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CK-12 Texas Instruments Algebra I

Student Edition

Texas Instruments Algebra I Student Edition

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Chapter 1 SE Introduction

Chapter Outline

1.1 Algebra I TI Resources Flexbook

Student Edition

Introduction

This flexbook contains Texas Instruments (TI) Resources for the TI-83, TI-83 Plus, TI-84, and TI-84 SE. All the activities in this flexbook supplement the lessons in the student edition. Teachers may need to download programs from www.timath.com that will implement or assist in the activities. All activities are listed in the same order as the Teacher's Edition. Each activity included is print-ready.

There are also corresponding links in the 1st Edition of Algebra I, 2nd Edition, and Basic Algebra.

- Algebra I, first edition: http://www.ck12.org/flexr/flexbook/1374
- Algebra I, second edition: http://www.ck12.org/flexr/flexbook/3659
- Basic Algebra: http://www.ck12.org/flexr/flexbook/2876

CHAPTER 2 SE Equations and Functions

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 1: http://www.ck12.org/flexr/chapter/4471
- Algebra I, second edition, Chapter 1: http://www.ck12.org/flexr/chapter/9560
- Basic Algebra, Chapter 1: http://www.ck12.org/flexr/chapter/9165

2.1 Back in Time?

This activity is intended to supplement Algebra I, Chapter 1, Lesson 5.

Definition

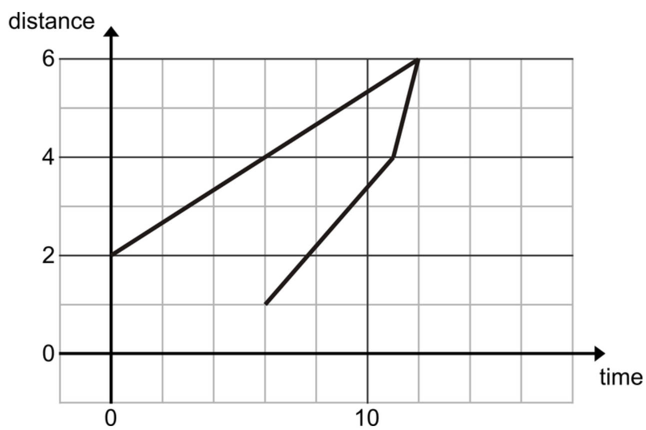
A *function* is a relation in which each input is paired with exactly **one** output.

For every value that goes into a function, the function outputs one unique result.

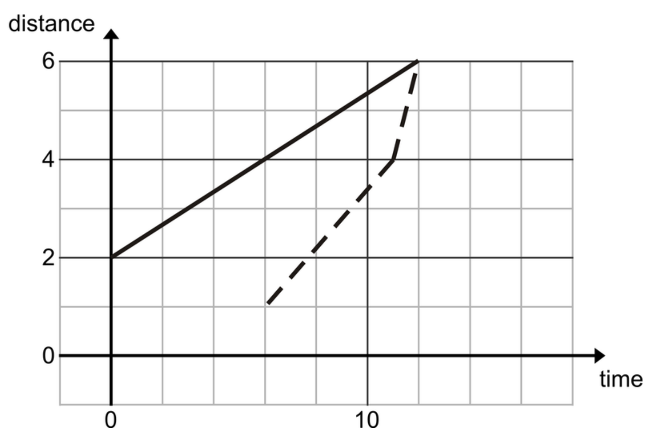
Problem 1 – Graphical

At time $t = 0$, Marty is at position $d = 2$.

1. Can the graph to the right describe Marty's position as a function of time? Explain.
2. What would have to happen for this graph to occur?



3. Redraw the dashed lines to make the graph a function.



Problem 2 – Set of Ordered Pairs

The first element of each ordered pair is the input value.

4. Which sets below describe a function? Explain why.

A. $\{(0,1),(1,4),(2,7),(3,6)\}$

B. $\{(-2,2),(-1,1),(0,0),(1,3),(2,4)\}$

C. $\{(3,2),(3,4),(5,6),(7,8)\}$

D. $\{(2,3),(3,2),(1,4),(4,1)\}$

Marty flies to Mars, where the acceleration of gravity is 0.375 of what it is on Earth. So with $a = 12ft/s^2$, use the distance formula $d = \frac{1}{2}at^2$ to compute the output when given the input.

5. Use the formula to compute d . Give the set or ordered pairs (t,d) when the input t is the set $\{0,1,2,6\}$.

6. Use the formula to compute t . Give the set of ordered pairs (d,t) if the input is d . The input set for d is $\left\{0,\frac{2}{3},6\right\}$.

7. Which of the two solutions sets from Questions 5 and 6 is a function? Why?

8. From solutions sets above, which is true?

A. d is a function of t

B. t is a function of d

C. both

D. neither

Problem 3 – Function Notation

If f is a function of x this can be written as $f(x)$.

For example, $f(x) = x^2$. So $f(3) = 9$.

To use the function ability of your graphing calculator, press $Y =$ and enter $x^2 - 2x + 3$.

Return to the Home screen.

To enter different values for x and observe what $f(x)$ equals, press **VARS**, arrow right to the **Y-VARS** menu, select **Function** and then choose $Y1$. Then enter **(#)**, replacing # with the $x-$ value.

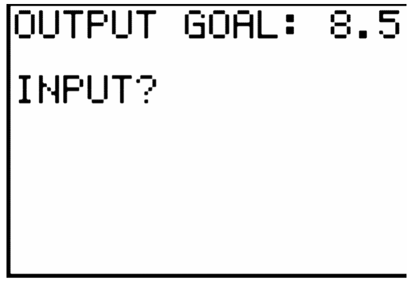
Press 2^{nd} **[ENTER]** to recall the last entry.

9. For $f(x) = x^2 - 2x + 3$, find $f(4)$ using the graphing calculator, then by substitution showing your work below.

10. For $f(x) = 3x^2 + 5x + 3$, find $f(2)$ using the graphing calculator, then by substitution showing your work below.

Problem 4 – Function Machine

Run the program **MACHINE** and select option **1**. The program will return an output for the input entered.



11. What is the input for the function $f(x)$ that gives an output of 8.5 ?
12. What is the unknown function?

Now select option **2**.

13. What is the input for the function $f(x)$ that gives an output of 6 ?
14. What is the unknown function?

Now select option **3**.

15. What is the input for the function $f(x)$ that gives an output of 83 ?
16. What is the unknown function?

2.2 Ordered Pairs

This activity is intended to supplement Algebra I, Chapter 1, Lesson 6.

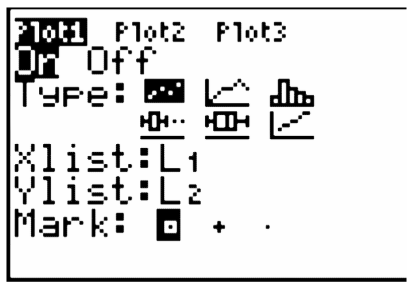
Problem 1 - Ordered Pairs

1. a. For the point $(-2,6)$, the first number, -2 , is the ____ -coordinate (or the abscissa).

b. For the point $(-2,6)$, the second number, 6 , is the ____ -coordinate (or the ordinate).

To graph a point, enter the coordinate in $L1$ and $L2$. Then turn the Stat Plot on and display the graph.

For example, to graph $(1,4)$, press **STAT**[Edit]. Then enter 1 in $L1$ and 4 in $L2$. Press 2^{nd} **[STAT PLOT]** and match the settings shown at the right. Press # and select **ZStandard**.



2. a. The point $(1,4)$ is in the first quadrant. In which quadrant is $(1,-4)$?

b. In which quadrant is $(-5,2)$?

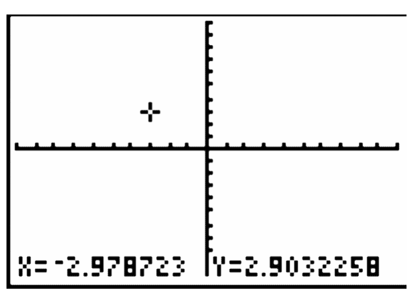
c. In which quadrant is $(-3,-2)$?

d. In which quadrant is $(4,4)$?

e. In which quadrant is $(-4,0)$?

f. In which quadrant is $(3,5)$?

To explore ordered pairs, press $Y =$ and make sure all the equations are cleared. Then, press **GRAPH** and use the arrow keys to move the cursor.



3. a. Where are the coordinates (negative, positive)?

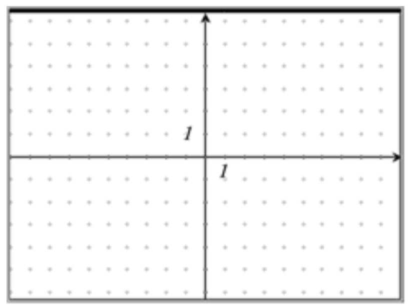
b. Where are the coordinates (positive, negative)?

c. Where is the ordered pair when it is (positive, positive)?

d. Where is the ordered pair when it is (negative, negative)?

Plot the following ordered pairs on the graph at the right. Label each pair with the appropriate letter.

$A(-1,3)$	$K(4,-2)$	$O(-2,-2)$
$C(1,-3)$	$M(-4,4)$	$R(-5,-1)$
$H(5,1)$	$S(6,-1)$	$T(2,2)$



4. What phrase do the points spell?

Problem 2 – Order Pairs

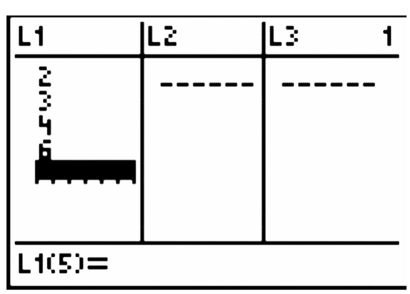
Math is everywhere. At the market, the equation $y = 1.5x$ represents the cost to buy x amount of pears, where y is the cost in dollars.

For example, you order 8 pears. The cost is $12 . This can be written as the ordered pair $(8,12)$.

5. Your order came to $3 . How many pears did you order?

6. Enter 5 ordered pairs for the cost of ordering pears using $L1$ and $L2$. If data already exists, arrow up to the top of the list and press **CLEAR[ENTER]** to clear the data.

Create the scatter plot and record your observation.



7. Press $Y =$. Graph the function $f(x) = x$ in $Y1$. Change the slope of the function until the line matches the points. What is the slope of your line? How does it relate to the problem?

Extension

We saw that values of a function can be written as a set of ordered pairs, listed in a table of values, and graphed as a scatter plot.

Extension 1: Find some other real-life data. Represent it as a set of ordered pairs, table, and scatter plot.

Extension 2: Come up with your own puzzle like the one at the bottom on page 1 of this worksheet. that you can share with a friend and your teacher.

CHAPTER 3 SE Real Numbers

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 2: http://www.ck12.org/flexr/chapter/4472
- Algebra I, second edition, Chapter 2: http://www.ck12.org/flexr/chapter/9561
- Basic Algebra, Chapter 2: http://www.ck12.org/flexr/chapter/9155

3.1 Factoring Composite Numbers

This activity is intended to supplement Algebra I, Chapter 2, Lesson 1.

Problem 1 - A

Write whatever you know about prime numbers in each section. Use words and numbers. Add more than one thing to a section if you know a lot about it!

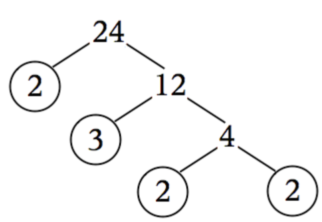
TABLE 3.1:

Definition:	**Fun facts:**
• a number greater than 1 that...	• 2 is the only even prime.
Examples	**Non-examples**
• $2,3,\ldots$	• $-7,0,\ldots$

Problem 2 – Exploring a Factor Tree for a Composite Number

Have you ever made a factor tree for a number? If so, then you already know what the prime numbers are. You may or may not know how to write the prime factorization using exponents rather than repeated multiplication. We will explore this skill using the TI-84 calculator together.

Here is a factor tree for the composite number 24 .

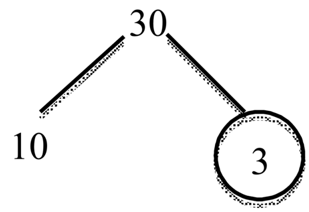


Its prime factorization is $24 = 2 \cdot 2 \cdot 2 \cdot 3$ or $24 = 2^3 \cdot 3$.

- Why is three the exponent for the factor 2 ?
- Is 24 a prime number? Explain.

Problem 3 – Exploring Division as a Means to Finding Prime Factors

You can use also division to find the prime factors of a number. Follow the steps to find the prime factorization of 30 .

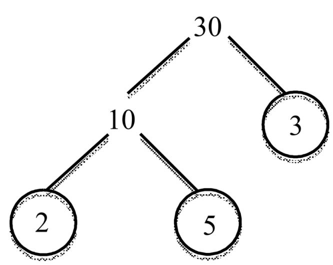


Think of a number that divides 30 evenly, like 3 . Divide 30 by 3 .

Draw a factor tree. Circle any factors that are prime (those cannot be divided further).

Think of a number that divides 10 evenly, like 2 . Divide 10 by 2 .

Expand the factor tree. Circle any factors that are prime. When none of the factors can be divided further, you have found the prime factorization. In this case, $30 = 2 \cdot 3 \cdot 5$.



- Why does the factorization of the number 30 NOT have any exponents?

Use division to find the prime factorization of 36 . Write your answer in exponent form. Show your work as a factor tree. $36 =$ _________

Problem 4 – Factoring on your own

Use the methods above to find the prime factorization of each number. Show your work as a factor tree and write the factorization in exponent form.

a. $27 =$ _________
b. $56 =$ _________
c. $72 =$ _________

3.2 Hot Air Balloon

This activity is intended to supplement Algebra I, Chapter 2, Lesson 3.

Problem 1 – Integer Addition

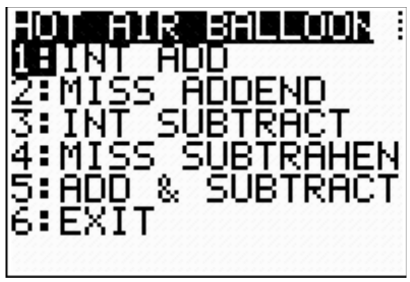
Press **PRGM** and find the **HOTAIR** program in the list.

Press **ENTER** to load the program.

Use the model to find the sum $2+(-6)$.

Press **ENTER** to choose option 1, Integer Addition.

Note: The right arrow adds a helium bag to the balloon, and the left arrow adds a sand bag.



Press **ENTER** to start and press $\rightarrow$ to move the balloon to ground level, the horizontal line.

The sum $2+(-6)$, in terms of the balloon:

- 2 signifies adding two *helium* bags, which raises the balloon 2 $units$. Press the right arrow twice.
- -6 signifies adding six *sand* bags, which lowers the balloon 6 $units$. Press the left arrow six times.

The balloon's resulting position is the sum.

With your partner, translate the following expressions into "balloon language" and use the model to find the sum. Remember to reset the balloon to ground level each time. When you are finished, press **ENTER** to return to the menu.

a. $-4+7=$ ___
b. $7+3=$ ___
c. $5+(-7)=$ ___
d. $-5+(-3)=$ ___

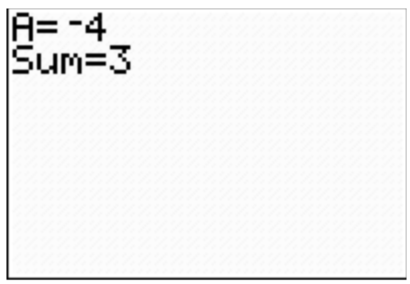
Problem 2 – Missing Addend

Find the value of b such that $-4+b=3$.

Arrow down to option 2 , Missing Addend, and press **ENTER.**

You are given the value of a and the value of the sum $a+b$, but the value of b is unknown.

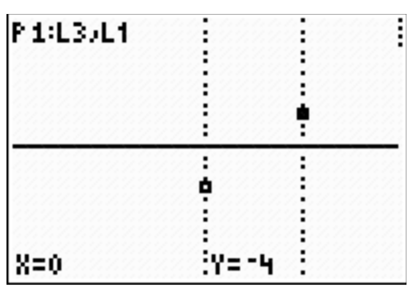
Input the value for A and press **ENTER.** Then input the sum and press **ENTER.**



Press **ENTER** to start and press $\rightarrow$ to move the balloon to ground level, the horizontal line.

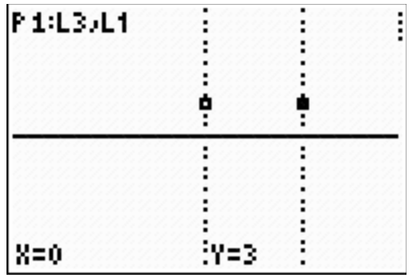
In terms of the balloon, there are four sand bags. Press the left arrow four times.

You can see the balloon at the bottom of the screen. Now there is also a target balloon to the right. The task is now to find the value of b that is needed to move the balloon on the right to the same position as the target balloon on the left.



The target balloon, on the right, is positioned at the value of the sum.

The target balloon is above your balloon's position, so you need to add helium bags. Press the right arrow to add helium bags until the balloons line up. Count as you press to find how many helium bags you added. For this example, $b=7$.



With your partner, translate the following equations into "balloon language" and use the model to find the missing addend. When you are finished, press **ENTER** to return to the menu.

1.

$$2+b=-3$$
$$b=\_\_\_$$

2.

$$-6+b=-1$$
$$b=\_\_\_$$

3.

$$5+b=1$$
$$b=\_\_\_$$

4.

$$-2+b=4$$
$$b=\_\_\_$$

Problem 3 – Integer Subtraction

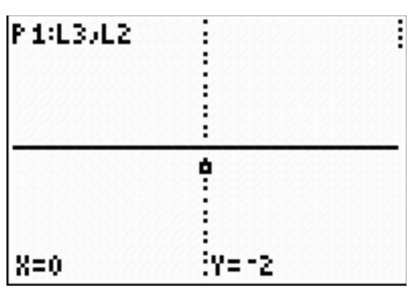
The model also provides a way of visualizing the subtraction of integers. As with addition, positive integers are represented by helium bags and negative integers by sand bags. However, subtraction is the operation of *taking off* a bag. For example, the expression $-2-5$ translates to "put on 2 sand bags and then take off 5 helium bags."

Arrow down to option 3, Integer Subtraction, and press **ENTER.**

Note: The right arrow **removes** a helium bag, and the left arrow **removes** a sand bag.

Reset the balloon at ground level by pressing the right arrow.

Move the balloon to its starting point, -2 .



To remove 5 helium bags, press the right arrow 5 *times* . The balloon falls 5 *units* to end at -7 , which is the difference $-2-5$.

With your partner, translate the following expressions into "balloon language" and use the model to find the difference. When you are finished, press **ENTER** to return to the menu.

a. $2-7=$ ___
b. $-3-1=$ ___
c. $5-(-2)=$ ___
d. $-4-(-7)=$ ___

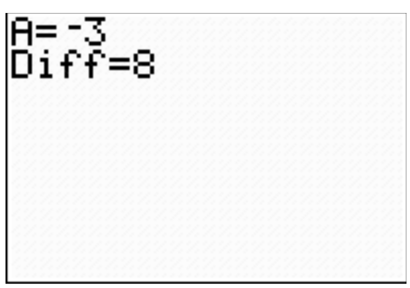
Problem 4 – Missing Subtrahend

This model shows two balloons side by side—like the model from Problem 2, except that it is used to find a missing *subtrahend* rather than a missing addend.

Arrow down to option 4, Missing Subtrahend, and press **ENTER.**

For example, find the value of b such that $-3-b=8$. In terms of the balloon, this means that the balloon ends up at 8 , and you have used 3 sand bags. You need to find how many and which type of bag you must *remove* to have a resulting position of 8 .

Input the value for A and press **ENTER.** Then input the difference and press **ENTER.**



Your balloon is at the top of the screen, and the target balloon to the right represents the difference.

Reset the balloon at ground level by pressing the right arrow.

Note: The right arrow **removes** a helium bag, and the left arrow **removes** a sand bag.

Move the balloon to its starting position, -3 .

The target balloon is above your balloon's position, so you need to remove sand bags. Press the left arrow to remove sand bags until the balloons line up. Count as you press to find how many sand bags you removed. For this example, you should find that $b = -11$.

With your partner, translate the following equations into "balloon language" and use the model to find the missing subtrahend. When you are finished, press **ENTER** to return to the menu.

1.

$$\begin{aligned} 6 - b &= 9 \\ b &= \_\_\_ \end{aligned}$$

2.

$$\begin{aligned} 5 - b &= -3 \\ b &= \_\_\_ \end{aligned}$$

3.

$$\begin{aligned} -4 - b &= -1 \\ b &= \_\_\_ \end{aligned}$$

4.

$$\begin{aligned} -2 - b &= 6 \\ b &= \_\_\_ \end{aligned}$$

CHAPTER 4

SE Equations of Lines

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 3: http://www.ck12.org/flexr/chapter/4473
- Algebra I, second edition, Chapter 3: http://www.ck12.org/flexr/chapter/9562
- Basic Algebra, Chapter 3: http://www.ck12.org/flexr/chapter/9156

4.1 One Step at a Time

This activity is intended to supplement Algebra I, Chapter 3, Lesson 1.

The following equations are examples of one step equations:

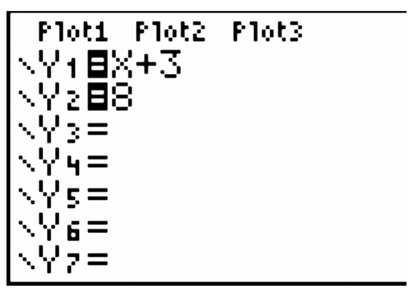
$$x+3=8 \qquad x-4=-2 \qquad 8x=40 \qquad \frac{x}{3}=2$$

Is there a rule for solving one-step equations? To find out, solve several one-step equations with your calculator and look for a pattern. To start, clear out any functions from the $Y=$ screen.

Problem 1 – Addition equations

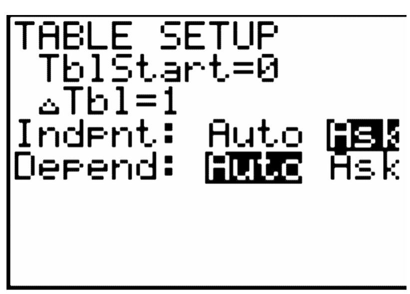
One way to solve an equation is by substitution, or trying different values for the variable until you find one that makes the equation true. Your calculator can help you solve the equation $x+3=8$ by substitution.

- Press $Y=$ to access lists.
- Enter the expression from the left side of the equation into $Y1$.
- Enter the expression from the right side of the equation into $Y2$.



Use the **Table** feature to test different values for x .

- Press 2^{nd} **[WINDOW]** to access the Table Settings menu.
- Change the independent (**Indpnt**) variable setting from **Auto** to **Ask**, as shown.
- Press 2^{nd} **[GRAPH]** to access the table.

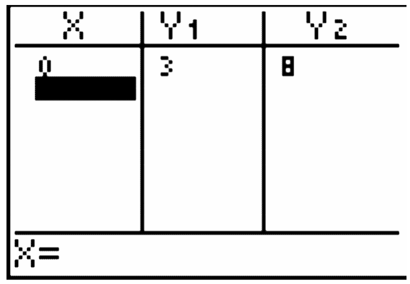


1^{st} column: values for x

2^{nd} column: value of the left side of the equation, $x+3$

3^{rd} column: value of the right side of the equation, 8

What value of x will make the two sides of the equation equal and the equation true? Enter guesses in the x column.



Use substitution to solve each equation. Enter the left side of each equation in $Y1$ and the right side of each equation in $Y2$. Then use **Table** to look for the value of x that makes the equation true. Enter these in like the first part of this problem.

1. a. $x+30=80$ $\quad x=$ ___

b. $x+(-3)=19$ $\quad x=$ ___

c. $73.3=4.3+x$ $\quad x=$ ___

d. $x+4.5=2.5$ $\quad x=$ ___

Write two one-step addition equations of your own. Use substitution to solve them.

2. _________ ___ = ____ _________ ___ = ____

Look for a pattern in the equations you solved and their solutions.

3. a. The solution to $x+3=8$ is $x=5$.

What operation can you perform with 8 and 3 to get 5 ?

b. Try this pattern on the other equations and solutions. Does it work? Give an example.

Listen as your teacher explains the **Subtraction Property of Equality**. This is what caused the pattern you found. You can use the Subtraction Property of Equality to solve addition equations.

$$\begin{aligned} m+5&=1\\ m+5-5&=1-5\\ m&=-4 \end{aligned}$$

Fill in the boxes to solve each equation.

4. a.

$$\begin{aligned} 2+q&=11\\ 2-\square+q&=11-\square\\ q&=\square \end{aligned}$$

b.

$$\begin{aligned} t+11 &= 10 \\ t+11-\square &= 10-\square \\ t &= \square \end{aligned}$$

c.

$$\begin{aligned} n+32 &= 5 \\ n+32-\square &= 5-\square \\ n &= \square \end{aligned}$$

d.

$$\begin{aligned} p+17 &= 0 \\ p+17-\square &= 0-\square \\ p &= \square \end{aligned}$$

Problem 2 – Multiplication equations

You can solve an addition equation by subtracting from both sides, because subtraction "undoes" addition. But what about other types of equations?

Use substitution to solve each equation. Enter the left side of each equation in $Y1$ and the right side of each quation in $Y2$. Then use the table to look for the value of x that makes the equation true.

5. a. $5x = 75 \quad x =$ ___

b. $-7x = 28 \quad x =$ ___

c. $4x = 52 \quad x =$ ___

d. $-5x = 48 \quad x =$ ___

Write two one-step multiplication equations of your own. Use substitution to solve them.

6.

_________ ___ = ____

_________ ___ = ____

Look for a pattern in the equations you solved and their solutions.

7. a. The solution to $5x = 75$ is $x = 15$.

What operation can you perform with 75 and 5 to get 15 ?

b. Try this pattern on the other equations and solutions. Does it work? Give an example.

Listen as your teacher explains the **Division Property of Equality**. This is what caused the pattern you found. You can use the Division Property of Equality to solve multiplication equations.

$$\begin{aligned} 7n &= 56 \\ \frac{7n}{7} &= \frac{56}{7} \\ n &= 8 \end{aligned}$$

Fill in the boxes to solve each.

8. a.

$$\begin{aligned} 8q &= 64 \\ \frac{8q}{\square} &= \frac{64}{\square} \\ q &= \square \end{aligned}$$

b.

$$\begin{aligned} 6t &= -120 \\ \frac{6t}{\square} &= \frac{-120}{\square} \\ t &= \square \end{aligned}$$

c.

$$\begin{aligned} 2n &= 2 \\ \frac{2n}{\square} &= \frac{2}{\square} \\ n &= \square \end{aligned}$$

d.

$$\begin{aligned} -3p &= 48 \\ \frac{-3p}{\square} &= \frac{48}{\square} \\ p &= \square \end{aligned}$$

Problem 3 – Inverse Operations

When one operation undoes another, they are called **inverse operations**. When two operations are inverse operations, either one undoes the other.

Inverse Operations

addition ⇔ subtraction

multiplication ⇔ division

9. a. What operation would you use to undo addition?

b. What operation would you use to undo subtraction?

c. What operation would you use to undo multiplication?

d. What operation would you use to undo division?

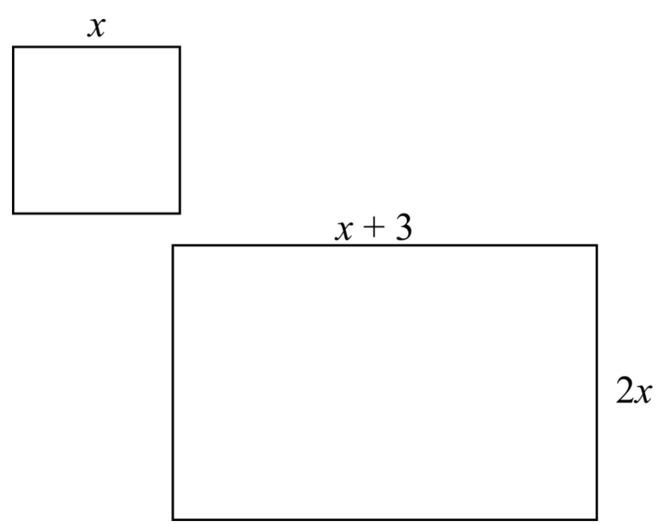
10. Write a rule to solve any one-step equation.

4.2 Variables on Both Sides

This activity is intended to supplement Algebra I, Chapter 3, Lesson 4.

Problem 1 – A Square and a Rectangle Have Different Perimeters.

A square has sides of length x . A rectangle has one side that is twice as long and another that is 3 $units$ longer than the sides of the square. Do these expressions reflect the description in the picture to the right?



- Write an algebraic expression for the perimeter of the square to the right.
- Write an algebraic expression for the perimeter of the rectangle to the right.
- If the rectangle has a perimeter that is 10 $units$ longer than the perimeter of the square, which of the following equations are true?

a. $4x+10=2(x+3)+2(2x)$

b. $4x-10=2(x+3)+2(2x)$

c. $4x=(x+3)+2x+10$

d. none of these

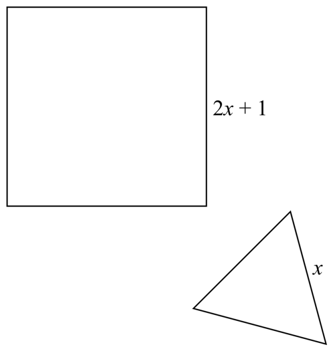
- What value of x will make the equation true?
- Check your answer using the **App4Math** application by pressing **APPS** and selecting **App4Math**. If your entered answer is correct, the calculator will display **true**.

Note: x, y, z , etc. can be entered using the alpha keys.

Use o for the equals sign.

Problem 2 – An Equilateral Triangle and a Square have Different Perimeters.

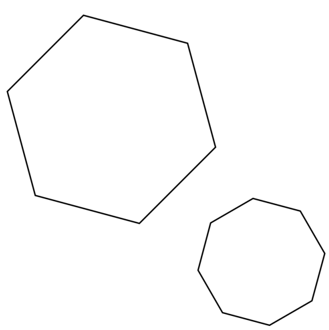
An equilateral triangle has sides of length x . A square has sides that are 1 more than twice that length. The perimeter of the square is 19 *centimeters* more than that of the triangle.



- How long are the sides of each polygon?
- Write an algebraic expression for the perimeter of the square.
- Write an algebraic expression for the perimeter of the triangle.
- Write an equation that shows the relationship if the perimeters of the square and triangle.
- Solve this equation and state the length of each side of the square.
- Check your answer using **App4Math**.

Problem 3 – A Regular Hexagon and a Regular Octagon

A regular hexagon has sides of length x . A regular octagon has sides that are half as long. The perimeter of the hexagon is 20 *inches* longer than that of the octagon.



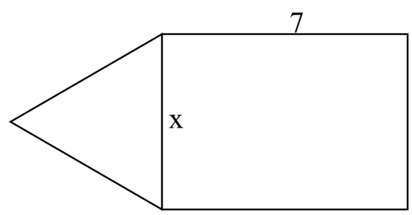
- If each side of the hexagon is of length $2x$, what is the length of each side of the octagon?

- Write an algebraic expression for the perimeter of the hexagon.
- Write an algebraic expression for the perimeter of the octagon.
- Write an equation shows the perimeter of the hexagon and octagon, then find the length of the sides of the hexagon.
- Check your answer using **App4Math**

Problem 4 – An Equilateral Triangle and a Rectangle

To the right is figure comprised of an equilateral triangle and a rectangle. The perimeter of the rectangle is 9 *centimeters* more than the perimeter of the triangle.

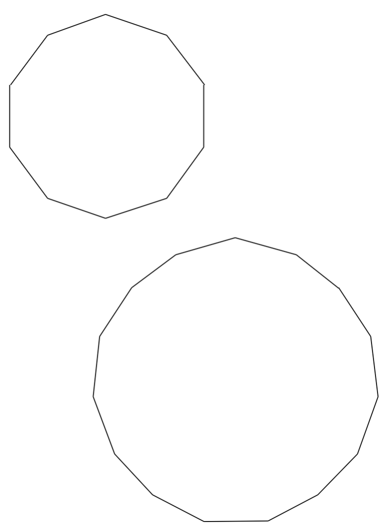
- Find the length, x , of each side of the triangle.



Problem 5 – Regular Decagon and 15-gon

The side lengths of the regular decagon and 15− gon to the right are equal.

- Find the difference in their perimeters.



4.3 Taxes Tips

This activity is intended to supplement Algebra I, Chapter 3, Lesson 7.

What does the word "per cent" mean?

Think of "per" as division, like miles per hour. "cent" $= 100$. You know there are 100 *years* in a CENTury or 100 CENTs in a dollar. So $6\% = \frac{6}{100} = 0.06 = 6$ one hundredths.

Problem 1 – Percent %

Write the decimal equivalent of the following percentages by dividing each value by 100 . For Questions 4 and 5, write a scenario with the percent.

a. You receive better than average service at a local restaurant and decide to tip 17% . ____
b. North Carolina raised their sales tax to 4.5% in 2008. ______
c. For good service the tip for eating at a local restaurant should be 15% before tax.
d. 5.75% . _________
e. 10% . ______

- Observe the pattern in the above percentage to decimal conversion. Explain the pattern. What happens to the 'decimal'?

Problem 2 - Using an Equation

The amount paid for taxes or tips is a percentage of the price.

The above sentence can be translated into a mathematical formula for ease of use.
"is" means equals and "of" means multiply

When using a formula, it is helpful to know what a variable represents.

Let T = amount of tax or tip paid, r = tax or tip rate given as a percent, p = price.

$$T = r \cdot p$$

TABLE 4.1:

Item number	**Price**
1. socks	\$4.79
2. hat	\$20.53
3. pants	\$45.88
4. TI-Nspire	\$131.97
5. shoes	\$149.99

TABLE 4.1: (continued)

Item number	Price
6. dress	$200.27
7. mp3 player	$250
8. laptop	$1000

Pick three items listed above and write their names and prices below. Then, choose a tax percentage and write it on each line of the "tax rate" column. Then, use the calculator to compute the taxes paid for each item by multiplying the price by the tax rate.

a. Item ___________ price = ____________ tax rate = _________ tax paid = ________
b. Item ___________ price = ____________ tax rate = _________ tax paid = ________
c. Item ___________ price = ____________ tax rate = _________ tax paid = ________

- What is the sum of the taxes paid on the three items?
- What is the tax paid after summing the prices of the three items?
- How do these two amounts compare?

Problem 3 – Mental Math and Estimation

Often you will only need a quick approximate answer for sales tax or the tip to leave at a restaurant.

Example:

The bill came to $\$28.85$, and you want to leave a 15% tip. One way to find 15% is to find 10% and 5% of the bill and add the two percentages together. Making the true values easier to work with helps a lot.

Step 1: Round $\$28.85 \approx \$30$

Step 2: Find 10% and 5% of the rounded amount. $\frac{30}{10} = 3 \rightarrow \frac{3}{2} = 1.50$

Step 3: Add the two percentage amounts. $\$3 + \$1.50 = \$4.50$.

- What actually is 15% of $\$28.85$? Was the estimate above a good one? Explain.
- Estimate the 15% tip if the bill before taxes was $\$17.97$.
- Approximately, what is a 20% tip on $\$51.12$? What was your thought process?
- Describe two ways to use mental math to determine the tax on a $\$1,000$ laptop if the sales tax is 4% .

Extension

You are eating at a restaurant in a state that has 7.25% sales tax. The bill for dinner is $\$1.98$ tax. You decide to leave a 15% gratuity.

- You leave a tip of how much? (Hint: 7.25% times 2 is close to 15%)
- How much was the original bill before tax and tip? Show your calculations.

CHAPTER 5 SE Graphs of Equations and Functions

Chapter Outline

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 4: http://www.ck12.org/flexr/chapter/4474
- Algebra I, second edition, Chapter 4: http://www.ck12.org/flexr/chapter/9563
- Basic Algebra, Chapter 4: http://www.ck12.org/flexr/chapter/9157

5.1 Points Lines Slopes

This activity is intended to supplement Algebra I, Chapter 4, Lesson 3.

Problem 1 – Coordinates of Points

Open the *CabriJr* app by pressing **APPS**. Open a new file and make sure the axes are displayed. Place a point, R , on the $x-$ axis and a point, S , on the $y-$ axis. Display the coordinates of the points.

1. Explain what is common to all points on the $x-$ axis.

2. Explain what is common to all points on the $y-$ axis.

Delete points R and S . Place two points, P and Q , in the top right section. Drag the points around into different sections.

Complete the sentences by writing *positive* or *negative*.

3. A point is in Quadrant 1 (top right) when its $x-$ coordinate is ___________ and its $y-$ coordinate is ___________.

4. A point is in Quadrant 2 (top left) when its $x-$ coordinate is ___________ and its $y-$ coordinate is ___________.

5. A point is in Quadrant 3 (bottom left) when its $x-$ coordinate is ___________ and its $y-$ coordinate is ___________.

6. A point is in Quadrant 4 (bottom right) when its $x-$ coordinate is ___________ and its $y-$ coordinate is ___________.

Draw two lines through point P , one perpendicular to the $x-$ axis and the other perpendicular to the $y-$ axis. Construct segments from point P to the axes, and then hide the lines. Measure the length of each segment. Drag point P and explore.

7. What is this relationship between the coordinates of point P and the distances to each axis?

Problem 2 – Lines, Equations, and Slopes

Delete or hide the segments and measurements. Draw a line connecting P and Q . Using tools from the Appearance menu find the equation and slope of the line.

Look for relationships between the slope and equation as you change the line by grabbing and dragging point P , and then by grabbing and dragging the line itself.

8. When dragging the line by point P , what is the relationship of the slope and the equation?

9. When dragging the line itself, what is changing in the equation?

10. Drag point Q to the $y-$ axis. What is the relationship between point Q and the equation of the line?

Problem 3 – Slopes of Parallel and Perpendicular Lines

Open the Cabri Jr. file **PARALLEL**. Drag the lines by points P and Q and examine the slopes.

11. What can you say about the slopes of two parallel lines?

Open the Cabri Jr. file **PERPENDI**. Again drag the lines to investigate the relationship between the slopes.

12. What can you say about the slopes of two perpendicular lines?

Use the **Calculate** tool to see what happens when the slopes of two perpendicular lines are multiplied together. Select one slope measurement, press $\times$, and select the other slope measurement. Move the product near the original expression.

Now, change the lines by grabbing and dragging point P .

13. What do you observe about the product of the slopes?

5.2 Math Man on the Slopes

This activity is intended to supplement Algebra I, Chapter 4, Lesson 4.

Problem 1- Visually estimating slopes

Press **APPS** and select Cabri Jr. Open the file **MATHMAN**.

Math Man is cross-country skiing from left to right.

- Which part(s) of the hill has the best "ski slope" for Math Man? Explain.

Now open the file **DIPPER**. You will see a representation of the "Big Dipper", a formation commonly recognized in the night sky.

The slopes of the lines of the segments are:

$$\{-0.1, -0.2, -0.4, -9.5, -1.4, 2.7\}$$

- Each segment is labeled with a letter. Match the slope with the segment. Record your answers below.
- How did you determine which slope belonged with which segment?

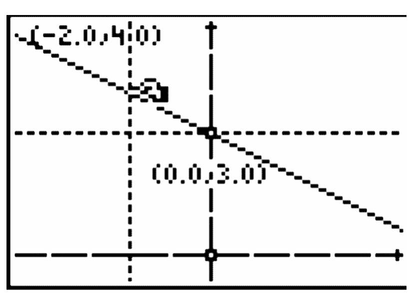
Self-Check Point

- I already know about $y = mx + b$ and what each letter means. True False

Problem 2 – Exploring precise slope

Open the file **SLOPE**.

Move the point at $(-2, 4)$, so the solid line has a slope of $\frac{2}{3}$.



- What are the coordinates of your point?
- How did you determine where to place your point?

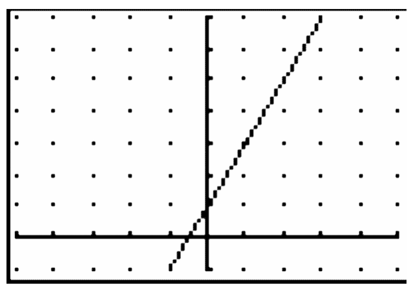
- What is the equation of the line in slope-intercept form?

Move the point at $(0,3)$ to $(1,0)$. Now move the other point so that you have the line $y = x - 1$.

- What is the slope of the line?
- What are the coordinates of your point?
- Did your method of placing the point change? Explain why or why not.

Problem 3 – Slope-Intercept Equation

Use the graph at the right to answer the following questions. The points $(0,1)$ and $(1,3)$ are on the line.



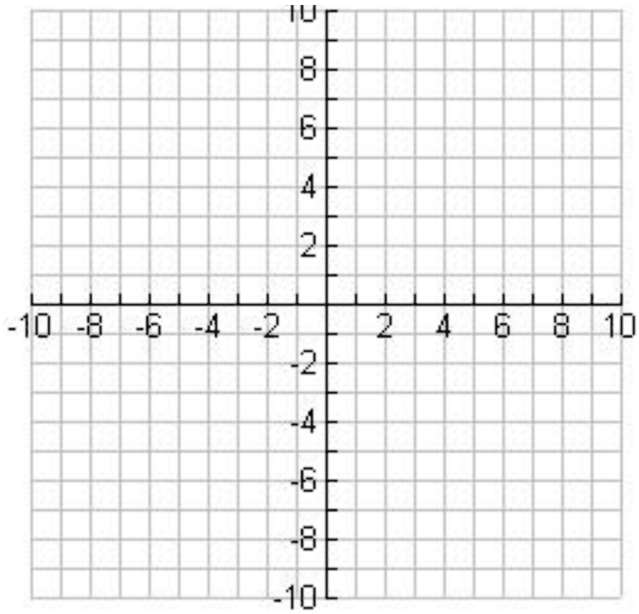
- What is the slope of the line?
- What is the y-intercept of the line?
- What is the equation of the line?

Problem 4 – Assessing Understanding

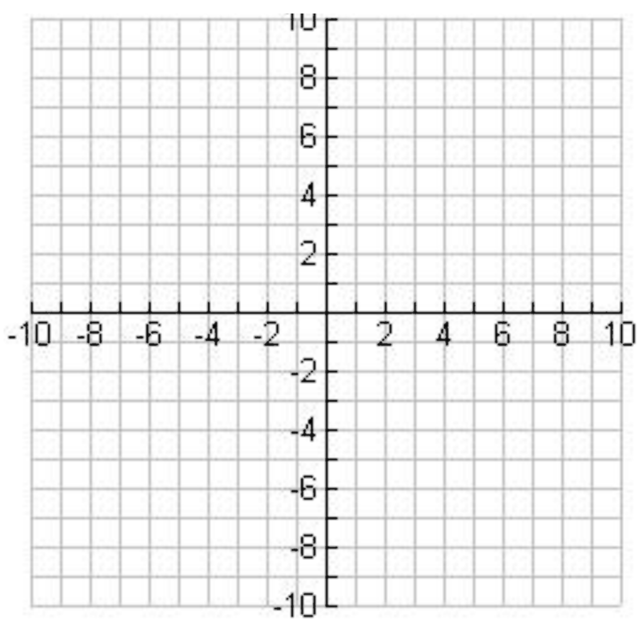
- What kind of line has a slope equal to 0 ?
- What is the slope and $y-$ intercept of $y = -3x + 1$?
- Name the slope and $y-$ intercept: $y = \frac{2}{5}x - 8$
- Name the slope: $y + x = 9$
- Name the slope: $y = -4$
- True or False: $(0,6)$ is the $y-$ intercept of $y = 2x - 6$.
- True or False: $(0,0)$ is the $y-$ intercept of $y = -3x$.
- True or False: $(0,4)$ is an $x-$ intercept since $x = 0$.

Extensions

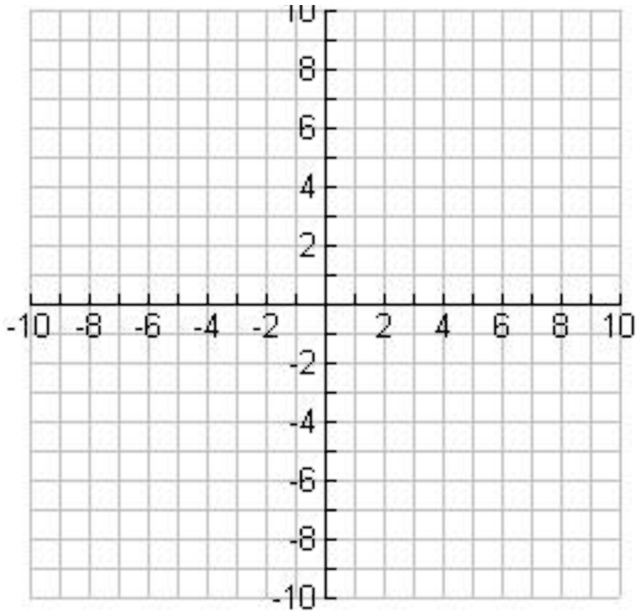
1. Draw a line on the graph at the right with $y-$ intercept $(0,4)$ and any positive slope. Write its equation.



2. Draw a line on this worksheet that goes through $(8,3)$ and has slope $m = 1$. Write its equation.



3. Draw a horizontal line that goes through $(4,-1)$. Write its equation.


10
8
6
4
2
-10 -8 -6 -4 -2 2 4 6 8 10
-2
-4
-6
-8
-10

5.3 Trains in Motion

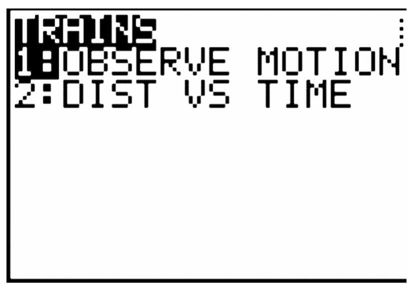
This activity is intended to supplement Algebra I, Chapter 4, Lesson 8.

Problem 1 – Observe Motion

Two trains are leaving the same train station but on different tracks. Time is measured in **hours** and distance is measured in **kilometers**.

Run the program **TRAINS** and select option 1, **OBSERVE MOTION**.

This shows the trains leaving the train station ($y-$ axis).



Train 1: top track

Train 2: bottom track

Press **TRACE** and use the arrow keys to help answer the following questions.

a. Write at least 2 complete sentences describing the motion. Compare train 1 to train 2.
b. When time $= 0$, what is the initial location of train 1? (include units) _________
c. What is the initial position of train 2? __________
d. Which train is traveling at a faster rate? ___________
e. What is the speed of the faster train? ___________
f. How far did the slower train go in 1 *hour* ? __________
g. What is the rate of motion of the slower train? ___________
h. At what distance are the trains the same distance from the station? ___________
i. What time are the trains the same distance from the station? _____________

Problem 2 – Distance-Time Graph

Press 2^{nd} **[MODE] ENTER**. Select option 2, **DIST VS TIME**.

For this graph, the $x-$ axis is time and the $y-$ axis is distance.

Press **TRACE** and use the arrow keys to help answer the following questions.

10. Which train has the graph with a steeper slope? _________

11. What quantity does the slope represent? __________

12. What is the $y-$ intercept of each graph?

a) $y-$ intercept for train 1: _______________

b) $y-$ intercept for train 2: _______________

13. What is the physical meaning of the $y-$ intercept for this distance-time graph?

14. Write an equation for the graph of each train.

a) train 1: ______________________

b) train 2: ______________________

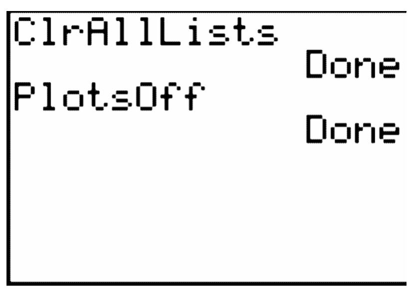
15. To algebraically solve for the time when the two trains are the same distance from the station, set the two equations equal to each other and solve for time. Substitute this time into either equation to find the distance. Show your work.

Extension: List of $d = r \cdot t$ Data

Clear all the lists. On the Home screen press 2^{nd} $[+]$, select **ClrAllLists**, and press **ENTER**.

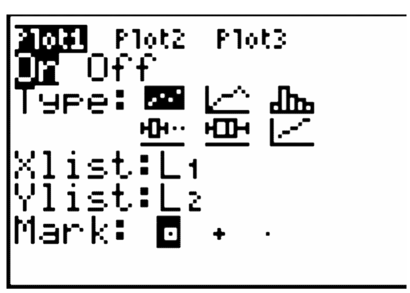
Also turn off all Plots. Press 2^{nd} $[Y =]$, select **PlotsOff**, and press **ENTER**.

Press **MODE** and press **ENTER** on **G-T** to display the graph and table side-by-side.



Set **Plot1** to be a scatter plot by pressing 2^{nd} $[Y =]$ **ENTER** and match the screen to the right.

Press Window and change the settings to $[-1,6]$ for x and $[-5,50]$ for y .

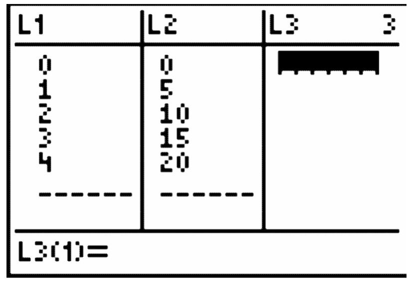


Now enter the following distance-time data into lists $L1$ and $L2$ by pressing **STAT** and selecting **Edit...**

$L1 : 0,1,2,3,4$

$L2 : 0,5,10,15,20$

Note: these distance are for a speed of 5 mi/hr .



Observe the table of values and their scatter plot by pressing **GRAPH**.

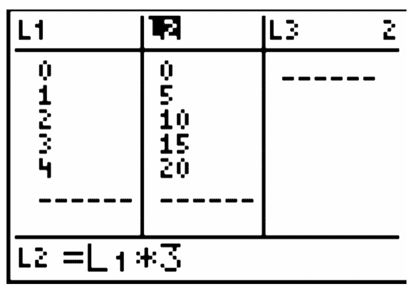
16. When the rate is 5 mi/hr , what is the distance when the time is 5 *hours* ? In other words, what should the next value be in $L2$?

Test this by entering 5 in $L1$ and your "guess" in $L2$. See if you are correct by pressing **GRAPH**.

To observe the changes other rates have on the resulting graph, in the List Editor move the cursor so that it is on top of $L2$ and enter $L1*3$.

Then view the graph to observe the resulting changes.

Enter as well as other values for the rate.



17. When r increases: Describe the slope. What happens to the distance?

When r decreases: Describe the slope. What happens to the distance?

CHAPTER 6 SE Writing Linear Equations

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 5: http://www.ck12.org/flexr/chapter/4475
- Algebra I, second edition, Chapter 5: http://www.ck12.org/flexr/chapter/9564
- Basic Algebra, Chapter 5: http://www.ck12.org/flexr/chapter/8158

6.1 Perpendicular Slopes

This activity is intended to supplement Algebra I, Chapter 5, Lesson 4.

In this activity, you will explore:

- *an algebraic relationship between the slopes of perpendicular lines*
- *a geometric proof relating these slopes*

Problem 1 – An Initial Investigation

Open the **Cabir Jr**. app by pressing **APPS** and choosing it from the menu. Press **ENTER**. Press any key to begin.

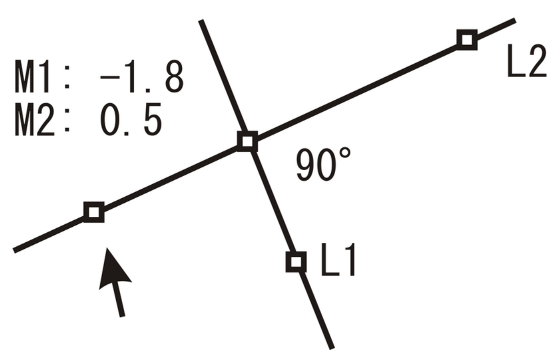
The calculator displays the Cabri Jr. window. Open the **F1: File** menu by pressing $Y =$. Arrow down to the **Open...** selection and press **ENTER**.

Choose figure **PERP1** and press **ENTER**.

Two lines are displayed:

line $L1$ with a slope of $m1$ and line $L2$ with a slope of $m2$.

Notice that the angle formed by the intersection of the lines measures 90° ; that is, the two lines are perpendicular.



Grab line $L1$ by moving the cursor over the point pressing **ALPHA** cursor turns into a hand to show that you have grabbed the point.

Rotate $L1$ by dragging the point using the arrow keys. Observe that as the slopes of the lines change, the two lines remain perpendicular. Explore the relationship between the slopes by answering the questions below.

a. Can you rotate $L1$ in such a way that $m1$ and $m2$ are both positive? Both negative?
b. Can you rotate $L1$ so that $m1$ or $m2$ equals 0 ? If so, what is the other slope?
c. Can you rotate $L1$ so that $m1$ or $m2$ equals 1 ? If so, what is the other slope?
d. Rotate $L1$ so that $m1$ is a negative number close to zero. What can be said $m2$?
e. Rotate $L1$ so that $m1$ is a positive number close to zero. What can be said about $m2$?

Problem 2 – A Closer Examination

Now that you have observed some of the general relationships between the slopes of two perpendicular lines, it is time to make a closer examination. Press 2^{nd} **[MODE]** to exit Cabri Jr.

Press **PRGM** to open the program menu. Choose **PERP2** from the list and press **ENTER** twice to execute it.

Enter a slope of 2 and press **ENTER**.

The program graphs a line $L1$ with the slope you entered and a line $L2$ that is perpendicular to $L1$. $m1$ is the slope of $L1$ and $m2$ is the slope of $L2$.

Press **ENTER** and the calculator prompts you for another slope. Use the graph to complete the following.

1. Enter 0 to make the slope of $L1$ equal to 0 . What is the slope of $L2$?

2. What is the slope of $L2$ when the slope of $L1$ is 1 ?

3. What is the slope of $L2$ when the slope of $L1$ is -1 ?

Enter other values for the slope of $L1$ and examine the corresponding slope of $L2$. For each slope that you enter, $m1$ and its corresponding value of $m2$ are recorded in the lists L_1 and L_2 . To see a history of your "captured" values, enter a slope of 86 to exit the program. Then press **STAT** and **ENTER** to enter the **List Editor**. The values of $m1$ are recorded in L_1 and the values of $m2$ are recorded in L_2 .

L1	L2	L3 1
2	2	-------
3	3	
.5	.5	
1	1	
−1	−1	
.25	.25	
------	------	

L1(1)=2

4. Conjecture a formula that relates the slope of two perpendicular lines. Enter your formula in the top of L_3 (with variable L_1) to test your conjecture.

Problem 3 – A Geometric Look

Start the Cabri Jr. app and open the file **PERP3**.

This figure shows another way to examine the slopes of perpendicular lines, geometrically. There should be two lines, $L1$ and $L2$, with a slope triangle attached to each of them.

Grab line $L1$, rotate it, and compare the rise/run triangles.

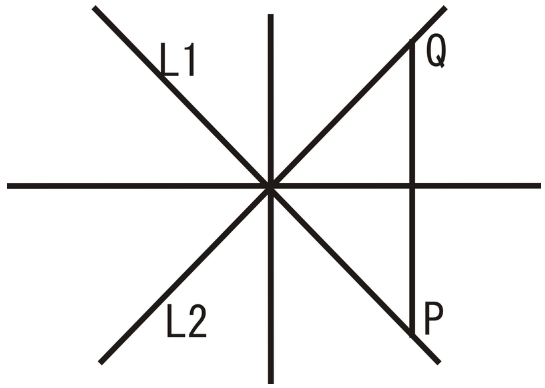
a. What do you notice about the two triangles?

Problem 4 – The Analytic Proof

We now will analytically verify that two lines with slopes $m1$ and $m2$ are perpendicular if and only if $m1 \cdot m2 = -1$.

(All of the following assumes $m1 \neq 0$. What can be said about the case when $m1 = 0$?)

Open the CabriJr file **PERP4**. This graph shows two perpendicular lines $L1$ and $L2$ with slopes $m1$ and $m2$ respectively, translated such that their point of intersection is at the origin. Refer to the diagram to answer the questions below.



a. What are the equations of these translated lines as shown in the diagram?
b. Let P be the point of intersection of line $L1$ and the vertical line $x = 1$ and let Q be the point of intersection of line $L2$ and the line $x = 1$. What are the coordinates of points P and Q ?
c. Use the distance formula to compute the lengths of $\overline{OP}, \overline{OQ}$, and $\overline{PQ}$.(Your answers should again be in terms of $m1$ and $m2$.)
d. Apply the Pythagorean Theorem to triangle POQ and simplify. Does this match your conjecture from Problem 2?

Problem 5 – Extension Activity #1

The CabriJr file **PERP5** shows a circle with center O and radius OR . Line T is tangent to the circle at point R .

The slopes of line T and segment OR are shown (mT and mOR , respectively.)

Your first task is to calculate $\frac{1}{mOR}$. Activate the **Calculate** tool, found in the **F5: Appearance** menu.

Move the cursor over 1 and press **ENTER**. Repeat to select mOR , the slope of the segment OR .

Press / to divide the two numbers. Drag the quotient to a place on the screen where you can see it clearly and press **ENTER** again to place it.

a. Grab point R and drag it around the circle. Observe the changing values of mT, mOR , and $\frac{1}{mOR}$. What can you conjecture about the relationship between a tangent line to a circle and its corresponding radius?

Problem 6 – Extension activity #2

The CabriJr file **PERP6** shows a circle with an inscribed triangle QPR . The segment QR is a diameter of the circle.

The slopes of segments PR and PQ are shown (mPR and mPQ , respectively.)

Compute $\frac{1}{mPQ}$ using the **Calculate** tool.

a. Grab point P , drag it around the circle, and examine the changing values. What can you conjecture about a triangle inscribed in a circle such that one side is a diameter?

6.2 Exploring Linear Equations

This activity is intended to supplement Algebra I, Chapter 5, Lesson 5.

In this activity we will

- Enter "life expectancy" data into lists and set up scatter plots.
- Trace the scatter plot to select two points. Use the points to calculate slope and write a linear equation.
- Use the APP to fit the data using a linear equation in slope-intercept form and analyze the meaning of the slope and the intercept in relationship to birth year and life expectancy.

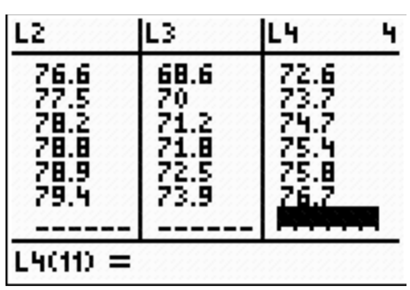
TABLE 6.1: U. S. Life Expectancy at Birth

Birth Year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.6	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
1998	79.4	73.9	76.7

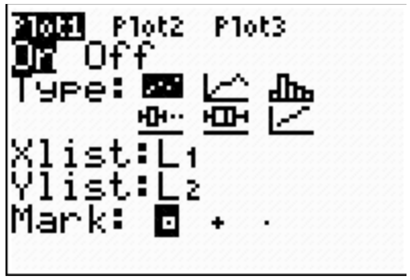
Problem 1 - Enter Data into the Table

Press **STAT** Edit.

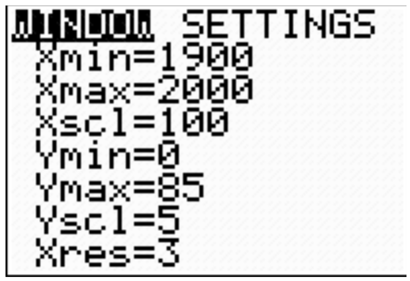
Enter Birth Year into L_1 , Female Life Expectancy into L_2 , Male Life Expectancy into L_3 , and Combined Life Expectancy into L_4 .



Set up a scatter plot of Birth Year versus Female Life Expectancy. Press 2^{nd} **[STAT PLOT]** for the **STAT PLOT** menu. Press **ENTER ENTER**. This will select **Plot 1** and turn it on. Because the defaults are "scatter plot, L_1 and L_2 " no other settings need to be changed.



Press **WINDOW** to set the window with the settings shown.



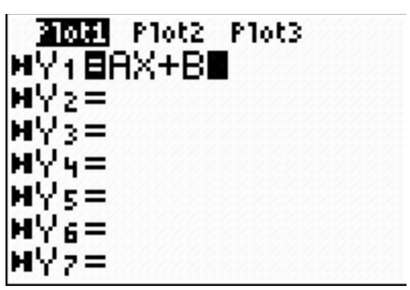
Press **TRACE**. Select two points that seem to fall on the line that would best fit the data.

Using the $x-$ and $y-$ values from the two points, calculate the slope. Then use the point slope form to calculate the $y-$ intercept. Discuss the meanings of both numbers (for each year after 1940 you were born, you should live about 2 tenths of a year longer. . . if the trend continued into the past, then in the year 0 people would have lived –327 years).

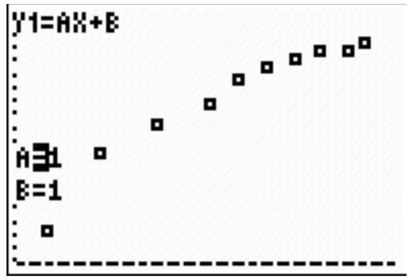
Press **APPS** and locate the *Transformation Graphing* **APP** (**Transfrm**). Press **ENTER**.

Press any key as directed. You will be sent to the home screen and it will appear as though nothing has happened.

Press $Y =$ to see the effect of engaging the **APP**. Enter the general form of a linear equation by pressing **ALPHA[MATH]** $x+$ **ALPHA[APPS]**.

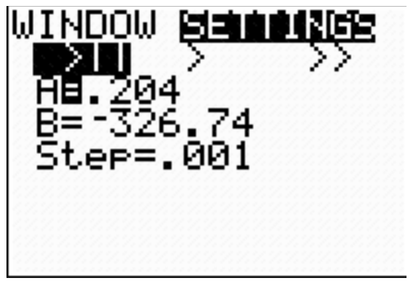


Press **GRAPH**. By default A will be set at 1 and B will be set at 1 unless the **APP** has been used since the calculator was reset.

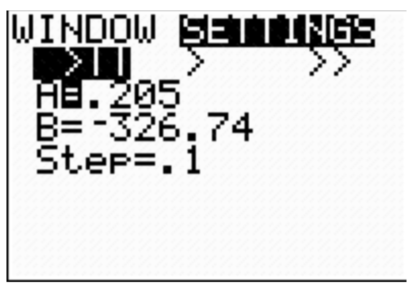


Type in the value of A found when calculating slope in an earlier step (.204 in the example). Press **ENTER**, then type in the value of *B* ([U+0080][U+0093]326.74 in the example). Press **ENTER**.

To adjust the slope of the line for fine-tuning, press **WINDOW** to view the settings. Reset the step to .001 . Return to the graph. Use the right and left arrows to adjust *A* to try to slightly to improve the fit.



To further adjust the line for fine-tuning, press **WINDOW** to view the settings. Reset the step to .1 .



Return to the graph. Use the right and left arrows to adjust *B* to further improve the fit.

Extension

Repeat for the life expectancy of males or for males and females combined. This allows students to practice using two points to calculate slope and write a linear equation and get immediate feedback on their accuracy.

6.3 Finding a Line of Best Fit

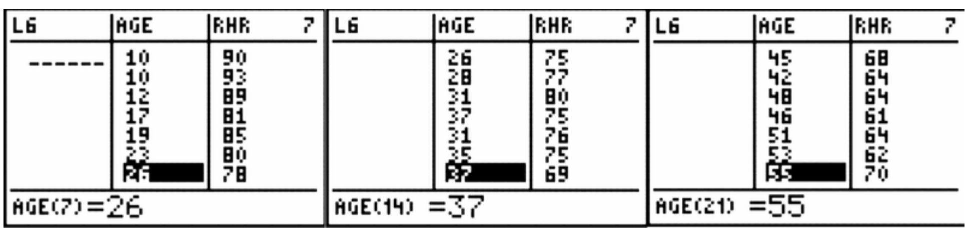
This activity is also intended to supplement Algebra I, Chapter 5, Lesson 5.

In this activity, you will explore:

- *Creating a scatter plot representing resting heart rates versus age.*
- *Graphing vertical and horizontal lines to show $Q1$ and $Q3$ for both the ages and the heart rates.*
- *Using the vertices of the $Q1$ and $Q3$ lines to calculate a line of best fit and graph it.*

Problem 1 - Entering the Data

Enter the 21 data points shown in the screens below in the **AGE** and **RHR** lists.



Plot this data as a scatter plot.

Problem 2 - Answer the Questions

- What are $Q1$ and $Q3$ of the ages? Graphs these as horizontal lines.
- What are $Q1$ and $Q3$ of the heart rates? Graph these as vertical lines.
- Where do these lines intersect? Write the coordinates of the four points.
- Identify the diagonal across the center rectangle that follows the direction of the points. What two intersection points does it connect?
- Use these two points to write the equation for the line that will form the diagonal using the point-slope form.

Problem 3 - Graph the Equation

Using the information from Problem 2, graph the equation of the line.

CHAPTER 7 SE Graphing Linear Inequalities

Chapter Outline

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 6: http://www.ck12.org/flexr/chapter/4476
- Algebra I, second edition, Chapter 6: http://www.ck12.org/flexr/chapter/9565
- Basic Algebra, Chapter 6: http://www.ck12.org/flexr/chapter/9159

7.1 Linear Inequalities

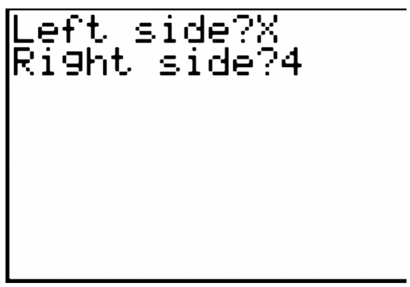
This activity is intended to supplement Algebra I, Chapter 6, Lesson 3.

Problem 1 – Table of Values

In this problem, you will explore and graph a simple inequality: $x \geq 4$. Press **PRGM** to access the **Program** menu and choose the **LINEQUA** program.

Enter the left side of the inequality, X , and press **ENTER**.

Then enter the right side of the inequality, 4 , and press **ENTER**.



Select an inequality symbol. Press 2 to choose $\geq$.

Choose **1:View Table** to see a table of values.

The calculator displays a table with several columns. The first column X shows the values of the variable, x . The second column $Y1$ shows the value of the left side for each $x-$ value. The third column $Y2$ shows the value of the right side for each $x-$ value.

1. Describe the numbers in the $Y1$ column. How do they compare to the $x-$ values? Explain.

2. Describe the numbers in the $Y2$ column. Are they affected by the $x-$ values? Explain.

Now look at the fourth column $Y3$. Each entry in this column is either a 1 or a 0 . Examine this column.

(***Note:*** To scroll up or down, return to the X column, scroll, and then return to the $Y3$ column.)

3. For what $x-$ values is there a 1 in the $Y3$ column?

4. Substitute one of these $x-$ values into $x \geq 4$. Is the inequality true for this value of x ?

5. For what $x-$ values is there a 0 in the $Y3$ column?

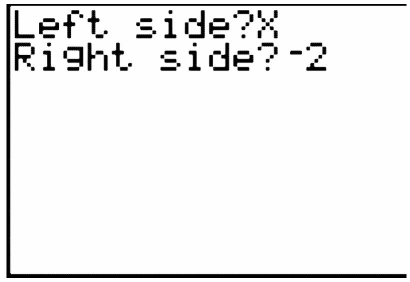
6. Substitute one of these $x-$ values into $x \geq 4$. Is the inequality true for this value of x ?

Press **ENTER** to exit the table, then press **ENTER** again to select **1:Another Ineq**.

This time enter the inequality $x < -2$.

Choose **1:View Table** from the menu. Look at each column. Again, each entry in the $Y3$ column is either a 1 or a 0 .

7. For what $x-$ values is there a 1 in the $Y3$ column?



8. Substitute one of these values in for x in $x < -2$. Is the inequality true for this value of x ?

9. For what $x-$ values is there a 0 in the $Y3$ column?

10. Substitute one of these values in for x in $x < -2$. Is the inequality true for this value of x ?

11. *Complete each statement.*

a) If the $x-$ value makes the inequality <u>true</u>, the entry in the $Y3$ column is _______.

b) If the $x-$ value makes the inequality <u>false</u>, the entry in the $Y3$ column is _______.

Problem 2 – Graphing

Now you are going to look at the graph of a simple inequality. Press **ENTER** to exit the table, and then press **ENTER** again to select **1:Another Ineq**. Enter the inequality $x > 2$. Choose **2:View Graph** from the menu.

The calculator draws a line above the $x-$ values on the number line where the inequality is true. The inequality is not true when $x = 2$, so an open circle is displayed there.

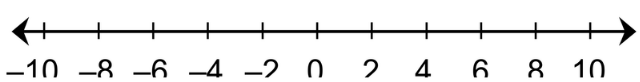
Press **ENTER** to exit the graph screen, and then press **ENTER** again to select **1:Another Ineq**. Graph $x \geq 2$.

12. Describe the difference between the graphs of $x > 2$ and $x \geq 2$.

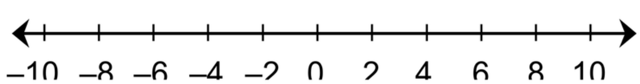
<u>Exercises</u>

13. Graph each inequality using your graphing calculator. Sketch the graphs here.

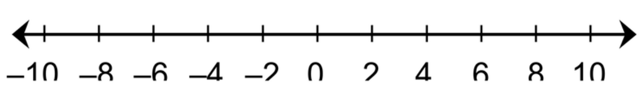
a) $t > 5$



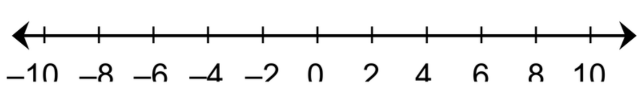
b) $p < -2$



c) $z \geq -2$



d) $y \leq 0$



Problem 3 – Solving Inequalities using Addition and Subtraction

You can use your calculator to check that two inequalities are equivalent. To see that $x-3>5$ and $x>8$ are equivalent, run the **LINEQUA** program. Enter the inequality $x-3>5$.

Choose **Compare Ineq**. to compare this inequality to another.

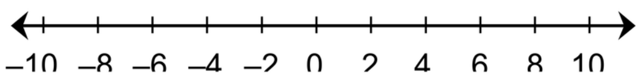
Enter $x>8$. The calculator displays the graphs of $x-3>5$ and $x>8$ on the same screen. The graphs are the same, so the inequalities are equivalent.

- **Caution:** In some graphs, the open circle will appear to be filled in. This is because of the size of the pixels on the graph screen. For this reason, a "closed circle" is shown as a cross, and an "open circle" as a dot.

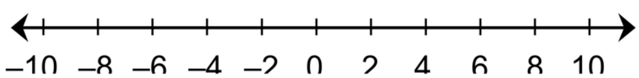
<u>Exercises</u>

14. Solve each inequality. Use your calculator to compare the original inequality with the solution. Then sketch the graph of the solution.

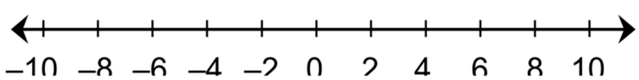
a) $f-5\geq 2$



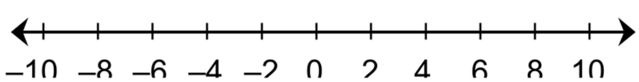
b) $-4>g-2$



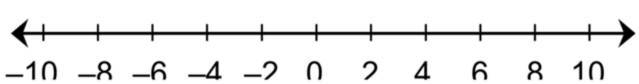
c) $u+1\leq 5$



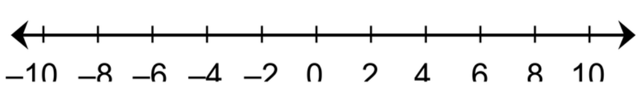
d) $1<8+v$



e) $-5>h-1$



f) $-5\leq 1+t$



Problem 4 – Solving Inequalities using Multiplication and Division

Use your calculator to compare the graphs of the given inequalities.

15. $\frac{x}{5} \le -1$ and $x \le -5$

a) Are these equivalent inequalities? Explain.

b) Can you multiply both sides of an inequality by 5 without changing its solutions?

16. $4x > 8$ and $x > 2$

a) Are these equivalent inequalities? Explain.

b) Can you divide both sides of an inequality by 4 without changing its solutions?

17. $-x > 4$ and $x > -4$

a) Are these equivalent inequalities? Explain.

b) Can you multiply both sides of an inequality by -1 without changing its solutions?

18. $-x > 4$ and $x < -4$

a) Are these equivalent inequalities? Explain.

<u>Exercises</u>

19. Compare graphs to find the inequality symbol that makes each pair of inequalities equivalent.

a) $\frac{v}{-4} \ge 2 \quad v\_\_\_\_ -8$

b) $-\frac{d}{3} < -3 \quad d\_\_\_\_9$

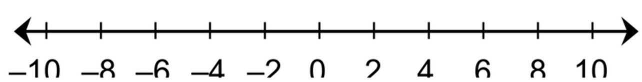
c) $-2h > -2 \quad h\_\_\_\_1$

d) $-5r \le 10 \quad r\_\_\_\_ -2$

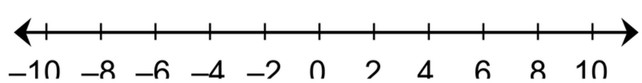
To solve an inequality using multiplication or division, multiply or divide both sides of the inequality by the same number. ***However***, if you multiply or divide both sides by a **negative** number, you must reverse the inequality symbol to obtain an equivalent inequality.

20. Solve each inequality. Use your graphing calculator to compare the original inequality with the solution. Then sketch its graph.

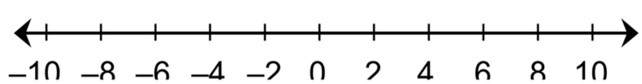
a) $\frac{c}{4} \ge 1$



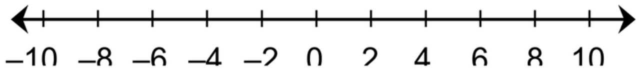
b) $2 < -\frac{d}{4}$



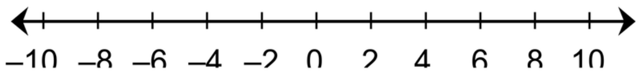
c) $3w \le -9$



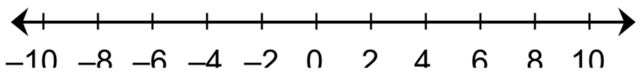
d) $20 > -5x$



e) $18d < -12$



f) $-\frac{5}{7}g > -5$



7.2 Introducing the Absolute Value Function

This activity is intended to supplement Algebra I, Chapter 6, Lesson 5.

In this activity, you will examine data by comparing individual data points to the mean by finding the difference (positive or negative) and the distance from the mean, plot the distances versus the differences to examine the shape of the plot, investigate the absolute value function in the $Y =$ register to model the relationship between the distances and the differences, and extend the investigation of absolute value equations by examining tables and graphs.

Problem 1 - Analyze the Data

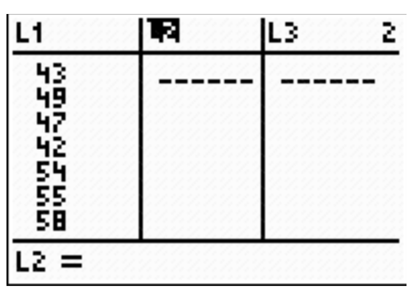
The high temperatures in the first twelve days of February were: $43, 49, 47, 42, 54, 55, 58, 58, 61, 62, 49, 46$.

Press **STAT ENTER**. Enter these 12 data points into L_1 .

Press 2^{nd} **[MODE]** to return to the home screen. Press 2^{nd} **[STAT]** $\rightarrow\rightarrow$ to the 'Math on Lists' menu. Press 3 to select **3:mean**(.

This will paste the command onto the home screen. Press 2^{nd} [1]) to complete the command to find the mean of L_1 . Press **ENTER** to execute.

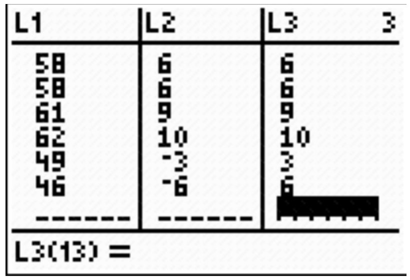
Now that you know the mean of the temperatures, press **STAT ENTER** to return to the 'statistics editor.' Arrow to the top of L_2 as shown.



Press 2^{nd} $[1] - 52$. This will command the calculator to subtract the mean of 52 from each of the temperatures in L_1 .

Press **ENTER** to execute. What do you notice about the numbers in L_2 ? What is the highest difference? What is the smallest difference? When are the differences negative? Positive?

Move over to L_3 . Examine each entry in L_1 and determine is DISTANCE from the mean (how far away). Enter the distances in L_3 . What is the relationship between the distances and the differences from L_2 ? Why is this so?

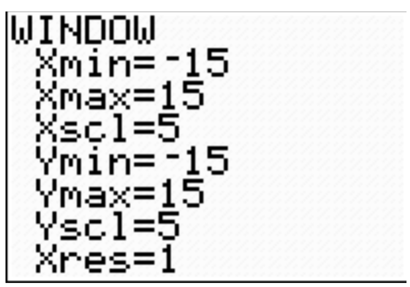


Set up a scatter plot to compare the distances to the differences (L_3 to L_2). Press 2^{nd} $Y =$. Press 1 to select **1:Plot 1**.

Press **ENTER** to turn the plot **On**. Arrow down to the **Xlist**.

Press 2^{nd} [2] to use L_2 (the differences) as the x list. Arrow down to the **Ylist**. Press 2^{nd} [3] to use L_3 (the distances) as the y list.

Press **WINDOW**. Set the window as shown.



Press **GRAPH**. Press **TRACE** to examine the relationships between the $x-$ and $y-$ coordinates of each point. When x is positive, what happens to y ?

When x is negative, what happens to y ? When will y be negative? Why? When is x negative?

(sample response: x is negative whenever the temperature was lower than the mean; y will not be negative because distances are positive)

Problem 2 - Compare Data Against Equations

Press $Y =$. Enter the equation $y = x$ into Y_1 .

Press **GRAPH**. What is the relationship between $y = x$ and the scatter plot?

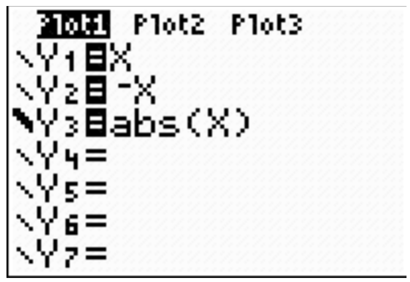
Return to $Y =$. Enter the equation $y = -x$ into Y_2 .

Press **GRAPH**. What is the relationship between $y = -x$ and the scatter plot?

Press 2^{nd} **[GRAPH]** to examine the tables for Y_1 and Y_2 . How are the values for X and Y_1 related? How are the values for X and Y_2 related? How are the values for Y_1 and Y_2 related? Where is each Y equal to zero?

Return to $Y =$. Arrow down to Y_3 . Press **MATH** $\rightarrow$ to find the absolute value command **1:abs(**. Press **ENTER**. This will paste the command into Y_3 .

Complete the function as shown. Arrow left of Y_3 . Press **ENTER** to change the graph to a 'thick line.'



Press **GRAPH**. What is the relationship between $y = abs(x)$ and the scatter plot? NOTE: In your textbook this function will be written as $y = |x|$.

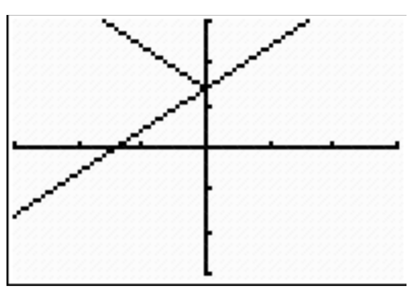
Press 2^{nd} **[GRAPH]** to examine the tables. How are the values for Y_3 related to Y_1 and Y_2 ? Where is Y equal to zero?

Extension

Examine another absolute value equation. First, clear all earlier functions in $Y =$ and enter $y = x + 7$ into $Y1$. To clear functions, put cursor over the equals sign and press **[CLEAR]**.

Press 2^{nd} **[GRAPH]** Examine the table. When are the Y_1 values positive? When are they negative? When is Y_1 zero?

Return to $Y =$. Enter the equation $y = abs(x) + 7$ into Y_2 using **[MATH]** $\rightarrow$ abs(Examine the graph. What seems to be the relationship between the graphs?



Examine the table. Is the relationship between Y_2 and Y_1 what you were expecting? Why or why not? Where are the Y values equal to zero?

Return to $Y =$. Enter the equation $y = abs(x + 7)$ into Y_2 as shown.

Examine the graph. What seems to be the relationship between the graphs? How is this picture different from the graph with $y = abs(x) + 7$?

Examine the table. Is the relationship between Y_2 and Y_1 what you were expecting? Why or why not? Where are the Y values equal to zero?

Compare $y = abs(x) + 7$ to $y = abs(x + 7)$. How are they similar? Different?

7.3 Can I Graph You, Too?

This activity is intended to supplement Algebra I, Chapter 6, Lesson 6.

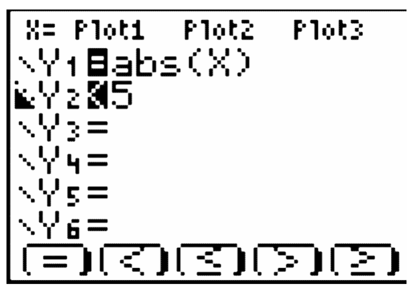
Probmem 1 - Introduction to Disjunction and Conjunction

Consider the equation $|x| = 5$. To solve, you would graph both sides of the equation as functions ($y = |x|$ and $y = 5$) and mark the solution as the area where the graphs intersect.

The same method can be applied to inequalities.

Press **APPS** and select the **Inequalz** app. Press any key to begin.

Example 1: $|x| < 5$



- Using $Y1 =$ graph the left side as $y = |x|$. The absolute value function is located by pressing **MATH** $\rightarrow$ and selecting **abs(**.
- Using $Y2 =$ graph the right side as $y < 5$. On the equals sign, press **ALPHA** $[F2]$ for the $<$ sign. Press **ZOOM** and select **ZoomStandard**.
- Find the intersection points by pressing 2^{nd} **[TRACE]** and selecting **intersect**. Now just move the cursor to the intersection point and press **ENTER** three times. The solution is where the shading overlaps the graph of the absolute value function.

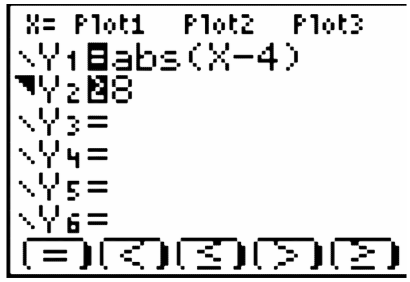
In this case, the solution is $-5 < x < 5$.

When an absolute value is less than a number, it is a **conjunction** because the solution is just one part of the graph.

$$|ax+b| < c \quad \rightarrow \quad -c < ax+b < c.$$

Example 2: $|x-4| \geq 8$

- Using $Y1 =$ graph the left side as $y = |x-4|$.
- Using $Y2 =$ graph the right side as $y \geq 8$. On the equals sign, press **ALPHA** $[F5]$ for the $\geq$ sign. Press **WINDOW** to choose appropriate window settings.
- Find the intersection points.



In this case, the solution is $x \le -4$ **or** $x \ge 12$.

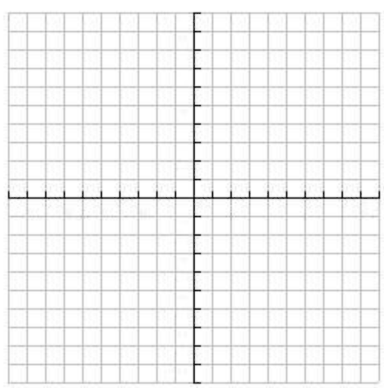
When an absolute value is greater than a number, it is a **disjunction** because the solution is two separate parts of the graph.

$$|ax+b| > c \quad \rightarrow \quad ax+b < -c \quad \text{or} \quad ax+b > c.$$

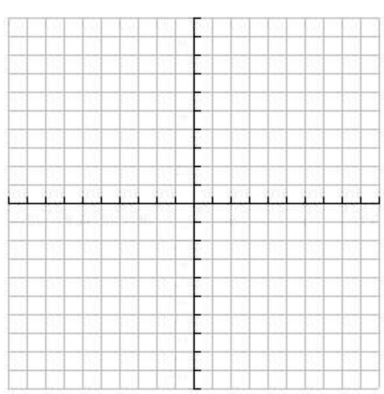
Problem 2 - Application of Disjunction and Conjunction

For the problems below, write the inequalities as either a conjunction or disjunction, then solve for x . Check your solution by graphing using the method described in Examples 1 and 2. Please use your graphing calculator to check your results.

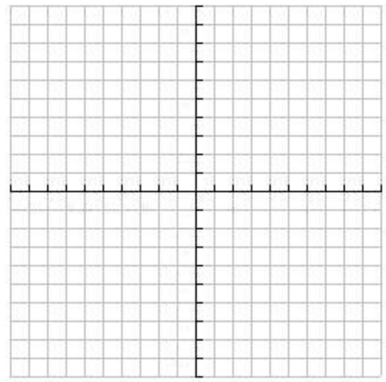
#1: $|2x-3| > 9$



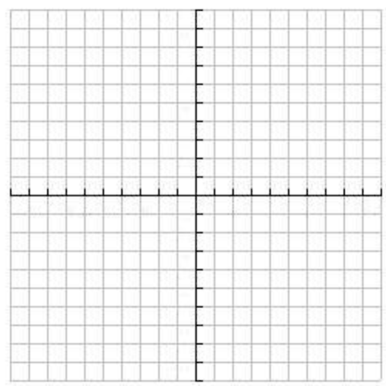
#2: $\left|\frac{1}{3}x-10\right| \le 11$



#3: $|3x| - 1 \geq 5$



#4: $2|4x-7| + 6 < 18$



Problem 3 - Real World Application

One application of absolute value inequalities is engineering tolerance. Tolerance is the idea that an ideal measurement and an actual measurement can only differ within a certain range.

A bolt with a 10 *mm* diameter has a tolerance range of 9.965 *mm* to 10 *mm* , while the hole that it fits into has a tolerance range of 10.05 *mm* to 10.075 *mm* .

How can you express the tolerances of both the bolt and the hole in terms of an absolute value inequality?

CHAPTER 8 SE Solving Systems of Equations and Inequalities

Chapter Outline

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 7: http://www.ck12.org/flexr/chapter/4477
- Algebra I, second edition, Chapter 7: http://www.ck12.org/flexr/chapter/9604
- Basic Algebra, Chapter 7: http://www.ck12.org/flexr/chapter/9160

8.1 Boots in Motion

This activity is intended to supplement Algebra I, Chapter 7, Lesson 2.

Problem 1 - Solving for Two Unknowns

From the town of Alton to Barnhart, the Mississippi River has an average surface speed of about 2 mph . Suppose it takes a boat 3 hours to travel downstream, but 5 hours to travel upstream the same distance between the two towns.

1. Let r be the rate of the boat in still water. How could the rate upstream and the rate downstream be expressed?

2. Use the above information to fill in the blank spaces.

distance $=$ rate $\times$ time

down $d = \_\_\_\_ \times \_\_\_\_$

up $d = \_\_\_\_ \times \_\_\_\_$

Note: The solution occurs when the distance d in both equations is the same.

3. Set the equations equal to each other and solve for r algebraically. Show your work here.

Solve the system of equations graphically. Press $Y =$ and enter both equations.

Note: x should replace r .

The solution is the intersection point. Adjust the window so that you can see the intersection of the two lines.

Find this intersection point by pressing 2^{nd} **[TRACE]** and selecting **intersect**. Select the first line, the second line, and make a guess. The coordinates of the intersection point will appear.

4. How does this point compare with your solution from Question 3?

Record the rate of the boat in still water and the distance between the towns. Include units.

Problem 2 - Distance-time graph, explore slopes

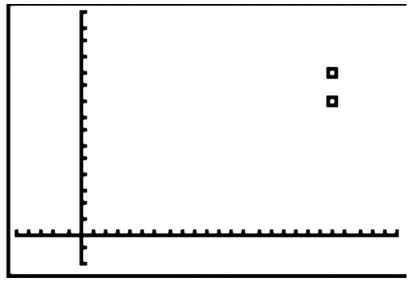
Velma is riding on a steam engine locomotive. As she walks forward, she travels 1.1 $miles$ in 2 $minutes$. As she walks back to her seat she travels 0.9 $miles$ in 2 $minutes$. How fast is the train moving and how fast was Velma walking inside the train?

To explore this situation graphically, we need to set up the graph.

Create a scatter plot of the data points.

Press **STAT[ENTER]** and enter the two pieces of the data (2, 1.1) and (2, 0.9) into lists $L1$ and $L2$.

Press 2^{nd} **[Y=]ENTER**, choose scatter plot, $L1$ for Xlist and $L2$ for Ylist.



Press **WINDOW**. Set x to $[-0.5, 2.5]$ and y to $[-0.2, 1.5]$.

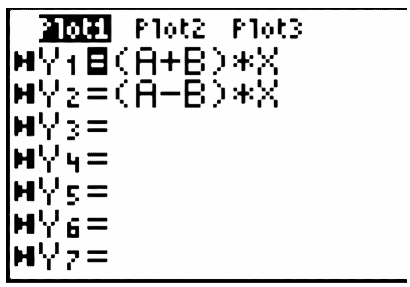
Graph the two $d = r \cdot t$ equations

Press **APPS** and select **Transform**. Enter the equations on the $Y =$ screen as shown at the right.

Then press **WINDOW** and arrow to **SETTINGS**.

Set $A = 0.1$, $B = 0.1$ and Step $= 0.01$

In these equations, A represents the rate of the train and B represent the rate of Velma walking.



Press **GRAPH**. Use the arrow keys to adjust the values of A and B so the line goes through the point. The $Y1$ line should go through the top point and the $Y2$ line should go through the bottom point.

5. What does the slope of the line in the distance-time graph represent?

6. Apply $d = r \cdot t$ to this situation. $r =$?

(Hint: r depends on A , the speed of the steam engine, and B , the velocity of Velma.)

distance $=$ rate $\times$ time

forward __ = ____ × __

back __ = ____ × __

7. Algebraically solve the equation.

Problem 3 - Extension/Homework

1. An airplane flew 3 hrs with a tail wind of 20 km/h . The return flight with the same wind took 3.5 hrs . Find the speed of the airplane in still air. Fill in the chart are answer and solve.

distance $=$ rate $\times$ time

west __ = (r + __) × __

east __ = (r − __) × __

2. Two cars leave town going in opposite directions. One travels 50 *mph* and the other travels 30 *mph* . In how many hours will they be 160 *miles* apart?

distance = rate × time

slow car __ = __ × __

fast car __ = __ × __

Hint: Distances will add up to 160

8.2 How Many Solutions?

This activity is intended to supplement Algebra I, Chapter 7, Lesson 5.

In this activity, you will explore:

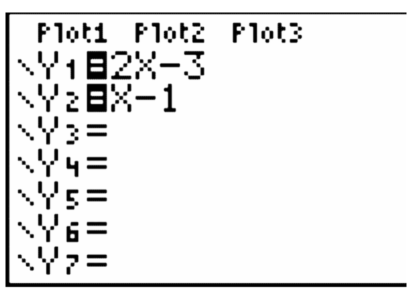
- *Graphing linear systems to determine the number of solutions*
- *Creating a linear system with a particular number of solutions*
- *The relationship between the coefficients of a linear system and the number of solutions*

Problem 1 - Graphing systems of linear equations

Do all linear systems have just one solution? In this problem, you will graph several different linear systems to see how many solutions they have.

Graph each system below by solving the equations for y and entering them into $Y1$ and $Y2$. View the graph in a standard viewing window. (If necessary, set the viewing window by going to **Zoom #62; ZStandard**.)

Sketch each graph. How many solutions does each system have?

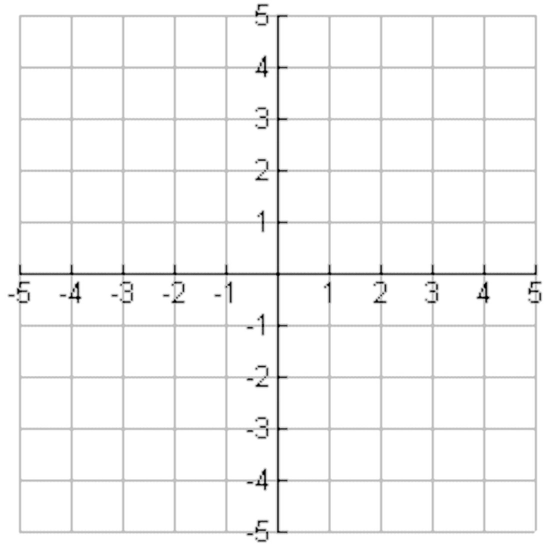


1. $\begin{cases} y = 2x - 3 \\ y = x - 1 \end{cases}$

2. $\begin{cases} y = -3x + 3 \\ y = -3x - 1 \end{cases}$

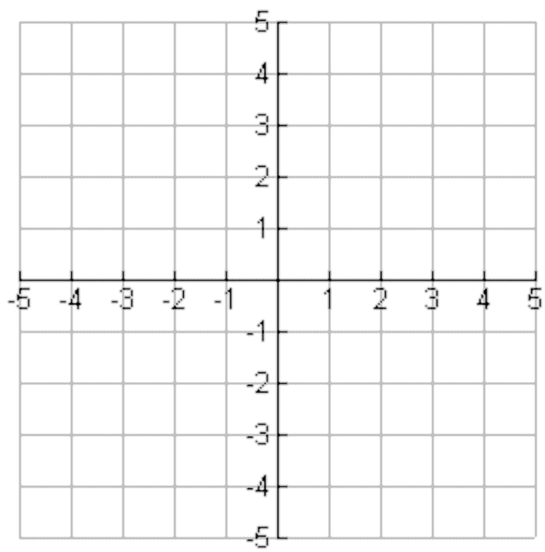
3. $\begin{cases} 4x + 2y = 6 \\ y = -2x + 3 \end{cases}$

Sketch:



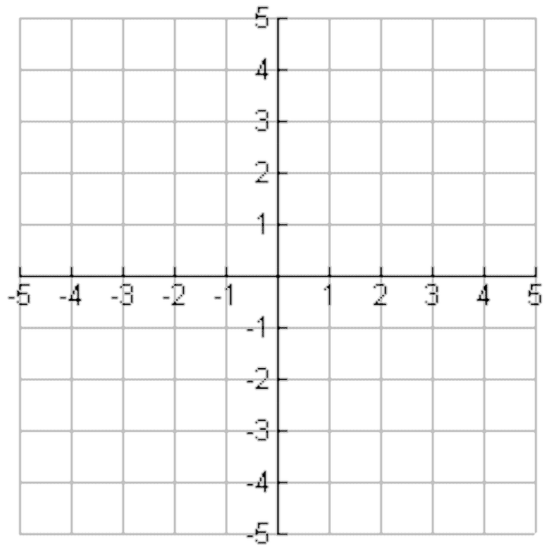
Number of solutions:

Sketch:



Number of solutions:

Sketch:



Number of solutions:

These three graphs show all the possible ways two lines can relate to each other.

If the two lines ...		Then the system has ...
• Cross at a single point	⟶	• One solution
• Never cross (are parallel)	⟶	• No solution
• Are really the same line	⟶	• Infinitely many solutions

Problem 2 - Create your own system

Can you create a system with one solution?

The **CabriJr** file **HOWMANY1** shows the graph of a linear equation. Press **[ALPHA]** $F1$ and scroll down to **Open** . . .

Use the **Line** tool (**[ALPHA]** $F2$ **ENTER Line ENTER**) to draw a second line on the graph so that the two lines form a linear system with exactly one solution.

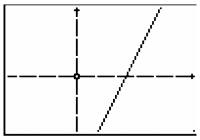
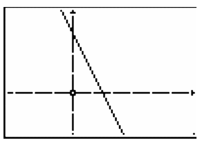
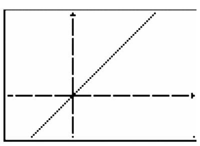
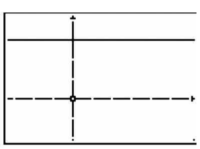
Use the **Coordinates and Equations** tool (**[ALPHA]** $F5$ **ENTER Coord. #38; Eq. ENTER**) to find the equations of the two lines. Record them in the table.

Delete the line you drew. Draw a new line to make a system with no solution. Record the equation of the line in the table.

Next, delete that line. Draw a new line to make a system with infinitely many solutions. Record the equation of the line.

Repeat this experiment with the lines you find in the **CabriJr** files **HOWMANY2**, **HOWMANY3**, and **HOWMANY4**. For each file, make a system with one solution, a system with no solutions, and a system with infinitely many solutions. Record all the equations in the table.

TABLE 8.1:

Original Line	One Solution	No Solutions	Infinitely Many Solutions
HOWMANY1  y =	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$
HOWMANY2  y =	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$
HOWMANY3  y =	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$
HOWMANY4  y =	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$	$\begin{cases} y = \\ y = \end{cases}$

Compare the equations for the lines you drew with the equations of the original line that was drawn for you.

4. Which equations have the same slope as the original equation? Those that form a system with one solution, no solution, or many solutions?

5. Which equations have the same $y-$ intercept as the original equation? Those that form a system with one solution, no solution, or many solutions?

6. Which equations are equivalent to the original equation?

7. Why is it sometimes hard to see that two equations in a linear system are equivalent? Give an example.

8. Complete each statement to create some rules about the number of solutions for a linear system of equations.

- A linear system has no solution if the equations have ____ slopes and ____ $y-$ intercepts.
- A linear system has infinitely many solutions if the equations have ____ slopes and ____ $y-$ intercepts.
- A linear system has one solution if the equations have ____ slopes and ____ $y-$ intercepts.

Determine how many solutions each system has without graphing.

9. $\begin{cases} y = x \\ y = 2x \end{cases}$

10. $\begin{cases} 3x + 4y = 12 \\ 2x + 4y = 8 \end{cases}$

11. $\begin{cases} y = \frac{1}{2}x + 1 \\ y = \frac{1}{2}x + 8 \end{cases}$

12. $\begin{cases} y = \frac{1}{2}x + 2 \\ -2y = -x - 4 \end{cases}$

8.3 Testing for Truth

This activity is intended to supplement Algebra I, Chapter 7, Lesson 6.

Problem 1 - Is a Point a Solution?

In this activity, you will be determining if random points are solutions to inequalities. Before beginning the activity, you will need to set up the random number generator. Change the random seed using the last 4 digits of your phone number. Enter the digits, then

↵

rand on the home screen and press **ENTER**.

Below, you are given the inequality $y > -x - 2$ and the coordinates of a point. Next, create 2 lists of 3 random numbers. On the home screen, enter **randInt(-5,10,3)ALPHA** $[L_1]$. Repeat, replacing L_1 with L_2 to generate the $y-$ list. Press **STAT** and select **1:Edit...** to see the $x-$ and $y-$ values in L_1 and L_2 . Using the table below, determine whether or not the point is a solution of the inequality.

TABLE 8.2:

Point A (x,y)	y	$-x-2$	$y > -x-2$	**T or F**
$(2,2)$	2	$-2-2$	$2 > -4$	T

You can check each equation on the home screen using the : menu. You can also check the solution by graphing the inequality, tracing to the given $x-$ value and checking to see if the $y-$ value corresponds.

Problem 2 - Generating Solutions

In this problem, you are given two inequalities, $y \leq 4$ and $y > -2$. Again, generate random numbers in L_1 (or x) and L_2 (or y). True is 1 and False is 0 .

Complete the table below. Generate coordinates until you find at least one solution to the inequality.

TABLE 8.3:

Point (x,y)	**Test:** $y \leq 4$ **(T or F)**	**Test:** $y > -2$ **(T or F)**	**Final answer? (T or F)**
ex: $(2,0)$	$0 \leq 4$ T	$0 \geq -2$ T	T

Problem 3 - Overlapping Regions

In this problem, three inequalities intersect to form a triangular region. Again, generate random numbers in L_1 and L_2 (for X and Y), and see if the coordinates are solutions to the system. Complete the table below, making sure you have found at least one solution.

You can test the equations on the home screen using the list of elements instead of each individual coordinate.

You can graph each inequality and test a point by moving a free-floating cursor to approximately each coordinate in the table to see if it falls inside the shaded region.

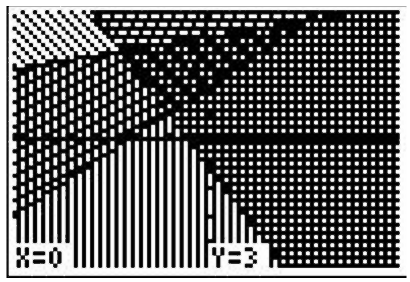


TABLE 8.4:

Point (x,y)	**Test:** $y \leq 0.25x+4$ **(T or F)**	**Test:** $y \geq -2x-1$ **(T or F)**	**Test:** $y \geq x+2$ **(T or F)**	**Final answer? (T or F)**
ex: $(2,0)$	$0 \leq 0.25(2)+4$ $0 \leq 0.5+4$ $0 \leq 4.5$ T	$0 \geq -2(2)-1$ $0 \geq -4-1$ $0 \geq -5$ T	$0 \geq 2+2$ $0 \geq 4$ F	F

CHAPTER 9 SE Exponential Functions

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 8: http://www.ck12.org/flexr/chapter/4478
- Algebra I, second edition, Chapter 8: http://www.ck12.org/flexr/chapter/9765
- Basic Algebra, Chapter 8: http://www.ck12.org/flexr/chapter/9161

9.1 Exponent Rules

This activity is intended to supplement Algebra I, Chapter 8, Lesson 2.

In this activity, you will explore:

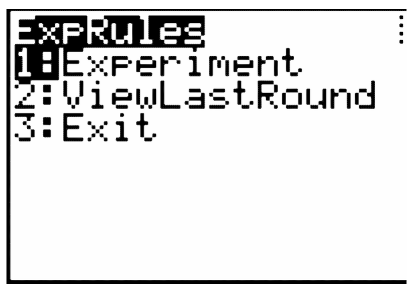
- *expressions involving products and quotients with exponents*
- *expressions with negative and zero exponents*

Before beginning the activity, clear out all functions from the $Y =$ screen and turn off all stat plots.

Problem 1 - Discovering Exponent Rules

Run the **ExpRules** program by pressing **PRGM** then choosing it from the menu and pressing **ENTER**.

This program allows you to explore 6 different rules of exponents by helping you evaluate exponential expressions for different values of x and y . To begin, choose **Experiment**, then type **1** to explore Rule 1.



The program displays the expression that you will be calculating to explore Rule 1, $2^x \, 2^y$. Calculate the expression several times, choosing values from 1 through 8 for x and y . Make and test a conjecture.

Repeat this process to explore rules $2-6$. Pay attention to the prompts, as some rules require you to enter negative values for the variables. Record your conjectures below.

- Rule 1: Make a rule for the product of two powers with like bases.
- Rule 2: Make a rule for the quotient of two powers with like bases.
- Rule 3: Make a rule for the power of a power.
- Rule 4: Make a rule for a power with a negative exponent.
- Rule 5: Make a rule for a power with a zero exponent.
- Rule 6: Make a rule for the power of a quotient.

Extension

Use your calculator to evaluate each of the expressions shown.

Then make a conjecture for m

1n .

12 = ____ 36 13 = ____ 8 12 = ____ 49 12 = ____ 16 14 = ____ 16

Complete: *m*

1n = ______

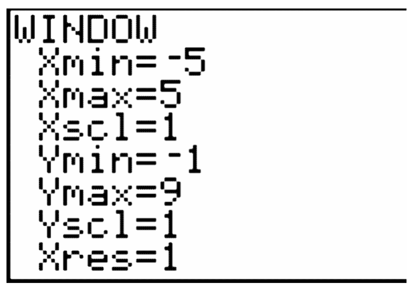
9.2 Exponential Growth

This activity is intended to supplement Algebra I, Chapter 8, Lesson 5.

Problem 1

Before beginning this activity, change your window settings to match those to the right.

Enter the function $f(x) = b^x$ with 5 different values of b (for $b > 0$). Choose some values that are greater than 1 and some values that are less than one. Then, press $Y =$ to graph the functions.



Use **TRACE** to observe how the value of b affects the shape of the graph. Use the up and down arrows to move among the curves. Use the left and right arrows to move along the curves.

- Write at least three observations about the effect of the value of b on the graph of $f(x)$.
- What value of b results in a constant function? Explain.
- Explain why the value of b cannot be negative.

Problem 2

Now you are going to graph function $f(x) = b^x$ along with its tangent line. Start by clearing the functions from the $Y =$ window. Enter the function $f(x) = 2^x$. Then, press **GRAPH** to view the graph of the function.

Press 2^{nd} **[DRAW]** to access the Draw menu. Select **5:Tangent** and press **ENTER**.

Enter an $x-$ value to choose a point where the line will be tangent with the graph of $f(x) = 2^x$. Press **ENTER**.

The calculator draws the tangent line and displays the equation of the line. Record the $x-$ value and the slope of the tangent line.

- x : _____
- slope of tangent: _____

Now find the value of the function $f(x) = 2^x$ at the same point. Press 2^{nd} **[CALC]** to open the Calc menu. Select **1:value**. Enter the $x-$ value you recorded. Press **ENTER**.

The calculator displays the $y-$ value of the function at this point. This is the value of the function for this value of x .

- $f(x)$: _____
- How does the slope of the tangent line at this point compare to the value of the function, $f(x)$?

Return to the $Y =$ screen. Change the value of b to a nonnegative number of your choice and graph the new function. Draw a tangent line at any point on the graph of $f(x)$.

Record the values of $b, x, f(x)$, and the slope of the tangent line at x in the table below along with your earlier observations.

TABLE 9.1:

b	x	$f(x)$	**slope of tanget at** x
2			
3			

Return to the $Y =$ screen and change the value of b again. Draw a tangent line for each curve and record your results in the table.

- Write at least two observations about the graph and/or the slope of its tangent at T .

Problem 3

Slope is a measure of rate of change in a function. In this example, sometimes the slope is ***less than*** y , and sometimes it is ***greater than*** y . There is only one value of b for which the rate of change of the function $y = b^x$ at any point is ***equal to*** the value of the function itself. Can you find an approximate value of this number?

When the rate of change of $y = b^x$ is ***equal to*** the value of the function, the ratio $\frac{\text{slope of tangent at } x}{f(x)}$ will equal one.

TABLE 9.2:

b	$\frac{\text{slope of tangent at } x}{f(x)}$
2	
3	

To begin the search for this value of b , use the data you have collected to complete the table.

Value of b that is closest to 1 and greater than 1 : _____

Value of b that is closest to 1 and less than 1 : ______

The value of b we are looking for must be between these two.

Choose some values of b that are between two numbers and repeat the process of graphing the function, drawing a tangent line, recording the value of the function and the slope of the tangent line at that point, and calculating the ratio. Narrow in on the value of b that yields a ratio of 1 as closely as you can.

TABLE 9.3:

	x	$f(x)$	**slope of tangent at** x	$\frac{\text{slope of tangent at } x}{f(x)}$

What is this value of b? $b \approx$ ____

Applications

The number you found is an approximation for the mathematical constant e . As you discovered, it is unique in that it is the only value of b such that $y = b^x$ changes at a rate that is equal to the value of the function itself. Some examples are: (a) the growth of populations of people, animals, and bacteria; (b) the value of a bank account in which interest is compounded continuously; (c) and radioactive decay.

The common feature is that the rate of growth or decay is proportional to the size of the population, account balance, or mass of radioactive material. Growth and decay situations can be modeled by equations of the form $P = P_0e^{kt}$, where P is the current amount or population, P_0 is the initial amount, t is time, and k is a growth constant. An amount is *growing* if $k > 0$ and *declining* if $k < 0$.

The following are examples of exponential growth or decay. For each exercise, write an equation to represent the situation and solve your equation to find the answer.

1. Suppose you invest $1,000 in a CD that is compounded continuously at the rate of 5% annually. (Compounded continuously means that the investment is always growing rather than increasing in discrete steps.) What is the value of this investment after one year?

Two years? Five years?

2. A colony of bacteria is growing at a rate of 50% per hour. What is the approximate population of the colony after *one day* if the initial population was 500?

3. Suppose a glacier is melting proportionately to its volume at the rate of 15% per year. Approximately what percent of the glacier is left after ten years if the initial volume is one million cubic meters? (This is an example of exponential decay.)

4. A snowball is rolling down a snow covered hill. Suppose that at any time while it is rolling down the hill, its weight is increasing proportionately to its weight at a rate of 10% per second. What is its weight after 10 *seconds* if its weight initially was 2 *pounds* ? After 20 *seconds* ? After 45 *seconds* ? After 1 *minute* ? What limitations might exist on this problem?

CHAPTER 10 SE Factoring Polynomials

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 9: http://www.ck12.org/flexr/chapter/4479
- Algebra I, second edition, Chapter 9:
- Basic Algebra, Chapter 9: http://www.ck12.org/flexr/chapter/9162

10.1 FOILed Again

This activity is intended to supplement Algebra I, Chapter 9, Lesson 2.

Problem 1 - Introduction to Area of a Rectangle

Run the **AREA** program (in **PRGM**) and select the option for Problem 1 (#1).

Enter 6 for W .

1. What are the lengths of the sides of the rectangle?

2. What is the area of the rectangle when $w = 6$?

Now, change the width of the side by running the program again and enter a new value for W .

3. What is the area of the rectangle when $w = 4$? When $w = 9$?

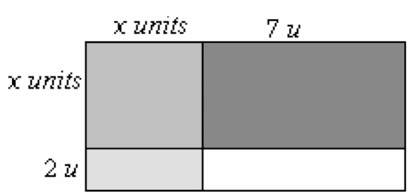
4. Explain how the expression for the area is simplified.

Problem 2 - Areas of Small Rectangles

The rectangle at the right has dimensions $(x+7)$ and rate of $(x+2)$. Each piece of the rectangle is a different color so that you can focus on its area.

5. What is the area of each small rectangle?

6. What is the total area of the rectangle?



Problem 3 - FOIL Method

Run the **AREA** program and select the option for Problem 3.

Enter $(x+7)(x+2)$ for $(AX+B)(CX+D)$. $(A = 1, B = 7, C = 1, D = 2)$

7. How do the areas of the small rectangles in Problem 2 relate to the expression shown on the bottom of the screen?

Practice finding the area of a rectangle and then check your answers with the program.

8. What is the expression of the area of a rectangle with dimensions $(3x+5)$ and $(6x+2)$?

9. a. $(4x+1)(3x+9)$

b. $(x+8)(7x+3)$

c. $(2x+(-3))(5x+8)$

Homework/Extensions

Practice finding the area. Record your answers here. Show each step of your work. Use the program to check your answer.

1. a. $(4x+2)(x+7) =$

b. $(3x-7)(2x+4) =$

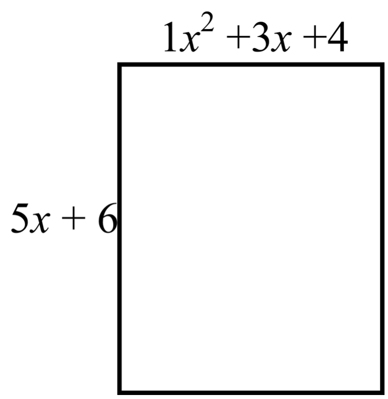
c. $(2x+5)(6x+1) =$

d. $(5x+3)(9x-2) =$

Next, you will be multiplying a trinomial (3 terms) times a binomial (2 terms) to find the area of a rectangle.

2. What method can you use to find the simplified expression for the area?

3. Use the letters a, b, c, d, and e to determine the formula used to find the 6 terms of area shown at the right.



4. What is the area of the rectangle with dimensions $(1x^2+3x+4)$ and $(5x+6)$?

5. a. $(2x^2+1x+7)(3x+(-6)) =$

b. $(4x^2+3x+8)(x+3) =$

c. $(2x^2+6x+4)(-3x+9) =$

10.2 Factoring Special Cases

This activity is intended to supplement Algebra I, Chapter 9, Lesson 6.

In this activity, you will explore:

- *factoring a perfect-square trinomial*
- *factoring a difference of squares*
- *using geometry to prove rules for factoring special quadratic expressions*

Problem 1 - Factoring a Perfect-Square Trinomial

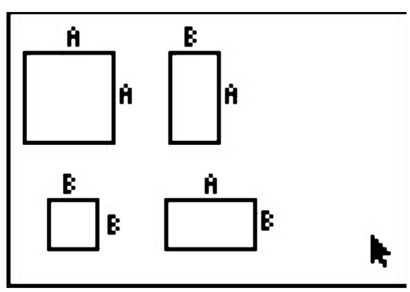
Any trinomial of the form $a^2+2ab+b^2$ is a perfect-square trinomial. If you recognize a perfect-square trinomial, you can factor it immediately as $(a+b)^2$.

To see why $a^2+2ab+b^2 = (a+b)^2$, start the **CabriJr** app by pressing the **APPS** button and choosing it from the menu.

Open the file **FACTOR1** by pressing **Y=** to open then **F1: File** menu, choosing **Open**, and choosing it from the list.

This file shows 2 squares and 2 rectangles, with their dimensions labeled.

What is the area of each shape? On the screenshot at right, label each shape with its area.



- Arrange the shapes to form a square. To move a shape, move the cursor over it (so that the entire shape becomes a moving dashed line) and press **ALPHA** to grab it, then move it with the arrow keys. When the shape is positioned where you want it, press **ENTER** to let it go.
- The area of this square is equal to the sum of the areas of the shapes that make it up. What is the area of the square? Have you seen this trinomial before?
- How long is one side of the square?
- Using the formula $A = s^2$ for the area of a square with side length s what is the area of this square?

You have shown that the area of this square is equal to $a^2+2ab+b^2$ and also equal to $(a+b)^2$. Therefore $a^2+2ab+b^2 = (a+b)^2$ You have proved the rule for factoring a perfect-square trinomial!

Problem 2 - Factoring a Difference of Squares

Any trinomial of the form $m^2 - n^2$ is a difference of squares. If you recognize a difference of squares, you can factor it immediately as $(m+n)(m-n)$.

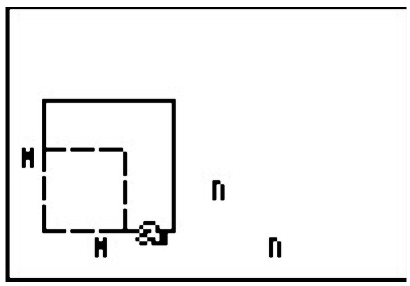
To see why $m^2 - n^2 = (m+n)(m-n)$, start the **CabriJr** app by pressing the A button and choosing it from the menu.

Open the file **FACTOR2** by pressing $Y =$ to open then **F1: File** menu, choosing **Open**, and choosing it from the list.

This file shows 2 squares with their dimensions labeled m and n .

What is the area of each square? On the screenshot at right, label each square with its area.

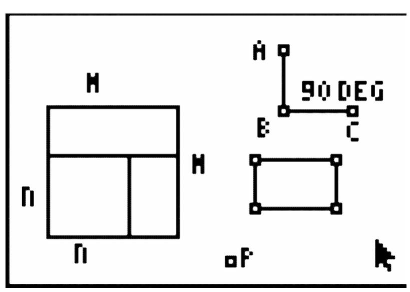
How can you represent the area $m^2 - n^2$ with these squares? Move the n^2 square on top of the m^2 rectangle so that their corners align. If you imagine cutting the smaller square out of the larger square, the $L-$ shaped area that remains is equal to $m^2 - n^2$.



We know that the area of the $L-$ shape is $m^2 - n^2$, but there is also another way to find its area: by taking it apart and rearranging the pieces into a single long rectangle.

Open the **CabriJr** file **FACTOR3**, which shows the same shapes, but with the $L-$ shaped area $(m^2 - n^2)$ divided into two rectangles.

Rotate the smaller rectangle about point P (at the bottom of the screen) clockwise 90° . Press **TRACE** to open the **F4: Transform** menu and choose **Rotation**. Move the cursor over the rectangle to highlight it and press e to choose it. Then move the cursor to the point you want to rotate around and press **ENTER**. Finally, mark the angle of rotation by choosing points A, B , and C in turn.



Hide the original small rectangle and the vertices of the rotated image. (Press **GRAPH** to open the **F5: Appearance** menu and choose **Hide/Show** $>$ **Objects**. Then choose the rectangle and vertices you want to hide.) Now there are two rectangles whose combined area is equal to the area of the original $L-$ shape.

Move the larger rectangle (the small rectangle cannot be moved) alongside the rotated image to form one long rectangle.

What are the dimensions of the long rectangle?

Using the formula $A = lw$ for the area of a rectangle and these dimensions, what is the area of this rectangle?

You have shown that the $L-$ shaped area is equal to $m^2 - n^2$ and also equal to $(m+n)(m-n)$. Therefore $m^2 - n^2 = (m+n)(m-n)$. You have proved the rule for factoring a difference of squares!

CHAPTER 11 SE Quadratic Equations and Quadratic Functions

CHAPTER OUTLINE

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 10: http://www.ck12.org/flexr/chapter/4480
- Algebra I, second edition, Chapter 10:
- Basic Algebra, Chapter 10: http://www.ck12.org/flexr/chapter/9163

11.1 Graphing Quadratic Equations

This activity is intended to supplement Algebra I, Chapter 10, Lesson 1.

In this activity, you will explore:

- *Graphing quadratic equations*
- *Comparing the value of constants in the equations to the coordinates and axes of symmetry on the graph*

Before beginning this activity, clear out any functions from the $Y =$ screen and turn all plots off.

Problem 1 – Vertex Form

Enter the equation $y = x^2$ into $Y1$. Press **ZOOM** and select **ZStandard** to view the graph in a standard size window.

1. Describe the shape of the curve, which is called a parabola.

The vertex form of a parabola is $y = a(x-h)^2 + k$.

For example, the equation $y = 2(x-3)^2 + 1$ is in vertex form. Graph this equation in $Y1$.

2. What is the value of a ?

3. What is the value of h ?

4. What is the value of k ?

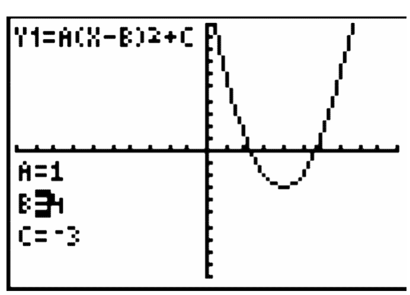
Next we will use the TI-84 application **Transformation Graphing** to see how the values of a, h , and k affect the characteristics of the parabola (such as the vertex, axis of symmetry, and maximum or minimum values).

To open the **Transformation Graphing** app, press **APPS** then **Transfrm** from the menu.

Go to the $Y =$ screen and enter $A(X-B)^2 + C$ in $Y1$. This is the equation of a parabola in vertex form. The **Transformation Graphing** application requires that we use the variables A, B , and C instead of a, h , and k .

Press **GRAPH**. The calculator has chosen values for A, B , and C and graphed a parabola. Note that the $=$ next to A is highlighted.

Press the down arrow to move to the $=$ next to B . Remember that B corresponds to h in the vertex form $y = a(x-h)^2 + k$.



Change the value of $B(h)$ and observe the effect on the graph. You can type in a new value and press e or use the left and right arrow keys to decrease or increase the value of B by 1 .

5. What happens when h is positive?

6. What happens when h is negative?

7. What happens as the absolute value of h gets larger?

8. What happens as the absolute value of h gets smaller?

9. What do you think will happen to the parabola if h is 0 ?

10. Change h to zero. Was your hypothesis correct?

11. Record the equation of your parabola.

$$a = A = \_\_\_\_ \quad h = B = 0 \quad k = C = \_\_\_\_ \quad y = a(x-h)^2 + k = \_\_\_\_(x-0)^2 + \_\_\_\_$$

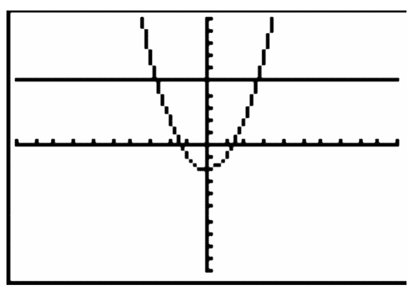
We will now turn our attention to a different feature of the graph of a quadratic function: the axis of symmetry. First, turn off the **Transformation Graphing** app. Go to **Apps** > **Transfrm** > **Uninstall**.

Next, enter the equation you recorded in question 11 in $Y1$.

Press **GRAPH** to view its graph.

Now we would like to draw a line parallel to the $x-$ axis that intersects the parabola twice, as in the graph shown. Experiment with different equations in $Y2$ until you find such a line.

Record the equation of the line. _______________



Use the **intersect** command to find the coordinates of the two points where the line intersects the parabola. Press 2^{nd} **[TRACE]** to open the **Calculate** menu, choose **intersect**, and follow the prompts. Record the coordinates of the intersection in the table on the next page.

TABLE 11.1:

Line	Left intersection	Distance from left intersection to $y-$ axis	Right intersection	Distance from right intersection to $y-$ axis
$y =$	(,)		(,)	
$y =$	(,)		(,)	
$y =$	(,)		(,)	

Choose a new line parallel to the $x-$ axis and find the coordinates of its intersection with the parabola. Repeat several times, recording the results in the table above.

12. What do you notice about the points in the table? How do their $x-$ coordinates compare? How do their $y-$ coordinates compare? Calculate the distance from each intersection point to the $y-$ axis.

13. What do you notices about the distances from each intersection point to the $y-$ axis?

The relationships you see exist because the graph is symmetric and the $y-$ axis is the *axis of symmetry*.

14. What is the equation of the axis of symmetry?

How do you think the graph will move if h is changed from 0 to 4 ? Change the value of h in the equation in $Y1$ from 0 to 4 . $Y1 = (X-4)^2 - 2$

As before, enter an equation in $Y2$ to draw a line parallel to the $x-$ axis that passes through the parabola twice, as shown. Find the two intersection points.

Left intersection: ______________

Right intersection: _______________

The axis of symmetry runs through the midpoint of these two points. Use the formula to find the midpoint of the two intersection points.

midpoint: _______________

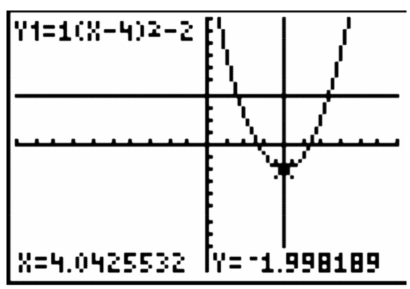
midpoint $(x1, y1)$ and $(x2, y2) = \left(\frac{x1+x2}{2}, \frac{y1+y2}{2}\right)$

Draw a vertical line through this midpoint. Press 2^{nd} **[MODE]** to return to the home screen. Then press 2^{nd} **[PRGM]** to open the **Draw** menu, choose the **Vertical** command, and enter the $x-$ coordinate of the midpoint. The command shown here draws a vertical line at $x = 4$.

This vertical line the axis of symmetry.

Use the **Trace** feature to approximate the coordinates of the point where the vertical line intersects the parabola. Round your answer to the nearest tenth. This point is the vertex of the parabola. vertex: _______________

15. Look back at the equation in $Y1$. How is the vertex related to the general equation $y = a(x-h)^2 + k$?



Since the vertex is the lowest point on the graph, it is also the minimum. Check your answer by pressing 2^{nd} **[TRACE]** for the **Calculate** menu and select the **minimum** command.

Now we will examine the effect of the value of a on the "width" of the parabola. Turn the **Transformation Graphing** app on again and enter $A(X-B)^2 + C$ in $Y1$.

Change the value of A (a) and observe the effect on the graph. You can type in a new value and press e or use the left and right arrow keys to decrease or increase the value of A by 1 .

16. What happens when a is positive?

17. What happens when a is negative?

18. What happens as the absolute value of a gets larger?

19. What happens as the absolute value of a gets smaller?

If the graph opens downward (a is negative), the vertex is a ***maximum*** because it is the highest point on the graph.

The vertex form of a parabola is $y = a(x-h)^2 + k$.

20. The coefficient ____ determines whether the parabola opens upward or downward, and how wide the parabola

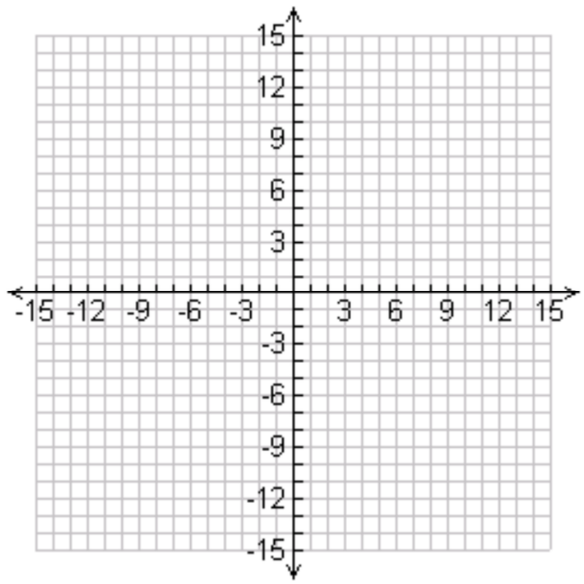
is.

21. The vertex of the parabola is the point with coordinates _______.

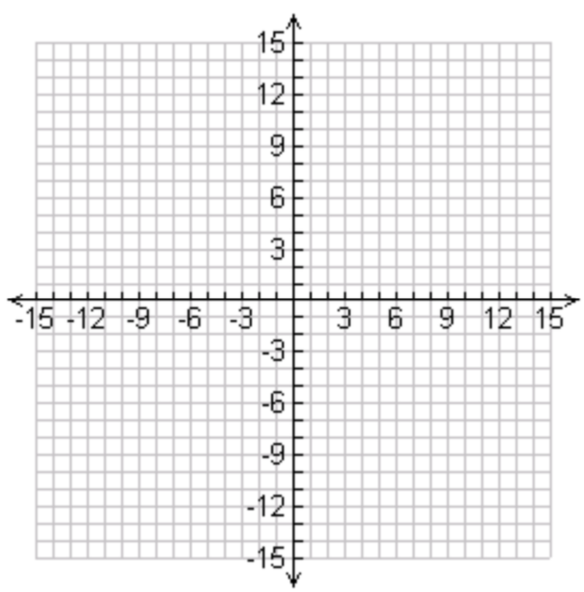
22. The equation of the axis of symmetry is $x =$ _____ .

Sketch the graph of each function. Then check your graphs with your calculator. (Turn off **Transformation Graphing** first.)

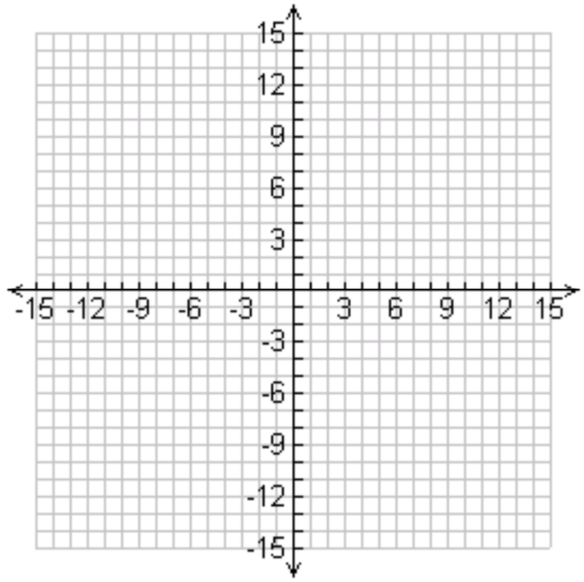
23. $y = x^2 - 3$



24. $y = (x-7)^2$



25. $y = -(x+5)^2 + 4$



Problem 2 – Standard Form

The standard form of a parabola is $y = ax^2 + bx + c$. Let's see how the standard form relates to the vertex form.

$$y = a(x-h)^2 + k$$
$$y = a(x^2 - 2xh + h^2) + k \qquad b = -2ah$$
$$y = \boxed{a}x^2 \boxed{-2ah}x + \boxed{ah^2 + k} \qquad h = -\frac{b}{2a}$$
$$y = \boxed{a}x^2 + \boxed{b}x + \boxed{c}$$

1. For the standard form of a parabola $y = ax^2 + bx + c$, the $x-$ coordinate of the vertex is _____.

The equation $y = 2x^2 - 4$ is in standard form. Graph this equation in $Y1$.

2. What is the value of a ?

3. What is the value of b ?

4. What is the value of c ?

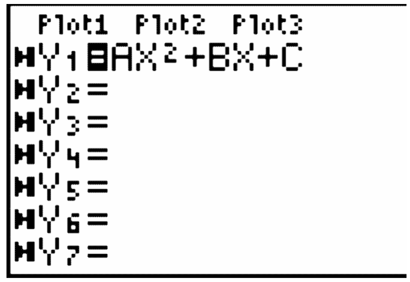
5. What is the $x-$ coordinate of the vertex?

6. Use the **minimum** command to find the vertex of the parabola.

vertex: _____________

How do you think changing the coefficient of x^2 might affect the parabola? Begin by turning on the **Transformation Graphing** app. (**APPS**, scroll down to **TRANSFRM**)

Enter the equation for the standard form of a parabola in $Y1$.



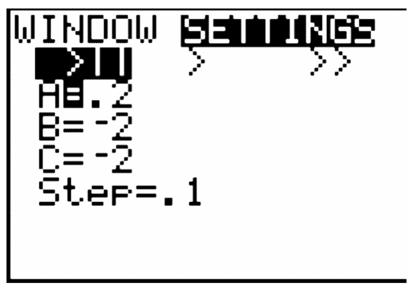
Try different values of A in the equation. You can type in a new value and press **ENTER** or use the left and right arrow keys to decrease or increase the value of A by 1 .

Make sure to test values of A that are between -1 and 1 . To do this, you can type in a value, as in the screenshot shown.

You can also adjust the size of the increase and decrease when you use the right and left arrows. Press **WINDOW** and arrow over to **Settings**. Then change the value of the step to 0.1 or another value less than 1 .

7. Does the value of a change the position of the vertex?

8. How does the value of a related to the shape of the parabola?



Next we will explore another feature of the parabola: the $y-$ intercept. To find the $y-$ intercept of the parabola, use the **value** feature, found in the **Calculate** menu, to find the value of the equation at $x = 0$.

Change the values of a, b , and/or c and find the $y-$ intercept. Repeat several times and record the results in the table below.

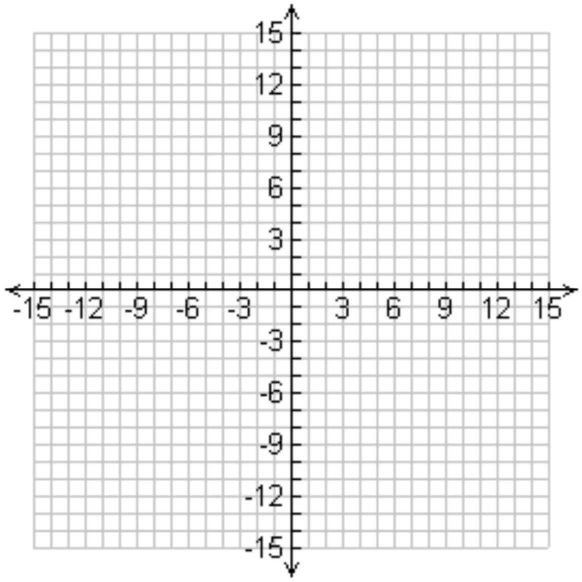
TABLE 11.2:

Equation	a	b	c	$y-$ **intercept**
$y = 2x^2 - 4$	2	0	-4	-4

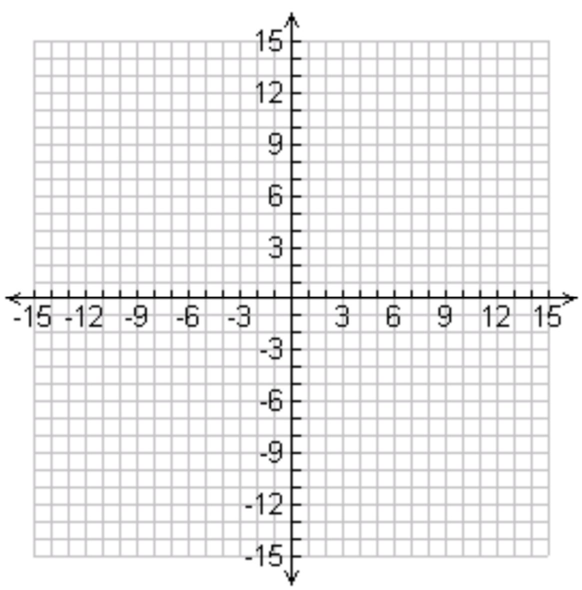
9. How does the equation of the parabola in standard form relate to the $y-$ intercept of the parabola?

Sketch the graph of each function. Then check your graphs with your calculator. (Turn off **Transformation Graphing** first.)

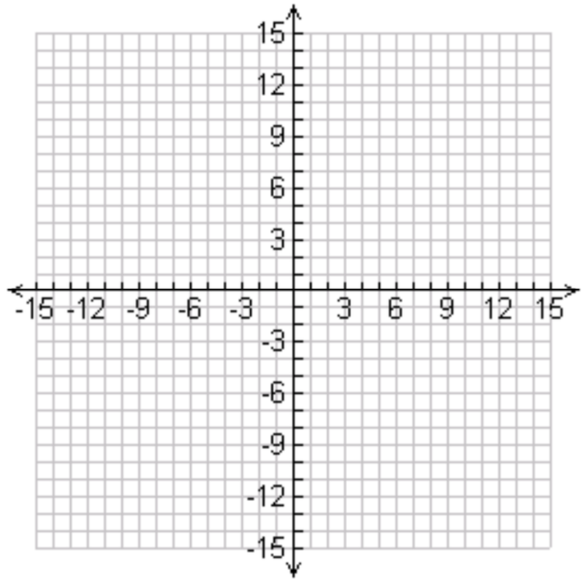
10. $y = x^2 + 6x + 2$



11. $y = -x^2 - 4x$



12. $y = -2x^2 + 8x + 5$



SUMMARY OF GRAPHING QUADRATICS

1. The vertex form of a parabola is __________________.

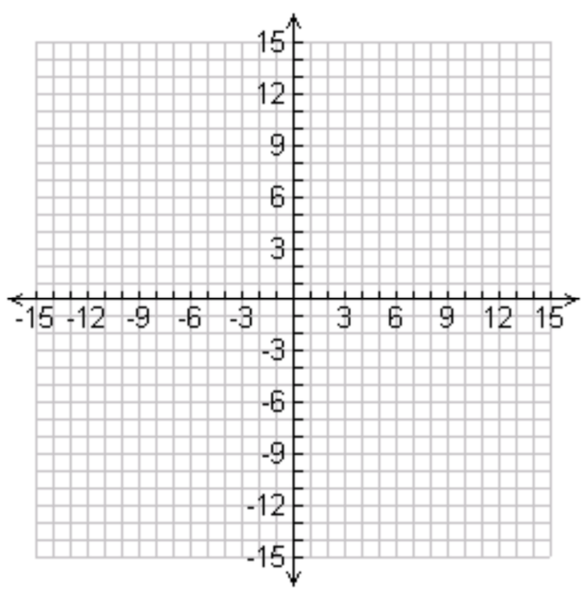
2. The coefficient ____ determines whether the parabola opens upward or downward, and how wide the parabola is. The vertex of the parabola is the point with coordinates _______. The equation of the axis of symmetry is $x =$ ______.

3. The standard form of a parabola is ____________________.

4. The $x-$ coordinate of the vertex is ______________. The equation of the axis of symmetry is $x =$ ______. The $y-$ intercept is ________.

Sketch the graph of each function. Identify the vertex and the equation of the axis of symmetry. Then check your graphs with your calculator.

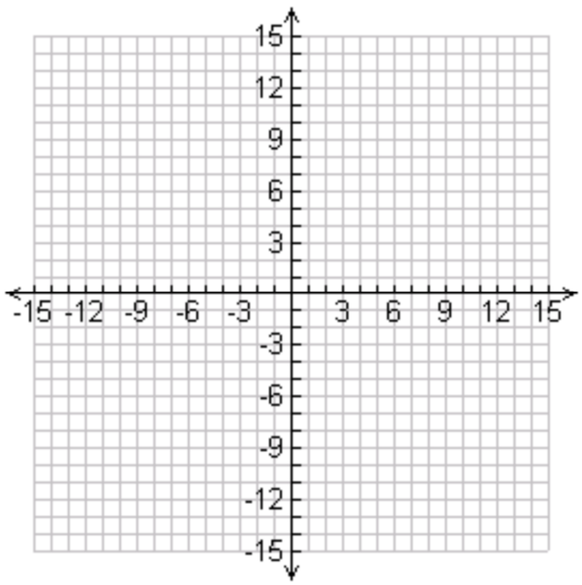
5. $y = x^2 + 4$



vertex _____

axis of symmetry _____

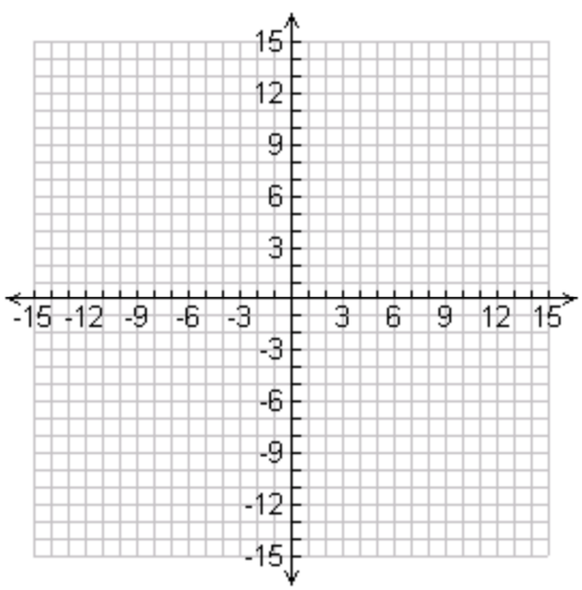
6. $y = (x-3)^2 + 5$



vertex _____

axis of symmetry _____

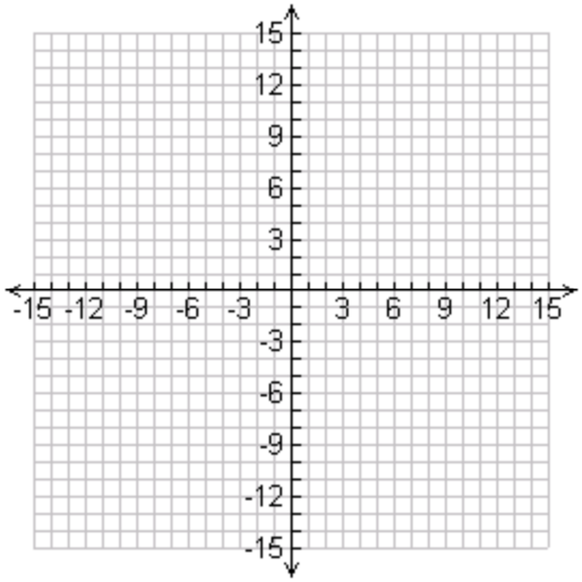
7. $y = -(x-2)^2$



vertex _____

axis of symmetry _____

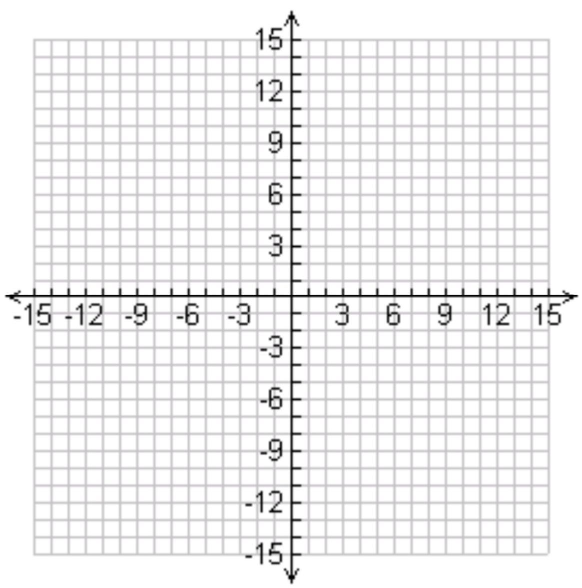
8. $y = x^2 + 6x + 9$



vertex _____

axis of symmetry _____

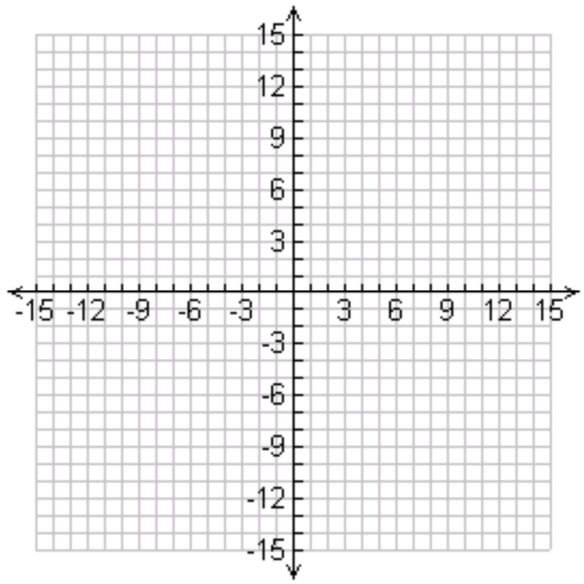
9. $y = -3x^2 + 6x + 1$



vertex _____

axis of symmetry _____

10. $y = x^2 + 1$



vertex _____

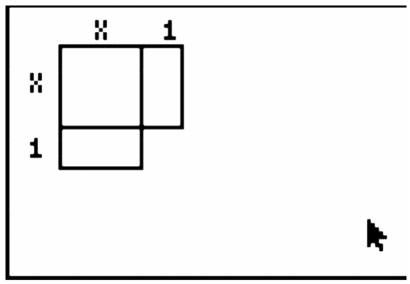
axis of symmetry _____

11.2 Area of the Missing Square

This activity is intended to supplement Algebra I, Chapter 10, Lesson 4.

Problem 1 – Introduction

Use this space for notes about the discussion of the model led by your teacher.



Area of the larger square

$$= x^2 + x + x + c$$
$$= x^2 + 2x + c$$

1. What is the area of the missing square that completes the larger square?

2. $(x+1)(x+1) =$

Problem 2 – Integer Lengths

Start the Cabri Jr. application by pressing **APPS** and selecting **CabriJr**. Open the file titled **SQUARE** by pressing $Y =$, selecting **Open** and then choosing it from the list.

Use the **ALPHA** key to grab the point on the side of the square and use the arrow keys to drag it down.

Change the displayed width values to 2 and then 3 . Observe the relationship between the coefficient of x , and the length of the little square that completes the (larger) square. Fill in the table as you work.

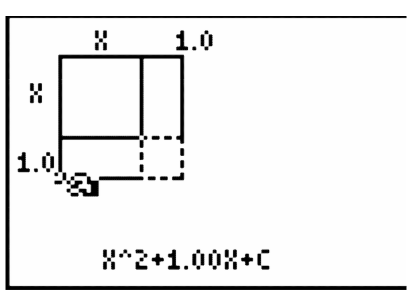


TABLE 11.3:

Width	(Side length) ^{2}	Area	b	c
1	$(x+1)^2$	x^2+2x+1	2	1
2				
3				

Problem 3 – Non-Integer Lengths

Use the **ALPHA** key to grab the point on the side of the square and use the arrow keys to drag it to change the displayed width values. Find the area of the small square and the larger square for each width value.

Observe the relationship between the coefficient of x and the length of the small square that completes the (larger) square. Fill in the table as you work.

TABLE 11.4:

Width	(Side length) ^{2}	Area	b	c
1.5				
2.1				
2.5				
3.1				
3.5				

3. How is the coefficient of x related to the length of the small square?

4. How is the coefficient of x related to the value of c ?

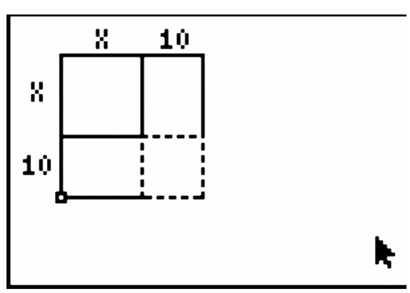
5. What is a formula to find the value of c ?

Problem 4 – Applying your Knowledge

Answer the questions below to apply your knowledge of completing the square.

6. Area $= x^2+20x+c$

What is the value of c ?



7. Area $= x^2+14x+c$

What is the value of c ?

8. Area $= x^2+5.4x+c$

What is the value of c ?

9. What is the value of c to complete a square with Area $= x^2 + 5x + c$?

- ☐ 10
- ☐ 25
- ☐ $\frac{25}{4}$
- ☐ $\frac{25}{2}$

10. In order to complete the square, which equation will have a $c-$ value of 8 ?

- ☐ $x^2 + 4x + c$
- ☐ $x^2 + 4\sqrt{2}x + c$
- ☐ $x^2 + 2\sqrt{2}x + c$

11. Which value below can you add to the equation $x^2 + 16x + 40$ to complete the square?

- ☐ 8
- ☐ 64
- ☐ 24
- ☐ -8

12. What must you add to the expression $x^2 + 4x + 1$ to complete the square? Why?

13. What must you add or subtract to the expression $x^2 + bx$ to complete the square? Why?

11.3 Quadratic Formula

This activity is intended to supplement Algebra I, Chapter 10, Lesson 5.

Problem 1

1. Identify the zeros of $y=x^2-4$ by graphing the equation in $Y=$. If needed, use the **zero** command found under 2^{nd} **[CALC]**. Write the zeros below.

2. You may already know the zero product property, and can demonstrate why the following are the solutions to the equation above:

$x+2=0$ and $x-2=0$

3. A program, ***QUAD***, is provided that has the Quadratic Formula defined. Use $A=1, B=0$, and $C=-4$. What are the solutions to the equation $y=x^2-4$?

Problem 2

4. Now, examine the graph of $y=x^2+x-6$. Graph the equation in $Y=$. Determine the zeros. Write the factored form below.

Use the ***QUAD*** program again. You only need to enter in the correct values for a, b , and c . This should confirm your answers for the $x-$ intercepts.

5. What are the solutions to the equation $y=x^2+x-6$?

Problem 3

6. Now, examine the graph of $y=x^2-4x+4$. Graph the equation in $Y=$ and determine the zeros. Write the factored form below.

7. Using the ***QUAD*** program, what are the solutions to the equation $y=x^2-4x+4$?

Exercise 4

8. Explore $y=x^2-2x-7$, which is not factorable with integers. You may ask why this quadratic function is not factorable and the previous examples were. Make a conjecture about why you think this could be true:

- "Some quadratic equations are not factorable with integers because…"

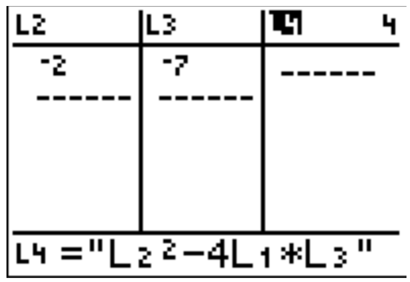
or

- "Quadratic equations are only factorable with integers when…"

9. Solve the following equations using the ***QUAD*** program.

- $y = x^2 - 2x - 7$
- $y = -3x + x + 3$

10. Finally, use **Lists** to calculate the value of the discriminant for the previous two problems, whose solutions were irrational. Enter the A coefficient in L_1, B in L_2 , and C in L_3 . Then, in L_4 , move to heading and enter the formula for the discriminant shown at the right.



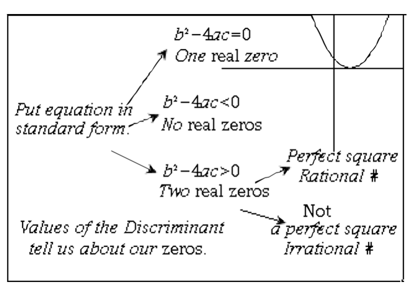
- $y = x^2 - 2x - 7$ Solution: ___________________
- $y = -3x + x + 3$ Solution: ___________________

Extensions/Homework

Use the formula in L_4 (above) to calculate the Discriminant for several other quadratics. Decide if the equation is factorable using integers, then solve it. Factor the quadratic if possible, if not, solve by the quadratic formula.

a. $y = x^2 - 6x + 9$
b. $y = 3x^2 + 4x + 5$
c. $y = -4x^2 + 2x + 2$
d. $y = 7x^2 + x - 8$
e. $y = 2x^2 - 5$

Look at the flow chart below and discuss with another student how to use it to answer these homework problems.



11.4 Manual Fit

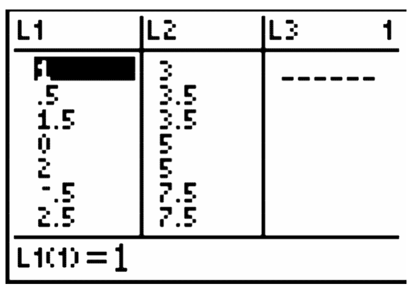
This activity is intended to supplement Algebra I, Chapter 10, Lesson 7.

Problem 1 – Match the Graph, Part 1

The vertex form for the equation of a parabola is $y = a(x-h)^2 + k$. If needed, graph $y = a(x)^2$ with various values of a and explore.

- In vertex form or in standard form, what happens when $0 < a < 1$?
- If $a > 1$, the graph will be narrow and open up. If $a < -1$, the graph will be what?

Enter the lists shown at the right. Create a scatter plot of $L1$ and $L2$. Then, enter the vertex form of the parabola in $Y1$ with an initial guess for each value for a, h , and k . See how the equation fits and then adjust the values to make the graph fit the data.



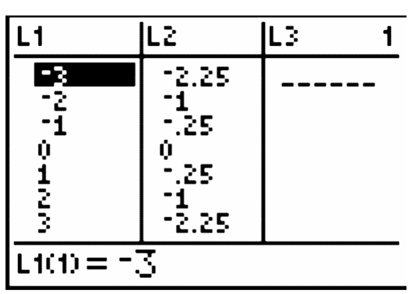
L1	L2	L3 1
1	3	------
.5	3.5	
1.5	3.5	
0	5	
2	5	
-.5	7.5	
2.5	7.5	

L1(1)=1

- What is the vertex of the parabola?
- What was your value of a for the parabola?
- What is the equation of the parabola you fit to the data?

Problem 2 – Match the Graph, Part 2

Repeat the process from Problem 1 to find the equation of a parabola that matches the data in $L1$ and $L2$.



L1	L2	L3 1
-3	-2.25	------
-2	-1	
-1	-.25	
0	0	
1	-.25	
2	-1	
3	-2.25	

L1(1)=-3

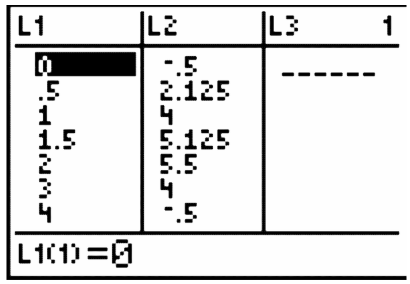
- To make the parabola open down, what must be true about the value of a ?
- To make the parabola wide, what must be true about the value of a ?
- What is the equation of your parabola that fits the data?

Problem 3 – Match the Double Arches

Change $L1$ and $L2$ to match the screenshot shown a the right. Now graph, $Y1 = \frac{(-1.5(X+2)^2+5.5)}{(-4 \leq X \text{ and } X \leq 0)}$

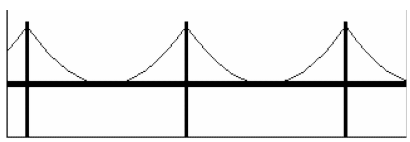
Next, match the second half of double arches.

- What do you notice about the two parabolas that formed the double arches?



- The vertex of the left arch is $(-2, 5.5)$. What is the vertex of the right arch?
- What is the equation of your parabola that matches the data?

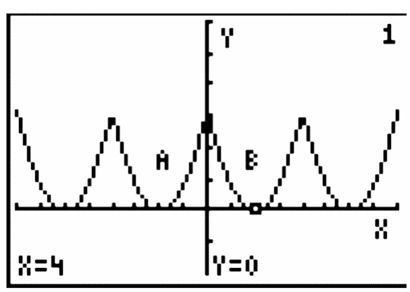
Problem 4 – The Main Cables of a Suspension Bridge



Here is a picture of a suspension bridge. Several loops of cable are represented. See the graph below to match an equation to a particular part of the graph.

The point where pieces A and B meet is $(0, 3.2)$.

- What is the equation of the piece of the graph labeled A ?
- What is the equation of the piece of the graph labeled B ?

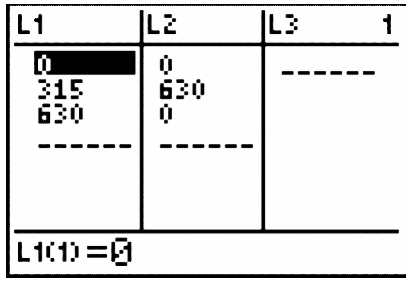


Extension – The Gateway Arch in St. Louis

The Gateway Arch in St. Louis, the "Gateway" to America, is a shape that looks like a parabola to the casual observer.

Use what you know about the vertex form to write an equation to match its shape and dimensions. Enter $L1$ and $L2$ shown and create a scatter plot with an appropriate window.

- What is the equation?



Using the same data, match the graph in standard form $(y = ax^2 + bx + c)$ by changing the $Y =$ equation. Important things to remember are; what does the value of a do to the graph, and what would your $y-$ intercept be (c in the equation)?

- What is your equation in standard form?
- How are the two equations similar?
- How are the two equations different?
- Expand the vertex form and convert it to standard form to make a final comparison.

Chapter 12 SE Algebra and Geometry Connection; Working with Data

Chapter Outline

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 11: http://www.ck12.org/flexr/chapter/4481
- Algebra I, second edition, Chapter 11:
- Basic Algebra, Chapter 11: http://www.ck12.org/flexr/chapter/9164

12.1 Radical Transformations

This activity is intended to supplement Algebra I, Chapter 11, Lesson 1.

Problem 1 – The General Radical Function

Graph the equation $y = \sqrt{x}$. Once graphed, use the **TRACE** key to observe the coordinate values for points on the graph.

a. What is the domain and range of the function?
b. Why does the graph "stop" at the origin?
c. When is the following statement true?

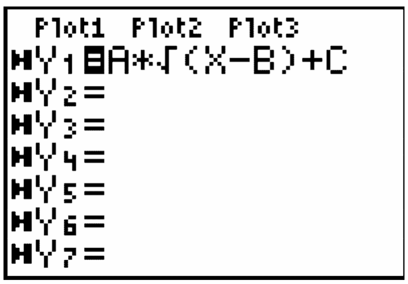
The graph of the square root function is completely in the first quadrant.

Sometimes Always Never

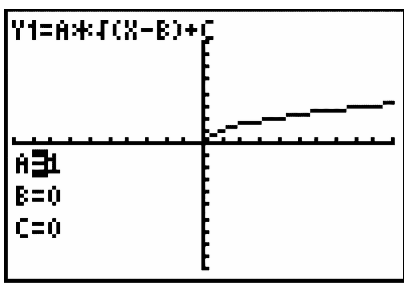
Problem 2 – Transformations

Start the **Transformation Graphing** application by pressing **APPS** and selecting **Transfrm**.

Now, press $Y =$ and enter $A\sqrt{(X-B)}+C$ into Y_1 .



Press **ZOOM** and select **6:ZStandard**. Notice the displayed equation. The values of A, B and C may be changed by using the arrow and number keys.



4. What does the graph look like when all three variables equal zero? Why?

5. Based on your exploration, when is the following statement true?

The graph of the square root function is completely in the first quadrant.

Sometimes	Always	Never

Continue to manipulate the values of A, B and C on the calculator to help answer Questions 6-16

6. Find two functions whose domain is $x \geq 3$.

7. What is the domain of the function $f(x) = 4\sqrt{x+2} - 3$? Check using the graph.

8. Changing which variable will create a horizontal shift?

9. Find two functions whose range is $y \geq -2$.

10. What is range of the function $f(x) = 4\sqrt{x+2} - 3$? Check using the graph.

11. Changing which variable will create a vertical shift?

12. What is the difference between the graphs of $f(x) = 4\sqrt{x+2} - 3$ and $g(x) = -4\sqrt{x+2} - 3$?

13. What is the difference between the graphs of $f(x) = 4\sqrt{x+2} - 3$ and $g(x) = 2\sqrt{x+2} - 3$?

14. What effect does the variable a have on the graph?

15. What is the domain of the function using the general equation $y = \sqrt{x-h} + k$?

16. What is the range of the function using the general equation $y = \sqrt{x-h} + k$?

Extension – Cube Root Functions

Press $Y =$ and enter $A\sqrt[3]{(x-B)} + C$ into Y_1 .

Change the values of the variables A, B , and C , and observe the effects of the changes on the graph.

17. What is the domain and range of the function in terms of the general equation?

18. Describe the effects of changing each variable on the graph.

12.2 Distances in the Coordinate Plane

This activity is intended to supplement Algebra I, Chapter 11, Lesson 5.

In this activity, you will explore:

- *finding the length of a segment using the Distance Formula*
- *finding the length of a segment using the Pythagorean Theorem.*

Problem 1 – The Distance Formula

Construct a segment. Find the coordinates of the endpoints and the measured length. Use the distance formula to calculate the length.

TABLE 12.1:

Endpoints	Measured Length	Calculated Length
(_____ , _____) and (_____ , _____)	_______________	_______________
(_____ , _____) and (_____ , _____)	_______________	_______________

What is important to remember when using the Distance Formula?

What happens to the Distance Formula when your segment is horizontal or vertical? Give an example using end-points.

(_____ , _____) and (_____ , _____)

Problem 2 – The Distance Formula and the Pythagorean Theorem

Find the length of all three sides of your triangle. Which side is the longest? Can two of the sides be equal lengths? Which two?

Use the Pythagorean Theorem to calculate the length of your segment in another way.

TABLE 12.2:

Endpoints	Measured Length	pythagorean Length
(_____ , _____) and (_____ , _____)	_______________	_______________
(_____ , _____) and (_____ , _____)	_______________	_______________

What is the relationship between the Pythagorean Theorem and the Distance Formula?

Problem 2 - Apply The Math

What formula gives the distance between the points (x_1, y_1) and (x_2, y_2) ?

Determine the length of the segment with the following endpoints:

1. $(1,2)$ and $(5,10)$
2. $(5,8)$ and $(9,5)$
3. $(7,4)$ and $(4,7)$
4. $(-2,3)$ and $(3,5)$
5. $(1,-9)$ and $(-2,-7)$
6. $(3,5)$ and $(3,-11)$

Given an endpoint and a length of a segment, find a possible other endpoint:

7. Endpoint: $(3,1)$; Length 5

12.3 Box Plots Histograms

This activity is intended to supplement Algebra I, Chapter 11, Lesson 8.

June collected the distances she drove each weekend for 30 weekends. The distances, stored in the list **WKND**, are listed below.

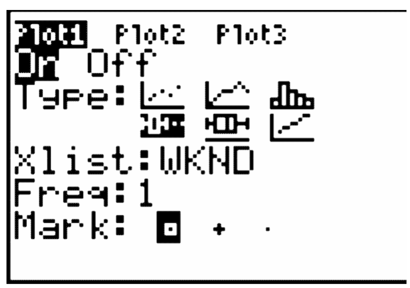
$$31, 8, 93, 69, 75, 2, 33, 194, 83, 17, 2, 207, 99, 32, 8,$$
$$2, 75, 126, 30, 9, 211, 93, 8, 75, 198, 25, 32, 71, 9, 98$$

Part 1 – Create a Box Plot

Create a box plot of the distances.

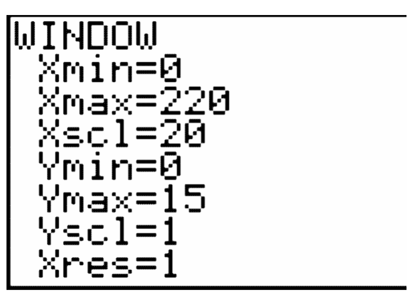
Press 2^{nd} $[Y =]$, and select **Plot1**. Press **ENTER** to turn the plot on. Select the box plot icon. Arrow down to **Xlist**.

To select **WKND** as the Xlist, press 2^{nd} **[STAT]**, arrow down **WKND** and press **ENTER**.



Press **WINDOW**. An appropriate window would include $x-$ values that range from 0 to 220 . The box plot is not affected by the y settings because it is not paired with a second set of numbers.

Press **GRAPH**.



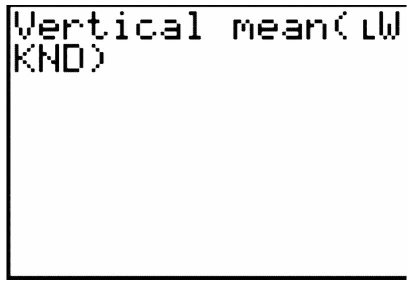
Press **TRACE** to view the values of each section of the plot.

1. Minimum: ____ Q1: ____ Median: ____ Q3: ____ Maximum: ____

2. Why is the first whisker so short? What does it mean for the other whisker to be so long?

3. What does the median value say about the distances traveled? Since this point is the "middle" point in the data, why is the box plot not balanced at this point?

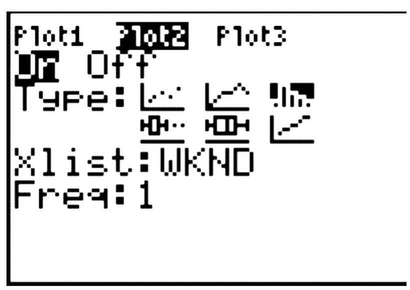
4. Plot the mean of the distances by entering the command shown at the right. Press 2^{nd} **[DRAW]** to access the **Vertical** command and press 2^{nd} **[LIST]** and arrow to the **MATH** menu for the **mean** command.



Where is the mean located on this plot?

Part 2 – Create a Histogram

Create a histogram of the distances.



Press 2^{nd} **[STAT PLOT]**, and select **Plot1**. Press **ENTER** to turn the plot off.

Press 2^{nd} **[STAT PLOT]**, and select **Plot2**. Press **ENTER** to turn the plot on. Select the histogram icon. Arrow down to **Xlist** and select **WKND**.

Press **GRAPH**. Press **TRACE** and use the arrow keys to view the number of entries per bar.

5. How many weekends did June drive between 20 and 40 *miles* ? ____

6. How many weekends did June drive less than 60 *miles* ? ____

7. How many weekends did June drive more than 120 *miles* ? ____

Plot the mean and median of the distances. Press 2^{nd} **[LIST]** and arrow to the **MATH** menu for the **median** command.

8. Where are the median and mean on this plot?

9. The interval from 40 to 60 should contain the median of 51 , but it shows zero entries. How is that possible?

Part 3 – Compare a Box Plot and a Histogram

To better understand the shape of the box plot, compare it to the histogram. Press 2^{nd} **[STAT PLOT]**, and select **Plot1**. Press **ENTER** to turn the plot on.

10. How does the shape of the histogram compare to the shape of the box plot?

11. How does the tallness of the first bar relate to the shortness of the first whisker?

12. What do you see now about why the other whisker is so long?

Chapter 13 SE Rational Equations and Functions; Topics in Statistics

Chapter Outline

Each activity below is intended to supplement our Algebra flexbooks.

- Algebra I, first edition, Chapter 12: http://www.ck12.org/flexr/chapter/4482
- Algebra I, second edition, Chapter 12:
- Basic Algebra, Chapter 12: http://www.ck12.org/flexr/chapter/9165

13.1 Inverse Variation

This activity is intended to supplement Algebra I, Chapter 12, Lesson 1.

Part 1 - Enter the Data

Enter the data from the table into lists.

Press **STAT ENTER**. Enter the x column in $L1$ and the y column in $L2$ as shown.

TABLE 13.1:

x	y
1	24
2	12
3	8
4	6
5	4.8
6	4

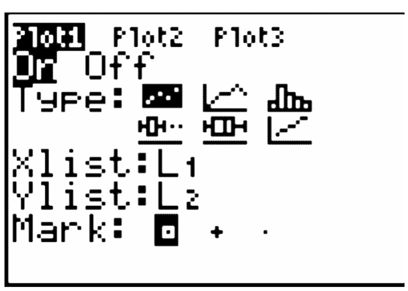
Press $Y =$, and select **Plot1**.

Press **ENTER** to turn the plot **On**. Select **scatter** as the type of plot, $L1$ for the Xlist, and $L2$ for the Ylist.

Press **WINDOW**. Set the window to the following:

$Xmin = 0, \ Xmax = 10, \ Xscl = 2$

$Ymin = 0, \ Ymax = 25, \ Yscl = 5$



Press **GRAPH**.

Part 2 - Questions

- How would you describe the relationship between x and y by examining this data?

Press **STAT ENTER** to return to the lists.

- What relationships can you see by examining the numbers in the lists?
- What is the product of each pair of numbers?

Arrow to the top of $L3$. Enter a formula to multiply the entries in $L1$ by the entries in $L2$. Press 2^{nd} $[L1]$ for $L1$ and press 2^{nd} $[L2]$ for $L2$. $L3 = L1 * L2$

Press **ENTER** to execute the formula. The product in each case is 24 . So, $L1 \cdot L2 = 24$ or $x \cdot y = 24$. This relationship, when x and y have a constant product, is called "inverse variation."

- What type of situation might this be a formula for?

Solve the equation $x \cdot y = 24$ for y . Press $Y =$. Enter the equation into $Y1$.

- What is your equation?

Press **GRAPH**.

- What other information can you find from the graph of the equation that you could not gather from the plot?
- Does this graph appear to be a function? Explain.

Press 2^{nd} **[TABLE]** to examine the function table.

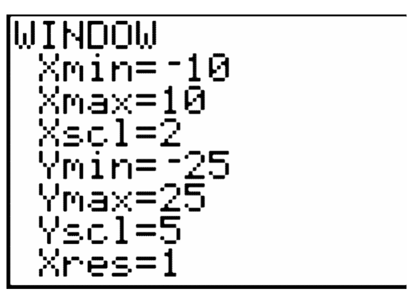
- What is happening when $x = 0$? Why?

Arrow up to the negative $x-$ values in the table.

- What do you notice about the $y-$ values?
- Why does this occur?
- What do you think the graph of your equation looks like to the left of the $y-$ axis?

Press **WINDOW**. Set the window as shown to examine the graph when x is negative.

Press **GRAPH**.



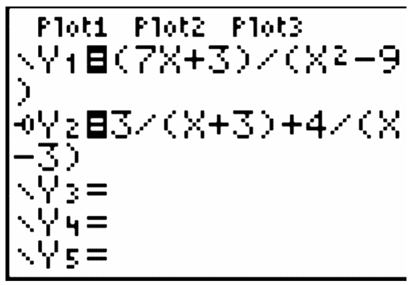
- What appears to be happening when $x = 0$?
- Why does the graph of the equation not appear in Quadrants II or IV?
- Do you think an inverse variation can ever be found in Quadrants II or IV? Why?
- Does this graph appear to be a function now? Explain.

13.2 Breaking Up is NOT Hard to Do

This activity is intended to supplement Algebra I, Chapter 12, Lesson 6.

Problem 1 – Introduction

Press $Y =$ and enter the two functions



$Y1(x) = \frac{7x+3}{x^2-9}$ and $Y2(x) = \frac{3}{x+3} + \frac{4}{x-3}$

Also, for the second expression, move the cursor to the left of $Y2 =$ and press **ENTER** until a circle appears. This will place a large circle in front of the graph as it is graphed on the handheld.

To view the graphs, press **ZOOM** and select **ZStandard**.

1. How do the graphs of the two given equations compare?

2. What do the graphic results tell us about the two functions?

Functions can often be expressed in several different ways. The second representation splits the initial rational function into fractional parts and is referred to as the **sum of partial fractions**.

3. How are the denominators in $\frac{3}{x+3} + \frac{4}{x-3}$, the partial fractions, related to the denominator of the original expression $\frac{7x+3}{x^2-9}$?

So, to begin understanding how these partial fractions are developed, begin by writing two fractions using the factors of the denominator of $Y1$. Let A and B represent the numerators yet to be determined.

$$Y1(x) = Y2(x)$$
$$\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$$

4. What is the LCD (least common denominator) for $\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$?

5. What is the result of multiplying both sides of $\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$ by the LCD?

6. Substitute in a convenient number for x and solve for A . What value did you obtain for A ?

7. Similarly substitute in a convenient number for x and solve for B . What value did you obtain for B ?

8. Now substitute the values you found for both A and B into the equation shown in Question 4 to show the equivalent rational function and sum of partial fractions.

9. How do your results for Question 8 support your answer to the Question 2 regarding what the graphs of the functions $Y1$ and $Y2$ tell us about the two functions?

Problem 2 – Practice

10. Express the rational function, $f(x) = \frac{7x-4}{x^2+x-6}$, as a sum of partial fractions.

11. Graph the initial function and your sum of partial fractions using the graphing calculator as outlined in Problem 1. How does this verify your results? Explain your reasoning.

Problem 3 – The Next Level

12. Express the rational function, $f(x) = \frac{5x-7}{4x^2-8x-12}$, as a sum of partial fractions.

13. Graph the initial function and your sum of partial fractions using the graphing calculator. How does this verify your results? Explain you reasoning.

Additional Practice Problems

Represent each of the following rational functions as a sum of partial fractions. Verify your results graphically.

14. $f(x) = \frac{-7x-11}{x^2+4x+3}$

15. $f(x) = \frac{2x+42}{x^2+2x-24}$

16. $f(x) = \frac{x}{x^2+2x-8}$