that the low bound and high bound are option, but if used must be placed in braces.

The expression needs to be equal to zero, but low and high bounds do not have to entered.



Press 2^{nd} **[CATALOG][LN]** and scroll down to select **solve(**.

Student Answers

4. Functions are A, B and D because for every input value there is only one output value.

5. $(0,0),(1,6),(2,24),(6,216)$

6. $(0,0),\left(\frac{2}{3},\frac{1}{3}\right),\left(\frac{2}{3},-\frac{1}{3}\right),(6,1),(6,-1)$

7. The first set is a function because it has only one output for every input.

8. A. d is a function of t

Problems 3 – Function notation

In this problem, students will explore using function notation on the calculator entering input values to find the output value.

Students will enter the function next to $Y1$ on the $Y =$ screen. Then to find $f(4)$, they will enter $Y1(4)$ on the Home screen. For Question 10, students will need to update the function next to $Y1$.

Student Answers

9. $f(4) = 4^2 - 2(4) + 3 = 11$

10. $f(2) = 3(2)^2 + 5(2) + 3 = 25$

Problem 4 – Function Machine

For this problem, students will need the program **MACHINE.** This program has 3 options, each option is a different function. When students enter an input value, it will return the output value.

Students need to determine which input will give the goal output and then determine the correct function that determines the given output.

Student Answers

11. 17

12. $f(x) = 0.5x$

13. -4

14. $f(x) = x + 10$

15. 20

16. $f(x) = 4x + 3$

2.2 Ordered Pairs

This activity is intended to supplement Algebra I, Chapter 1, Lesson 6.

ID: 11638

Time Required: 15 minutes

Activity Overview

In this activity, students will investigate ordered pairs. They will graphically explore the coordinates of a point on a Cartesian plane, identifying characteristics of a point corresponding to the coordinate. Students will plot ordered pairs of a function, list these in a table of values, and graph them in a scatter plot.

Topic: Functions #38; Relations

- *Cartesian coordinate system*
- *Characteristic of ordered pairs in a quadrant*
- *Graph ordered pairs on a scatter plot*

Teacher Preparation and Notes

- *Before beginning the activity, students should clear all lists and turn off functions. To clear the lists, press* **STAT [EDIT]** *and scroll down until the arrow is in front of* ***ClrAllLists*** or ***ClrList***. *Press enter twice. To clear any functions, press* $Y =$ *and then press* **CLEAR** *when the cursor is next to each* $Y =$ *equation.*
- *This activity can serve as an introduction to ordered pairs, quadrants, graphing points and see the connection between a function and a graph.*

Associated Materials

- Student Worksheet: Ordered Pairs http://www.ck12.org/flexr/chapter/9611, scroll down to second activity.

Problem 1 – Ordered Pairs

First, students explore the coordinates of a point in the various quadrants. They will enter a given coordinate into lists $L1$ and $L2$, where $L1$ is the $x-$ value and $L2$ is the $y-$ value. There should only be one number in each list at all times. Then students will graph the coordinate as a scatter plot using **Plot1**.

Students should press **ZOOM** and select **ZStandard** to see the standard viewing window. After selecting **ZStandard** for the first coordinate, they can then press **GRAPH** for the other coordinates.

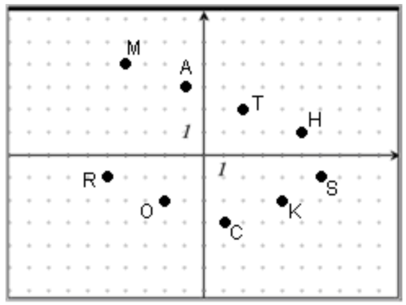
Explain to students that a point on the $x-$ or $y-$ axis, when the first or second number in an ordered pair is equal to zero, is not in a **quadrant** since it is on the boundary between quadrants.

After students answer the questions about what quadrant an ordered pair is in, they will explore where ordered pairs are in general. They need to turn off **Plot1** and then press **GRAPH**. When students move the cursor, the coordinates will not be integers, but they should still be able to conjecture where the $x-$ and $y-$ values are positive and negative.

Ask students to tell you where specific points will fall, without using the calculator.

- Where will $(5,1)$ fall? *Quadrant 1* If it is in Quadrant 1, where will $(-5,1)$ fall? *Quadrant 2* $(5,-1)$? *Quadrant 4* $(-5,-1)$? *Quadrant 3*
- Continue with similar questioning until all students feel comfortable with the four quadrants.

Then students are to apply what they learned by plotting points to solve a puzzle. The solution of the puzzle is "**MATH ROCKS**."



Problem 2 – Order Pairs

Students are given a function for the cost of ordering pears. They need to enter 5 ordered pairs into lists $L1$ and $L2$. Then they will set up **Plot1** to display the scatter plot of the pairs.

To set an appropriate window, students can press **WINDOW** and change the settings individually or press **ZOOM** and select **ZoomStat.**

Pressing **TRACE** and using the arrow keys will allow students to see the $x-$ and $y-$ value of a point.

Lastly, students will graph the line $y = x$ and then adjust the slope so that it goes through the ordered pairs of the scatter plot. This means that they will need to add a number before x and then change that number as needed.

Students should see that the slope of the line is the same as the coefficient of the function given for the cost of ordering pears.

The **Manual-Fit** regression can also be used to allow students to adjust the line of fit on the graph screen. Press **STAT**, arrow to the **CALC** menu and select **Manual-Fit**. Press **ENTER** when Manual-Fit appears on the home screen. Students can then adjust the slope and $y-$ intercept until the equation fits their data. Once students are satisfied with the line, they can press $Y =$ to see the equation.

Extension

Extension 1

Students are to find some other real-life data and then represent it as a set of ordered pairs, table, and scatter plot. Teachers can show students how to use the Graph-Table split (found in the **MODE** screen) to see the graph and table at the same time.

Extension 2

Students are to come up with their own puzzle like the one page 1 of the worksheet, which spelled "math rocks". They can share their puzzle with a friend or the class. Or students can draw a picture on a coordinate grid and identify key coordinate pairs. They then create two lists of $x-$ and $y-$ values to exchange with a partner. The partner will then redraw the image. Also, students could be given an image or two for practice. Trees and leaves make good examples.

CHAPTER 3 TE Real Numbers - TI

CHAPTER OUTLINE

3.1 Factoring Composite Numbers

This activity is intended to be used with Algebra I, Chapter 2, Lesson 1.

ID: 10891

Time required: 15 minutes

Activity Overview

In this activity, students will work with composite numbers to find their prime factorizations in exponent form. They will create factor trees and use division as a means to finding prime factors. An extension involving finding a common denominator for two fractions is given at the end of the activity.

Topic: From Arithmetic to Algebra

- *Prime factorization*
- *Factor tree and prime factors*
- *Prime factorization in exponent form*

Teacher Preparation and Notes

- *If an interactive white board is available, create a number sort with a Venn diagram. For example, intersecting circles for Primes and Multiples of 8. The students would be given a list of a few numbers to place into the proper part of the diagram. In this case, there would be no intersection. For another example, use Primes and Numbers less than* 40 *. There would be quite a few possibilities for numbers contained in the intersection. (credit: Teaching Reading in Mathematics, M. Barton and C. Heiderma, 2000)*

Associated Materials

- Student Worksheet: Factoring Composite Numbers, http://www.ck12.org/flexr/chapter/9612

Problem 1 – A Frayer Square for

Discuss students' Frayer Squares as a class, recording their responses on the board or an overhead transparency. Use this activity to assess students' knowledge of the concepts *prime* and *composite.*

A **prime number** can be divided evenly only by itself and 1. A **composite number** is a non-zero number that is not prime.

Problem 2 – Exploring a factor tree for a composite number

Examine the factor tree of 24 with students and assess their prior knowledge of factorization and factor trees.

The number at the top of the tree is the number being factored. Moving up the factor tree represents multiplication and moving down it represents division. The circled numbers, or leaves of the tree, are prime. They make up the prime factorization.

Discussion Questions:

- *What if the factor tree starts out with two different factors than the ones shown? Will you still get the same answer?* Challenge students to change the factor tree using two different starting factors.
- *Does it matter in what order I list the final prime factors? Why might a teacher prefer that the numbers be listed in ascending order?*
- *Why is the exponent form used for the prime factorization?*

Problem 3 – Exploring division as a means to finding prime factors

Demonstrate how to find the prime factorization of 30 while students follow along on their calculators. The factor trees on their worksheets help students track their progress.

Students will follow this example to complete the prime factorization and factor tree for the number 36 .

Things students can do after working through this problem:

- *How are prime factors used to create GCF and LCM? Demonstrate with an example.*
- *Explain how Eratosthenes' Sieve works to a younger student, to a peer, or to an adult.*

Problem 4 - Factoring on your own

Students can demonstrate the use of prime factors in selecting a common denominator for fraction addition or subtraction problems.

They can use either method from the activity, factor tree or division, to find the prime factorization of each denominator.

Students can use the common denominator to simplify the expressions.

$$\frac{a}{126}+\frac{a}{84}$$
$$\frac{a}{2\cdot3\cdot3\cdot7}+\frac{a}{2\cdot2\cdot3\cdot7}$$
$$\left(\frac{2}{2}\right)\frac{a}{2\cdot3\cdot3\cdot7}+\left(\frac{3}{3}\right)\frac{a}{2\cdot2\cdot3\cdot7}$$
$$\frac{2a}{2\cdot2\cdot3\cdot3\cdot7}+\frac{3a}{2\cdot2\cdot3\cdot3\cdot7}$$
$$\frac{5a}{2\cdot2\cdot3\cdot3\cdot7}=\frac{5a}{252}$$

$$\frac{5x}{78}-\frac{x}{66}$$
$$\frac{5x}{2\cdot3\cdot13}-\frac{x}{2\cdot3\cdot11}$$
$$\left(\frac{11}{11}\right)\frac{5x}{2\cdot3\cdot13}-\left(\frac{13}{13}\right)\frac{x}{2\cdot3\cdot11}$$
$$\frac{55x}{2\cdot3\cdot11\cdot13}-\frac{13x}{2\cdot3\cdot11\cdot13}$$
$$\frac{42x}{2\cdot2\cdot11\cdot13}=\frac{42x}{858}=\frac{7x}{143}$$

$$\frac{n}{30}-\frac{n}{63}$$
$$\frac{n}{2\cdot3\cdot5}+\frac{n}{3\cdot3\cdot7}$$
$$\left(\frac{3\cdot7}{3\cdot7}\right)\frac{n}{2\cdot3\cdot5}+\left(\frac{2\cdot5}{2\cdot5}\right)\frac{n}{3\cdot3\cdot7}$$
$$\frac{21n}{2\cdot2\cdot3\cdot3\cdot7}+\frac{10n}{2\cdot2\cdot3\cdot3\cdot7}$$
$$\frac{31n}{2\cdot3\cdot3\cdot5\cdot7}=\frac{31n}{630}$$

Solutions

Problem 1

TABLE 3.1:

Definition:	Fun facts:
• a number greater than 1 that... has only one pair of factors, 1 and itself.	• 2 is the only even prime. • 1 is **NOT** a prime • Eratosthenes came up with a "sieve" to find the primes.
Examples	**Non-examples**
• $2, 3, 5, 7, 11, 13$	• $-7, 0, 1, 4, 12, 100$

Problem 2

- The exponent is three because there are three 2*s* .
- The number 24 is not a prime number because it has factors other than 1 and itself.

Problem 3

- There are no exponents in this example because there is only one of each factor.
- $36 = 2^2 \cdot 3^2$; factor tree will vary

Problem 4

Factor trees will vary.

1. $27 = 3^3$
2. $56 = 2^3 \cdot 7$
3. $72 = 2^3 \cdot 3^2$

3.2 Hot Air Balloon

This activity is intended to be used with Algebra I, Chapter 2, Lesson 3.

ID: 8613

Time required: 30 minutes

Activity Overview

In this activity, students use a dynamic, electronic manipulative to perform integer addition and subtraction. The goals of the activity are to (1) provide students with a visual for adding and subtracting integers and (2) help students understand that subtraction can be thought of as "adding the opposite" or "adding the inverse."

The model given in this activity shows a hot air balloon that begins at ground level with a certain number of helium bags (providing lift) and the same number of sand bags (providing weight). The vertical position of the hot air balloon is determined by adding or removing a number of helium bags (representing positive integers) and sand bags (representing negative integers).

Topic: From Arithmetic to Algebra

- *Use technology to verify that adding the number $-x$ is equivalent to subtracting x*
- *Solve one-step linear equations of the form $x+a=b$ where a and b are real numbers*

Teacher Preparation and Notes

- *This activity may be used to introduce or review integer addition and subtraction. You may choose to use the activity in its entirety or break it up into separate activities by sets.*
- *It is very important that you thoroughly describe the model to students prior to them exploring the model on their own. Use a projector or a real balloon to demonstrate the model in a whole class, teacher-led setting. Some students will catch on very quickly and wean themselves from using the model. Others will prefer and/or need to stay a longer time with the model.*
- ***To download the calculator file, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=8613 and select "HOTAIR.8xp" to download.***

Associated Materials

- Student Worksheet: Hot Air Balloon, http://www.ck12.org/flexr/chapter/9612, scroll down to the second activity.
- HOTAIR.8xp (program)

In this activity, students will explore:

- *adding and subtracting integers using a model of a hot air balloon*
- *the relationship between addition and subtraction*

Explain how the model of the hot air balloon works. Students will need to download the **HOTAIR** program to their handheld and work with a partner to complete the activity.

Things to Remember...

This model is similar to a hot air balloon and provides a way to visualize adding and subtracting integers.

- *Positive* integers are represented by helium bags. They *raise* the balloon.

  Negative integers are represented by sand bags. They *lower* the balloon.

- Addition is the operation of *putting bags on* the balloon.

  Subtraction is the operation of *taking bags off* of the balloon.

- Always reset the balloon's vertical position to 0 before beginning another calculation.

To begin, press **PRGM** and find the **HOTAIR** program in the list. Press **ENTER** to load the program.

Problem 1 – Integer addition

Students will now use the model to find the sum $2+(-6)$. Press **ENTER** to choose option 1, Integer Addition.

The program displays instructions. The right arrow adds a helium bag to the balloon, and the left arrow adds a sand bag.

Students can see the balloon at the bottom of the screen. The horizontal line represents ground level. The balloon's vertical position is displayed at the bottom of the screen. Right now, the position is -12 .

To view this addition $a+b$ using the model, students are to first reset the balloon to ground level by pressing the right arrow.

To find the sum $2+(-6)$, students need to understand what the sum represents in terms of the balloon. The 2 signifies adding two helium bags to the balloon, which raises it 2 *units* . They should press the right arrow twice to add 2 helium balloons.

Adding -6 means you then add six sand bags, which lowers the balloon 6 *units* . Press the left arrow 6 *times* to add 6 sand bags. The balloon's resulting position is the sum.

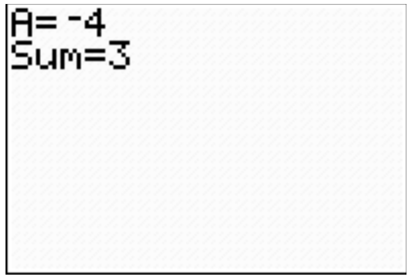
Together with a partner, students need to translate the following expressions into "balloon language" and use the model to find the sum. Remind them to reset the balloon to ground level each time. When they are finished, they can press **CLEAR** to return to the menu.

1. $-4+7=\underline{3}$
2. $7+3=\underline{10}$
3. $5+(-7)=\underline{-2}$
4. $-5+(-3)=\underline{-8}$

Problem 2 – Missing addend

In this problem, students are given the value of a and the value of the sum $a+b$, but the value of b is unknown.

Students are to arrow down to option 2, Missing Addend, and press **ENTER**.



Suppose you want to find the value of b such that $-4+b=3$. Ask students to first think about what this means in terms of the balloon. They know that 4 sand bags are added, and that the balloon ends up at 3 . Ask students: How many and which type of bag do you need to add to have a resulting position of 3 ?

They are to input the value for A and press **ENTER**. Then input the sum and press **ENTER**.

The program displays instructions. Remind students that the right arrow adds a helium bag to the balloon, and the left arrow adds a sand bag, as with Integer Addition.

They can see the balloon at the bottom of the screen. Now there is also a target balloon to the right. The target balloon is positioned at the value of the sum. The task for students is to now to find the value of b that is needed to move the balloon on the right to the same position as the target balloon on the left.

To find b using the model, students should first use the right arrow to reset the balloon's vertical position to 0 .

Add 4 sand bags by pressing the left arrow 4 *times* .

The target balloon is above the balloon's position, so students need to add helium bags. They should press the right arrow to add helium bags until the balloons line up. Instruct them to count as they press to find how many helium bags they added. For this example, $b=7$.

Together with a partner, students are to translate the following equations into "balloon language" and use the model to find the missing addend. When they are finished, they can press **ENTER** to return to the menu.

1.
$$\begin{aligned} 2+b &= -3 \\ b &= \underline{-5} \end{aligned}$$

2.
$$\begin{aligned} -6+b &= -1 \\ b &= \underline{5} \end{aligned}$$

3.
$$\begin{aligned} 5+b &= 1 \\ b &= \underline{-4} \end{aligned}$$

4.
$$\begin{aligned} -2+b &= 4 \\ b &= \underline{6} \end{aligned}$$

Problem 3 – Integer subtraction

The model also provides a way of visualizing the subtraction of integers. As with addition, positive integers are represented by helium bags and negative integers by sand bags. However, subtraction is the operation of *taking off* a bag. For example, the expression $-2\ -5$ translates to "put on 2 sand bags and then take off 5 helium bags."

Students are to now arrow down to option 3, Integer Subtraction, and press **ENTER.**

Use the model in the same manner as in Problem 1, with one difference: In this model, the right arrow **removes** a helium bag, and the left arrow **removes** a sand bag.

The model is the same as before. The balloon is at the top of the screen.

Students need to reset the balloon at ground level by pressing the right arrow.

Now they can move the balloon to its starting point, -2 .

To remove 5 helium bags, students should press the right arrow 5 $times$. Explain to students that the balloon falls 5 $units$ to end at -7 , which is the difference $-2\ -5$.

Together with a partner, students are to translate the following expressions into "balloon language" and use the model to find the difference. When you are finished, press $\subseteq$ to return to the menu.

1. $2-7=\underline{-5}$
2. $-3-1=\underline{-4}$
3. $5-(-2)=\underline{7}$
4. $-4-(-7)=\underline{3}$

Problem 4 – Missing subtrahend

This model shows two balloons side by side—like the model from Problem 2, except that it is used to find a missing *subtrahend* rather than a missing addend.

Students can now arrow down to option 4, Missing Subtrahend, and press **ENTER**.

For example, find the value of b such that $-3-b=8$. Explain to students that in terms of the balloon, this means that the balloon ends up at 8 , and you have used 3 sand bags. They need to find how many and which type of bag they must *remove* to have a resulting position of 8 .

Students need to input the value for A and press **ENTER.** Then input the difference and press **ENTER.**

Use the model in the same manner as in Problem 2, with one difference: In this model, the right arrow **removes** a helium bag, and the left arrow **removes** a sand bag.

The balloon is at the top of the screen, and the target balloon to the right represents the difference.

Students need to first reset the balloon at ground level by pressing the right arrow.

Then they can move the balloon to its starting position, -3 .

The target balloon is above their balloon's position, so they need to remove sand bags. Press the left arrow to remove sand bags until the balloons line up. Remind students to count as they press to find how many sand bags they removed. For this example, students should find that $b=-11$.

Together with a partner, students are to translate the following equations into "balloon language" and use the model to find the missing subtrahend. When they are finished, they can press **ENTER** to return to the menu.

1. $$\begin{aligned}6-b&=9\\b&=\underline{-3}\end{aligned}$$

2. $$\begin{aligned}5-b&=-3\\b&=\underline{8}\end{aligned}$$

3. $$\begin{aligned}-4-b&=-1\\b&=\underline{-3}\end{aligned}$$

4. $$\begin{aligned}-2-b&=6\\b&=\underline{-8}\end{aligned}$$

Problem 5 (Extension) – Addition and subtraction exploration

Students are to arrow down to option 5, Addition and Subtraction, and press **ENTER.**

The balloon on the left is for addition and the one on the right is for subtraction. Students can use the up and down arrows to move between the two models.

For each of the following expressions, students are to use what they've learned from Problems 1 and 3 to translate into "balloon language" and then find each sum or difference.

1. $-2-4=$ ___

2. $-2+(-4)=$ ___

3. $5-(-6)=$ ___

4. $5+6=$ ___

For each of the following equations, students use what they've learned from Problems 2 and 4 to translate into "balloon language" and then find each missing addend or subtrahend.

5.

$$-1-b=5$$
$$b=\_\_\_$$

6.

$$-1+b=5$$
$$b=\_\_\_$$

7.

$$3-b=-4$$
$$b=\_\_\_$$

8.

$$3+b=-4$$
$$b=\_\_\_$$

Now students are to complete the following statements.

9. Taking off 8 sand bags is the same as putting on 8 ___________ bags.

10. Taking off 5 helium bags is the same as putting on 5 ___________ bags.

11. If a and b are any two integers, then $a-b=a+$ ___ . That is, subtracting a number is equivalent to adding its ___________.

To exit the program, students can arrow down to option 6 and press **ENTER**.

Solutions

1. -6
2. -6
3. 11
4. 11
5. -6
6. 6
7. 7
8. -7
9. helium
10. sand
11. $-b$; opposite (or additive inverse)

CHAPTER 4 TE Equations of Lines - TI

CHAPTER OUTLINE

4.1 One Step at a Time

This activity is intended to supplement Algebra I, Chapter 3, Lesson 1

ID: 8683

Time required: 30 minutes

Activity Overview

In this activity, students solve one-step equations involving addition and multiplication by substituting possible values of the variable. The equations they solve and their solutions become data as the students are guided to formulate and test a hypothesis about solving one step equations. The students use their result to solve several one-step equations algebraically. The activity closes with a discussion of inverse operations and a general rule for solving one-step equations.

Topic: Linear Equations

- *Solve "one-step" linear equations of the form $x+a=b$ and $ax=b$ where a and b are real numbers.*
- *Verify the solution to a linear equation by substitution.*

Teacher Preparation

- *This activity is designed for use in an Algebra 1 or Pre-Algebra classroom. It uses a numerical and empirical approach to help students discover one of the basic techniques of algebra on their own. These concepts can also be presented via manipulatives such as the balanced scale or algebra tiles, or as consequences of the Properties of Equality. This activity is not intended to replace those approaches, but to supplement them.*
- *Prior to beginning the activity, students should know how to evaluate algebraic expressions, perform basic operations with integers, and be familiar with the terms variable, expression, and equation.*
- *One-step equations involving subtraction and division are not covered in this lesson. This allows the teachers to choose how to present these types of equations (either as further examples of addition and multiplication equations or as operations in their own right, or both.)*
- *This activity is designed to be* ***student-centered*** *with the teacher acting as a facilitator while students work cooperatively and brief periods of teacher-led, whole class discussion. The student worksheet is intended to guide students through the main ideas of the activity and provide a place to record their observations.*

Associated Materials

- Student Worksheet: One Step at a Time, http://www.ck12.org/flexr/chapter/9613

An equation is like a statement in mathematical language. The solution to an equation is the value that makes the statement true. The statement is true when one side of the equation equals the other.

Problem 1 – Addition equations

Students begin by testing values for x in the equation $x+3=8$, looking for the value of x that makes the equation true. They will set up the **Table** feature to perform the substitution automatically. Students are prompted to enter other addition equations into the $Y =$ screen and repeat the process of entering values in the Table to find a solution.

Note: When students change the expressions in the $Y =$ screen, the $x-$ values they entered previously in the Table will remain. To delete the $x-$ values, when on the Table screen, use the arrow keys to highlight the value and then press **CLEAR**.

By observing the solutions to many equations of the same form, students gather data to form a hypothesis about the solving an equation of this form.

After students write and solve their own equations is a good point to introduce solving one-step addition equations with algebra tiles. The action of taking ones tiles away from both sides reinforces the pattern that students have observed.

Discuss and demonstrate the **Subtraction Property of Equality** and its application to solving one-step addition equations in a whole-class setting before having students individually complete the equations for Question 4.

Problem 2 – Multiplication equations

Students now turn their attention to one-step equations involving operations other than addition. This example focuses on equations of the form $ax = b$.

As before, students use the **Table** feature to solve several one-step multiplication equations, formulate a hypothesis about the solution to a multiplication equation, and test the hypothesis by looking back at the equations they solved.

Discuss and demonstrate the **Division Property of Equality** and its application to solving one-step multiplication equations in a whole-class setting before having students individually complete the equations for Question 8.

Problem 3 – Inverse operations

Wrap up the activity with a discussion of **inverse operations** as operations that "undo" each other. With the class, formulate a general rule for solving any one-step equation.

Solutions – Student worksheet

Problem 1

1. a. $x = 50$

b. $x = 22$

c. $x = 69$

d. $x = -2$

2. Answers will vary. Check that students' equations are solved correctly.

3. a. Subtract 3 from 8 to get 5

b. Yes. The solution to $x+30 = 80$ is $x = 50$, and you can subtract 30 from 80 to get 50 .

4. a.

$$\begin{aligned}2+q&=11\\2-2+q&=11-2\\q&=9\end{aligned}$$

b.

$$\begin{aligned}t+11&=10\\t+11-11&=10-11\\t&=-1\end{aligned}$$

c.

$$\begin{aligned}n+32&=5\\n+32-32&=5-32\\n&=-27\end{aligned}$$

d.

$$\begin{aligned}p+17&=0\\p+17-17&=-17\\p&=-17\end{aligned}$$

Problem 2

5. a. $x = 15$

b. $x = -4$

c. $x = 13$

d. $x = -9.6$

6. Answers will vary. Check that students' equations are solved correctly.

7. a. Divide 75 by 5 to get 15

b. Yes. The solution to $-7x = 28$ is $x = -4$, and you can divide 28 by -7 to get -4 .

8. a.

$$\begin{aligned}8q&=64\\\frac{8q}{8}&=\frac{64}{8}\\q&=8\end{aligned}$$

b.

$$\begin{aligned}6t&=-120\\\frac{6t}{6}&=\frac{-120}{6}\\t&=-20\end{aligned}$$

c.

$$\begin{aligned} 2n &= 2 \\ \frac{2n}{2} &= \frac{2}{2} \\ n &= 1 \end{aligned}$$

d.

$$\begin{aligned} -3p &= 48 \\ \frac{-3p}{-3} &= \frac{48}{-3} \\ p &= -16 \end{aligned}$$

Problem 3

9. a. subtraction

b. addition

c. division

d. multiplication

10. To solve a one-step equation, apply the inverse operation to both sides.

4.2 Variables on Both Sides

This activity is intended to supplement Algebra I, Chapter 3, Lesson 4.

ID: 11131

Time Required: 15-20 minutes

Activity Overview

In this activity, students will encounter various scenarios involving perimeter of polygons. The students will write equations and solve them in order to answer the questions provided.

Topic: Solving Equations with Variables on Both Sides

- *The student will use algebraic expressions to form equations relating two different perimeters to each other.*
- *The student will solve equations with variables on both sides.*
- *The students will use the App4Math to check their answers.*
- *The student will answer a deeper-level inquiry question regarding the relationship between two regular polygons and the difference in their perimeters for different lengths of sides.*

Teacher Preparation and Notes

- *Teacher preparation must include having students set up and solve equations with variables on both sides. These equations include the Distributive Property.*
- *Students should also be encouraged to show their work, whether it be on paper or in the document itself. If the teacher prefers paper, then use a prepared handout labeled by tab number to correspond to the tabs in the .tns file.*
- *Be sure the App4Math is installed on all calculators.*
- ***To download the App4Math application, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?i and select "App4Math.8xk" to download.***

Associated Materials

- Student Worksheet: Variables on Both Sides, http://www.ck12.org/flexr/chapter/9613, scroll down to the second activity.
- App4Math Application

Introduction to App4Math

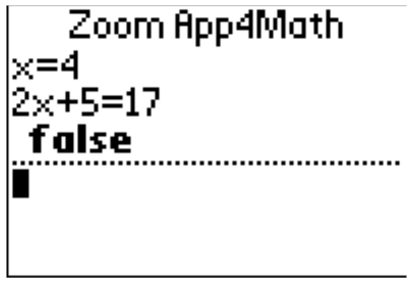
You may want to take a few minutes introducing **App4Math** to your students.

A few quick notes:

- x, y, z , etc. can be entered using the alpha keys or by repeatedly pressing X, T, θ, n .
- Use 2^{nd} **[MATH]** for the equals sign.

- The up and down arrows can be used to cycle through previously written equations. This will help eliminate a lot of repeated key entry.

Have the students use the app4math to check if $x = 4$ is a solution to $2x + 5 = 17$.



Now have them check to see if $x = 6$ is the solution. (The arrow keys will be helpful here to recall the equation $2x + 5 = 17$.)

Problem 1 – A Square and a Rectangle Have Different Perimeters

Student should observe the labels for the length of the sides, and be certain to understand the translation from the text on the left side to the labels. Then, students are to write expressions for the perimeter of the square and of the rectangle. (*Sample answers: Perimeter of the square:* $4x$; *Perimeter of the rectangle:* $6x + 6$).

Students will use the perimeter expressions to create and solve an equation that shows the relationship between the perimeters of the figures. (*Answer:* a. $4x + 10 = 2(x + 3) + 2(2x); x = 2$)

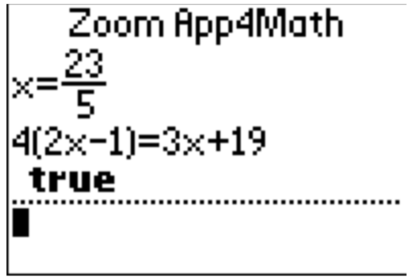
Students should use the **App4Math** app to check their answer.

Problem 2 – An Equilateral Triangle and a Square with Different Perimeters

Students should again observe the labels on the polygons, and be sure that the labels agree with the written description. Then, students will write expressions for the perimeter of the square and triangle. Then, students will create an equation show the relationship between the perimeters and solve the equation to find the length the sides of the triangle and square.

Student worksheet solutions

- *Side of the triangle is x and side of the square is* $2x + 1$.
- *The perimeter of the square is* $4(2x + 1)$.
- *The perimeter of the triangle is* $3(x)$.
- $4(2x - 1) = 3x + 19$
- $x = \frac{23}{5}$; *side of square is* $\frac{51}{5}$ *cm* or 10.2 *cm*
- *The correct **App4Math** screen is below.*



Problem 3 – A Hexagon and an Octagon with Sides that are Related by a Scale Factor

Student worksheet solutions

- x
- $6(2x)$
- $8(x)$
- $12x = 8x + 20$, $x = 5$, $hexagon\ side\ length = 10$

Problem 4 – An Equilateral Triangle and a Rectangle that Share a Common Side

Student worksheet solutions

- $3x + 9 = 14 + 2x,\ x = 23$

Problem 5 – A Regular Decagon and 15-gon

Student worksheet solutions

- *Because the side lengths for both polygons all equal x , the perimeter of the* $15-$ *gon will be longer by the length* 5x *because it has five more sides.*

4.3 Taxes Tips

This activity is intended to supplement Algebra I, Chapter 3, Lesson 7.

ID: 11356

Time Required: 15 minutes

Activity Overview

In this activity, students will increase their understanding of the use of the formula $T = r \cdot p$ *, which is encountered both in the real world and in most Algebra 1 class. They will calculate the amount of taxes and tips exactly, and then use estimation.*

Topic: Linear Equations

- *Percents*
- *Using formulas*
- *Estimation and mental math*

Teacher Preparation and Notes

- *This activity serves as a good introduction to using formulas that involve percentages.*
- *The student worksheet provides instructions and questions to guide inquiry and focus observations.*

Associated Materials

- Student Worksheet: Taxes #38; Tips, http://www.ck12.org/flexr/chapter/9613, scroll down to the third activity.

Problem 1 – Percentage

As the students divide their numbers by 100 , they are to observe the pattern and describe how the decimal point moves. They should see that the decimal point moves to the left by two places.

Problem 2 – Using an Equation

Some students may not be aware of how the tax on an item is calculated. For this activity, the students will use the formula $T = r \cdot p$. For clarity and to reinforce the concept, have the students re-read what the variables represent.

The student activity gives some hypothetical prices for various items that may be of interest to them. Students will select three of these items.

First, students will use the calculator to find the taxes paid on three items listed on their worksheets and find the sum of those taxes. Remind students to round to 2 decimal places because the situation involves money.

Second, students are to sum up the three individual prices and multiply this sum by the tax rate they chose to use in this example.

Students should see that these two amounts are almost the same, varying only by a penny.

Note that the amounts are almost the same because of the distribution property.

$$price \cdot tax + price \cdot tax + price \cdot tax = (price + price + price) \cdot tax$$

Note: Make sure the students have converted the tax rate to a non-percentage before finding the taxes paid.

Discussion questions:

Are there different tax rates for the state in which you live for products and services?

Examine a receipt from a store (other than a grocery store, since there is often no tax on food) and determine if the tax rate charged is correct.

Do an internet search of "sales tax by state" to find the real tax information from various states and use this information to compare the tax paid if the item was purchased in another state. (www.taxadmin.org/fta/rate/sales.html).

Problem 3 – Mental math and estimating

The student worksheet gives an example of estimating 15% of an amount. They will use estimation or mental math to solve three other real-world tip questions.

Students should see that the estimate in the example is a relatively good estimate because it is less than 20 *cents* away from the actual amount.

If students' estimates of 15% of 17.97 is anywhere near $2.70 , they have estimated correctly.

To estimate 20% on a bill, students should round up to the nearest ten and then multiply the first number by 2 .

Two ways to determine 4% tax on $1000 :

a) $4 \cdot 1000 = 4000 \rightarrow 1\%$ of $4000 = 40$

b) 1% of $1000 = 10 \rightarrow 10 \cdot 4 = 40$

Extension

To solve the problem, students should multiply the tax rate by 2 , using 15% instead of 7.25% to estimate the tip.

In some states, like Illinois, the tax rate is 7.5% in a restaurant. If the taxed amount on a bill is doubled, the 15% tip amount will be calculated. This also avoids what some people dread – "tipping on the tax."

CHAPTER 5 TE Graphs of Equations and Functions - TI

CHAPTER OUTLINE

5.1 Points Lines Slopes

This activity is intended to supplement Algebra I, Chapter 4, Lesson 3.

ID: 8891

Time required: 45 minutes

Activity Overview

In this activity students will explore the relationship between coordinates of points and locations on the coordinate plane, the relationships of lines with their equations, slopes and $y-$ intercepts, and lastly, the slopes of parallel and perpendicular lines.

Topic: Linear Functions

- *Graph an equation of the form $y = mx$ and describe how the value of m changes as the graph is rotated.*
- *Graph an equation of the form $y = mx + b$ and describe how b changes under vertical translations.*
- *Prove that lines with equal slope are parallel.*
- *Graph lines whose slopes are negative reciprocals.*
- *Deduce that the slopes of perpendicular lines are negative reciprocals.*

Teacher Preparation and Notes

- *This investigation offers many possible extensions depending on the level of the student. The activity can be used in the middle grades and even in later elementary grades.*
- *This activity can review coordinates and then go on to introduce or review the relationships of points, lines, slopes, and equations. The graph could also be divided into different zones by graphing $f(x) = x$ and $f(x) = -x$ and having students generalize the slope for each of these zones.*
- *After graphing lines and looking at relationships, another point could be placed on the plane and students could look at the coordinates of the point with respect to the line. This could be used to introduce inequalities.*
- ***To download Cabri Jr., PARALLEL.8xv, and PERPENDI.8xv, go to http://www.education.ti.com/calculators/downloads and select each file. If your calculator already has the Cabri Jr. application, you may still need to download PARALLEL.8xv and PERPENDI.8xv.***

Associated Materials

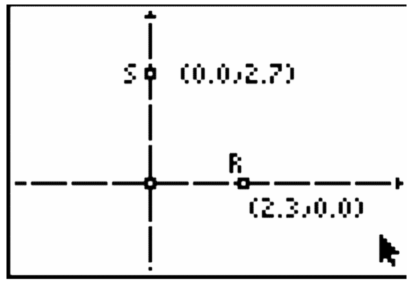
- Student Worksheet: Points #38; Lines #38; Slopes, http://www.ck12.org/flexr/chapter/9614
- PARALLEL and PERPENDI (Cabri Jr. files)

Three focus questions define this activity (Some discussion should follow the posing of these questions):

1. *What are the relationships of the coordinates of points with various locations in the Cartesian plane?*
2. *What is the relationship of a line with its slope, equation, and $y-$ intercept?*
3. *What is the relationship between the slopes of parallel or perpendicular lines?*

Problem 1 – Coordinates of points

Students are to open a new Cabri Jr. file. If the axes are not displayed, they should select **F5: Appearance #62; Hide/Show #62; Axes**.



Step 1 : Students will use the **Point on** tool (**F2: Creation #62; Point #62; Point on**) to place and label a point R on the $x-$ axis and a point S on the $y-$ axis. (The labels P and Q may be typed by choosing the **Alpha-Num** tool from the **F5: Appearance** menu directly after placing each point, pressing **ENTER**, and typing the letter.) Then have students label the coordinates of each point, by selecting **F5: Appearance #62; Coord. #38; Eq**.

Have students now drag point R and answer questions from the worksheet about $x-$ intercepts. (This foreshadows two important concepts later: $x-$ intercepts of graphs and equations of vertical lines.) To drag a point, move the cursor over it and press **ALPHA**. Then use the arrow keys to move the point. Press **ALPHA** to let the point go.

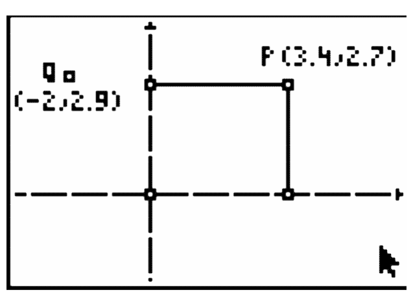
Next, have them drag point S and answer questions about $y-$ intercepts. (This foreshadows the concept of the $y-$ intercept of a graph.)

Step 2: Instruct students to delete points R and S (move the cursor over each point and press **DEL**) and place and label points P and Q anywhere in the coordinate system using the **Point** tool from the **F2: Creation** menu. Then have students label the coordinates of each point, by selecting **F5: Appearance #62; Coord. #38; Eq**.

Step 3: Have students once again drag points P and Q around the plane and answer the questions on their worksheet about the sign of the $x-$ and $y-$ coordinates in each quadrant.

Step 4: Students should now be able to make sense of exactly what the coordinates should be in different quadrants as well as on each axis. Give students the coordinates of a point and ask which quadrant it is in and visa versa. This simple activity is good for students to develop their technology skills and establish sign patterns for the four quadrants.

Step 5: Have students use the **Perpendicular** tool (**F3: Construction #62; Perp.**) to construct perpendicular lines through point P to each axis. After the perpendicular lines are in place, direct students to construct segments from point P to each axis (**F2: Creation #62; Segment**) and then hide the "excess" perpendicular lines using the **Hide/Show** tool, available from **F5: Appearance #62; Hide/Show**.



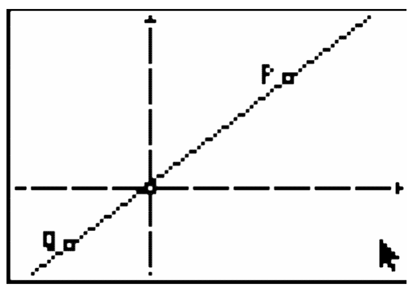
Note: When using the **Segment** tool, to select the intersection point of the perpendicular line and the axis, both of these lines must flash and then students should press **ENTER**.

Step 6: Now have students measure the length of each segment using the **D. #38; Length** tool from the **F5: Appearance #62; Measure** menu. Drag point P and have students conjecture about the distances and the coordinates. (There is a one-to-one correspondence between the distances from P to each axis and the absolute value of the coordinates of P .)

Problem 2 – Lines, equations, and slopes

Step 1: Have students delete or hide the coordinates, segments, and measurements. Only points P and Q with their labels should appear.

If this is students' first introduction to slope, provide a brief description of the concept, including "rise over run." Ask students to visualize a line between points P and Q and conjecture as to whether the slope is positive or negative, small or large, etc. Then have them draw a line through points P and Q using the **Line** tool from the **F2: Creation** menu.



Step 2: Next, have students find the slope (**F5: Appearance #62; Measure #62; Slope**) and equation (**F5: Appearance #62; Coord. #38; Eq.**) of the line. Have them label the slope measurement as shown at right by choosing **Alph-Num** from the **F5: Appearance** menu and typing **SLOPE:** in the appropriate place.

Step 3: Now students can investigate the relationship of the line with its slope and equation. First have students drag point P or Q and notice what changes and what does not. Next drag by the line itself and observe what changes. Students should also drag point Q to the $y-$ axis to investigate the relationship between a line's $y-$ intercept and its equation.

Problem 3 – Slopes of parallel and perpendicular lines

Step 1: In the Cabri Jr. file **PARALLEL**, students are given two parallel lines, their equations, and their slopes. (This construction is done for them. An alternative would be for the students to construct the parallel lines, show the slopes, and explore.)

Students should explore the relationships of parallel lines and slopes while they drag the original line by one of its defining points, P or Q .

Step 2: In the Cabri Jr. file **PERPENDI**, students are shown two perpendicular lines, their equations, and their slopes. Again, students could be led through this construction, if desired.

Encourage students to drag the lines around and try to identify the relationship between their slopes.

Step 3: Another way to look at the relationship of the slopes is to find the product of the two slopes. Have students select **F5: Appearance #62; Calculate**. Next, they should position the cursor over the first slope, and press **ENTER**. Students should then press $\times$ to specify the operation and move the cursor to the second slope and press **ENTER** again. This will yield the product of the two slopes—have students drag it to a convenient location on the screen.

Finally, have students drag the line through points P or Q by grabbing and dragging either point. Encourage them to notice what happens to the product as the lines change!

Extensions

As stated in the teacher preparation, there are many extensions to this activity depending on the level of the student. This activity can easily be manipulated to lead students into a deeper study of slopes, parallel lines, and proofs.

5.2 Math Man on the Slopes

This activity is intended to supplement Algebra I, Chapter 4, Lesson 4.

ID: 11748

Time required: 15-30 minutes

Activity Overview

In this activity, students will practice identifying slopes with informal pictures, and can self-check their understanding with one of the measurement tools. The students will also identify the slope and intercept of a given graph and will choose the correct equation in a multiple choice format.

Topic: Slope and Slope-intercept

- *Visual ideas about slope*
- *Finding the intercept*
- *Determining the slope*
- *Writing an equation from a line*

Teacher Preparation and Notes

- *Teacher preparation should include instruction about the basics of identifying positive and negative slopes, as well as intercepts. The students will need to have exposure and practice with the form $y = mx + b$, but can also gain confidence through self assessment items embedded in the file.*
- *Before beginning the activity, students should clear all lists and turn off functions. To clear the lists, press* **STAT** [Edit] *and scroll down until the arrow is in front of* ***ClrAllLists***. *Press enter twice. To clear any functions, press $Y =$ and then press* **CLEAR** *when the cursor is next to each $Y =$ equation.*
- ***To download Cabri Jr., MATHMAN.8xv, DIPPER.8xv, and SLOPE.8xv go to http://www.education.ti.com/calculators/d and select each file. If your calculator already has the Cabri Jr. application, you may still need to download the .8xv files.***

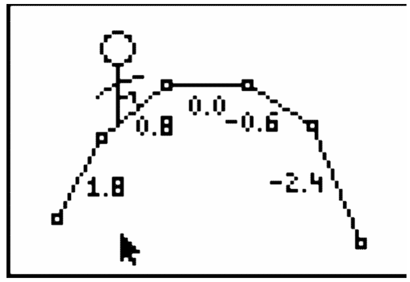
Associated Materials

- Student Worksheet: Math Man on the Slopes, http://www.ck12.org/flexr/chapter/9614, scroll down to the second activity.
- MATHMAN, DIPPER, SLOPE (Cabri Jr. files)

Problem 1 – Visually Estimating Slope

Students are introduced to the visual idea of slopes uphill and downhill as one might see when looking at a ski slope. With the Cabri Jr. file **MATHMAN** open, discuss with students where Math Man should ski. Ask questions of which lines are the steepest and have them discuss what impact this might have on his speed.

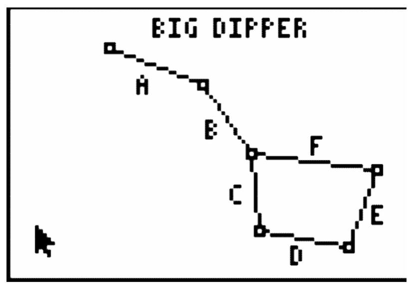
Math Man skis best on the slopes of -0.6 and -2.4 .



With the file **DIPPER**, students will see a representation of the "Big Dipper", a formation commonly recognized in the night sky. Let students work in pairs to discuss which slope best fits each segment.

Helpful hint for students:

Choose the most obvious answers first, and leave the "tricky" ones (A, D, and F) for last.



To check their answers, students can use the **Slope** tool (**GRAPH** #62; Measure #62; Slope). Move to a segment and when the arrow appears and the segment flickers, press **ENTER**.

Problem 2 – Exploring Precise Slope

Students are given a line with 2 *points* . They are to move the point at $(-2, 4)$ find a new line with one of the points $(0, 3)$ remaining the same. The new line should have a slope of $\frac{2}{3}$. They can determine the slope by either looking at the slope triangle or by counting the spaces on the $x-$ and $y-$ axes (rise over run). Each space represents one unit.

Facilitate a discussion on the methods used to determine where to move the point to make a slope of $\frac{2}{3}$. Also discuss the coordinates of the point, the $y-$ intercept, and the equation of the line. Students should understand that no matter what point they choose to make the slope $\frac{2}{3}$, the equation will remain the same.

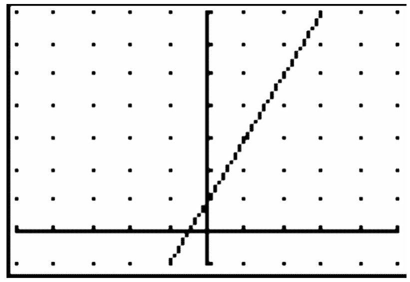
Students are to move the point at $(0, 3)$ to $(1, 0)$. They are to move the other point so the line has a slope of 1 .

Ask students if their method of determining where to place the point changed. Challenge students to find another point that will give the same line.

Problem 3 – Slope-Intercept Equation

This problem is intended for students to tie everything, slope, $y-$ intercept, and equation, together.

The first question asks students to identify the slope of the line. Students can use the grid to count the slope triangle in the form of "rise over run" or they can mentally use the formula for slope since the points are given on the worksheet.



The second question asks students to identify the $y-$ intercept of the line. They may need to be reminded that a $y-$ intercept is on the vertical $y-$ axis, and its ordered pair would be $(0, \ b)$.

Students are to use both pieces of information to determine the slope-intercept form of the equation of the line.

Problem 4 – Assessing Understanding

Several questions are given on the student worksheet to assess the understanding of students on slope-intercept form.

5.3 Trains in Motion

This activity is intended to supplement Algebra I, Chapter 4, Section 8.

ID: 11194

Time required: 15 minutes

Activity Overview

In this activity, students will make observations about the motion of two objects. They will compare and contrast this motion and consider how it corresponds to a graph representing distance as a function of time. As an extension, students will explore the relationship between the slope and the rate.

Topic: Linear Equations #38; Functions

- *Motion, distance = rate · time*
- *Slope, graphically expressing data*

Teacher Preparation and Notes

- *This activity can serve as a nice introduction to using formulas and graphs to represent a real-life situation.*
- *The student worksheet provides instructions and questions to guide the inquiry and focus the observations.*
- *Be sure to add the program to each student's graphing calculator.*
- ***To download the TRAINS program go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=11 and select "TRAINS.8xp" to download.***

Associated Materials

- Student Worksheet: Trains in Motion, http://www.ck12.org/flexr/chapter/9614, scroll down to the third activity.
- TRAINS (program)

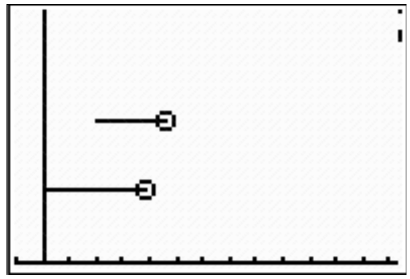
In this activity, students use the program **TRAINS** to view two graphs. Make sure that students have loaded the program before beginning the activity. The first graph is of two trains leaving a train station. The second is the distance-time graph of this motion.

Problem 1 – Observe Motion

After selecting the first option of the program **OBSERVE MOTION,** students will see two lines and a circle that travels each line. The circle represents the trains. Explain to students that even though the lines (tracks) stop, in the real world the train tracks would continue on. Also, students should pretend that there is track connecting the first train to the station.

When students press **TRACE**, three values will appear at the bottom of the screen. The $T-$ **value** represents time and the $X-$ **value** represents distance. Students can disregard the $Y-$ value.

Students can press 2^{nd} **[MODE]** to return to the Home screen after answering the questions on the worksheet. Pressing **ENTER** right away will return them to the program menu.



Answers to the worksheet questions.

1. Answers will vary, but the following question should complement their answers.

2. When $t = 0$, train 1 is at 80 km .

3. Train 2 starts at the origin.

4. Train 2 is moving faster.

5. Train 2 is traveling at 120 km/hr .

6. Train 1 traveled 80 km in one hour.

7. Train 1 's speed is 80 $km/hr.$

8. They are at the same distance from the station at 240 km .

9. This occurs at 2 $hours$.

Problem 2 – Distance-Time Graph

After selecting the second option of the program **DIST VS TIME**, students will see two lines will positive slopes, which intersect each other.

When students press **TRACE**, two values will appear at the bottom of the screen. The $X-$ **value** represents time and the $Y-$ **value** represents distance. In the upper right corner of the screen, $P1$ or $P2$ will appear. Students will determine that $P1$ represents train 1 and $P2$ represents $P2$.

Students should understand that the intersection point of the two lines is the time at which the trains are equal distance from the station.

Pressing 2^{nd} **[MODE]** will return students to the Home screen.

Answers to the worksheet questions.

10. Train 2 has a steeper slope.

11. The slope is the speed or *rate* of change of distance.

12. a) $y-$ intercept of train 1 is 80 .

b) $y-$ intercept of train 2 is zero.

13. This indicates their starting location.

14. train 1 : $d = 80t + 80$, train 2 : $d = 120t$

Also acceptable would be $y = 80x + 80$ and $y = 120x$

15. $80t + 80 = 120t \rightarrow 80 = 40t \rightarrow t = 2\ hr$

Extension: Problem 2 – List of $d = r \cdot t$ Data

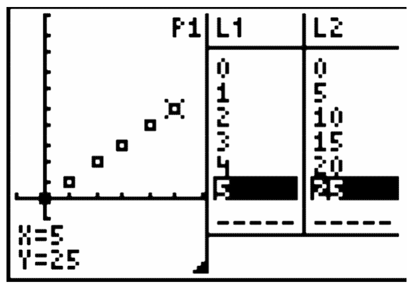
This extension introduces a numeric and graphical exploration. Students begin by determining what the next time and distance values would be in the list.

16. 5 hr , 25 km/hr

Students then are instructed to go to the data lists and change the data depending upon a different r value. They are instructed to observe both the graph and the data when the r value is changed.

17. When r gets larger, the slope gets steeper.

When r decreases, the slope is less steep.



CHAPTER 6 TE Writing Linear Equations - TI

CHAPTER OUTLINE

6.1 Perpendicular Slopes

This activity is intended to supplement Algebra I, Chapter 5, Lesson 4.

ID: 8973

Time required: 45 minutes

Topic: Linear Functions

- *Graph lines whose slopes are negative reciprocals and measure the angles to verify they are perpendicular.*

Activity Overview

In this activity, students investigate the "negative reciprocal" relationship between the slopes of perpendicular lines. The final phase of the activity is appropriate for more advanced students as they are led through an algebraic proof of the relationship. Optional geometric activities (Problems 5 and 6) use the result to verify that (1) the radius of a circle and its tangent line are perpendicular and (2) a triangle inscribed in a circle with the diameter as one side is a right triangle.

Teacher Preparation

This activity is appropriate for students in Algebra 1. It is assumed that students have recently been introduced to the notion of slope and perhaps the fact that two lines are parallel if and only if they have the same slope.

- ***To download Cabri Jr. and its files, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=89 and select Cabri Jr., PERP1.8xv, PERP2.8xv, PERP3.8xv, PERP4.8xv, PERP5.8xv, and PERP6.8xv. If your calculator already has the Cabri Jr. application, you may still need to download the .8xv files".***

Classroom Management

- *This activity is designed to have students explore* ***individually and in pairs.*** *However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on the slope of perpendicular lines.*
- *This worksheet is intended to guide students through the main ideas of the activity. You may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *Information for an optional extension is provided at the end of this activity; information for students is provided in the CabriJr Files* ***PERP5*** *and* ***PERP6.***

Associated Materials

- Student Worksheet: Perpendicular Slopes, http://www.ck12.org/flexr/chapter/9615
- Cabri Jr. Application
- PERP1, PERP2, PERP 3, PERP4, PERP5, PERP6 (Cabri Jr. files)

See the student worksheet for specific directions on how to complete the activity.

6.2 Exploring Linear Equations

This activity is intended to supplement Algebra I, Chapter 5, Lesson 5.

ID: 8189

Time required: 30 minutes

Topic: Functions #38; Relations

- *Graph a set of ordered pairs as a scatter plot.*
- *Given two points on a line, calculate the slope and write the equation of the line.*

Activity Overview

In this activity we will

- *Enter "life expectancy" data into lists and set up scatter plots.*
- *Trace the scatter plot to select two points. Use the points to calculate slope and write a linear equation.*
- *Use the Transformation Graphing APP to fit the data using a linear equation in slope-intercept form and analyze the meaning of the slope and the intercept in relationship to birth year and life expectancy.*

Teacher Preparation

This activity is designed to be used in a middle school or high school Algebra 1 classroom.

- *Students should already be familiar with the meaning of slope and how to find the slope of a line given two points on the line (the ratio: difference of $y-$ values and difference of $x-$ values).*

- ***To download the Transformation Graphing Application, go to http://www.education.ti.com/calculators/downloads/US/A and select Transformation Graphing under the Applications header.***

Classroom Management

- *This activity is intended to be mainly* ***teacher-led****, with breaks for individual student work. Use the Student Worksheet to present the material to the class and encourage discussion. Students will follow along using their graphing calculators.*
- *The student worksheet is intended to guide students through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

Associated Materials

- Student Worksheet: Exploring Linear Equations, http://www.ck12.org/flexr/chapter/9615, scroll down to the second activity.
- Transformation Graphing Application

See the student worksheet on specific directions on how to complete this activity.

6.3 Finding a Line of Best Fit

This activity is intended to supplement Algebra I, Chapter 5, Lesson 5.

ID: 8192

Time required: 35 minutes

Topic: Data Analysis #38; Probability

- *Represent and interpret data displayed in line graphs and scatterplots.*
- *Display univariate data in a spreadsheet or table and determine the mean, mode, standard deviation, extrema and quartiles.*

Activity Overview

Students make a scatter plot of heart rate versus age data and draw lines of best fit using three different methods—by hand, using the upper and lower quartiles, and using the calculator's regression feature.

Teacher Preparation

- *This activity is appropriate for an Algebra 1 classroom. Students should have experience with sequences of positive exponents.*

Classroom Management

- *This activity is intended to be* ***teacher-led*** *with students in* ***small groups.*** *You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the Student Worksheet to present the material to the class and encourage discussion.*
- *The student worksheet is intended to guide students through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

Associated Materials

- Student Worksheet: Exploring Linear Equations, http://www.ck12.org/flexr/chapter/9615, scroll down to the third activity.

See the student worksheet on specific directions on how to complete this activity.

CHAPTER 7 TE Graphing Linear Inequalities - TI

CHAPTER OUTLINE

7.1 Linear Inequalities

This activity is intended to supplement Algebra I, Chapter 6, Lesson 3.

ID: 8773

Time required: 60 minutes

Activity Overview

In this activity, students first look at tables of values to see that inequalities (in a single variable) are true for some values of the variable and not for others. They are guided to connect "true" with a value of 1 and "false" with a value of 0, creating a bridge from the table of values to a graph of the linear inequality. In Problem 2, students solve one-step linear inequalities using addition or subtraction and compare the graph of the original inequality with that of the simplified form, finding they are the same. In Problem 3, students perform a similar exploration solving one step linear inequalities using multiplication or division.

Topic: Linear Inequalities

- *Graph a simple linear inequality of the form* $x < a,\ x \leq a,\ x > a$ *, or* $x \geq a$ *on the real line.*
- *Use addition or subtraction to solve a "one step" linear inequality in a single variable.*
- *Use multiplication or division to solve a "one step" linear inequality in a single variable.*

Teacher Preparation and Notes

- *This activity is designed for use in an Algebra 1 or Pre–Algebra classroom.*
- *Prior to beginning the activity, students should download the* ***LINEQUA*** *program to their calculators. They should also have some experience solving simple one-step and multiple-step linear equations.*
- *This activity is intended to be a combination of* ***teacher-led*** *and* ***student-centered*** *activity. It is recommended that you introduce each problem by guiding students through the step-by-step instruction in a whole-class setting, and then allow them to complete the exercises individually or in small groups with your assistance.*
- *If time constraints prevent you from completing the activity in one class period, you may choose to complete Problem 1 one day and Problems 2 and 3 the next day.*
- ***To download the calculator program, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id= and select "LINEQUA.8xp".***

Associated Materials

- Student Worksheet: Linear Inequalities, http://www.ck12.org/flexr/chapter/9616
- LINEQUA.8xp (program)

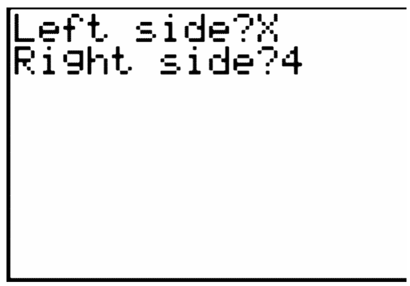
Introduction

An inequality is a mathematical sentence that shows the relationship between two quantities using these signs: $>, \geq, <, \leq$, or $\neq$.

Solving inequalities in a single variable is similar to solving equations in a single variable, but there are some important differences. One big difference occurs when you multiply or divide by a negative number. In this activity, you will practice solving linear inequalities and explore these differences using algebraic and graphing techniques.

Problem 1 – Table of Values

First students will use the **LINEQUA** program to explore the table of values of a simple inequality: $x \geq 4$. Directions are given on the student worksheet for how to enter the inequality. Students will enter the left side, enter the right side, select a symbol, and choose to view the table. They can either press the number associated with each choice or use the arrow keys to move to the choice and press enter.

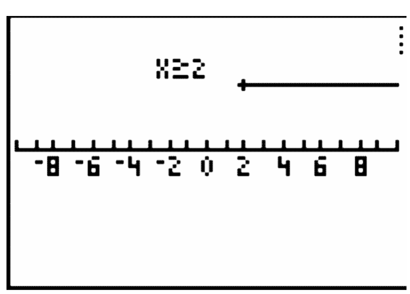


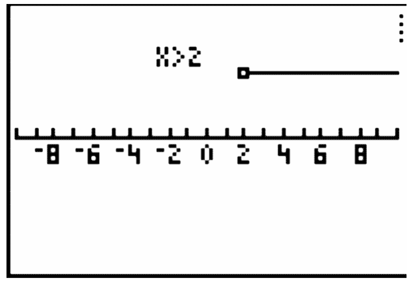
When students first view the table, they will only see X, $Y1$, and $Y2$. The first column X shows the values of the variable, x. The second column, labeled $Y1$, shows the value of the left side for each $x-$ value. The third column, labeled $Y2$, shows the value of the right side for each $x-$ value.

Students can use the right arrow key to move to the fourth column $Y3$. Each entry in this column is either a 1 or a 0 . After investigating the second inequality $x < -2$, students should realize that a 0 represents when the inequality is **false**, and a 1 represents when the inequality is **true**.

Problem 2 – Graphing

Students are to graph the inequalities $x > 2$ and $x \geq 2$ one at a time, and then describe the differences they see. This is a preliminary investigation into open and closed circles.

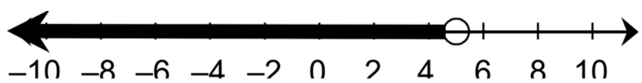




Examples

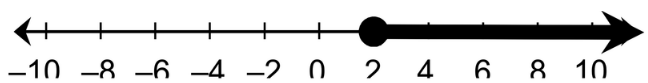
The following examples show how to sketch the graph of an inequality on paper. Explain the two inequalities to students before doing the exercises.

$x < 5$



The open circle at 5 means that $x = 5$ is not a solution to the inequality. Since x is less than 5 , shade to the left of the circle.

$w \geq 4$



The closed circle at 2 means that $x = 2$ is a solution to the inequality. Since w is greater than or equal to 5 , shade to the right of the circle.

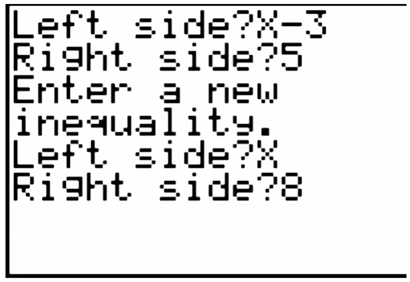
Problem 3 – Solving inequalities using addition and subtraction

Equivalent inequalities are inequalities with the same solutions. For example, $x - 3 > 5$ and $x > 8$ are equivalent inequalities. Explain to students that they can add or subtract the same number from both sides of an inequality without affecting the solutions, just as they can do with equations.

Examples

$$\begin{aligned} x-3 &> 5 \\ x-3+3 &> 5+3 \\ x &> 8 \end{aligned} \qquad \begin{aligned} x+6 &\leq 10 \\ x+6-6 &\leq 10-6 \\ x &\leq 4 \end{aligned}$$

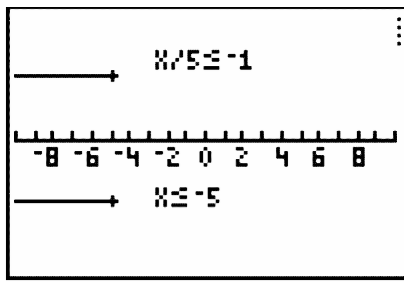
Students will compare the starting inequality and the final inequality using the Compare Ineq choice in the **LINEQUA** program. The first inequality entered will appear in the top half of the screen. The second inequality entered will appear in the bottom half of the screen.

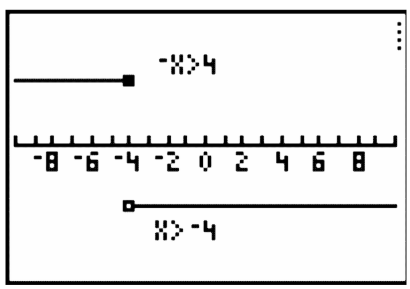


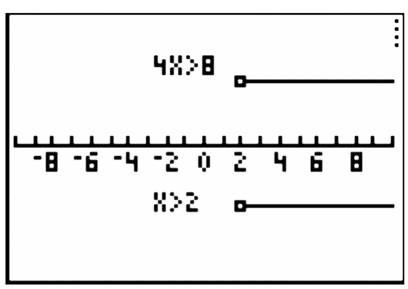
Caution: In some graphs, the open circle will appear to be filled in. This is because of the size of the pixels on the graph screen. For this reason, a "closed circle" is shown as a cross, and an "open circle" as a dot.

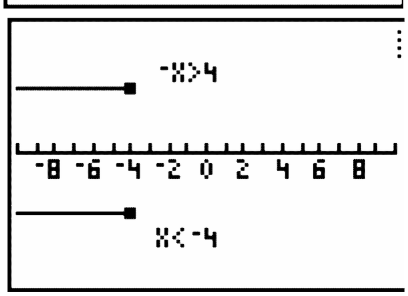
Problem 4 – Solving inequalities using mulitplication and division

Students are to graph four different pairs of inequalities that involve multiplication or division. This will lead them an investigation of when the inequality symbol is flipped or reversed.









Solutions

Problem 1

1. The numbers in the $Y1$ column are the same as the $x-$ values because the left side of the inequality is x .

2. The numbers in the $Y2$ column are not affected by the $x-$ values, because the right side of the inequality is a constant, 4 .

3. For all of the $x-$ values greater than or equal to 4

4. Sample: $x = 5 \rightarrow x \geq 4 \rightarrow 5 \geq 4$; yes

5. For all of the $x-$ values less than 4 .

6. Sample: $x = 3 \rightarrow x \geq 4 \rightarrow 3 \geq 4$; no

7. For all $x-$ values less than -2 .

8. Sample: $x = -4 \rightarrow x < -2 \rightarrow -4 < -2$; yes

9. For all $x-$ values greater than or equal to -2 .

10. Sample: $x = -1 \rightarrow x < -2 \rightarrow -1 < -2$; no

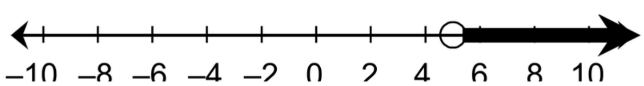
11. a) 1

b) 0

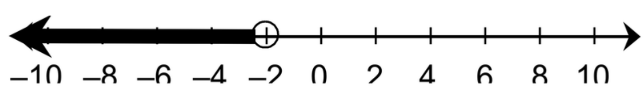
Problem 2

12. The graphs of $x > 2$ has an open circle at $x = 2$, and the graph of and $x \geq 2$ has a cross at $x = 2$.

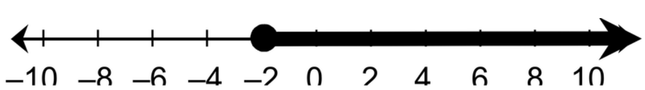
13. a) $t > 5$



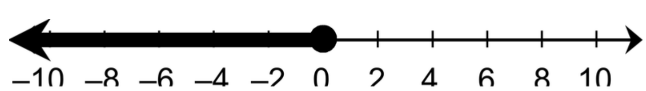
b) $p < -2$



c) $z \geq -2$

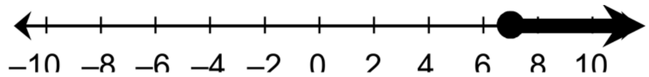


d) $y \leq 0$

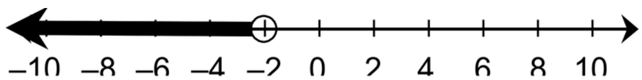


Problem 3

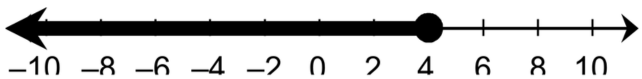
14. a) $f \geq 7$



b) $-2 > g$

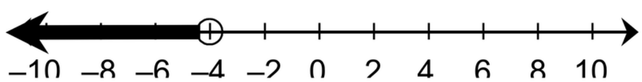


c) $u \leq 4$

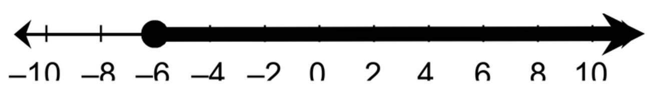


d) $-7 < v$

e) $-4 > h$



f) $-6 \leq t$



<u>Problem 4</u>

15. a) yes; they have the same solutions

b) yes

16. a) yes; they have the same solutions

b) yes

17. a) no; they do not have the same solutions

b) no

18. a) yes; they have the same solutions

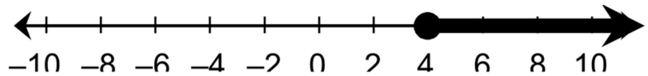
19. a) $\leq$

b) $>$

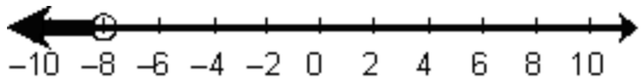
c) $<$

d) $\geq$

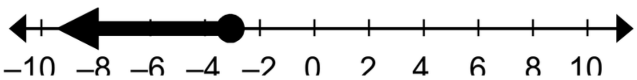
20. a) $c \geq 4$



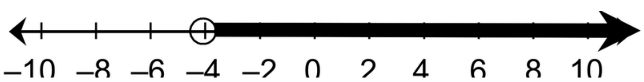
b) $-8 > d$



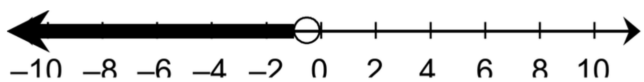
c) $w \leq -3$



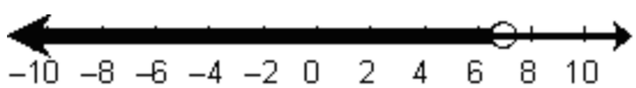
d) $-4 < x$



e) $d < -\frac{2}{3}$



f) $g < 7$



7.2 Introduction to Absolute Value Function

This activity is intended to supplement Algebra I, Chapter 6, Lesson 5.

ID: 8196

Time required: 30 minutes

Topic: Linear Equations

- *Solve a linear equation involving absolute value.*

Activity Overview

Students will examine data by comparing individual data points to the mean by finding the difference (positive or negative) and the distance from the mean, plot the distances versus the differences to examine the shape of the plot, investigate the absolute value function in the $Y =$ *register to model the relationship between the distances and the differences, and extend the investigation of absolute value equations by examining tables and graphs.*

Teacher Preparation

- *This activity is designed to be used in an Algebra 1 classroom.*
- *Prior to beginning this activity, students should have experience graphing linear functions and analyzing simple graphs and function tables.*

Classroom Management

- *This activity is intended to be mainly* ***student-centered,*** *with breaks for whole-class discussion following each problem. The student worksheet guides students through the activity and provides a place for students to record their answers.*
- *Information for an optional extension is provided at the end of this activity on the student worksheet. Should you not wish students to complete the extension, you may have students disregard that portion of the student worksheet.*
- *Prior to beginning the activity, students should clear out any functions from the* $Y =$ *screen and any data from the Lists.*

Associated Materials

- Student Worksheet: Introducing the Absolute Value Function, http://www.ck12.org/flexr/chapter/9616, scroll down to the second activity.

See the student worksheet on how to complete this activity.

7.3 Can I Graph You, Too?

This activity is intended to supplement Algebra I, Chapter 6, Lesson 6.

ID: 11479

Time Required: 30 minutes

Activity Overview

In this activity, students will explore absolute value inequalities graphically, numerically, and algebraically. They will rewrite absolute value inequalities as compound inequalities without absolute value and solve.

Topic: Absolute Value Inequalities

- *Compound Inequalities*
- *Disjunctions and Conjunctions*

Teacher Preparation and Notes

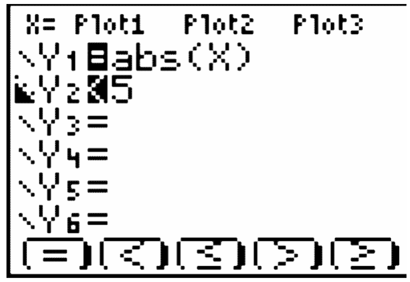
- *Students should know how to graph linear inequalities by shading the appropriate half-plane. They should also know how to graph a linear absolute value function which produces a "v" shape.*
- *Teachers may want to give more guidance regarding isolating the absolute value expression on the left-hand side of the inequality before writing the disjunction or the conjunction.*
- *Students will be using the Inequality Graphing App in this activity.*
- ***To download the calculator program, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id= and select "Inequality Graphing" under the Applications header.***

Associated Materials

- Student Worksheet: Can I Graph You, Too?, http://www.ck12.org/flexr/chapter/9616, scroll down to the third activity.
- Inequality Graphing Application

Introduction to Disjunction and Conjunction

In this activity, students will explore absolute value inequalities graphically and numerically. **Example 1** describes how students would graphically solve the equation $|x| < 5$. This may be a different approach than your students have seen, but it helps them visualize what is going on.



Teaching Notes:

You may want to further explain the terms conjunction and disjunction.

Students should notice that the solution is an interval, not (x, y) points.

When using the **Inequalz** app, students can select the symbol that they need by pressing a and then the key below where the symbol is shown. This will automatically shade the graph and display the horizontal line as dashed or solid.

Challenge the students to graph the inequality on a piece of paper and to draw a segment on the $x-$ axis representing the solution, like they would on a number line.

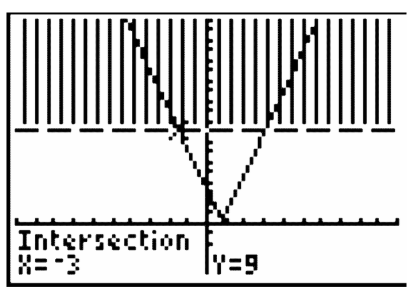
Make sure that they understand that for a disjunction, the solution goes to negative and positive infinity.

Application of Disjunction and Conjunction

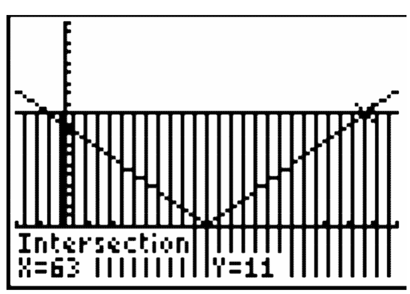
Students will use what they've learned to solve inequalities graphically and algebraically. The absolute values are by themselves for the first two problems, but for the second two problems, students will need to get the absolute value by itself on the left hand side before writing as a conjunction or disjunction.

When confirming their answer by graphing, students should graph the left side and right side as they appear originally, not after getting the absolute value by itself.

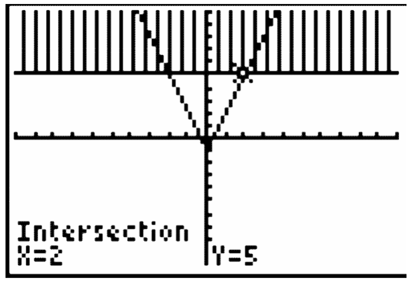
The graphs and the resulting answers appear to the right and on the next page.



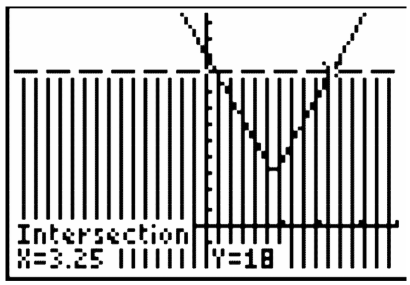
Problem 1: $x < -3$ or $x > 6$



Problem 2: $-3 \leq x \leq 63$



Problem 3: $x \leq -2$ or $x \geq 2$



Problem 4: $0.25 < x < 3.25$

Real World Application

Students are introduced to the idea of engineering tolerance and how absolute value inequalities are used to express this concept. They are asked to express the given bolt and hole tolerances as absolute value inequalities.

To write these inequalities, students will have to find the median of the two values and distance from the median to an end value.

Student Answers

bolt: $|x - 9.9825| \leq 0.0175$

hole: $|x - 10.0625| \leq 0.0125$

Chapter 8 TE Solving Systems of Equations and Inequalities - TI

Chapter Outline

8.1 Boots in Motion

This activity is intended to supplement Algebra I, Chapter 7, Lesson 2.

ID:11237

Time Required: 15 minutes

Activity Overview

In this activity, students will explore the motion of a boat going up and down the river. They will be instructed to solve the resulting system of equations algebraically and graphically. The transformation graphing application will allow students to explore the slope of a distance-time graph. Additional problems are provided for further practice.

Topic: Linear Equations

- *Motion,* $distance = rate \times time$
- *Slope and graphically solving equation*

Teacher Preparation and Notes

- *The student worksheet provides instructions and question to guide the inquiry and focus the observations.*

Associated Materials

- Student Worksheet: Boats in Motion, http://www.ck12.org/flexr/chapter/9617.

Problem 1 – Boat Motion #38; Graphically Solve

Ask students why the boat goes faster downstream than upstream. They should know that the boat goes with the current downstream making it travel more distance in less time. When the boat goes upstream, it has to fight the current.

1. Downstream rate $= r+2$; Upstream rate $= r-2$

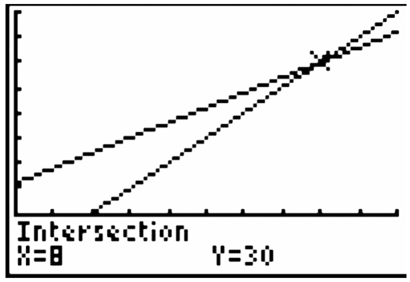
2.

$$\begin{aligned} d &= 3(r+2) \\ d &= 5(r-2) \end{aligned}$$

3.

$$\begin{aligned} 3r+6 &= 5r-10 \\ 2r &= 16 \\ r &= 8\ mph \end{aligned}$$

Substituting this into either equation gives a distance of 30 $miles$. Alton and Barnhart are about 30 $miles$ apart along the Mississippi in the St. Louis area.



Problem 2 – Distance-Time Graph, Explore Slopes

Using $d = r \cdot t$ for this situation gives the following equations:

$$1.1 = (A+B)2$$
$$0.9 = (A-B)2$$

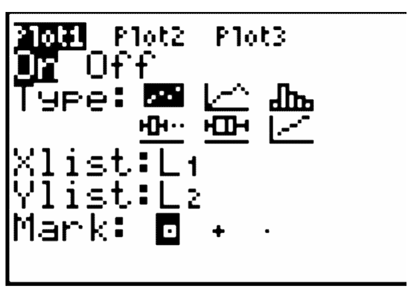
where A is the rate (speed) of the steam engine and B is the rate (velocity) of Velma's walking.

Students should set up lists $L1$ and $L2$ and the scatter plot settings as shown at right.

On the graph, students are to use the arrows keys to change the rates of the train (A) and Velma's walking (B) so that the lines go through the points $(2,1.1)$ and $(2,0.9)$. The slopes of the lines represent the rates of A and B . Using the **Transformation Graphing** App, students will only be able to graph one equation at a time, but they should see that once one equation has been graphed correctly, the other will be correct as well (no need to change the values of A or B).

To solve this system of equations, students are to distribute and then add the first equation to the second.

The solution is the train moves at 0.5 $miles/min$ and Velma walks 0.05 $miles/min$.



Extension/Homework

1.

$$D = (r+20) \cdot 3$$
$$D = (r-20)\ 3.5$$
$$\text{So } 3r+60 = 3.5r-70$$
$$0.5r = 130$$
$$r = 260\ km/h$$

So, the speed of the airplane in still air is 260 km/hr .

2. This problem can only be solved algebraically.

$$\begin{aligned} d_{slow} &= 30t \quad d_{fast} = 50t \\ 30t + 50t &= 160 \\ t &= 2 \ hours \end{aligned}$$

So, the two cars will be 160 $miles$ apart in 2 $hours$.

8.2 How Many Solutions?

This activity is intended to supplement Algebra I, Chapter 7, Lesson 5.

ID: 9283

Time required: 60 minutes

Topic: Linear Systems

- *Graph a system of linear equations to determine whether they have no solutions, one or infinitely many.*
- *Examine the coefficients of a pair of linear equations to determine how many solutions for the system.*

Activity Overview

In this activity, students graph systems of linear functions to determine the number of solutions. Once acquainted with each of the three possibilities—one solution, zero solutions and infinitely many solutions—they use their experience with the graphs to investigate the relationship between the coefficients of a pair of linear equations and the number of solutions. In the investigation, students are given one line and challenged to draw a second line that creates a system with a particular number of solutions. By repeating this experiment and recording the equations of the line, students gather data that they use to write rules about the number of solutions of a linear system based on its coefficients.

Teacher Preparation

- *This activity is appropriate for students in Algebra 1. It is assumed that students are familiar with linear functions, their graphs, and have solved linear systems algebraically.*

- ***To download Cabri Jr. and its files, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=92 and select Cabri Jr., HOWMANY1, HOWMANY2, HOWMANY3, and HOWMANY4. If your calculator already has the Cabri Jr. application, you may still need to download the .8xv files.***

Classroom Management

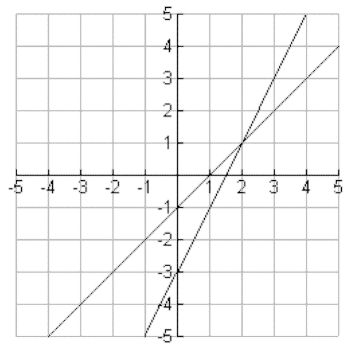
- *This activity is designed to have students explore* ***individually and in pairs****. However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on the slope of perpendicular lines.*
- *The student worksheet guides students through the main ideas of the activity. You may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

Associated Materials

- Student Worksheet: How Many Solutions?, http://www.ck12.org/flexr/chapter/9617, scroll down to the second activity.
- Cabri Jr. Application
- HOWMANY1.8xv, HOWMANY2.8xv, HOWMANY3.8xv, HOWMANY4.8xv

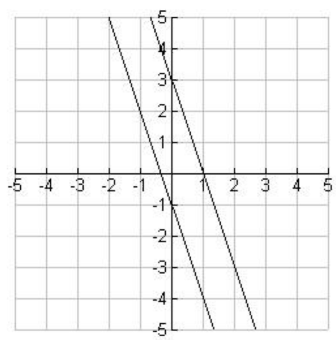
Solutions

1.



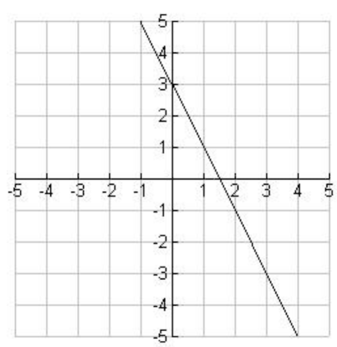
one solution

2.



no solutions

3.



infinitely many solutions

4. The equations that form systems with no solutions or infinitely many solutions have the same slope.

5. The equations that form systems with infinitely many solutions have the same $y-$ intercept.

6. The equations that form systems with infinitely many solutions are equivalent.

7. Sometimes equations are written differently although they equivalent. One equation may have been multiplied by a constant on both sides or terms moved around. For example, $\begin{cases} 4x+2y=6 \\ y=-2x+3 \end{cases}$.

8. A linear system has no solution if both equations have the same slope but different $y-$ intercepts.

A linear system has infinitely many solutions if both equations have the same slope and same $y-$ intercepts.

A linear system has one solution if the two equations have different slopes and different $y-$ intercepts.

9. one solution

10. one solution

11. no solution

12. infinitely many solutions

8.3 Testing for Truth

This activity is intended to supplement Algebra I, Chapter 7, Lesson 6.

ID: 12175

Time required: 20 minutes

Activity Overview

In this activity, students will identify whether points lie within a shaded region that is bounded by linear inequalities. The focus is on testing the points for truth in the inequality. Students will use a graph to verify their answers.

Topic: Inequalities

- *Students will visually determine the location of a point with regard to shading or the line itself. (Above, below, or on the line itself)*
- *Students will answer True or False as to whether a point satisfies multiple conditions established by linear inequalities.*

Teacher Preparation and Notes

- *Students will utilize a random number generator to create unique scenarios and answer questions about the points plotted as a result.*
- *Students will organize their thinking by using a table structure on the worksheet to answer True or False as to whether a point lies within a shaded region.*
- *Students will use the : menu to evaluate inequalities as true or false.*

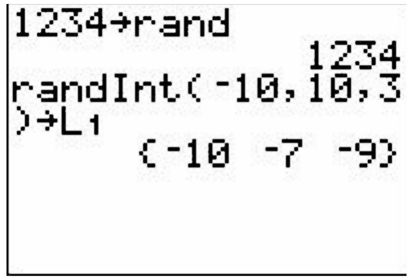
Associated Materials

- Student Worksheet: Testing for Truth, http://www.ck12.org/flexr/chapter/9617, scroll down to the third activity.

Problem 1 - Is a Point a Solution?

Students begin this activity by setting the random seed, so that students will not get the same sets of points throughout the activity. Instructions are given to use the last 4 digits of their phone numbers, but other numbers could be used, such as house or apartment number or birthday (in the form **MMDD**). Enter 1 2 3 4 **STO** $\rightarrow$ **MATH** $\leftarrow$ **ENTER ENTER** to save the random seed.

Students will then generate random numbers in L_1 and L_2 as $x-$ and $y-$ coordinates. Press **MATH** $\leftarrow 5(-)10, 10, 3$) **STO** $\rightarrow 2^{nd}$ $[L1]$ **ENTER**. Repeat replacing L_1 with L_2 to generate the $y-$ coordinates.



The table on the student worksheet will make it easier for students to organize their thinking and to show their work.

What will students write if the point lands on the line itself? On the line is not included in the shaded region for this example, since the inequality is $y > -x - 2$.

Students can also test the inequalities on the home screen to see if they are true or false. To test the given example, press 2^{nd} $[L1]$ 2^{nd} **[TEST]** 3 $(-)$ 2^{nd} $[L1] - 2$ The calculator will return either 1 for **True** or 0 for **False**. In this case, the first two coordinates do not satisfy the inequality but the third coordinate does.

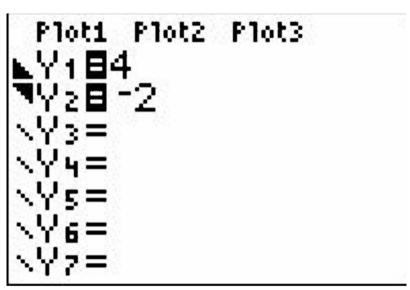
Problem 2 - Generating Solutions

Have students use the table to record their answers on the student handout.

The emphasis here can be on dotted and solid lines, and $<, \leq, >, \geq$ symbols.

Again, have students generate random numbers and store to L_1 and L_2 . Students can use the home screen to test but they can also graph the inequalities and determine if a point is a solution graphically.

The equations should look like the screen at the right. To change the “style” of the graph, move the cursor to the left of $Y =$ and press **ENTER** until you see either



Problem 3 - Overlapping Region

The students must regenerate points until at least one lies within the triangle.

The students should use the table to record their answers on the student handout.

Encourage the students to continue generating ordered pairs if they can not easily see the grid in order to record the correct numbers. If the students can’t clearly determine whether the point lands on a line, or within or outside of the shaded region, don’t use that point for simplicity of discussion.

CHAPTER 9 TE Exponential Functions - TI

CHAPTER OUTLINE

9.1 Exponent Rules

This activity is intended to supplement Algebra I, Chapter 8, Lesson 2.

ID: 9730

Time required: 30 minutes

Topic: Polynomials

- *Use technology to verify for various values of a and n that* $a^{-n} = \frac{1}{a^n}$ *where n is an integer.*
- *Evaluate simple numerical expressions raised to integral exponents (including zero exponents).*
- *Use technology to evaluate more complex numerical expressions involving exponents.*

Activity Overview

This activity allows students to work independently to discover rules for working with exponents, such as the Power of a Power rule. Students also investigate the value of a power whose exponent is zero or negative. As an optional extension, students investigate the value of a power whose exponent is a fraction with a numerator of one.

Teacher Preparation

- *This activity is designed to be used in an Algebra 1 classroom. It can also be used in a Pre-Algebra classroom, or by any student learning the rules for operating with exponents.*
- *Students should already be familiar with basic powers, exponents, and bases, such as* $2^3 = 8$.
- *While students can use the Calculator application at any time, you may wish to review the positive powers of two before beginning this activity* $(2, 4, 8, 16, 32, 64\ldots)$.
- ***To download the EXPRULES program, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id and select EXPRULES.8xp.***

Classroom Management

- *This activity is intended to be mainly* ***student-led****, with breaks for the teacher to introduce concepts or bring the class together for a group discussion. Each student should have their own graphing calculator to work with.*
- *Information for an optional extension is provided at the end of this activity. Should you not wish students to complete the extension, you may have students disregard that portion of the worksheet.*

Associated Materials

- Student Worksheet: Exponent Rules, http://www.ck12.org/flexr/chapter/9618
- EXPRULES (program)

9.2 Exponential Growth

This activity is intended to supplement Algebra I, Chapter 8, Lesson 5.

ID: 9467

Time required: 45 minutes

Activity Overview

The main objective of the activity is to find an approximation for the value of the mathematical constant e and to apply it to exponential growth and decay problems. To accomplish this, students are asked to search for the base, b , that defines a function $f(x) = b^x$ with the property that at any point on the graph, the slope of the tangent line (instantaneous rate of change) is equal to $f(x)$. The result is approximating the value of e—Euler's number and the base of the natural logarithms.

Topic: Exponential #38; Logarithmic Functions

- *Graph exponential functions of the form $f(x) = b^x$.*
- *Evaluate the exponential function $f(x) = b^x$ for any value of x .*
- *Calculate the doubling time or half-life in a problem involving exponential growth or decay.*

Teacher Preparation and Notes

- *Students encounter the exponential constant e at various levels in their mathematics schooling. It may happen well before they reach calculus, and it is often used without an appreciation for where it originates (or why it is important). A good time to use this activity is when students first encounter e, but it is also appropriate for Precalculus and Calculus students when they are studying derivatives and instantaneous rate of change.*
- *Prerequisites for the students are: familiarity with graphing and tracing functions on the calculator; an understanding of functions and function notation (both " $y =$ " and " $f(x) =$ "); and an intuitive understanding of rate of change.*

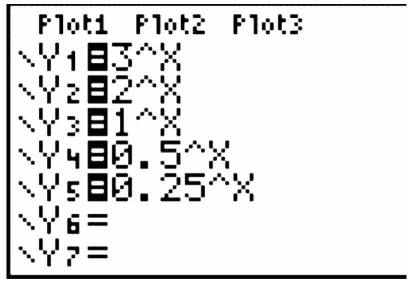
Associated Materials

- Student Worksheet: Exponential Growth, http://www.ck12.org/flexr/chapter/9618, scroll down to the second activity.

Problem 1

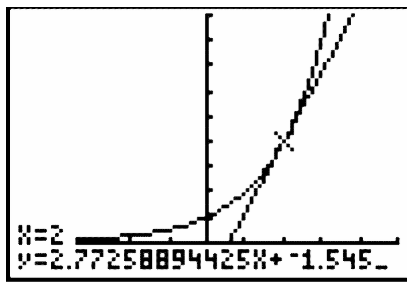
The activity begins with an investigation of how the value of b affects the shape of the graph of $y(x) = b^x$. Students will enter several equations with different values of b and examine the graphs of these functions. They are asked to make observations and draw conclusions.

The last question posed in this problem asks students to explain why the value of b cannot be negative. It may be worthwhile to discuss this in a whole-class setting.



Problem 2

In this problem, students work specifically with the graph of $y(x) = 2^x$. Students will draw a tangent line to the curve. Students will explore the relationship between the slope of the tangent line and the value of the function at that point. They should examine several different points and different values of b .



Problem 3

Students are again asked to observe the changing values of the slope of the tangent line and the value of the function—and how they are related.

Students will discover that there is *exactly* one value of b for which the slope of the tangent and value of the function are equal—and that this value is a very interesting number!

Applications

Students are given a series of application problems to apply the knowledge about what they have learned by doing completing this activity.

Student Solutions

Problem 1

- Answers may vary. Possible observations: graph gets "steeper" as b increases and "flatter" as b decreases; always passes through the point $(0,\ 1)$; increasing when $b > 1$ and decreasing when $0 < b < 1$

- $b = 1$
- Answers may vary. Possible explanation: Even roots of negative numbers are not real numbers. Consider, for example, $(-1)^{0.5} = \sqrt{-1}$, which is not a real number.

Problem 2

- x , $f(x)$, and slope will vary
- the slope is less than $f(x)$
- Answers may vary. Possible observations: slope is always positive; as x increases, the slope increases; curve never reaches the $x-$ axis

TABLE 9.1: Sample table

b	x	$f(x)$	**slope of tangent at** x
2	3	8	5.545
3	1	3	3.296
0.5	0	1	-0.693
0.25	2	0.063	-0.087

Problem 3

- Answers will vary. Possible answer: 3
- Answers will vary. Possible answer: 2
- $b \approx 2.718$
- Answers will vary.

TABLE 9.2: Sample table

b	$\frac{\text{slope of tangent at } x}{f(x)}$
2	0.693
3	1.099
0.5	-0.693
0.25	-1.381

TABLE 9.3: Sample table

b	x	$f(x)$	**slope of tangent at** x	$\frac{\text{slope of tangent at } x}{f(x)}$
2.2	0	1	0.788	0.788
2.4	1	2.4	2.101	0.875
2.6	0	1	0.956	0.956
2.8	2	7.84	8.072	1.030
2.7	0	2.7	0.993	0.993
2.75	0	2.75	1.0116	1.0116

Applications

1. Modeling equation: $P = 1,000e^{0.05t}$ (where P is the value and t is the time in years); one year: $\$1,051.27$; two years: $\$1,105.17$; five years: $\$1,284.03$

2. Modeling equation: $P = 500e^{0.5 \cdot 24}$ (where P is population); about $81,000,000$
3. Modeling equation: $P = 1,000,000e^{-0.15 \cdot 10}$ (where P is the volume); about 22.3%
4. Modeling equation for growing snowball: $P = 2e^{0.1t}$ (where P is the weight and t is the time in seconds); $10\,seconds : 5.43\,pounds\,; 20\,seconds : 14.78\,pounds\,; 45\,seconds : 180.03\,pounds\,; 1\,minute : 806.86\,pounds$

Possible limitations: the modeling equation might not be appropriate after too long a period of time, for example—the snowball may break apart if it gets too big, or it might reach the end of the hill.

CHAPTER 10 TE Factoring Polynomials - TI

CHAPTER OUTLINE

10.1 FOILed Again

This activity is intended to supplement Algebra I, Chapter 9, Lesson 2.

ID: 12431

Time required: 20 minutes

Activity Overview

In this activity, students will practice finding rectangular areas with algebraic expressions for the lengths of the sides. In the first problem, students use the distributive property and in the second problem, students use the FOIL method.

Topic: Polynomial Multiplication

- *Distribution Property*
- *FOIL method*
- *Adding together rectangular areas to find the total area of the figure*

Teacher Preparation and Notes

- *Students will practice finding areas of rectangular figures by using either the Distributive property or FOIL. Some instruction on these methods is assumed.*
- *Since the problems are presented as dimensions of rectangles, most of the numbers used (as coefficients and constants) are positive. This may make more sense to students than using negative numbers as part of those expressions. Emphasize, however, that those expressions simply represent the relationship between the two numbers that are the length and width of the given rectangle. For example, if a rectangle has sides of length* 7 *and* 12 *, the two could be described algebraically as* $4x-5$ *and* $6x-6$ *. The negative numbers in those expressions do NOT mean the rectangle has any negative lengths!*
- *In option 1 of the program AREA, students can only enter positive values for* W *. In option 2, students can only enter positive values for* A *and* C *, but can enter negative and positive values for* B *and* D *.*
- ***To download the calculator file, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=12431 and select AREA.8xp.***

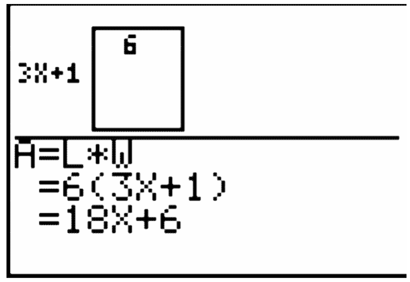
Associated Materials

- Student Worksheet: FOILed Again, http://www.ck12.org/flexr/chapter/9619
- AREA (program)

Problem 1 – Introduction to area of a rectangle

Using option 1 of the program **AREA**, students will change the width of the rectangle, and will find the change in the calculation for area. Several problems are provided on the student worksheet. Pressing **ENTER** after the calculation is shown will return students to the main menu of the program.

Students should see that when one of the sides of a rectangle is a number or a monomial, the **distributive property** can be used to simplify the expression for the area.



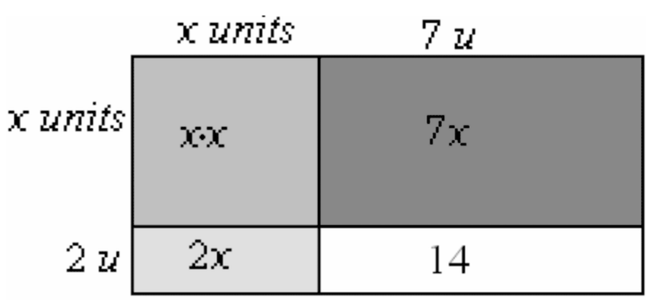
Student worksheet answers

1. $3x+1$ and 6
2. $18x+6$
3. when $w=4$, $A=12x+4$ and when $w=9$, $A=27x+9$
4. the value of the width is multiplied by the x term and the constant term of the length (distributive property)

Problem 2 – Areas of small rectangles

At the right, the rectangle is shaded so that the students will see the four different pieces that must be added together to find the total area. The sides of the entire rectangle are of length $x+7$ and $x+2$.

Students should find the area of each of the parts of the rectangle and then add them together. The whole area is $x^2+9x+14$.



Student worksheet answers

5. x^2, $2x$, $7x$, and 14
6. $x^2+9x+14$

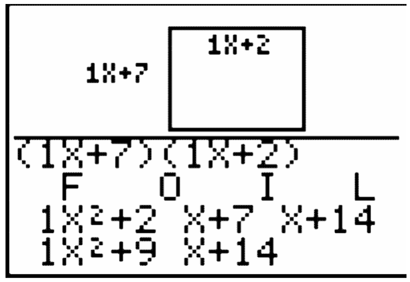
Problem 3 – FOIL method

Using option 2 from the program AREA, students will enter the coefficients and constants of the binomial dimensions.

Help students make the connection between the 4 terms shown in the middle step of the calculation and the area of the four smaller rectangles shown in Problem 2.

Pressing **ENTER** after the calculation is shown will return students to the main menu of the program.

Selecting option **3:QUIT** in the main menu will exit students out of the program and the split-screen mode.



Student worksheet answers

7. Each term under the letters F, O, I, and L is connected with one of the smaller rectangles in the diagram in Problem 2.

8. $18x^2+36x+10$

9. a. $12x^2+39x+9$

b. $7x^2+59x+24$

c. $10x^2+x-24$

Homework/Extensions

Exercise 1 presents an area problem for students to do: $(4x+2)\ (x+7)$. Students may be encouraged to show each step if necessary.

Again, students can check any binomial multiplication using option 2 of the program.

Students can enter the binomail $(2x-3)$ in the program by entering A as 2 and B as -3 .

In Exercise 2, a rectangle is shown with a trinomial as the length of one side, and a binomial for the other side. You may wish to have students use the FOIL model of showing their distributive steps. Underlining like terms is also a good idea.

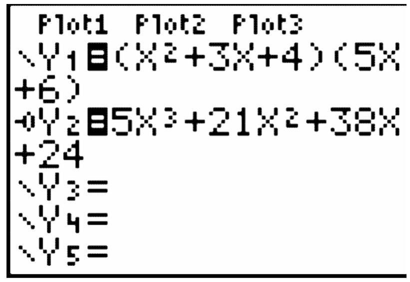
Students are to determine the formula used to find the 6 terms of the expression for area before like terms are combined. The formula or pattern is:

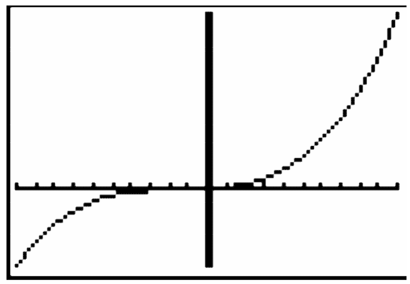
$$(ax^2+bx+c)(dx+e)=$$
$$(a\cdot d)x^3+(a\cdot e)x^2+(b\cdot d)x^2+(b\cdot e)x+(c\cdot d)x+(c\cdot e)$$

This formula invokes the use of the distributive property.

Another way to check any multiplication problem is by graphing. Students should enter the multiplication problem in $Y1$ and then enter the simplified expression in $Y2$. They need to change the graph style of $Y2$, by arrowing to the left of the $=$ sign and pressing enter until -0 appears.

Then students can press **ZOOM** and select **ZoomFit** to view the graph. Ensure they understand that two graphs that coincide are equivalent equations.





<u>*Other Possible Extension Ideas:*</u>

A rectangular prism could be introduced with algebraic expressions for length, width and height. Other geometric figures could also be created with various algebraic expressions as dimensions.

10.2 Factoring Special Cases

This activity is intended to supplement Algebra I, Chapter 9, Lesson 6.

ID: 9604

Time required: 30 minutes

Topic: Quadratic Functions and Equations

- *Express a trinomial square of the form $a^2+2ab+b^2$ as the binomial squared $(a+b)^2$.*
- *Express a difference of squares of the form x^2-a^2 as $(x-a)\ (x+a)$ and display as a difference of areas.*

Activity Overview

In this activity, students explore geometric proofs for two factoring rules: $a^2+2ab+b^2=(a+b)^2$ and $x^2-a^2=(x-a)(x+a)$. Given a set of shapes whose combined areas represent the left-hand expression, they manipulate them to create rectangles whose areas are equal to the right-hand expression.

Teacher Preparation

This activity is appropriate for students in Algebra 1. Prior to beginning this activity, students should be familiar with factoring quadratic expressions. The activity should be followed by practice applying the rules discussed.

- *This activity requires students to drag, rotate, and hide objects in CabriJr. If students are not familiar with these functions of the CabriJr, extra time should be taken to explain them.*
- ***To download Cabri Jr. and the calculator files, go to http://www.education.ti.com/calculators/downloads/US/Activities/D and select Cabri Jr, FACTOR1, FACTOR2, FACTOR3. If you already have Cabri Jr, you will still need to download the calculator files.***

Associated Materials

- Student Worksheet: Factoring Special Cases, http://www.ck12.org/flexr/chapter/9619, scroll down to the second activity.
- Cabri Jr.
- FACTOR1, FACTOR2, FACTOR3

Classroom Management

- *This activity is designed to be performed by* ***individual students*** *or* ***small groups*** *with teacher assistance. By using the computer software and the questions found in this document, you can lead an interactive class discussion on solving quadratic equations.*
- *This document guides students through the main ideas of the activity and provides a place for students to record their work. You may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

CHAPTER 11 TE Quadratic Equations and Quadratic Functions - TI

CHAPTER OUTLINE

11.1 Graphing Quadratic Equations

This activity is intended to supplement Algebra I, Chapter 10, Lesson 1.

ID: 9406

Time required: 60 minutes

Topic: Quadratic Functions #38; Equations

- *Graph a quadratic function* $y = ax^2 + bx + c$ *and display a table for integral values of the variable.*
- *Graph the equation* $y = a(x-h)^2$ *for various values of a and describe its relationship to the graph of* $y = (x-h)^2$.
- *Determine the vertex, zeros, and the equation of the axis of symmetry of the graph* $y = x^2 + k$ *and deduce the vertex, the zeros, and the equation of the axis of symmetry of the graph of* $y = a(x-h)^2 + k$

Activity Overview

In this activity, students graph quadratic functions and study how the constants in the equations compare to the coordinates of the vertices and the axes of symmetry in the graphs. The first part of the activity focuses on the vertex form, while the second part focuses on the standard form. Both activities include opportunities for students to pair up and play a graphing game to test how well they really understand the equations of quadratic functions.

Teacher Preparation

This activity is designed to be used in an Algebra 1 classroom. It can also be used as review in an Algebra 2 classroom.

- *Problem 1 introduces students to the vertex form of a quadratic equation, while Problem 2 introduces the standard form. You can modify the activity by working through only one of the problems.*
- *If you do not have a full hour to devote to the activity, work through Problem 1 on one day and then Problem 2 on the following day.*

Classroom Management

- *This activity is intended to be mainly **teacher-led,** with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*

Associated Materials

- Student Worksheet: Graphing Quadratic Equations, http://www.ck12.org/flexr/chapter/9620

Solutions

Problem 1

1. Answers will vary. Sample answer: the curve appears symmetric, and becomes less steep as x increases or decreases.
2. 2
3. 3
4. 1
5. The lowest point of the graph is to the right of the $y-$ axis.
6. The lowest point of the graph is to the left of the $y-$ axis.
7. The graph moves away from the $y-$ axis.
8. The graph moves closer to the $y-$ axis.
9. The graph will center on the $y-$ axis.
10. Answers will vary.
11. Answers will vary.
12. The $x-$ coordinates of the points are opposites of each other. The $y-$ coordinates of the points are the same.
13. The left and right points are equidistant from the $y-$ axis.
14. $x = 0$
15. The vertex is $(h,\ k)$.
16. The parabola opens up.
17. The parabola opens down.
18. The parabola becomes "narrower."
19. The parabola becomes "wider."
20. a
21. $(h,\ k)$
22. h
23. Check students graphs.
24. Check students graphs.
25. Check students graphs.

Problem 2

1. $-\frac{b}{2a}$
2. 2
3. 0
4. -4
5. 0
6. $(0,\ 4)$
7. No
8. When a is positive, the parabola opens up. When a is negative the parabola opens down. The greater the absolute value of a , the "narrower" the parabola. The smaller the absolute value of a , the "wider" the parabola.
9. c is the $y-$ intercept of the parabola
10. Check students graphs.
11. Check students graphs.
12. Check students graphs.

Solutions for the Summary

1. $y = a(x-h)^2 + k$

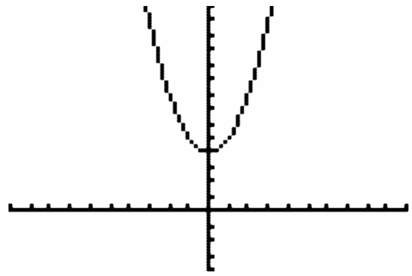
2. $a; (h,\ k); h$

3. $y = ax^2 + bx + c$

4. $-\frac{b}{2a};\ x = -\frac{b}{2a};\ c$

Sketch the graph of each function. Identify the vertex and the equation of the axis of symmetry. Then check your graphs with your calculator.

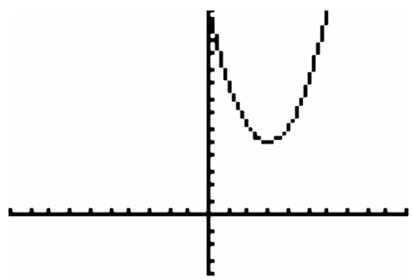
5. $y = x^2 + 4$



vertex $(0,\ 4)$

axis of symmetry $x = 0$

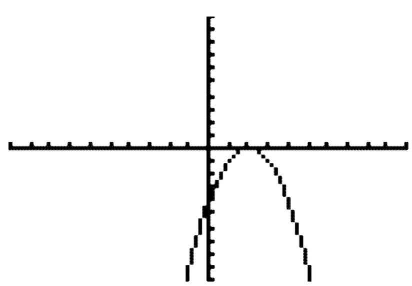
6. $y = (x-3)^2 + 5$



vertex $(3,\ 5)$

axis of symmetry $x = 3$

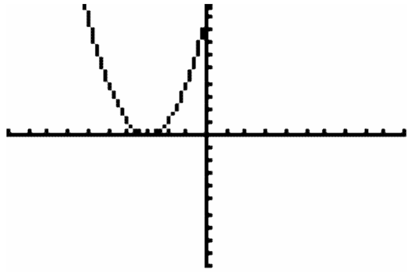
7. $y = -(x-2)^2$



vertex $(2,\ 0)$

axis of symmetry $x = 2$

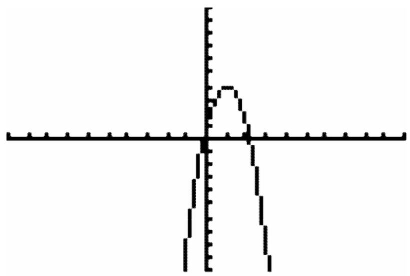
8. $y = x^2 + 6x + 9$



vertex $(-3,\ 0)$

axis of symmetry $x = -3$

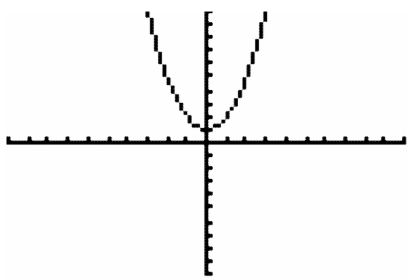
9. $y = -3x^2 + 6x + 1$



vertex $(1,\ 4)$

axis of symmetry $x = 1$

10. $y = x^2 + 1$



vertex $(0,\ 1)$

axis of symmetry $x = 0$

11.2 Area of the Missing Square

This activity is intended to supplement Algebra I, Chapter 10, Lesson 4.

ID: 11750

Time required: 40 minutes

Activity Overview

Students will be introduced to an area model for representing a quadratic equation. Students will explore the relationship between the value of b and c , in $y = x^2 + bx + c$, form of the quadratic equation. The relationship will be examined with integer and non-integer values in order to help students recognize a pattern. Students will then apply their knowledge by answering several questions using the relationship.

Topic: Quadratic Equations

- *Completing the Square*
- *Factoring*
- *Perfect Square Trinomials*

Teacher Preparation and Notes

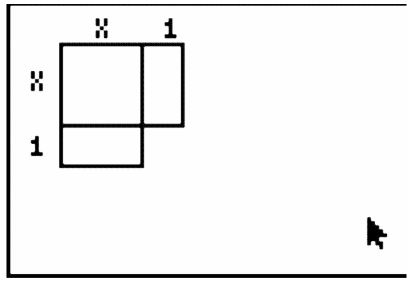
- *The teacher should be prepared to introduce the first problem of the document so students understand the "algebra tile" area representation of a quadratic equation. Teachers could use algebra tiles to introduce the activity if available.*
- *This activity explores completing the square when the coefficient of x^2 is one. The teacher should use the pattern recognition by the students to extend the activity to coefficient values other than one after the activity.*
- ***To download Cabri Jr. and the calculator file, go to http://www.education.ti.com/calculators/downloads/US/Activities/De and select the Cabri Jr Application and SQUARE.8xv. If you already have Cabri Jr. on your calculator, you may still need to download SQUARE***

Associated Materials

- Student Worksheet: Area of the Missing Square, http://www.ck12.org/flexr/chapter/9620, scroll down to the second activity.
- Cabri Jr.
- SQUARE (program)

Problem 1 – Introduction

Introduce the area model of a quadratic equation. The use of squares for x^2 and integers, and rectangles for x terms should be explained. Discuss the length of each side of the figures and the area of each figure. Also note to students that when the square is completed it can be factored into a perfect square.

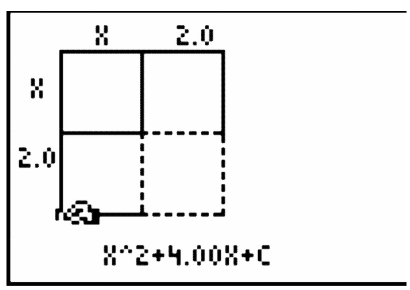


Discussion Questions:

- What is the area of each figure?
- How can the area of the missing square be found?
- How are the side lengths of each figure related?
- How does the factored form relate to the length and width of the completed square?
- How does the quadratic relate to the area of the square?

Problem 2 – Integer Lengths

In this problem, students will use the **ALPHA** key to grab the point in the lower left corner of the square to increase the length of the square. They will find the area of the square needed to complete the square. Students should observe the relationship between the coefficient of x and c , completing the table on the worksheet.



Discussion Questions:

- How does the coefficient of x in the area equation change each time the width increases by one?
- How is the area of the square or missing square related to the width?

Problem 3 – Non-integer Lengths

In this problem, students will use the **ALPHA** key to increase the length of the square to non-integers. They will find the area of the square needed to complete the square. Students should observe the relationship between the coefficient of x and c . Students will answer questions about the pattern they found.

Discussion Questions:

- How does the coefficient of x in the Area equation change each time the width increases?
- How is the area of the square in the lower left corner or missing square related to the displayed width?

- How is the coefficient of x related to the length of the square in the lower right corner?
- How is the coefficient of x related to the value of c ?
- What is a formula or method for finding the value of c without using the calculator?

Problem 4 – Applying Your Knowledge

In this problem, students will answer questions applying their knowledge for completing the square.

The discussion after reviewing the solutions to the application problems should prepare to extend the activity, at a future time, to completing the square for quadratics with coefficients of x^2 not equal to one.

Discussion Questions:

How would you complete the square if the coefficient of x^2 is not equal to one?

Solutions – student worksheet

1. A square of length and width 1 completes the square.
2. x^2+2x+1
3. The coefficient is double the length of the grey square.
4. If you take half of the coefficient and square it you get c .
5. $c=\left(\frac{b}{2}\right)^2$
6. 100
7. 49
8. 7.29
9. $\frac{25}{4}$
10. $x^2+4\sqrt{2}x+c$
11. 24
12. Add three. Half of 4 squared is 4 and we already have 1 so add three to get 4 .
13. $\left(\frac{b}{2}\right)^2$

11.3 Quadratic Formula

This activity is intended to supplement Algebra I, Chapter 10, Lesson 5.

ID: 12383

Time required: 20 minutes

Activity Overview

In this activity, students will make connections between the visual ways to find zeros of a parabola and algebraic ways. The quadratic formula is heavily emphasized in this activity, and is utilized in both a calculator program and in the Lists screen.

Topic: Quadratic Formula

- *Students will see the graph of a parabola, and identify its zeros ($x-$ intercepts).*
- *Students will solve the quadratic by using the quadratic formula, with the discriminant being calculated in a formula in lists.*
- *Students will use a program to solve the quadratic completely.*

Teacher Preparation and Notes

- *Students will need to enter a formula into a list.*
- *Students will learn to store a formula in a list. The students will then use the same formula for several problems with different values for a , b , and c $(L_1,\ L_2,\ \text{and}\ L_3)$.*
- ***To download the calculator file, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=12383 and select QUAD.8xp.***

Associated Materials

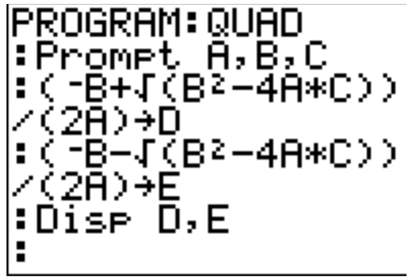
- Student Worksheet: Quadratic Formula, http://www.ck12.org/flexr/chapter/9620, scroll down to the third activity.
- QUAD (program)

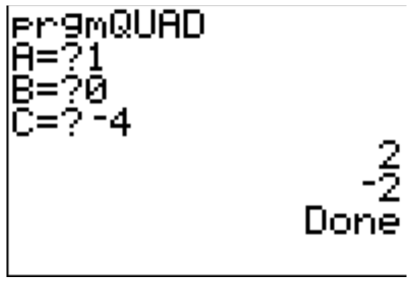
Exercise 1

Students are asked to graph the equation $y = x^2 - 4$, and identify the zeros by looking at the graph or the equation. Students may know the zero product property, and thus may be able to solve by that method on their papers.

Students can also use the **zero** command, 2^{nd} **[TRACE]**, the **CALC** function. When prompted for a "**Left Bound?**," move the cursor to the left of the zero. For "**Right Bound?**," move the cursor to the right of the zero. For "**Guess?**," move the cursor near the zero. This process will be repeated if there is more than one zero.

The program ***QUAD*** has been provided to evaluate the quadratic formula, both the positive and negative solutions. If you would prefer to use this as a teaching opportunity for programming, you can have the students enter the program.





The program can be sent to student calculators using either TI-Connect ^{TM} or TI-Navigator ^{TM} .

You can also instruct students to enter their own program following your cues.

Exercise 2

The students again graph an equation, determine the zeros, and determine the factored form of the equation $y = x^2 + x - 6$.

Now, the procedure is familiar. The ***QUAD*** program can be used and the students only needs to enter in the correct values for a , b , and c .

Exercise 3

Students draw the graph, determine the zeros, and write the factored form for $y = x^2 - 4x + 4$.

Students can use the ***QUAD*** program to confirm their answers, if obtained using the **zero** command.

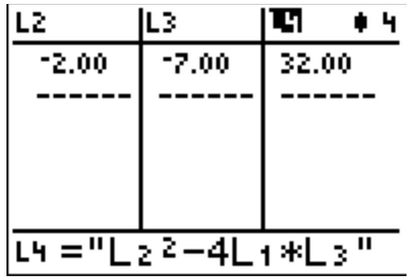
Exercise 4

Students may ask why this quadratic $(y = x^2 - 2x - 7)$ is not factorable using integers and the previous examples were. A discussion of the discriminant and what it tells us about the roots (zeros) would be appropriate here. However, students will discover in the next few exercises that the value of the discriminant for this example is not a perfect square, and will thus create an irrational number in the solution.

Students will first obtain the solutions for the functions in Question 9 by graphing and finding the zeros or by using the ***QUAD*** program.

Finally, lists are used to calculate the value of the discriminant for the previous two problems, whose solutions were irrational.

Now, the lists and formula are available for students to use to calculate the discriminant for other problems provided on the student worksheet as homework, or it can be used to rework any previous problems.



Homework/Extensions

Students will use the formula provided in Question 10 to calculate the discriminant for several other quadratics.

Then, students are to examine the flow chart and discuss with students how to use it to answer their homework problems. Ask students to sketch a different graph for each scenario. One is already shown for them.

11.4 Manual Fit

This activity is intended to supplement Algebra I, Chapter 10, Lesson 7.

ID: 12274

Time required: 20 minutes

Activity Overview

In this activity, students will manipulate parabolas in vertex form so that the curve matches a set of data points graphed as a scatter plot. This activity will serve to reinforce understanding of the vertex form for a parabola. In the extension, students will find an equation in vertex form and standard form that matches points from the Gateway Arch in St. Louis.

Topic: Quadratic Functions

- *Graph the parabola so that its vertex and shape match a set of plotted points.*
- *Understand the value of a and its contribution to shape and direction of opening.*
- *Apply knowledge of parabolas to parabolic shapes in real world problems.*

Teacher Preparation and Notes

- *This activity is intended for an Algebra 1 or Algebra 2 class.*
- *Students will need to be able enter data into lists and graph as a scatter plot. They will also need to be able to graph a function and adjust window settings.*
- *Students will answer questions about the vertex, direction of opening, and the relative width of opening for a particular shape.*

Associated Materials

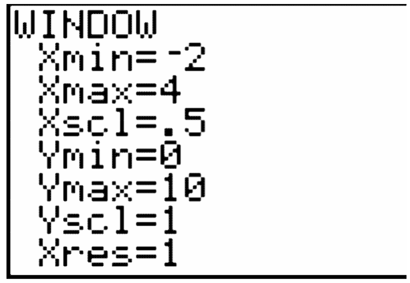
- Student Worksheet: Manual Fit, http://www.ck12.org/flexr/chapter/9620, scroll down to the fourth activity.

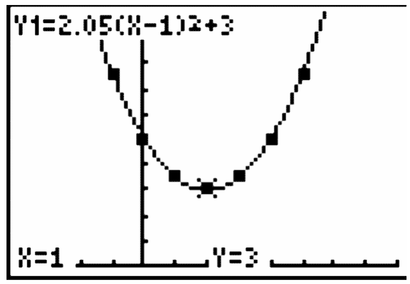
Problem 1 – Match the graph, Part 1

Vertex form for the equation of a parabola is shown on the worksheet. Students will use this information in the next few questions to help them answer questions.

Instruct students that to change the graph, they can substitute different values into the $Y =$ equation. Students should enter initial values (non-zero) into the vertex form of the equation to begin their exploration.

Notice that the students may not get the "exact" answer that they wish. Tell them that they will find more exact methods of finding matching equations for data in later classes.





Acceptable equation: $y = 2(x-1)^2 + 3$

You may also want to cover the effects of changing the window on the appearance of the graph. Stress the importance of knowing the minimum, maximum, and scale to determine the equation.

Problem 2 – Match the graph, Part 2

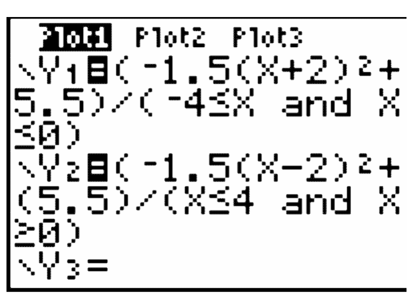
Students are given another set of plotted points and asked to grab and move the parabola to match the graph. Encourage discussion of the placement of the vertex, and the relative width of the curve. This time, a negative value for a is required. $y = -0.25x^2$

Problem 3 – Match the Double Arches

After matching the data well, the [U+0080][U+009C]*M*[U+0080][U+009D] double arches appear quite nicely. Discussion could follow about reflections, symmetry, and the design of company logos using mathematical or geometric figures that are pleasing to the eye.

Note: Students can enter the less than or equal to $(\leq)$ and the greater than or equal to $(\geq)$ symbols by pressing 2^{nd} [0] , the Catalog menu, and selecting from the list.

To enter the word *and*, students can press 2^{nd} **[MATH]** and move to the **LOGIC** menu.



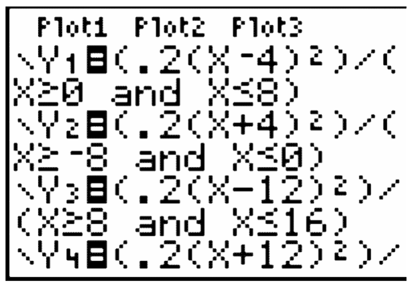
Problem 4 – The Main Cables of a Suspension Bridge

Several loops of cable are represented here. Students will be matching an equation to a particular piece of the graph. What the students have learned about vertex form should be of help in this problem.

Section A: $y = 0.2(x+4)^2$

Section B: $y = 0.2(x-4)^2$

To graph the given screen, see the equations to the right. Conditional statements are used to limit the domain of the function.



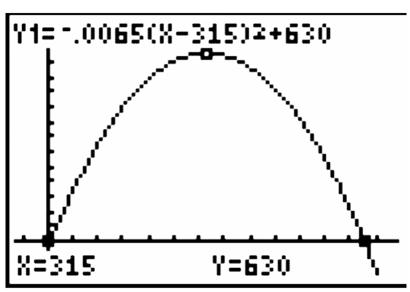
Extension – The Gateway Arch in St. Louis

The Gateway Arch in St. Louis, the "Gateway" to America, is a shape that looks like a parabola to the casual observer (It is actually called a catenary curve.).

Students will create an equation in vertex form to match the data given in $L1$ and $L2$.

Using the same data, students are asked to match the graph in *standard form.* Important things to remember are; what does the value of a do to the graph, and what would your $y-$ intercept be (c in the equation)?

Discussion that follows includes how the equations are the same, and different. Assist the students in expanding the vertex form so that a direct comparison can be made for the two equations.



Other Arches

This section gives students a few real-world situations where they can find parabolas. Students can find the equations that model these situations.

- Hang a chain (or necklace) against a piece of graph paper and trace its graph (or take a digital photo). Write an equation in vertex form to match the shape of the curve.

- Place a laminated piece of graph paper behind a drinking fountain and take a digital photo. Write an equation to match the shape of the curve.

Chapter 12 TE Algebra and Geometry Connections; Working with Data - TI

Chapter Outline

12.1 Radical Transformations

This activity is intended to supplement Algebra I, Chapter 11, Lesson 1.

ID: 11574

Time Required: 15 minutes

Activity Overview

In this activity, students will use the transformational graphing application to examine how the square root function is transformed on the coordinate plane. As an extension, students will examine similar transformations on a cube root function.

Topic: Radical Functions

- *Domain, Range*
- *Square Root*
- *Transformations*

Teacher Preparation and Notes

- *Students will need the Transformation Graphing application installed on their graphing calculators.*
- ***To download the calculator application, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?i and select Transformation Graphing under the Applications header.***

Associated Materials

- Student Worksheet: Radical Transformation, http://www.ck12.org/flexr/chapter/9621
- Transformation Graphing

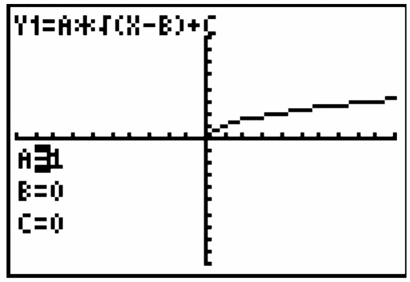
Problem 1 – The General Radical Function

Students will examine the graph of $y = \sqrt{x}$ and determine the domain and range of the function.

Students will also conjecture if the graph is always in the first quadrant.

Problem 2 – Transformations

In Problem 2, students will change values in the general equation of a square root graph. Students will determine the domain and range of a square root function based on the general equation. They will revisit the question of where the graph lies in the plane. They will also describe the transformation performed on the graph by changing each variable.



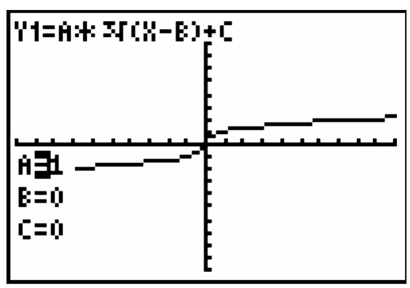
Discussion Questions:

- How does each variable affect the graph of the function?
- How can we algebraically show the domain and range of the function?
- Why does the graph "stop" (no longer exist) on one side?
- How does the variable a affect the graph?

Extension – Cube Root Functions

In this problem, students will repeat the exploration above for the cube root function and determine domain, range, and transformations on the graph.

Note: The cube root can be entered by pressing **MATH** and selecting 5: $\sqrt[3]{\ }$.



Solutions

1. Domain: $x \geq 0$; Range: $y \geq 0$
2. Sample Response: The function is not defined for values less than zero because the square root becomes negative.
3. Sometimes (Incorrect answer is okay. This is a conjecture question.)
4. Sample response: The graph is a straight line because the square root is multiplied by zero, making the function $y = 0$.
5. Sometimes (Now students should have the correct answer.)
6. Sample responses must have $\sqrt{x-3}$
7. $x \geq -2$
8. h
9. Sample response must have -2 after square root
10. $y \geq 3$
11. k

12. Sample response: Positive 4 opens down (concave down); Negative 4 opens up (concave up).
13. Sample response: Positive 4 is steeper than positive 2 .
14. Sample response: Flips the graph open up or open down. Makes the graph steeper as $|a|$ gets larger.
15. $x \geq h$
16. $y \geq k$
17. Domain and range: All real numbers
18. h is a horizontal shift; k is a vertical shift; a makes the branches steeper and flips the graph.

12.2 Distances in the Coordinate Plane

This activity is intended to supplement Algebra I, Chapter 11, Lesson 5.

ID: 8685

Time required: 40 minutes

Topic: Points, Lines #38; Planes

- *Given the coordinates of the ends of a line segment, calculate its length.*

Activity Overview

In this activity, students will explore distances in the coordinate plane. After finding the coordinates of a segment's endpoints, students will substitute these values into the distance formula and compare the results to the measured length of the segment. Then students will find the distance between the endpoints using the Pythagorean Theorem.

Teacher Preparation

- *This activity is designed to be used in a high school or middle school geometry classroom.*
- *The Distance Formula for the distance between two points* (x_1, y_1) *and* (x_2, y_2) *is* $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
- *The Pythagorean Theorem for a right triangle with legs a and b and hypotenuse c is* $a^2+b^2=c^2$. *Solving for the hypotenuse, the equation becomes* $c=\sqrt{a^2+b^2}$.
- *Cabri Jr. does not allow the* $x-$ *values and* $y-$ *values of points to be used separately in calculations. The construction in Problem 1 uses a perpendicular line to project the coordinate on the appropriate axis, and then the distance from* $(0, 0)$ *is used to find the* $x-$ *values and* $y-$ *values.*
- *The screenshots on pages 1–6 demonstrate expected student results.*
- ***To download Cabri Jr. and the calculator files, go to http://www.education.ti.com/calculators/downloads/US/Activities/D and select Cabri Jr., DISTNC1.8xv, and DSTNC2.8xv.***

Associated Materials

- Student Worksheet: Distances in the Coordinate Plane, http://www.ck12.org/flexr/chapter/9621, scroll down to the second activity.
- Cabri Jr.
- DISTNC1, DISTNC2

Classroom Management

- *This activity is designed to be* ***student-centered*** *with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.*
- *The student worksheet GeoWeek04_Distance_Worksheet_TI-84 helps guide students through the activity and provides a place for students to record their answers and observations.*
- *Depending on student skill level, you may wish to use points with integer coordinates, or only positive values.*
- ***Note:*** *The coordinates can display* 0 , 1 , *or* 2 *decimal digits. If* 0 *digits are displayed, the value shown will round from the actual value. To ensure that a point is actually at an integer value rather than a* <u>*rounded*</u> *decimal value, do the following:*

1. *Move the cursor over the coordinate value so it is highlighted.*
2. *Press + to display additional decimal digits or - to hide digits.*

Problem 1 – The Distance Formula

Note: If the file *Distncl* is distributed to student calculators, skip Steps 1 and 2. Proceed with Step 3.

Step 1: Students should open a new Cabri Jr. file. If the axes are not currently showing, they should select **Hide/Show #62; Axes**.

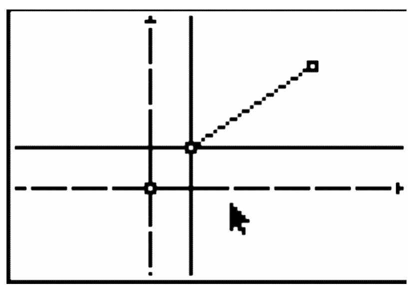
Students are to construct a segment in the first quadrant using the **Segment** tool.

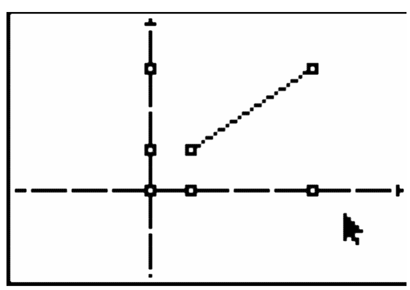
Step 2: In order to use the $x-$ values and $y-$ values of the endpoints separately in calculations, students will use a perpendicular line to project the coordinates on the appropriate axis.

Direct students to select the **Perp.** tool to construct a line through one endpoint of the segment perpendicular to each axis.

Find the intersection of the perpendicular line with the axis using the **Point #62; Intersection** tool. Then, hide the perpendicular line with the **Hide/Show #62; Object** tool.

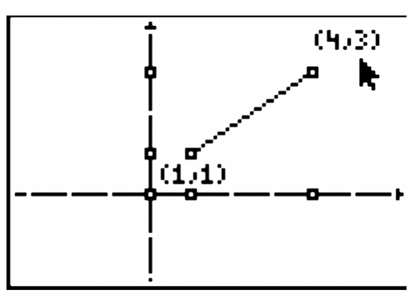
Repeat this process for the second endpoint.





Step 3: Students will select **Coord. #38; Eq**. and show the coordinates for the endpoints of the segment.

If the coordinates of the endpoints are not integers, they need to use the **Hand** tool to drag the endpoints until the coordinates are integers.



Step 4: Students should measure the distance from each axis point to the origin using the **Measure #62; D. #38; Length** tool. These distances should match the $x-$ values and $y-$ values of the coordinates.

Students can now drag the endpoints and observe that the axis point distances change as the segment endpoints move, but still match the coordinates.

Step 5: Have students measure the length of the segment using the **Measure #62; D. #38; Length** tool.

Step 6: The **Calculate** tool can perform calculations on pairs of numbers. The Distance Formula calculations will be broken down into individual steps. Be sure to select coordinates in the proper order!

Students should complete the following steps in order:

a. **Subtract** the two $x-$ values.

b. **Multiply** this result by itself.

c. **Subtract** the two $y-$ values.

d. **Multiply** this result by itself.

e. **Add** the two products.

f. **Square root** this sum.

Note: If desired, have students do the calculations directly on the worksheet rather than within the Cabri Jr. file.

Now students can drag the segment endpoints and observe the calculation results as they update.

Discuss how these calculation results relate to the measured length of the segment.

Problem 2 – Apply the Math

Note: If the file *Distnc2* is distributed to student calculators, skip Steps 1, 2 and 3. Proceed with Step 4.

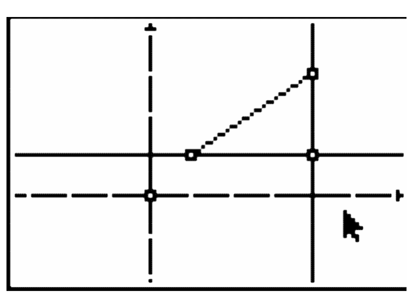
Step 1: Students should open a new Cabri Jr. file. If the axes are not currently showing, they should select **Hide/Show #62; Axes.**

Students are to construct a segment in the first quadrant using the **Segment** tool.

Step 2: Students will be constructing a small right triangle for the segment such that the segment is the hypotenuse of the right triangle.

Select the **Perp.** tool and construct a line through the upper segment endpoint perpendicular to the $x-$ axis. Construct a second line through the lower segment endpoint perpendicular to the $y-$ axis.

Have students find the intersection of the perpendicular lines by selecting **Point #62; Intersection.**

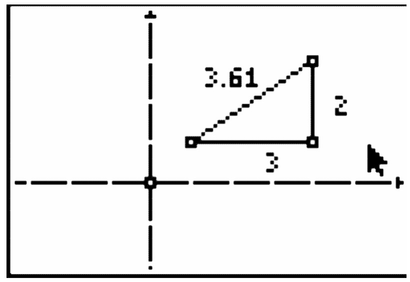


Step 3: Direct students to hide the perpendicular lines with the **Hide/Show #62; Object** tool. Do not hide the intersection point.

They should then construct segments for the legs of the right triangle with the **Segment** tool.

Step 4: Tell students to find the length of each side of the triangle.

Discuss the lengths of the three sides of the triangle and decide which is longest. Identify the legs and the hypotenuse.



Step 5: Use the **Calculate** tool to perform the following calculations:

Students should complete the following steps in order:

a. **Multiply** the length of one leg by itself.

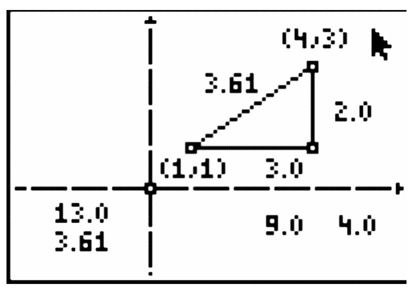
b. **Multiply** the length of the other leg by itself.

c. **Add** the two values.

d. **Square root** this sum.

Note: If desired, have students do the calculations directly on the worksheet rather than within the Cabri Jr. file.

Step 6: Students should show the coordinates of the segment endpoints and calculate the length using the Distance Formula on their worksheet.



Students should drag the endpoints of the segment and observe the relationship between the 3 values:

- Measured length
- Calculated length (Distance Formula) on worksheet
- Calculated Pythagorean distance

Discuss the connections between the Distance Formula and the Pythagorean Theorem. Challenge students to explain how $\sqrt{a^2+b^2}$ is derived from the Pythagorean Theorem.

12.3 Box Plots Histograms

This activity is intended to supplement Algebra I, Chapter 11, Lesson 8.

ID: 8200

Time required: 30 minutes

Activity Overview

Students create and explore a box plot and histogram for a data set. They then compare the two data displays by viewing them together and use the comparison to draw conclusions about the data.

Topic: Data Analysis and Probability

- *Represent and interpret data displayed in data graphs including bar graphs, circle graphs, histograms, stem-and-leaf plots and box-and-whisker plots.*
- *Display univariate data in a spreadsheet or table and determine the mean, mode, standard deviation, extrema and quartiles.*

Teacher Preparation and Notes

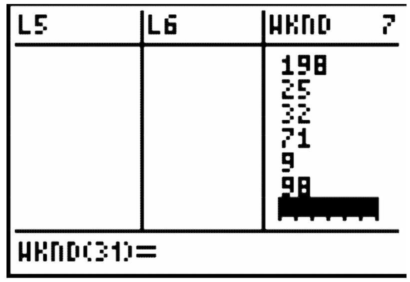
- *This activity is appropriate for students in Algebra 1. It assumes that students are familiar with mean, median, minimum, interquatile, etc.*
- *This activity is intended to be* ***teacher-led*** *with students in* ***small groups.*** *You should seat your students in pairs so they can work cooperatively on their calculators. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators.*
- *The student worksheet is intended to guide students through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*
- ***To download the data list, go to http://www.education.ti.com/calculators/downloads/US/Activities/Detail?id=8200 and select WKND.8xl.***

Associated Materials

- Student Worksheet: Compare a Box Plot and Histogram, http://www.ck12.org/flexr/chapter/9621, scroll down to the third activity.
- WKND (data list)

Before beginning the activity, the list **WKND** needs to be transferred to the students' calculators or students need to store the numbers given on the worksheet to a list titled **WKND.**

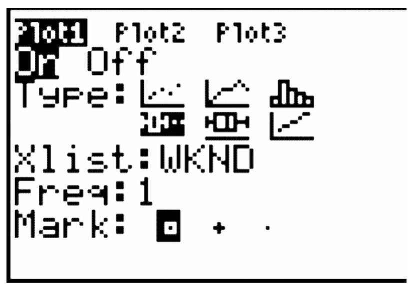
If you choose to have students store the numbers manually, they need to press **STAT ENTER** to access the List Editor. Then arrow to the top of $L1$ and arrow over to the top of the 7th list which has no heading. The calculator will be in Alpha Mode, so students are to type the heading **WKND.** Then enter the numbers as usual.



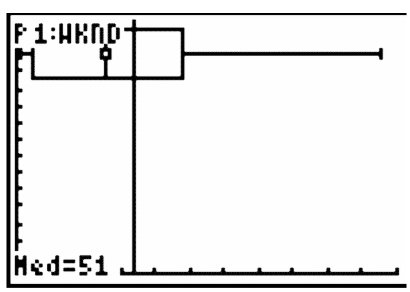
Part 1 – Create a box plot

First students will set up and investigate a box plot of the distances in the **WKND** list using **Plot1.** They should make sure that all other plots and equations have been turned off.

The window settings that are given on the worksheet will enable students to view the box plot and the histogram later in the activity without having to change the settings.



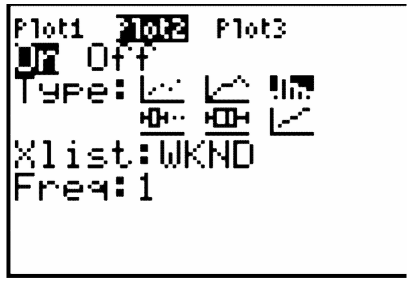
After answering the questions on the worksheet students will draw a vertical line on the graph to show where the mean is located in the box plot compared to the median.



Part 2 – Create a histogram

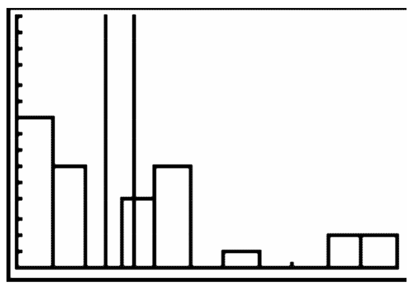
Now students will set up and investigate a histogram of the distances in the **WKND** list using **Plot2.** They need to turn off Plot1 (the box plot) before viewing the graph.

The **Xscl** of the graph is 20 . Explain to students that this means each bar is an interval of 20 (i.e., 0 to 19 , 20 to 39).



After answering the questions on the worksheet, students will draw two vertical lines, one for the mean and one for the median of the distances.

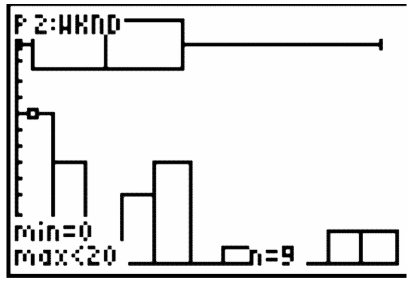
Using the commands **mean(LWKND)** and **median(LWKND)** on the Home screen will allow students to see the exact values of the mean and median (67.83 and 51 , respectively).



Part 3 – Compare a box plot and a histogram

Students are directed to turn **Plot1** (box plot) back on so that they can view the box plot and histogram on the same screen.

Use this view to guide a discussion about the relationship between the box plot and the histogram. Begin with more obvious connections—a taller bar in the histogram corresponds with more points on the box plot than a shorter bar, for example. Then lead students to make deeper conclusions about the shape of the data. (For example, it is grouped mostly in the lower values; the data contains many distances close together on the low end of the scale and relatively few larger distances; the larger distances are more spread out.)


P2:WKND
min=0
max<20
n=9

CHAPTER 13 TE Rational Equations and Functions; Topics in Statistics - TI

CHAPTER OUTLINE

13.1 Inverse Variation

This activity is intended to supplement Algebra I, Chapter 12, Lesson 1.

ID: 8203

Time required: 30 minutes

Activity Overview

Students will examine a set of ordered pairs that vary inversely. They will plot the ordered pairs and explore the graph and table for relationships. Students will also graph the inverse variation and determine whether it is a function.

Topic: Rational Functions #38; Equations

- *Given the values of two variables that vary inversely, determine the rational function that relates them.*

Teacher Preparation

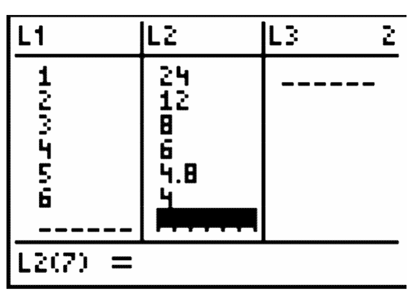
- *This activity is appropriate for an Algebra 1 classroom or review for an Algebra 2 class.*

Associated Materials

- Student Worksheet: Inverse Variation, http://www.ck12.org/flexr/chapter/9622

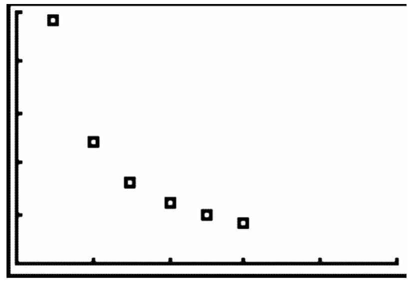
Students are to enter the data from the table on the worksheet into lists $L1$ and $L2$.

Be sure that students have cleared all lists before beginning. To do this, press **STAT**, then select **ClrAllLists.**



Students will then create a scatter plot of the data using **Plot1.**

They are to describe the relationship of x and y . As the $x-$ values increase, the $y-$ values decrease. The rate is not constant; the amount of change in y decreases as x increases.



For list $L3$, students will enter the formula $L1 * L2$, multiplying the $x-$ values by the $y-$ values.

The formula $x \cdot y = 24$ might be used for the area of 24 *units* , where x is the length and y is the width.

When students solve the equation $x \cdot y = 24$ for y , they should get $y = \frac{24}{x}$ and enter this in $Y1$.

As students view the function table, they should see that the function is undefined at $x = 0$.

They should also see that the negative $x-$ values have the same $y-$ values, although negative, as the positive $x-$ values.

When students view the entire graph, they should confirm that x is undefined at zero (vertical and horizontal asymptotes at zero).

The graph does not appear in Quadrants II or IV because the $x-$ and $y-$ values have the same sign.

An inverse variation could be in Quadrants II or IV, if the coefficient is negative.

13.2 Breaking Up is NOT Hard to Do

This activity is intended to supplement Algebra I, Chapter 12, Lesson 6.

ID: 11934

Time Required: 20 minutes

Activity Overview

In this activity, students will split rational functions into sums of partial fractions. Graphing is utilized to verify accuracy of results and to support the understanding of functions being represented in multiple ways.

Topic: Rational Functions #38; Equations

- *Least common denominator*
- *Sum of partial fractions*
- *Equivalent functions*

Teacher Preparation and Notes

- *Problems 1-3 should be done in class as guided practice or small group work. Several problems are provided on the student worksheet for additional practice.*
- *As an extension, the teacher could include a discussion of the placement of vertical asymptotes.*
- *Before beginning the activity, make sure that all plots have been turned off and all equations have been cleared from the $Y =$ screen.*

Associated Materials

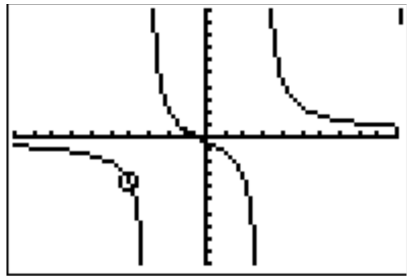
- Student Worksheet: Breaking Up is NOT Hard to Do, http://www.ck12.org/flexr/chapter/9622, scroll down to the second activity.

Problem 1 – Introduction

This part of the activity involves an exploration of equivalent ways to express a rational function. Students will generate function graphs from which they will learn that a rational function can be represented as the sum of individual fractions, known as partial fractions.

To make it clear to students that the graphs of $Y1$ and $Y2$ are identical, show students how to place the tracing circle in front of $Y2 =$ by using the arrows to move to the left of $Y2 =$ and pressing **ENTER** until the tracing circle appears.

Students will answer questions regarding their observations of the graphs of the given equations. They are also asked to observe the denominators of the two functions.



The denominators of the fractions in $Y2$ are the factors of the denominator in $Y1$.

Since the graphic results show that the two functions are equivalent, they are set equal to each other and a framework is established for finding the numerators of the partial fractions of a rational function. Directions are provided to help students through the process.

$$
\begin{aligned}
Y1(x) &= Y2(x)\\
\frac{7x+3}{x^2-9} &= \frac{A}{x+3}+\frac{B}{x-3}
\end{aligned}
$$

Students proceed to solve for A and B by substituting in values for x that will simplify the work to be done. For example, substituting -3 for x will eliminate the B term and simplify the process of solving for A . Similarly, substituting 3 for x will simplify solving for B .

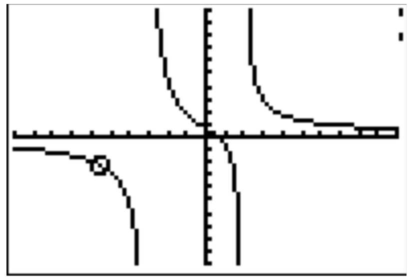
Discuss with students why it might be helpful to decompose a rational expression into a sum of partial fractions. Students may note that the partial fractions, being less complex, will be easier to work with for certain mathematical applications.

$$
\begin{aligned}
(x+3)(x-3)\left(\frac{7x+3}{x^2-9} = \frac{A}{x+3}+\frac{B}{x-3}\right)&\\
7x-3 &= A(x-3)+B(x+3)\\
\text{Let } x=3; 7(3)-3 &= A(3-3)+B(3+3)\\
18 &= 6B\\
3 &= B\\
\text{Let } x=-3; 7(-3)-3 \ &= A(-3-3)+B(-3+3)\\
24 &= -6A\\
-4 &= B
\end{aligned}
$$

Problem 2 – Practice

Students apply what was learned in Problem 1 to find a sum of partial fractions equivalent to a given rational function.

Once the algebraic work is completed, students can verify the equivalence of their solution to the original function via graphing the two functions. Remind students to use the show/hide feature to the left in the function entry bar of the graph page to be certain that the graphs of the two functions are identical.



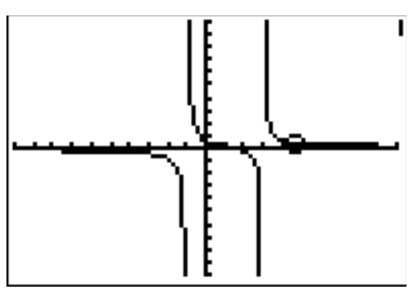
$$\frac{7x-4}{x^2+x-6} = \frac{A}{(x+3)} + \frac{B}{(x-2)}$$
$$(x+3)(x-2)\left(\frac{7x-4}{x^2+x-6} = \frac{A}{(x+3)} + \frac{B}{(x-2)}\right)$$
$$7x-4 = A(x-2) + B(x+3)$$
$$\text{Let } x = 2; 7(2) - 4 = A(2-2) + B(2+3)$$
$$10 = 5B, \quad B = 2$$
$$\text{Let } x = -3; 7(-3) - 4 = A(-3-2) + B(-3+3)$$
$$25 = -5B, \quad B = -5$$

Problem 3 – The Next Level

Students again apply what has been learned, but the challenge level increases.

In this situation, the denominator has a constant factor in addition to two binomial factors. A hint is given to prompt students to use the algebraic binomial factors as denominators for the partial fractions to be determined.

When students solve for A and B , they will find that the value for B is a fraction, which will result in a need for simplification of the partial fractions obtained.



$$\frac{5x-7}{4x^2-8x-12} = \left(\frac{A}{(x-3)} + \frac{B}{(x+1)}\right)$$

$$4(4x-3)(x+1)\left(\frac{5x-7}{4x^2-8x-12} = \frac{A}{(x-3)} + \frac{B}{(x+1)}\right)$$

$$5x-7 = 4A(x+1)+4B(x-3)$$

$$\text{Let } x=-1;\ 5(-1)-7 = 4A(-1+1)+4B(-1-3)$$

$$-12 = -16B, \quad B = \frac{3}{2}$$

$$\text{Let } x=3;\ 5(3)-7 = 4A(3+1)+4B(3-3)$$

$$8 = 16A; \quad A = \frac{1}{2}$$

Solutions

1. The graphs are the same.
2. The functions appear to be equal.
3. The denominators of $f2$ are factors of the denominator of $f1$.
4. x^2-9 or $(x-3)(x+3)$
5. $7x+3 = A(x-3)+B(x+3)$
6. 3
7. 4
8. $\frac{7x+3}{x^2-9} = \frac{3}{x+3} + \frac{4}{x-3}$
9. The results verify algebraically that the two functions are equivalent.
10. $\frac{7x-4}{x^2+x-6} = \frac{2}{x-2} + \frac{5}{x+3}$
11. Yes; Same graph result verifies equivalent algebraic result.
12. $\frac{5x-7}{4x^2-8x-12} = \frac{1}{2x-6} + \frac{3}{4x+4}$
13. Yes; Same graph result verifies equivalent algebraic result.
14. $\frac{-7x-11}{x^2+4x+3} = \frac{-2}{x+1} - \frac{5}{x+3}$
15. $\frac{2x+42}{x^2+2x-24} = \frac{5}{x-4} - \frac{3}{x+6}$
16. $\frac{x}{x^2+2x-8} = \frac{2}{3x+12} + \frac{1}{3x-6}$